

# Theory

A. De Rújula, M. Lusignoli, M. Masetti, A. Pich,  
S. Petrarca, J. Prades, A. Pugliese, H. Steger.

## 1. Introduction and Motivation.

Once upon a time, Nicola Cabibbo summarized a talk [1] by showing a cartoon of a nuclear mushroom labelled “CP-violation” and two ostriches with their proverbially buried heads. In the caption they tell each other “we fully understand the weak interactions”. The situation has improved but the moral has not changed.

In the three-generation Standard Model, CP-violation originates from the single phase naturally occurring in the weak-quark-current Cabibbo-Kobayashi-Maskawa (CKM) matrix. The observations in the neutral kaon system, as well as all the limits gathered elsewhere, agree with the expectations: aren’t these grounds for total satisfaction? No. For one reason, we have no understanding of why nature has chosen the number and properties of fundamental fields *just so* that CP-violation may be possible, but only large enough to be observable, so far, in the kaon sector. Moreover, our empirical information on CP-violation could hardly be more meager.

Most studies of CP-violation rely on the hope that the observable effects be much larger than the Standard Model (SM) would predict. The notable exception is CP-violation in the beauty analog to the  $K_0\bar{K}_0$  system, wherein the SM predicts very large asymmetries in suitably chosen channels with, a price must be payed, low branching ratios. A dedicated  $e^+e^-$  collider may be incomparably clean, but access to the luminosity required for these studies is an unresolved problem, particularly in those cases that require the analysis of  $B_s^0$  decays. In a  $pp$  collider, contrarywise, the maximal luminosity would be too large to handle, but a reduced luminosity suffices to access the channels with large CP-odd effects, in which the fake asymmetries induced by the  $pp$  non-symmetric initial state are not expected to be an untractable problem. Similar considerations apply to relatively modest extracted or internal beams hitting a solid or a gas-jet target. In the no-longer-bearable scenario that the SM be correct, and if Higgs scalars are stubbornly sneaky, CP violation in the beauty sector may be the *only* novel and assured result of the LHC.

Let the CKM matrix be parametrized in the Maiani-Wolfenstein way:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (1.1)$$

where  $\lambda \simeq \sin \theta_C \simeq 0.22$ ,  $|A| \sim 0.7 \rightarrow 1.1$  is determined from ratios of semileptonic B-decays, and  $\eta \neq 0$  if CP is violated. Differences of rates that signal CP-violation are proportional to the small product  $A^2 \lambda^6 \eta$ , but the corresponding asymmetries (difference / sum) are enhanced in B-decay relative to K-decay by the fact that the decay widths involve CKM elements that are much smaller in the B realm ( $|V_{cb}|^2$  or  $|V_{ub}|^2 \ll |V_{us}|^2$ ).

The unitarity of the  $3 \times 3$  CKM matrix implies, to leading order in  $\lambda$ , the relation:

$$V_{ub}^* + V_{td} \simeq \lambda V_{cb} \quad (1.2)$$

which can be simply visualized as a triangle in the complex plane. This is shown in fig. 1 where the triangle of (1.2) has been scaled by dividing its sides by  $\lambda V_{cb}$ . In what follows the CP asymmetries are expressed in terms of the angles  $\alpha$ ,  $\beta$ , and  $\gamma$  of this unitarity triangle which, in absence of CP violation, degenerates into a segment along the real axis.

We concentrate on those CP-odd effects that are expected to be sizable and that are amenable to a fair theoretical treatment, namely the asymmetries [2] involving the decays of  $B^0$  and  $\bar{B}^0$  (or  $B_s^0$  and  $\bar{B}_s^0$ ) into a common final state  $f$  that is a CP-eigenstate. An example of the type of process one would like to observe is depicted in fig. 2. A pair consisting of a neutral and a charged  $B$  ( $B^0 B^-$  or  $\bar{B}^0 B^+$ ) is produced in a  $pp$  interaction. The nature of the newly born  $B^0$  is determined by the charge of the companion charged  $B$ , that must be detected as a *tag* (the extent to which this gedanken tagging is realistic is discussed in Section 3). The  $B^0$  (or  $\bar{B}^0$ ) can decay directly to  $f$ , as in fig. 2a, or do it after the meson has been transmogrified into its antiparticle via the ‘‘mixing’’ process, as in fig. 2b. For the observable appearance of the final state  $f$  as a function of time, the interference between these two paths produces a deviation from an exponential decay law. If only a single amplitude contributes to the decay process,  $|\bar{\rho}(f)| \equiv |A(\bar{B}^0 \rightarrow f)/A(B^0 \rightarrow f)| = 1$ , and the modified decay law has the simple form:

$$\Gamma [B^0(t) \rightarrow f] \propto e^{-\Gamma t} \{1 - \text{Im}(\Lambda) \sin(\Delta M t)\}, \quad (1.3)$$

$$\Gamma [\bar{B}^0(t) \rightarrow f] \propto e^{-\Gamma t} \{1 + \text{Im}(\Lambda) \sin(\Delta M t)\}, \quad (1.4)$$

with  $\text{Im}(\Lambda)$  a function of the CKM parameters. In some cases, such as those listed in Table 1,  $\text{Im}(\Lambda) \simeq \sin(2\Phi)$ , with  $\pm\Phi$  one of the angles of the unitarity triangle.

Decay mode	CKM factor (Direct decay)	CKM factor (Penguin)	Exclusive channels	$\Phi$
$b \rightarrow c\bar{c}s$	$A\lambda^2$	$-A\lambda^2$	$\bar{B}_d^0 \rightarrow J/\psi K_S, J/\psi K_L$	$\mp\beta$
$b \rightarrow s\bar{s}s$	-	$-A\lambda^2$	$\bar{B}_d^0 \rightarrow K_S\phi, K_L\phi$	$\mp\beta$
$b \rightarrow c\bar{c}d$	$-A\lambda^3$	$A\lambda^3(1 - \rho + i\eta)$	$\bar{B}_d^0 \rightarrow D^+D^-, J/\psi\pi^0$	$\approx \mp\beta$
$b \rightarrow u\bar{u}d$	$A\lambda^3(\rho - i\eta)$	$A\lambda^3(1 - \rho + i\eta)$	$\bar{B}_d^0 \rightarrow \pi^+\pi^-, p\bar{p}, \rho^0\pi^0, \omega\pi^0$ $\bar{B}_s^0 \rightarrow \rho^0K_S, \omega K_S, \pi^0K_S,$ $\rho^0K_L, \omega K_L, \pi^0K_L$	$\approx \pm\alpha$ $\approx \mp\gamma$

TABLE 1

CKM factors and relevant angle  $\Phi$  for some B-decays into CP-eigenstates. Upper (lower) signs in the last column refer to CP-even (odd) final states, with the convention (1.4).

