Electroweak Radiative Corrections at LEP + LHC

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Abstract

We give a review of the $\mathcal{O}(\alpha)$ electroweak radiative corrections in deep inelastic electron proton scattering at LEP+LHC. These corrections include one-loop contributions and single-photon bremsstrahlung. A major contribution to the radiative corrections is due to real photon bremsstrahlung $ep \rightarrow e\gamma X$. The Monte Carlo event generator HERACLES is used to study event distributions and the observability of radiative events is discussed.

1 Introduction

The knowledge of the detailed features of electroweak radiative corrections is indispensable for the interpretation of any high energy experiment. For HERA it is known that these corrections can be very large for large y and small x [1,2]. We will show that these large corrections are due to the emission of hard photons, mainly from the lepton line. If these potentially visible events could be excluded from the data sample used for a physics analysis, the remaining corrections which are due to unidentified radiative events would be smaller. After cutting out radiative events, standard unfolding procedures could be applied over a larger range of x and y than one could have expected from a study of the fully inclusive bremsstrahlung corrections.

2 Inventory of Radiative Corrections for $ep \rightarrow eX$

We are not going to present the complete set of formulas for the $\mathcal{O}(\alpha)$ radiative corrections. They can be found *e.g.* in [1,2]. Instead we only discuss some important features:

- The Born cross section for eP scattering is expressed as a sum over quark flavors and over the type of the exchanged boson. Each contribution gets different corrections. They depend on both the external quark line and on the type of the exchanged boson. The $\mathcal{O}(\alpha)$-corrected cross section has the general form

$$\left.\frac{d^2\sigma}{dxdy}\right|_{eP\rightarrow eX} = \sum_f \sum_{B=\gamma,Int,Z} (1+\delta_{f,B})\hat{\sigma}^{Born}_{f,B}(x,y) + \int d^3PS(\vec{k})R_{f,B}(x,y,\vec{k})\hat{\sigma}^{Born}_{f,B}(\hat{x},\hat{y}). \qquad (1)$$

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The factorized part $\delta_{f,B}$ contains virtual one-loop contributions and soft photonic corrections. The hard bremsstrahlung part results from a convolution of the Born cross sections $\hat{\sigma}_{f,B}^{Born}(x,y) = d^2\sigma/dx\,dy|_{f,B}^{Born}(x,y)$ taken at rescaled kinematic variables $\hat{x}, \hat{y}$ which are functions of the photon momentum k with radiation functions $R_{f,B}(x,y,\vec{k})$.

The various contributions to $\delta_{f,B}$ and $R_{f,B}$ can be further separated according to gauge invariant subsets of diagrams, namely into:
i) the leptonic corrections that are described by diagrams containing an additional photon attached to the lepton line, *i.e.* the photonic correction to the lepton gauge boson vertex, the photonic contribution to the self energies of the external fermion lines, and the photon emission from the lepton line.
ii) the quarkonic corrections described by diagrams with an additional photon at the quark line analogous to *i*).
iii) The lepton-quark interference part consisting of the $\gamma\gamma$ and γZ box diagrams and the interference of leptonic and quarkonic bremsstrahlung.
iv) the purely weak corrections consisting of all the other diagrams that do not contain an additional photon. This part is IR finite and contains the diagonal γ and Z self energies, the γZ mixing, the weak lepton and quark vertex corrections, and the boxes with two heavy gauge bosons.

- The self energy diagrams contain loop diagrams that are built with all particle degrees of freedom that couple to the gauge bosons. Therefore they contain information on the whole theory. They depend on the top mass, the Higgs mass, and on the masses and couplings of eventually existing other unknown particles.
- The self energies are dominated by the fermion loops. This contribution is sometimes refered to as a QED part. The photon self energy can be accounted for by the use of the running fine structure constant $\alpha(Q^2) = \alpha(0)/(1 - \Pi^\gamma(Q^2))$, where $\alpha(0) = 1/137.036$ and Π^γ is the vacuum polarization. At $Q^2 \simeq M_Z^2$ its value is $\Pi^\gamma \simeq 0.06$. The prescription to use the running fine structure constant together with the leptonic QED corrections (discussed below) gives the $\mathcal{O}(\alpha)$ corrected cross section already with a precision of a few %. The Z self energy can be included approximately by normalizing the Z exchange part with the help of the μ decay constant.
- **Radiation from the lepton line**. Numerical results for the leptonic corrections show a very pronounced increase at small x and large y and can become also large but negative for small y and large x. This behaviour can be understood from the following observations:

