

MEASUREMENT OF THE F_2^Y PHOTON STRUCTURE
FUNCTION AT LEP

A. CORDIER

LAL - ORSAY (ALEPH)

INTRODUCTION

Since the first predictions on electron-photon deep inelastic scattering in the early 70's^[1] a lot of experimental measurements have been done at PETRA and PEP^[2]. Also theorists have done detailed calculations, particularly on the hadronic contribution and on high order QCD corrections, leading to a rather controversial situation^[3]. The lack of statistics and the limited W, Q^2 range accessible with PETRA and PEP do not actually allow these recent predictions to be tested. It is interesting to consider what progress could be made at LEP in the F_2^Y structure function measurement. Some aspects of this subject have been studied by M. DAVIER^[4] and J. FIELD^[5] a few years ago, but now the LEP detectors are under construction and it is useful to consider counting rates in detail as well as review the theoretical situation.

I - THEORETICAL SITUATION

The scattering of an e^+ or e^- on a real photon target in the reaction $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ can be describe by the diagram of fig. 1. Then the differential cross-section is :

$$\frac{d\sigma}{dx dy d\phi/2\pi} = \frac{4\pi\alpha^2 p.k}{Q^4} [1+(1-y)^2] * \left[2x F_T^Y + \epsilon(y) F_L^Y + \epsilon(y) \epsilon(E_Y/E) F_X^Y \cos 2\phi \right]$$

where x and y are the usual Bjorken scaling variables :

$$x = \frac{Q^2}{2p.k} = \frac{Q^2}{Q^2 + W^2} \quad y = \frac{q.k}{p.k} = 1 - \frac{E'}{E} \cos^2 (\theta/2)$$

$$\text{and } \epsilon(y) = \frac{2(1-y)}{1+(1-y)^2}, \quad E_Y = E - E'$$

F_T^Y and F_L^Y are respectively the transverse and longitudinal structure functions of the photon.

F_X^Y is the structure function for a transversely polarized photon and is only measurable by double tagging experiments.

Similar to the lepton-nucleon structure function one defines :

$$\begin{aligned} F_1(x, Q^2) &= F_T^Y(x, Q^2) \\ F_2(x, Q^2) &= 2x F_T^Y(x, Q^2) + F_L^Y(x, Q^2) \\ F_3(x, Q^2) &= F_X^Y(x, Q^2). \end{aligned}$$

Then the differential cross-section can be written as:

$$\frac{d\sigma}{dx dy} = \frac{16\pi\alpha^2 EE_Y}{Q^4} \left[(1-y) F_2(x, Q^2) + xy^2 F_1(x, Q^2) \right]$$

In the usual experimental conditions xy^2 is small and only F_2 is measured.

Measurement of $F_L(x, Q^2)$ has also been studied by D. MILLER.

We now discuss various evaluations of the structure function $F_2(x, Q)$ available in the literature.

1) the Quark Parton Model (QPM)

The $F_2^{\text{QPM}}(x, Q^2)$ is obtained in a similar way to the QED calculation. Quarks are treated as free particles with an effective mass m as the only free parameter.

One complete expression is given by Hill and Ross [6] :

$$F_2^{\text{QPM}}(x, Q^2, k^2, m^2) = \frac{\Sigma e_i^4}{4\pi^2} \left[-2x(1 - 3x + 3x^2) + (x - 2x^2 + 2x^3) \ln \frac{Q^2(1-x)}{x(m^2 + k^2(x^2-x))} + \frac{m^2(x + 2x^2 - 2x^3)}{m^2 + k^2(x^2 - x)} \right] \quad (1)$$

For light free quarks of mass m and electric charge e . Ref. 7 gives also the expression for heavy quarks.

For a real photon target ($k \approx 0$), $F_2^{\text{QPM}}(x, Q^2, m^2)$ becomes [7] :

$$F_2^{\text{QPM}}(x, Q^2, m^2) = \frac{\Sigma e_i^4}{16\pi^2} \left[x[1 - 2x(1-x)] \ln \left(\frac{Q^2}{m^2} \frac{1}{x} - 1 \right) - x + 8x^2(1-x) \right]$$

The equivalent expression for $F_L^{\text{QPM}}(x)$ is :

$$F_L^{\text{QPM}}(x) = \frac{\Sigma e_i^4}{4\pi^2} x^2(1-x)$$

We can observe that F_2 grows logarithmically as a function of Q^2 while F_L displays canonical Bjorken scaling.

ii) Asymptotic leading order QCD

Leading order calculations of gluon emission have been carried out by many authors using the following two methods :

- the Operator Product Expansion (OPE) method using the Renormalization group equation and Wilson's operators [6,7],
- evolution equation similar to the Altarelli-Parisi equations (AP) [8].