The b-quark is heavy enough, one hopes, to ascertain the underlying quark transition from the final states in a B-meson decay. The leading contributions to  $b \rightarrow q'\bar{q}'q$  decay amplitudes are either “direct” (Fermi) or generated by gluon exchange (“penguin”). Although of higher order in the strong coupling constant, penguin amplitudes are logarithmically enhanced, due to the virtual W-loop, and are potentially competitive. Table 1 contains the CKM factors associated with the direct and penguin diagrams for different B-decay modes into CP-eigenstates (CKM unitarity and  $m_b^2 \gg m_c^2, m_u^2$  imply that the phase of penguins is that of  $V_{tb}V_{tq}^*$  ( $q = d, s$ ) [3]). The  $b \rightarrow c\bar{c}s$  quark decays are theoretically unambiguous: the direct and penguin amplitudes have the same weak phase  $\sin(2\beta)$ . Ditto for  $b \rightarrow s\bar{s}s$ , where only the penguin mechanism is possible. The  $b \rightarrow c\bar{c}d$  and  $b \rightarrow u\bar{u}d$  decay modes are not so simple, the two decay mechanisms have the same Cabibbo suppression ( $\lambda^3$ ) and different weak phases, but the penguin amplitudes are down by  $(\alpha_s/6\pi)\ln(m_W/m_b) \approx 3\%$ : these decay modes can be used as approximate measurements of the angles of the unitarity triangle. We have not considered doubly Cabibbo-suppressed decay amplitudes, such as  $b \rightarrow u\bar{u}s$ , for which penguin effects can be important and spoil the simple estimates based on the direct decay mechanism.

Presumably the most realistic channels for the measurement of the angles  $\Phi = (\beta, \alpha, \gamma)$  are  $B_d^0 \rightarrow J/\psi K_S^0$ ,  $B_d^0 \rightarrow \pi^+ \pi^-$ ,  $B_s^0 \rightarrow \rho^0 K_S$ , respectively. The first of these processes is no doubt the one with the cleanest signature and the most tractable background. Most of the following discussion refers to this mode of B decay.

## 2. Current constraints on the unitarity triangle.

To derive the present constraints on the angles of the unitarity triangle, we follow the analysis in [4], updating the experimental [5] and theoretical inputs. As in [4], we are conservative, and allow for generous error bars in the input parameters. The allowed region for the vertex in the  $(\rho, \eta)$  parameters defined in (1.1) is presently explained, and shown in fig. 3 for three different values of the top-quark mass,  $m_t = 100, 140$  and  $180$  GeV.

We use [5]  $|V_{cb}| = 0.044 \pm 0.009$ . With  $\lambda = \sin \theta_c = 0.221$ , this implies  $|A| = 0.90 \pm 0.18$ . Three experimental inputs constrain the position of the  $\rho, \eta$  vertex:

i) The constraint [5]

$$|V_{ub}/V_{cb}| = 0.09 \pm 0.04 \quad (2.1)$$

forces the point  $(\rho, \eta)$  to lie between the two (dashed) circles centered at the origin.

ii) The measured [6]  $B^0 - \bar{B}^0$  mixing parameter,  $x_d \equiv \Delta M/\Gamma = 0.66 \pm 0.11$ , can be translated on information about the CKM-matrix, provided definite values are taken for  $m_t$  and  $\xi_B \equiv |f_B \sqrt{|B_B|}|$  (which parametrizes the hadronic matrix element of the  $\Delta B = 2$  four-quark operator between the  $B^0$  and  $\bar{B}^0$  mesons).

The actual size of  $\xi_B$  has been controversial for some time. Since the  $c$  and  $b$  quarks are quite heavy, many have used the infinite mass limit relation  $f_B/f_D \sim \sqrt{m_c/m_b}$  to extrapolate the value of  $f_D$  (computed either via QCD-sum rules or lattice simulations) to the bottom-mass scale. Moreover,  $B_B = 1$  has been usually assumed. But calculations of the bottom decay constant in the context of QCD-sum rules [7] [8] often result in  $f_B \geq f_D$ , also the prediction of recent lattice computations [9]. In a direct calculation [10] of the  $B^0 - \bar{B}^0$  matrix element (i.e.  $\xi_B$  instead of  $f_B$ ) a large value was also found, the result depending on the input b-quark ‘‘pole’’ mass. Using the presently favoured [8] value  $m_b = (4.6 \pm 0.1)$  GeV, one obtains from [10] the range that we shall adopt:

$$\xi_B \equiv |f_B \sqrt{|B_B|}| = (1.7 \pm 0.4) f_\pi, \quad (2.2)$$

with which, assuming  $B_B = 1$ , lattice estimates also agree.

Using the experimental input [11]  $\tau_b |V_{cb}|^2 = (3.5 \pm 0.6) \times 10^9 \text{ GeV}^{-1}$ , the  $x_d$ -constraint forces the vertex  $(\rho, \eta)$  to the region between the two (dash-dotted) circles centered at the point  $(1,0)$ . The bigger circle corresponds to the smaller  $\xi_B$ .

iii) The third constraint is imposed by the measured CP-violating contamination [12] in the  $K^0 - \bar{K}^0$  mixing matrix,  $|\epsilon| = 2.27 \times 10^{-3}$ . The main uncertainty here is the size of the hadronic matrix element of the  $\Delta S = 2$  four-quark operator between the  $K^0$  and  $\bar{K}^0$  mesons, which is usually characterized by the so called  $B_K$ -parameter. Chiral symmetry arguments [13] [14] and QCD-sum rules calculations [14] [15] give  $B_K$ -values in the range  $1/3$  to  $1/2$ . A value around  $3/4$  is obtained [16] with  $1/N_c$ -expansion techniques, and lattice calculations favour  $B_K \sim 1$  [17]. We use

$$1/3 \leq B_K \leq 1. \quad (2.3)$$

The resulting allowed domain for the vertex  $(\rho, \eta)$  of the unitarity triangle is limited by the two hyperbolas (solid curves) in the figures. The smaller values of  $B_K$  correspond to bigger values of  $\eta$ .

The intersection of the three regions resulting from the constraints in i) ii) and iii) gives the final allowed domain (shaded area). The permissible values of the angles of the unitarity triangle can be read from the figure. For  $90 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$ , we find:

$$-1 \leq \sin 2\alpha \leq 1, \quad (2.4a)$$

$$0.16 \leq \sin 2\beta \leq 1, \quad (2.4b)$$

$$-1 \leq \sin 2\gamma \leq 1. \quad (2.4c)$$

While nothing can be said at the moment on the expected size of  $\sin 2\alpha$  and  $\sin 2\gamma$ , the magnitude of the CP-violating interference term is auspiciously guaranteed to be bigger than 0.16, for those asymmetries governed by  $\sin 2\beta$ . This lower bound depends on  $m_t$ . For  $m_t = 100, 140$  and  $180 \text{ GeV}$ , the result is  $\sin 2\beta \geq 0.16, 0.21$  and  $0.24$ , respectively.