i) The order of magnitude is determined by the factor

$$\frac{\alpha}{\pi} \ln \frac{Q^2}{m_e^2} \simeq (0.24\%) \times 25 \simeq 6\,\%.$$

This number is multiplied by logarithms of ratios of the maximal photon energy and the center of mass energy which can also become large for small values of x and large y.
ii) The emission of an energetic photon can shift the value of the momentum transfer $\hat{Q}^2$ seen from the quark line to very small values. x, y and $Q^2 = -(p_e - p_{e'})^2 = xyS$ are determined from the momenta p_e and $p_{e'}$ of the incoming and outgoing electron. But $\hat{Q}^2 = -(p_e - p_{e'} - k)^2$ is also determined by the momentum

k of the emitted photon and $\hat{Q}^2$ can be very small compared to Q^2 if the emitted photon takes away a large energy. In this case the bremsstrahlung contribution gets enhanced through the photon propagator $1/\hat{Q}^4$. This effect is similar to the radiative tail effect above the peak of a resonant cross section. It is responsible for the large increase of the corrections at large y.

iii) At small values of y and large x the photon phase space volume shrinks and for $x \to 1$ and $y \to 0$ the virtual and soft real corrections dominate and lead to large negative contributions.

- **Radiation from the quark line.** The quarkonic corrections contain mass singularities due to the initial quark masses. These can be factorized from the cross section and absorbed into the definition of the distribution functions. The only effect of the photonic quark line corrections is to introduce an additional Q^2 dependence which can be described in complete analogy to the Q^2 dependence arising from gluonic corrections, *e.g.* with the help of the Altarelli-Parisi equations. Numerically the corrections are then at most of the order of 2% in the range of x and Q^2 accessible at LEP + LHC. In addition to these leading logarithmic corrections there are also non-logarithmic contributions. But they are even smaller and can be neglected if one aims at not more than an accuracy of 1 %.
- **The lepton-hadron interference** contributions do not contain a logarithmic dependence neither on the lepton nor on the quark mass and therefore remain numerically small except at extreme values of x and y. They also can safely be neglected if one contents oneself with a precision at a few percent level.