### 3. Fake CP Violation in B Decays at pp Colliders

The initial state at a  $pp$  collider is not a CP eigenstate. The question arises that this CP-asymmetry could percolate, via ‘‘leading particle effects’’, into the final state and

induce fake asymmetries mimicking CP violation in the B sector. This question has also been addressed by Fridman [18], whose treatment differs from ours in points of detail.

It must be **vehemently** emphasized from the very start that the physics connected to the details of the production of specific final states in high energy  $pp$  collisions is not understood to a depth sufficient to answer with confidence questions such as the fake asymmetries. An insufficiently uncommon attitude consists in believing results from Monte Carlo generators that are not trustworthy enough to address this type of question. Our attitude will be to use those generators, when unavoidable, to help establish extreme worst-case scenarios. Only in these extremes do the fake asymmetries turn out to size up with the true asymmetries expected in the Standard Model. To deal with the unlikely case that nature is equally extreme and perverse, and in an attempt to reach an empirical resolution of the problem, we discuss related measurable asymmetries that are expected to be exclusively “fake”, and their use as “background subtractions”.

Consider the gold-plated channel  $B_d^0 \rightarrow K_S J/\psi$ , followed by  $K_S \rightarrow \pi^+ \pi^-$  and  $J/\psi \rightarrow l^+ l^-$ . The ideal case is that in which the second B is perfectly tagged, in the sense that the  $b$  or  $\bar{b}$  nature of its defining quark is established (this is possible for example for charged B’s flying long enough for their charged nature to be observed, for instance via their decay into a lepton the sign of whose charge is measured). Moreover, CP-conjugate  $b$ -flavoured hadron pairs must be produced in equal amounts, as in  $e^+ e^-$  or  $p\bar{p}$  collisions. The time-integrated asymmetry has in this ideal case the simple form:

$$A_{J/\psi K_S} = \frac{N(J/\psi K_S l^+) - N(J/\psi K_S l^-)}{N(J/\psi K_S l^+) + N(J/\psi K_S l^-)} = \frac{x_d}{1 + x_d^2} \sin 2\beta, \quad (3.1)$$

with  $x_d/(1 + x_d^2) \approx 0.46$  the dilution factor due to the oscillation of the neutral B.

In a  $pp$  collider mode the ideal case may not be realistic. First, we must study the potential consequences of not knowing the precise identity of the tagging B-particle. The complication arises because, via their mixing, neutral B mesons may yield leptons of the “wrong” sign. Define the probability for an initially produced  $B^0$  meson to decay as a  $\bar{B}^0$  (or vice versa) by  $W$  (for the *W*rong sign of the lepton) and let  $R = 1 - W$  be its complement:

$$W = \frac{N(B^0 \rightarrow \bar{B}^0)}{N(B^0 \rightarrow \bar{B}^0) + N(B^0 \rightarrow B^0)} = \frac{N(\bar{B}^0 \rightarrow B^0)}{N(\bar{B}^0 \rightarrow B^0) + N(\bar{B}^0 \rightarrow \bar{B}^0)} = \frac{x^2}{2 + 2x^2}. \quad (3.2)$$

The possible semileptonic tags are  $B^+ \rightarrow l^+ \nu_l X$  or  $\bar{\Lambda}_b \rightarrow l^+ \nu_l X$  (fully efficient)<sup>1</sup>,  $B_d^0 \rightarrow l^+ \nu_l X$  (less efficient because of  $B_d^0 - \bar{B}_d^0$  mixing) and  $B_s^0 \rightarrow l^+ \nu_l X$  (inefficient: mixing is expected to be maximal). Because of this tag hierarchy, it is convenient to introduce the quantities:

$$\begin{aligned}
\tilde{n}^+ &= N(B^+ \bar{B}_d^0) \text{Br}(B^+ \rightarrow l^+ \nu_l X) + N(\bar{\Lambda}_b \bar{B}_d^0) \text{Br}(\bar{\Lambda}_b \rightarrow l^+ \nu_l X), \\
\tilde{n}^- &= N(B^- B_d^0) \text{Br}(B^- \rightarrow l^- \bar{\nu}_l X) + N(\Lambda_b B_d^0) \text{Br}(\Lambda_b \rightarrow l^- \bar{\nu}_l X), \\
\tilde{n}^0 &= N(B_d^0 \bar{B}_d^0) \times \frac{1}{2} [\text{Br}(B_d^0 \rightarrow l^+ \nu_l X) + \text{Br}(\bar{B}_d^0 \rightarrow l^- \bar{\nu}_l X)], \\
\tilde{n}_s &= N(B_s^0 \bar{B}_d^0) \text{Br}(B_s^0 \rightarrow l^+ \nu_l X), \\
\tilde{n}_{\bar{s}} &= N(\bar{B}_s^0 B_d^0) \text{Br}(\bar{B}_s^0 \rightarrow l^- \bar{\nu}_l X), \\
\tilde{n} &\equiv \tilde{n}^+ + \tilde{n}^- + 2\tilde{n}^0 + \tilde{n}_s + \tilde{n}_{\bar{s}},
\end{aligned} \tag{3.3}$$

in terms of the numbers  $N$  of the various produced pairs. Up to an overall (and ultimately irrelevant) common factor, the number of decays into  $J/\psi K_S l^+ X$  and  $J/\psi K_S l^- X$  is obtained by combining (3.2) and (3.3), with the result:

$$\begin{aligned}
N(J/\psi K_S l^+ X) &\propto \{ \tilde{n}^+ + \tilde{n}^0 R_d + \tilde{n}_s R_s \} \left\{ 1 + \frac{x_d}{1+x_d^2} \sin(2\beta) \right\} \\
&\quad + \{ \tilde{n}^0 W_d + \tilde{n}_{\bar{s}} W_s \} \left\{ 1 - \frac{x_d}{1+x_d^2} \sin(2\beta) \right\}, \tag{3.4a}
\end{aligned}$$

$$\begin{aligned}
N(J/\psi K_S l^- X) &\propto \{ \tilde{n}^- + \tilde{n}^0 R_d + \tilde{n}_{\bar{s}} R_s \} \left\{ 1 - \frac{x_d}{1+x_d^2} \sin(2\beta) \right\} \\
&\quad + \{ \tilde{n}^0 W_d + \tilde{n}_s W_s \} \left\{ 1 + \frac{x_d}{1+x_d^2} \sin(2\beta) \right\}. \tag{3.4b}
\end{aligned}$$