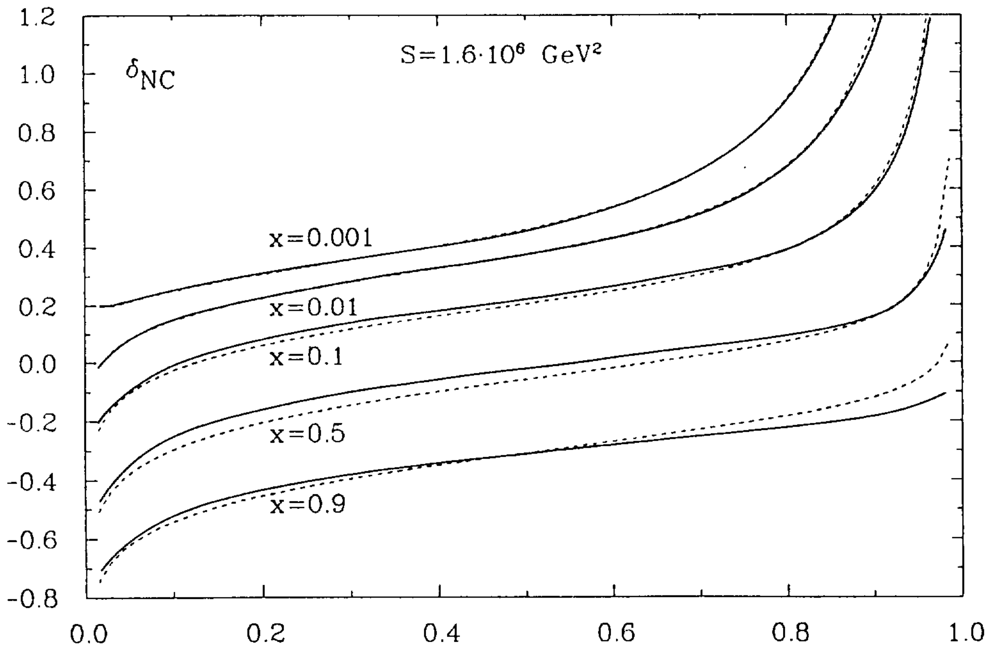

Fig. 1a

Fig. 1a shows a comparison of numerical results of a calculation including the complete $\mathcal{O}(\alpha)$ electroweak corrections except the quarkonic QED part (full lines) with an approximation which includes the leptonic QED contributions in the leading logarithmic approximation [3] and the photon and Z boson self energies (dashed lines). Based on this approximation we have calculated contours of constant radiative corrections $\delta_{NC}(x, Q^2) = const$ for the neutral current process at LEP + LHC in the x–Q^2 plane. The results in Fig. 1b show that requiring the corrections to stay below 50 % would mean to restrict the x–Q^2 region considerably. We will show now that the accessible region can be enlarged if radiative events are identified and rejected from event samples used for a physics analysis.

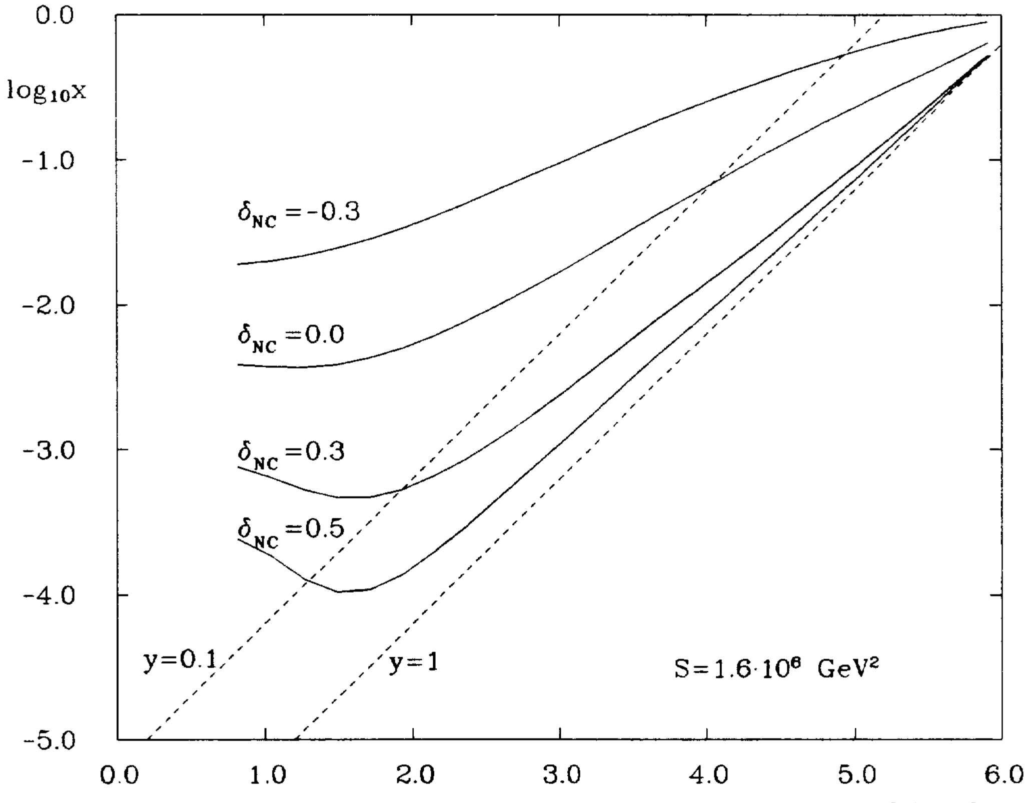

Fig. 1b

3 Results of a Monte Carlo Study

The event generator HERACLES [4], originally designed for deep inelastic scattering at HERA, was used to study the characteristics of radiative events in ep collisions at LEP+LHC. HERACLES includes the leptonic corrections as well as the complete one-loop virtual corrections and is thus able to give a good description of the neutral current reaction, including radiative effects. The event generation is performed on the parton level and the events are described by the 4-momenta of the final state particles electrons, quarks, and photons (and the flavor of the scattered quark), but the hadronic final state is not generated.

Fig. 2a shows the distribution of events with $0.75 \times 10^{-2} \leq x \leq 1.25 \times 10^{-2}$, $0.85 \leq y \leq 0.90$, $E_\gamma \geq 2\,GeV$ versus the emission angle of the photon θ_γ which is measured with respect to the electron direction. In the figure one recognizes three peaks:

- Events with $\theta_\gamma \simeq 0$, *i.e.* events where the photon is emitted collinearly with the

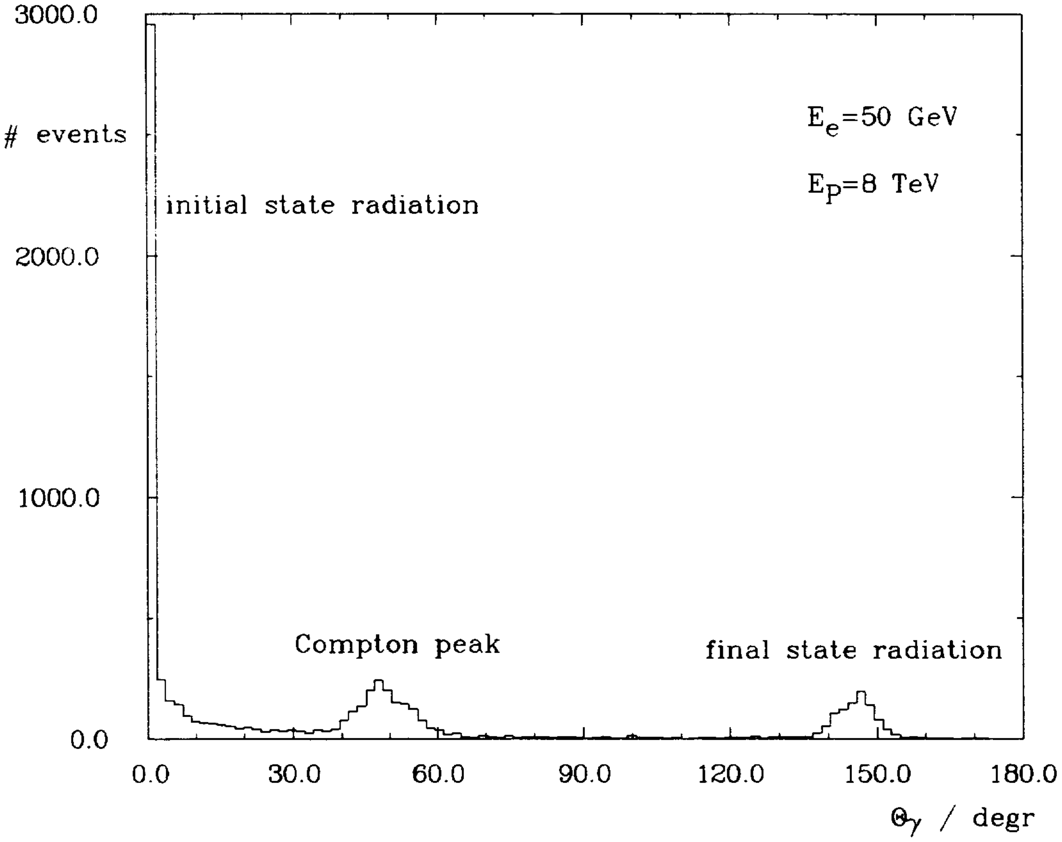

Fig. 2a

incoming electron (initial state radiation). Events of this type are the main source of the large corrections at small x and large y.