This corresponds to a time-integrated asymmetry:

$$\begin{aligned}
A_{J/\psi K_S} &= \frac{N(J/\psi K_S l^+) - N(J/\psi K_S l^-)}{N(J/\psi K_S l^+) + N(J/\psi K_S l^-)} \\
&= \frac{\tilde{n}^+ - \tilde{n}^- + \frac{\tilde{n}_s - \tilde{n}_{\bar{s}}}{1+x_d^2} + \sin(2\beta) \frac{x_d}{1+x_d^2} (\tilde{n}^+ + \tilde{n}^- + \frac{2}{1+x_d^2} \tilde{n}^0 + \frac{\tilde{n}_s + \tilde{n}_{\bar{s}}}{1+x_d^2})}{\tilde{n} + \sin(2\beta) \frac{x_d}{1+x_d^2} (\tilde{n}^+ - \tilde{n}^- + \tilde{n}_s - \tilde{n}_{\bar{s}})}, \tag{3.5}
\end{aligned}$$

that would, in the half-ideal case of “perfect” tagging, simplify to:

$$A_{J/\psi K_S} = \frac{\tilde{n}^+ - \tilde{n}^- + \sin(2\beta) \frac{x_d}{1+x_d^2} (\tilde{n}^+ + \tilde{n}^-)}{\tilde{n}^+ + \tilde{n}^- + \sin(2\beta) \frac{x_d}{1+x_d^2} (\tilde{n}^+ - \tilde{n}^-)}. \tag{3.6}$$

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<sup>1</sup> We denote by  $\bar{\Lambda}_b$  any  $\{\bar{b}\bar{q}\bar{q}'\}$  baryon that decays only weakly.

The asymmetry  $A_{J/\psi K_s}$  does not vanish in the CP-conserving limit ( $\beta = 0$ ) if  $\tilde{n}^+$  differs from  $\tilde{n}^-$ , as it may be expected in a  $pp$  collision. To gauge the incidence of fake asymmetries it suffices to neglect in (3.5) the  $\sin(2\beta)$  term in the denominator, and the terms containing  $x_s$  (the SM expectation is  $x_s^2 \sim 80$  or larger). The asymmetry can then be simplified to read:

$$A_{J/\psi K_s} \approx a \cdot (F + \sin(2\beta)), \quad (3.7)$$

with

$$a \approx \frac{x_d}{1+x_d^2} \frac{1}{\tilde{n}} \left( \tilde{n}^+ + \tilde{n}^- + \frac{2}{1+x_d^2} \tilde{n}^0 \right) \approx \frac{2}{3} \frac{x_d}{1+x_d^2} \approx 0.30 \quad (3.8)$$

the coefficient of  $\sin(2\beta)$  in the true asymmetry and

$$F \equiv \frac{\tilde{n}^+ - \tilde{n}^-}{\frac{x_d}{1+x_d^2} (\tilde{n}^+ + \tilde{n}^- + \frac{2}{1+x_d^2} \tilde{n}^0)} \quad (3.9)$$

representing the “fake” addition to the true CP-odd parameter  $\sin(2\beta)$ .

We proceed to attempt to estimate  $F$ . The  $\bar{b}$  antiquarks may catch an incoming valence quark to result in more  $\bar{b}u$  and  $\bar{b}d$  than  $b\bar{u}$  and  $b\bar{d}$  mesons. Balancing this behaviour, more  $b$  baryons than antibaryons are to be expected. Unlike  $\sin(2\beta)$ ,  $F$  should peak at large  $x_F$  and small  $p_T$ . Let  $p_u = p_d$ ,  $p_s$  and  $p_\Lambda$  be the fragmentation probabilities (adding up to unity) for a  $\bar{b}$  quark to end up as  $B^+$ ,  $B^0$ ,  $B_s^0$  and  $\bar{\Lambda}_b$ , in  $e^+e^-$  annihilation, wherein the leading particle effects are absent. To describe the difference between these probabilities and the corresponding ones in  $pp$  collisions (dubbed  $P$ ) we introduce two parameters  $l_m$  and  $l_b$  (for mesons and baryons) implicitly defined via the following equations:

$$\begin{aligned} P(B^+) &= p_d(1 - l_m) + 2l_m/3, & P(B^-) &= p_d(1 - l_b), \\ P(B_d^0) &= p_d(1 - l_m) + l_m/3, & P(\bar{B}_d^0) &= p_d(1 - l_b), \\ P(B_s^0) &= p_s(1 - l_m), & P(\bar{B}_s^0) &= p_s(1 - l_b), \\ P(\bar{\Lambda}_b) &= p_\Lambda(1 - l_m), & P(\Lambda_b) &= p_\Lambda(1 - l_b) + l_b. \end{aligned} \quad (3.10)$$

For  $l_m$  and/or  $l_b \neq 0$  the above particle-antiparticle asymmetries clearly yield a nonvanishing fake asymmetry  $F$ . Using PYTHIA Fridman [18] finds, for  $p_T < 5$  GeV,  $l_m \sim 1\%$ ;  $l_b \sim 3\%$ . Assuming  $p_d = 0.38$ ,  $p_s = 0.14$  and  $p_\Lambda = 0.10$ , these values correspond to  $F \approx -3\%$ . Our own estimates based on the HERWIG 5.0 generator [19] indicate even smaller fake asymmetries, but strongly depend on the choice of undecidable free parameters in the program. To illustrate a wide range of parameter space, we show in fig. 4a



the values of  $A_{J/\psi K_S}$  for the case  $\sin(2\beta) = 0$ , as a function of  $l_m$  and  $l_b$ , both in the interval  $0 \rightarrow 0.1$ , keeping  $p_d$ ,  $p_s$  and  $p_\Lambda$  at the values given before. For b-hadron decays the spectator model should hold, and we have in fig. 4a assumed

$$R_{\Lambda_b} \equiv \frac{\text{Br}(\Lambda_b \rightarrow l\nu X)}{\text{Br}(B \rightarrow l\nu X)} = 1. \quad (3.11)$$

To test the sensitivity to this assumption, we show in fig. 4b a similar plot using a value of  $R_{\Lambda_b} \sim 0.37$ , the measured result for charmed hadrons. The sensitivity to  $l_b$  and  $R_{\Lambda_b}$  is considerable: the role of semileptonic b-baryon decays is significant. In these plots we assumed an 8% probability of double versus single b-pair production [18], no doubt a pessimistically large choice (the single pair multiplicity is estimated to be less than  $\sim 0.5\%$  in collider mode and  $\approx 10^{-4}$  in fixed target mode). We did also neglect the possibility of distinguishing experimentally single from double pair production.