- Events with $\cos\theta_\gamma \simeq \cos\theta'_e$, *i.e.* events where the photon is emitted collinearly with the scattered electron (final state radiation); the scattering angle of the electron is determined by

$$\cos\theta'_e = \frac{(1-y)E_e - xyE_P}{(1-y)E_e + xyE_P}. \tag{2}$$

(E_e and E_P are the energies of the incoming electron and of the proton. Fermion masses have been neglected here). In the (x, y) bin considered here the electron scattering angle varies from 138° to 154°. Consequently also the peak from final state emission is smeared out over this range.

- A third peak is due to events with $\hat{Q}^2 \simeq 0$. The condition $\hat{Q}^2 = 0$ fixes the energy and the emission angle of the photon as functions of x and y:

$$E_\gamma^C = yE_e + x(1-y)E_P, \quad \cos\theta_\gamma^C = \frac{yE_e - x(1-y)E_P}{yE_e + x(1-y)E_P}. \tag{3}$$

These expressions are identical to the relations determining the energy $E_{q'}^0$ and the angle $\theta_{q'}^0$ of the final quark from x and y in the case of non-radiative scattering, *i.e.* for the $2 \to 2$ process $eq \to eq$. From eq. (3) one finds that for $\hat{Q}^2 \simeq 0$ the transverse momentum of the photon and of the electron are balanced: $k_T^C = p'_{e,T}$. This third contribution to the bremsstrahlung cross section is called the Compton part because it can be viewed as resulting from the emission of a quasireal photon from the quark line with subsequent Compton scattering $e\gamma \to e\gamma$. This peak is more pronounced for smaller values of x and large values of y but disappears at

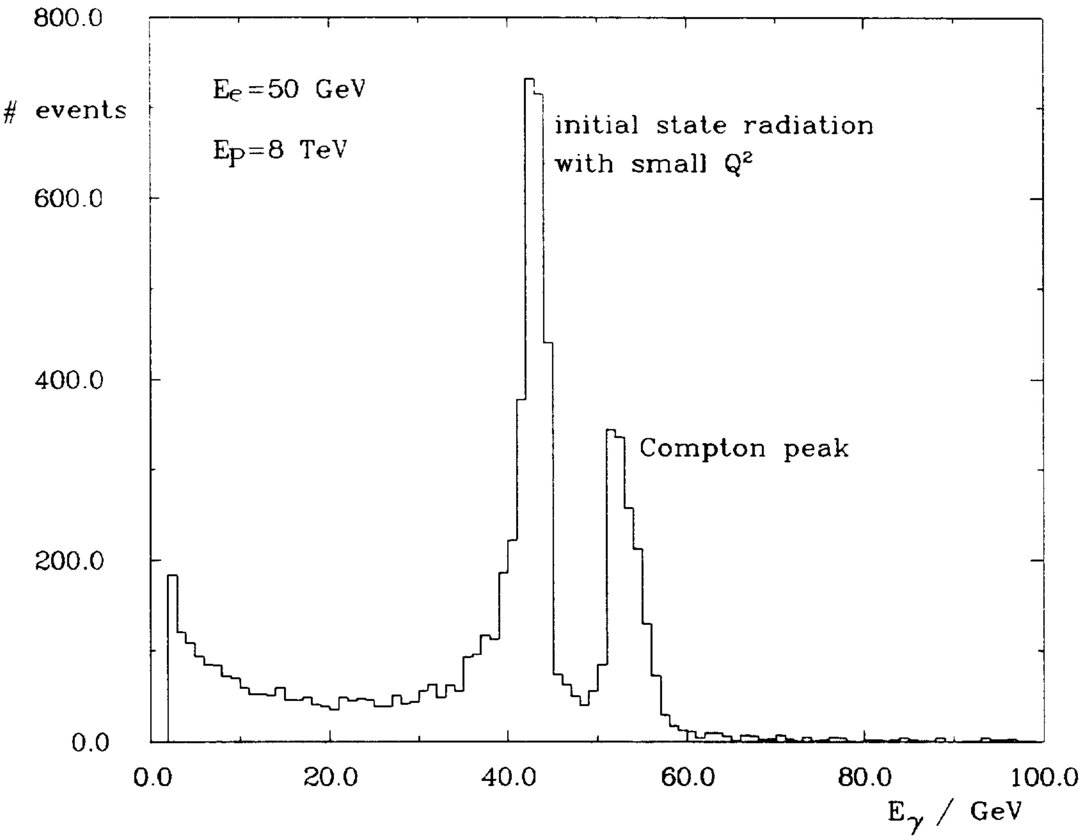

Fig. 2b

large x and small y. It is also visible in fig. 2b which shows the same sample of events distributed over the energy of the photon E_γ. The soft photon peak at $E_\gamma = 0$ is cut at the lower end of the spectrum by the condition $E_\gamma \geq 2\,GeV$. In addition to these peaks one finds also an accumulation of events with energies E_γ between 40 and $45\,GeV$. This enhancement of the cross section at a rather large photon energy is due to the combined effect of the factors $1/kp_e$ and $1/\hat{Q}^2$ in the differential cross section. Its position is determined by the maximal photon energy allowed for emission parallel to the incoming electron:

$$E_\gamma^{max}(\theta_\gamma = 0) = y\frac{1-x}{1-xy}E_e. \tag{4}$$

The aim of the following discussion is to present first ideas of how radiative events could eventually be identified and radiative corrections be reduced thereby. A complete investigation would have to start with a Monte Carlo which also simulates hadronization effects in order to include photons occuring during the evolution of the quark cascade, photons from hadron bremsstrahlung and hadron decays, as well as broadening of the current jet and systematic shifts of the angle and energy of the original parton from which the current jet is emerging. Eventually, it will also be necesseray to perform a detector simulation. This was not done here and the results shown below should be understood as showing up directions for further studies.

Photons can be identified if they have enough energy and if they are emitted with an angle being large enough. Our Monte Carlo study showed that cutting out the phase space region characterized by $6^o \leq \theta_\gamma \leq 174^o$ and $E_\gamma \geq 2\,GeV$ would reduce the corrections already by typically 30 % to 50 % except at small y where the corrections are dominated by soft photons. At $x \simeq 10^{-2}$ for instance the corrections would reach 50% only above $y \simeq 0.9$ (without cut for $y \leq 0.7$). Somewhat smaller reductions could be reached by leaving out events with $\theta_\gamma \leq 2\,mrad$ and $E_\gamma \leq 2\,GeV$. It might be possible that events of this type can be identified with the help of a luminosity monitor.

In addition to directly identifying a bremsstrahlung photon there is also the possibility to observe a photon indirectly because the emission of momentum by a photon disturbs the relation of energies and scattering angles of the electron and the hadron jet as it would be expected for events without (or only soft) photons.

A cross-check of this kinematical relation could be performed by comparing the results of the electron measurement of the scaling variables x_e, y_e with the values x_h, y_h which are obtained by using the Jaquet-Blondel method via the measurement of the total hadron flow. We assume that the difference of the polar angles between the actually emitted quark q' and that of the expected quark q'_0 as it is calculated from the electronic measurement of x and y using non-radiative kinematics eq. (3) is a measure of $|x_e - x_h|$ and $|y_e - y_h|$.