In fig. 4c we show  $F$ , defined in (3.9), for  $R_{\Lambda_b} = 1$ .  $F$  is very insensitive to  $l_m$  and may be as large as  $-14\%$  for  $l_m = 0$ ,  $l_b = 0.1$ . Such a large fake asymmetry may seriously hinder the search for a true CP-odd effect, but is difficult to imagine on fairly general grounds. A value  $l_b = 0.1$  corresponds to a 10% probability of a b-quark spousing the original diquark in one of the incident protons. A typical LHC collider-mode event may have a particle multiplicity of  $\sim 100$  or a  $q + \bar{q}$  multiplicity of  $\sim 200$ . It is hard to believe that in 10% of the cases the b-quark is the one to stick to the diquark. Complete satisfaction, however, may require more than educated guesswork, and we proceed to discuss two previous and one new empirical ways to tackle the problem of fake asymmetries:

i) Reconstruct and measure the decays of  $B^+$ ,  $B^-$ ,  $B^0$ ,  $\bar{B}^0$ ,  $\Lambda_b$  and  $\bar{\Lambda}_b$  produced in pp collisions, to obtain information on the parameters  $l_m$  and  $l_b$  [18]. This procedure involves absolute measurements of cross sections and is more sensitive to systematics than the (normalized) asymmetries are. Even without leading-particle effects  $\tilde{n}^+$ ,  $\tilde{n}^-$  and  $\tilde{n}^0$  must be separately measured to extract the CP-odd effect. The method calls for an excellent vertex detector, and large statistics ( $10^{10} \div 10^{11}$  b pairs per year), not a trivial task.

ii) The measurement and subtraction of asymmetries involving final states with opposite CP, e.g.  $B \rightarrow J/\psi K_S$  and  $B \rightarrow J/\psi K_L$  [20]. In this example

$$\begin{aligned} \Delta &\equiv A_{J/\psi K_S} - A_{J/\psi K_L} \\ &= 2 \sin(2\beta) \frac{x_d}{1+x_d^2} \left\{ \frac{\tilde{n}^+ + \tilde{n}^- + 2\frac{\tilde{n}^0}{1+x_d^2}}{\tilde{n}} + O\left(\left[\frac{\tilde{n}^+ - \tilde{n}^-}{\tilde{n}}\right]^2\right) \right\}. \end{aligned} \quad (3.12)$$

Again, one must measure  $\tilde{n}^+$ ,  $\tilde{n}^-$  and  $\tilde{n}^0$  separately, and since  $K_L$ 's might often escape undetected, there are severe requirements on statistics.

iii) Express the fake contribution to the asymmetry in a channel where one may expect a large CP-odd effect in terms of asymmetries measured in channels wherein the putative effects are expected to be exclusively fake. As an example, consider our favorite ‘‘signal’’ channel  $J/\psi K_S$ , and the ‘‘fake’’ or calibration channels  $D^+\pi^-1^+X$ ,  $D^+\pi^-1^-X$ ,  $D^-\pi^+1^-X$  and  $D^-\pi^+1^+X$ , for which the standard CP-odd effects are doubly Cabibbo suppressed ( $\sin^2(\theta_C)$  in the amplitude). The relevant products of branching ratios [12] favourably compare with those in the  $J/\psi K_S$  mode:  $\text{Br}(B^0 \rightarrow D^-\pi^+) \times \text{Br}(D^- \rightarrow K^+\pi^-\pi^-) \approx 3 \cdot 10^{-4}$  versus  $\text{Br}(B^0 \rightarrow J/\psi K_S) \times \text{Br}(J/\psi \rightarrow 1^+1^-) \times \text{Br}(K_S \rightarrow \pi^+\pi^-) \approx 3 \cdot 10^{-5}$  (the reconstruction efficiency of  $D^-$  from the  $K^+\pi^-\pi^-$  final state must also be compared with the product of those for  $J/\psi \rightarrow 1^+1^-$  and  $K_S \rightarrow \pi^+\pi^-$ ). For the four possibilities of charge combinations we find:

$$\begin{aligned}
N(D^+\pi^-1^+) &= \tilde{n}^+ R_d + \tilde{n}^0 \cdot (R_d^2 + W_d^2) + \tilde{n}_{\bar{s}} W_d W_s + \tilde{n}_s R_d R_s, \\
N(D^+\pi^-1^-) &= \tilde{n}^- W_d + 2\tilde{n}^0 R_d W_d + \tilde{n}_{\bar{s}} W_d R_s + \tilde{n}_s R_d W_s, \\
N(D^-\pi^+1^-) &= \tilde{n}^- R_d + \tilde{n}^0 \cdot (R_d^2 + W_d^2) + \tilde{n}_{\bar{s}} R_d R_s + \tilde{n}_s W_d W_s, \\
N(D^-\pi^+1^+) &= \tilde{n}^+ W_d + 2\tilde{n}^0 R_d W_d + \tilde{n}_{\bar{s}} R_d W_s + \tilde{n}_s W_d R_s.
\end{aligned} \tag{3.13}$$

These may be combined into six asymmetries, three of which are linearly independent. Introduce the ratios:

$$\begin{aligned}
A_1^{D\pi} &= (N(D^+\pi^-1^+) - N(D^-\pi^+1^-))/\tilde{n} \\
&= ((\tilde{n}^+ - \tilde{n}^-)R_d + (\tilde{n}_s - \tilde{n}_{\bar{s}})(R_d R_s - W_d W_s))/\tilde{n}, \\
A_2^{D\pi} &= (N(D^-\pi^+1^+) - N(D^+\pi^-1^-))/\tilde{n} \\
&= ((\tilde{n}^+ - \tilde{n}^-)W_d - (\tilde{n}_s - \tilde{n}_{\bar{s}})(R_d W_s - W_d R_s))/\tilde{n}, \\
B_1^{D\pi} &= (N(D^+\pi^-1^+) - N(D^-\pi^+1^+))/\tilde{n} \\
&= (\tilde{n}^+ + \tilde{n}^0(R_d - W_d) + \tilde{n}_s R_s - \tilde{n}_{\bar{s}} W_s)(R_d - W_d)/\tilde{n}, \\
B_2^{D\pi} &= (N(D^-\pi^+1^-) - N(D^+\pi^-1^-))/\tilde{n} \\
&= (\tilde{n}^- + \tilde{n}^0(R_d - W_d) - \tilde{n}_s W_s + \tilde{n}_{\bar{s}} R_s)(R_d - W_d)/\tilde{n}
\end{aligned} \tag{3.14}$$

and notice that only  $A_1^{D\pi}$ ,  $A_2^{D\pi}$  and  $(B_1^{D\pi} - B_2^{D\pi})$  vanish for  $\tilde{n}^+ = \tilde{n}^-$  and  $\tilde{n}_s = \tilde{n}_{\bar{s}}$ , but not  $B_1^{D\pi}$  and  $B_2^{D\pi}$  separately. The fake part  $F$  of the asymmetry in  $J/\psi K_S$  decays can

be expressed in terms of asymmetries in the  $D\pi$  decays of  $B$  mesons (expected to be pure “fakes”) and of the mixing parameter  $x_d$  of the  $B_d^0 - \bar{B}_d^0$  system:

$$\begin{aligned}
 F &= \frac{1}{x_d} \frac{A_1^{D\pi} + A_2^{D\pi}}{B_1^{D\pi} + B_2^{D\pi}} \\
 &= \frac{1}{x_d} \frac{N(D^+\pi^-1^+) - N(D^-\pi^+1^-) - N(D^+\pi^-1^-) + N(D^-\pi^+1^+)}{N(D^+\pi^-1^+) - N(D^-\pi^+1^+) + N(D^-\pi^+1^-) - N(D^+\pi^-1^-)}.
 \end{aligned}
 \tag{3.15}$$