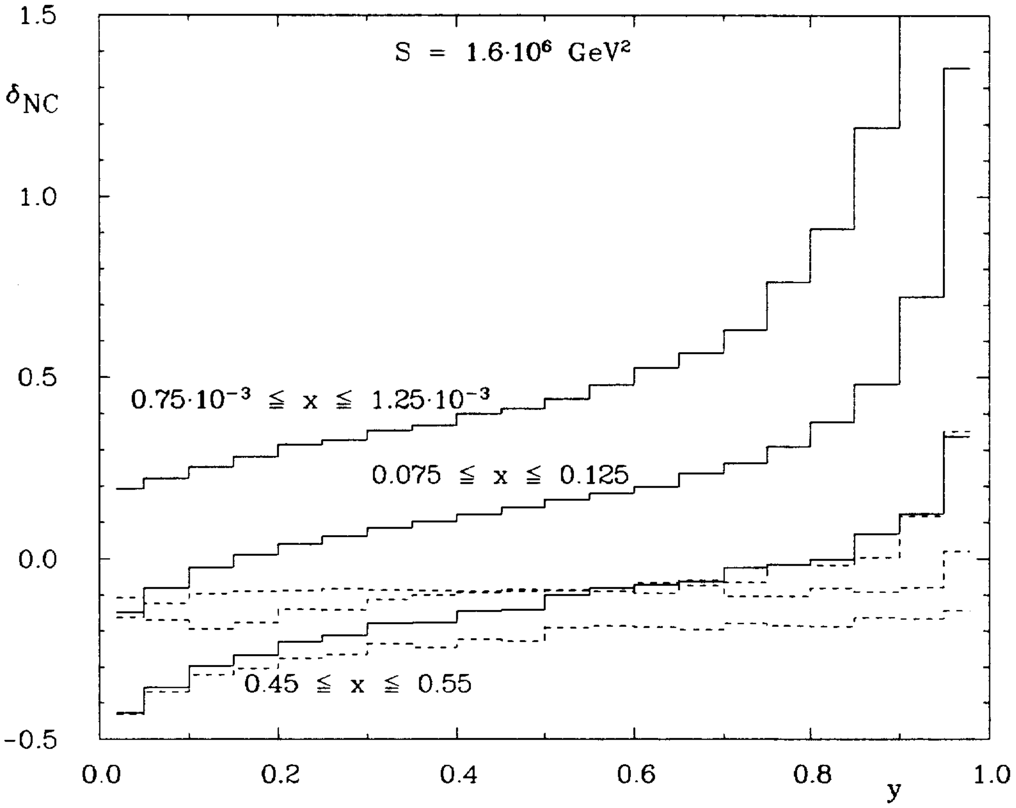

Fig. 3

In Fig. 3 the effect of a cut is shown which combines the possibility of cross-checking the electronic and the hadronic measurements via $\theta_{q'}$ with that of directly observing a photon. The cut is defined by the conditions

$$\begin{array}{ll} (i) & E_\gamma \geq 1\,GeV \text{ and } 50\,mrad \leq \theta_\gamma \leq \pi - 150\,mrad, \\ & \text{but } \angle(\vec{k}, \vec{p}_e') \geq 150\,mrad, \\ (ii) & E_\gamma \geq 5\,GeV \text{ and and } \theta_\gamma \leq 5\,mrad, \\ (iii) & |\theta_{q'} - \theta^0_{q'}| \geq 10^\circ. \end{array} \tag{5}$$

Results are shown for the fully inclusive corrections, *i.e.* without any cut (full lines) and for the corrections that remain after applying the cut (5). We find huge reductions down to values below 0% even at very large y. The results for the corrections after cut don't depend very strongly on the actual minimal value of $|\theta_{q'} - \theta^0_{q'}|$ which means that a good accuracy of the jet angle measurement is not essential. Note, that we did not use the energy of the scattered quark for the cross-checking of kinematics. An additional cut on $\Delta E_{q'} = |E_{q'} - E^0_{q'}|$ could lead to a further reduction of the radiative corrections. The results obtained here with the help of a Monte Carlo treatment of the exact $O(\alpha)$ leptonic corrections are in good agreement with a leading-log calculation [5].

As a prerequisite of the applicability of a cut on the jet angle, the jet has of course to come out with an energy big enough so that it can be identified as a jet. This is the case for larger values of x. Requiring a minimal jet energy of $20\,GeV$ would not change the results shown in Fig. 3b essentially. Only in the last bin $0.95 \leq y \leq 0.98$ and for $x \simeq 10^{-3}$ the corrections would be bigger by a few % than whithout this additional condition. For smaller values of x however, the jet energies are generally smaller and the additional condition $E_{q'} \geq E_{q',min}$ prevents from reaching similarly big reductions.