One may also express the total asymmetry for  $J/\psi K_S$  in terms of the  $A$ 's,  $B$ 's,  $x_d$  and  $\sin(2\beta)$ :

$$A_{J/\psi K_S} = \frac{(A_1^{D\pi} + A_2^{D\pi}) + x_d \sin(2\beta) \cdot (B_1^{D\pi} + B_2^{D\pi})}{1 + x_d \sin(2\beta) \cdot (A_1^{D\pi} - A_2^{D\pi})}.
 \tag{3.16}$$

From this it is easy to calculate the asymmetry difference  $\Delta$  of (3.12):

$$\Delta = 2 x_d \sin(2\beta) \frac{B_1^{D\pi} + B_2^{D\pi} - (A_1^{D\pi})^2 + (A_2^{D\pi})^2}{1 - [x_d \sin(2\beta)(A_1^{D\pi} - A_2^{D\pi})]^2}.
 \tag{3.17}$$

It is therefore possible in the presence of fake asymmetries to measure the phase of the CKM matrix, even without measuring absolute cross sections. Naturally, the requirements on statistics are stiffer than for CP invariant initial states.

To conclude, we have argued (but not proven) that fake asymmetries induced by the leading-particle effects characteristic of a  $pp$  collider should not seriously hinder the search for CP violation in  $B$  decays. There are empirical ways to tackle the threat of forgery. The obvious one is to check the  $p_T$ - and  $x_F$ -independence of the potential signal. Another one is the search for asymmetries that (in the Standard Model) are not expected to reflect CP-violation. These asymmetries, if found not to vanish, can also be used to subtract the fake contribution to an honest-to-goodness CP-odd signal.

#### 4. Search for $B_c$ mesons

Besides CP violation in the beauty sector, there are other items of interest in  $b$ -physics: rare decays,  $B_s \bar{B}_s$ , mixing, etc. Many of these may be investigated at LEP, CESR or future  $e^+e^-$  Beauty factories before LHC starts operation at 23:59, Dec. 31st, 1997. An exception could be the discovery of long-lived particles containing two or more heavy quarks, such as mesons carrying both beauty and charm quantum numbers, no doubt the most promising example.

The spectrum of  $b\bar{c}$  states can be predicted in potential models [21], that successfully describe the charmonium and bottomonium families. The lightest state is expected to be a ( $\bar{b}c$ ) pseudoscalar,  $B_c^+$ , with a mass  $\sim 6.3$  GeV. The first excited state, a vector meson, should be heavier by only  $70 \rightarrow 100$  MeV: it would decay emitting a photon which, if detected, could help in identifying the subsequent weak  $B_c$  decay.

The decays of  $B_c^+$  are of three types: spectator  $c$  quark, spectator  $\bar{b}$ , and  $c\bar{b}$  annihilation. The total rate due to processes of the first class is expected to be equal to the decay rate for a  $B^+$  (or  $B_d^0$ ) meson. Contrarywise, the naive expectation should fail that the spectator-model prediction for the  $D$  decay rate correctly describe the rate of spectator  $\bar{b}$  decays, since the heavy (and less strongly bound) final  $B_s$  (or  $B_{d,u}$ ) considerably reduces phase space: a  $\sim 40\%$  suppression factor is expected [22]. Accepting the potential-model  $B_c$  decay constant ( $f_{B_c} \simeq 570$  MeV), the annihilation contribution should be rather important. The three decay types contribute respectively 36%, 40% and 24% to the total rate; the lifetime is estimated to be similar to that of  $D^0$ 's, namely  $\tau_{B_c} \simeq 4.4 \cdot 10^{-13}$  s.

A clear signature could be the decay  $B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu$ , followed by  $J/\psi \rightarrow \mu^+ \mu^-$ . An estimate of the branching ratio is  $0.03 \times 0.07 \simeq 2 \cdot 10^{-3}$ , to be multiplied by four if the analogous electron channels are also observed. Complete reconstruction of the  $B_c$  mass would be possible in the decay  $B_c^+ \rightarrow J/\psi \pi^+$ , whose branching ratio is expected to be an order of magnitude below the previous one.

Predictions for  $B_c$  production cross section are very uncertain. For  $e^+e^-$  colliders, a naive nonrelativistic model [23] predicts a meagre 600  $B_c^+$  events in  $10^7$  s at the  $Z^0$  peak with the project LEP luminosity  $\mathcal{L} = 1.7 \cdot 10^{31}$  cm $^{-2}$  s $^{-1}$ .

Existing estimates for  $pp$  interactions [24] depend on three unknowns: the probability  $\lambda_c(\lambda_b)$  to produce a charmed (beautiful) quark pair in the fragmentation of a beauty (charm) quark and the probability  $\mu$  for the  $c$  and  $\bar{b}$  to recombine in a  $B_c^+$ . The guess in [24], corresponding to  $\lambda_c \simeq 10 \cdot \lambda_b \simeq 0.04$  and  $\mu \simeq 0.1$  is  $\sigma(pp \rightarrow B_c^+ X) \simeq 5 \cdot 10^{-3} \sigma(pp \rightarrow b\bar{b} X)$ .

To have a firmer estimate, HERWIG 5.0 [19] has been used. The result depends on parameters describing hadronization, in particular the probability of  $c\bar{c}$  production in the non-perturbative splitting of a heavy colour-singlet cluster. Keeping this probability on the high side, within the constraints from ISR data, we obtain

$$\sigma(pp \rightarrow B_c^+ X) \leq 3 \cdot 10^{-4} \sigma(pp \rightarrow b\bar{b} X). \quad (4.1)$$

This result is much smaller than the guesstimate in [24], mainly because of the smaller  $\mu \simeq 5 \cdot 10^{-3}$ . One  $B_c^+$  produced every  $10^4$   $b\bar{b}$  pairs with at least  $10^{10}$   $b\bar{b}$  pairs produced in

one year in a *low luminosity* ( $10^{31} \text{cm}^{-2} \text{s}^{-1}$ ) intersection at LHC energies [25], corresponds to one million  $B_c$ 's yearly produced. Even a tenth of this yield, not unthinkable within the uncertainties, should suffice to discover the  $B_c$  and to make a first study of its decays.

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### Figure Captions

- Fig. 1. The unitarity triangle serving to define the phases  $\alpha$ ,  $\beta$  and  $\gamma$ .
- Fig. 2. CP violation in tagged B decays, with the neutral B decaying into a CP-eigenstate  $f$ .
- Fig. 3. Constraints from  $|V_{ub}/V_{cb}|$  (dashed circles),  $x_d$  (dot-dashed circles) and  $\epsilon$  (solid hyperbolas) on the unitarity triangle of fig. 1, for  $m_t = 100, 140$  and  $180$  GeV. The shaded region is allowed.
- Fig. 4. [a] The fake asymmetry  $A = aF$  for  $R_{\Lambda_b} = 1$  as a function of  $l_m$  and  $l_b$ . [b] The same for  $R_{\Lambda_b} = 0.37$ . [c] The fake contribution  $F$  to the asymmetry, for  $R_{\Lambda_b} = 1$ .

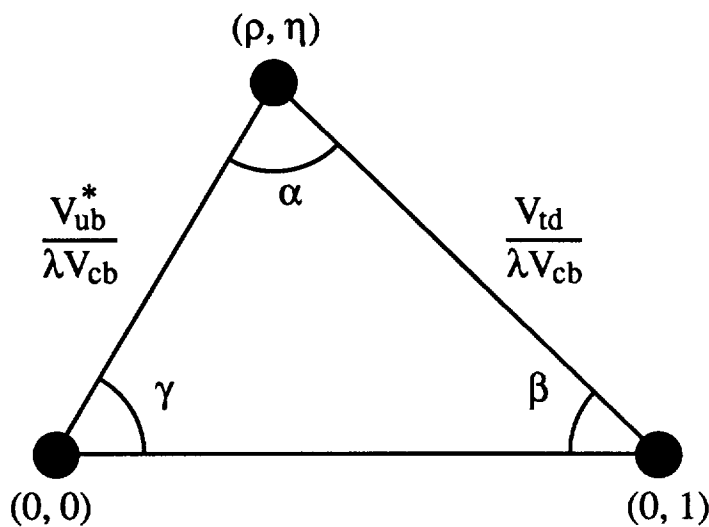


Fig. 1

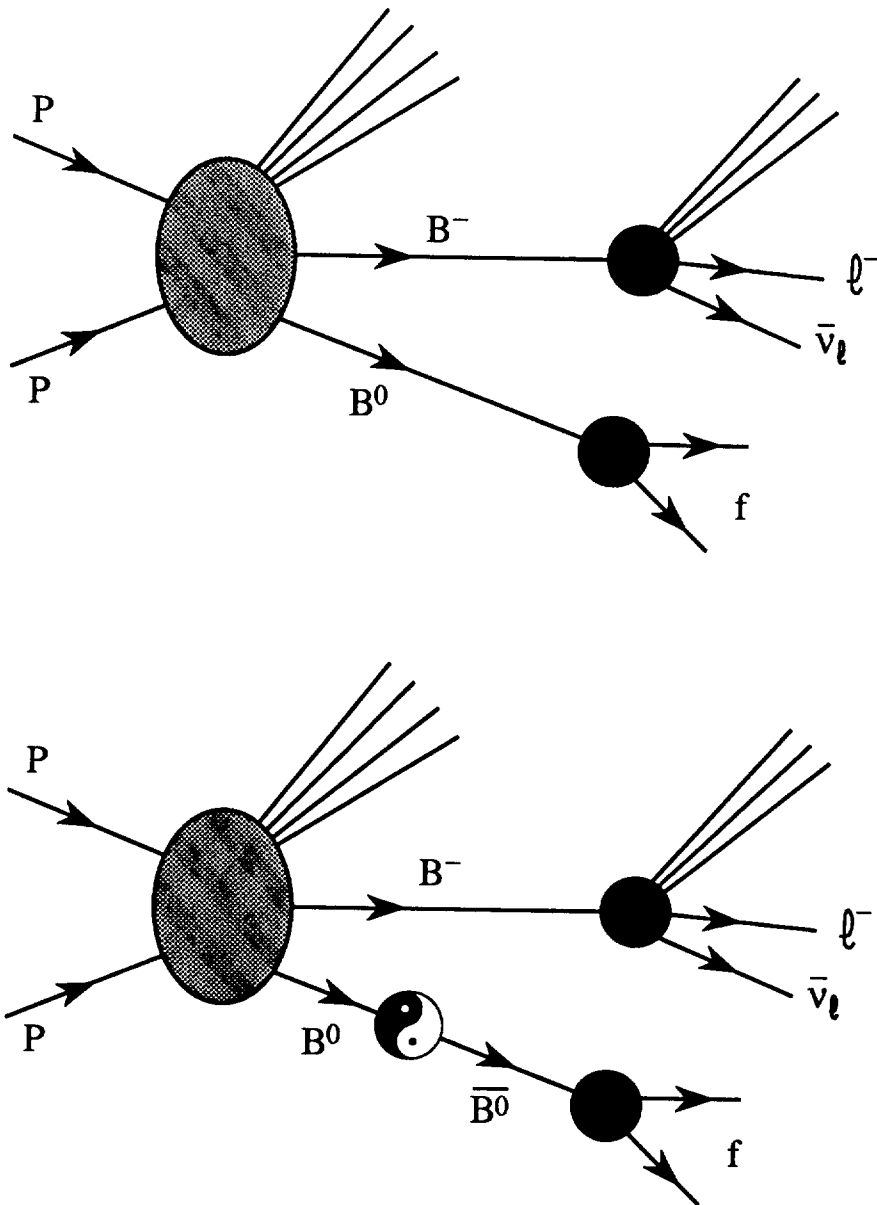


Fig. 2

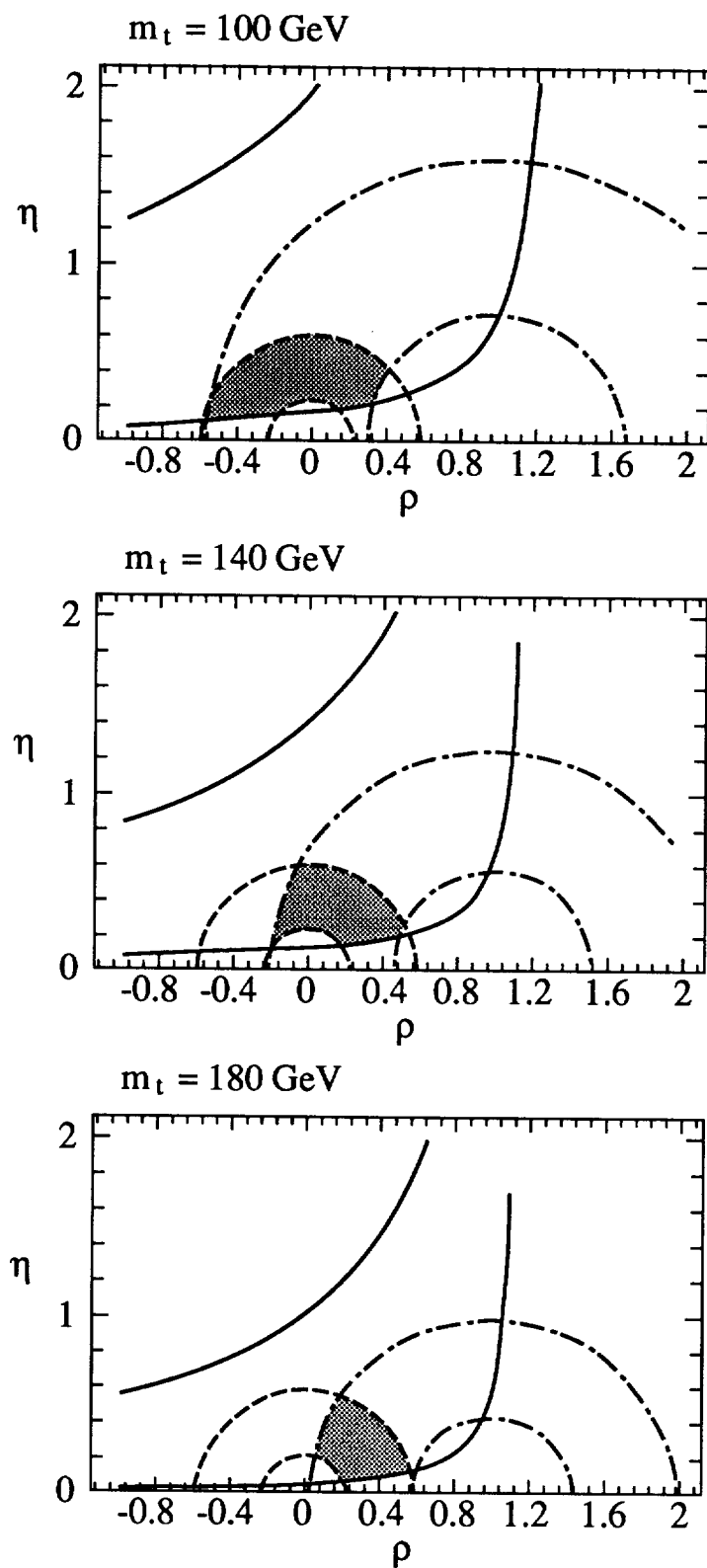
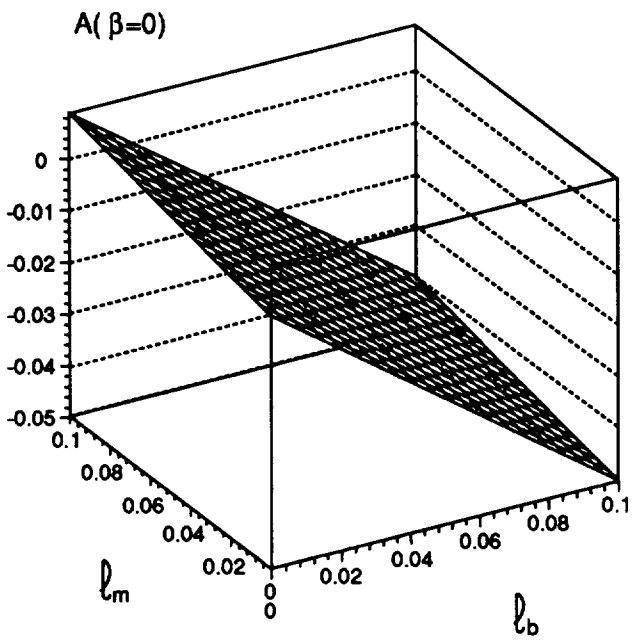
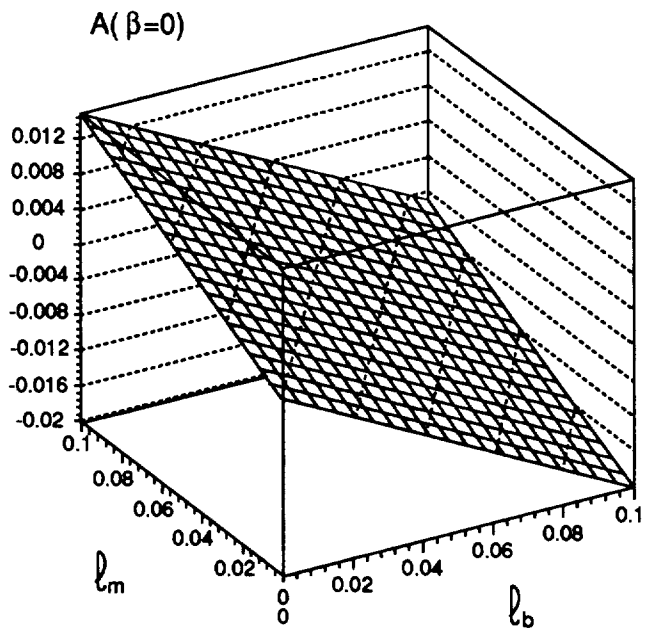


Fig. 3

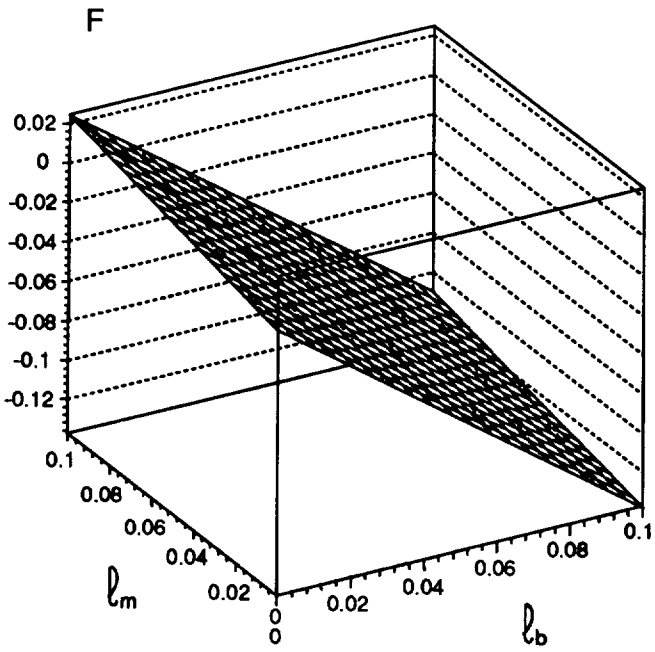




(a)



(b)



(c)

Fig. 4