The experimental feasibility of these cuts is due to the fact that many events have a hard bremsstrahlung photon which turns the scattered quark into the central region of the detector so that radiative events can be identified because they have a clearly visible jet although from the electron measurement there was none expected. Also, photon emission allows for scattering with larger cms-energy and therefore higher energetic outgoing quarks are also possible.

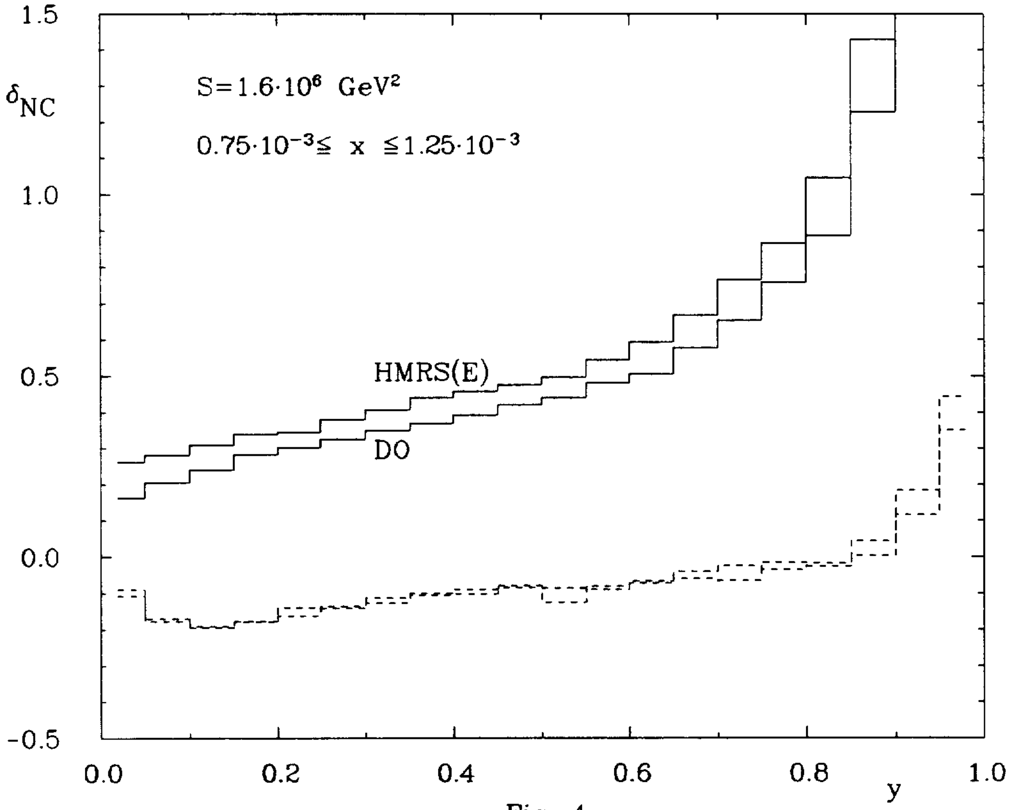

Fig. 4

Radiative corrections should not be seen as a source of theoretical uncertainties but rather as a generic ingredient in the calculation of reliable predictions from theory. However, as is the case for Born level calculations, they are subject to uncertainties from the parton distribution functions. Fig. 4 shows results of a comparison of two different input distributions. We have applied the cut (5) on two samples of events in the same x-region $0.75 \times 10^{-3} \leq x \leq 1.25 \times 10^{-3}$. For the first sample the parton parametrizations of [6] (set E) is used, while for the second we took the parton distributions of [7] as input. The total corrections are clearly distinct. The results for the corrections after cutting out observable radiative events, however, are very similar to each other. This fact should simplify the physics analysis of experimental data considerably. In turn it also means that the observable radiative events themselves have a potential for obtaining information on the structure functions.

Finally, we would like to comment on the influence of higher order corrections. $\mathcal{O}(\alpha^2)$ corrections in the leading logarithmic approximation have been calculated in [5] also for LEP + LHC. Only at extremely large y and at small x these corrections can reach a level of several percent.

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