One usual $F_2^{LO}(x, Q^2)$ expression is :

$$F_2^{LO}(x, Q^2) = \frac{3\alpha}{\pi} \sum_i e_i^4 \left[f(x) \ln \frac{Q^2}{\Lambda^2} \right] \text{ with } f(x) = 0.24x + 0.12 \quad (2)$$

this expression scales also with $\ln Q^2$.

iii) Asymptotic high order QCD

Many calculations of the high order corrections have been performed in the early 80's, using either the OPE method [9] or the AP method [8]. They lead to an expression of the type :

$$F_2^{HO}(x, Q^2) = \frac{3\alpha}{\pi} \sum_i e_i^4 \left[f(x) \cdot \ln \frac{Q^2}{\Lambda_{MS}^2} + g(x) \cdot \ln \left(\ln \frac{Q^2}{\Lambda_{MS}^2} \right) + h(x) \right] \quad (3)$$

Unfortunately these calculations lead to a negative F_2 for $x < 0.2$

iv) Regularized LO and HO-QCD

The inadequacy of the asymptotic formulas has been pointed out by Bardeen [3], and later by Glück and Reya [10]. They showed that the difficulties were intimately related to the nonasymptotic and previously ill-treated hadronic components of the photon.

These hadronic non-point-like pieces were then estimated using the parametrization of $F_T^\pi(x)$ as extracted from the reaction $\pi p \rightarrow \mu^+ \mu^- + X$ and calculated using VDM arguments :

$$F_2^{\text{Had}}(x, Q^2) \approx \left(\frac{e}{f_\rho}\right)^2 \cdot F_2^\pi(x) \approx \frac{\alpha}{\pi} \left(\frac{4\pi}{f_\rho^2}\right) (1-x) \approx \alpha(0.2 \pm 0.05) (1-x) \quad (4)$$

This scale invariant parametrization has been tested at low and moderate Q^2 and except for $x < 0.1$ seems to be acceptable [11].

Nevertheless, simply adding this contribution to asymptotic LO and HO-QCD is not satisfactory, and in particular does not remove the HO-QCD singularity at low x .

One solution for solving this problem has been proposed by Antoniadis and Grunberg [12]. They write F_2^{HO} as follows :

$$F_2^{\text{HO}}(x, Q^2) = F_2^{\text{R}}(x, Q^2, \Lambda_{\overline{\text{MS}}}) + \Delta(x, t) + F_2^{\text{Had}}(x)$$

F_2^{R} is the regular part of the point-like contribution depending only on $\Lambda_{\overline{\text{MS}}}$ at any given value of x and Q^2 .

$\Delta(x, t)$ is the term of regularization which removes the unphysical pole in the second moment of the point-like component, by mixing with the singular hadronic piece.

F_2^{Had} is what is left of the non perturbatively calculable hadronic matrix elements, and usually identified with equation (4).

The actual cancellation of the two poles depends on the parameter t . If $t=0$, F_2^{HO} reduces to the asymptotic solution. For $x \geq 0.3$, F_2^{HO} is not very sensitive to t variations (fig. 2), giving a possibility of an eventual normalization and allowing a $\Lambda_{\overline{\text{MS}}}$ measurement.

Another way to calculate F_2^{HO} has been proposed by Glück, Grassie and Reya [13]. They solve in Bjorken $-x$ space the inhomogeneous evolution equations satisfied by the parton distribution in the photon $q_i^Y(x, Q^2)$.

This solution needs, as in the deep inelastic lepton-nucleon case, an input distribution $q_i^Y(x, Q_0^2)$ at a given $Q^2 = Q_0^2$. This input includes the non point-like hadronic piece of the photon and has to be extracted from experiment. Fig. 3 shows that F_1^Y depends largely not only on the model but also the input $q_i^Y(x, Q_0^2)$. However, if an absolute normalization is not possible, the increase with $\ln Q^2/\Lambda^2$ for $Q^2 \geq 20 \text{ GeV}^2$ is rather insensitive to this input, providing a measurement of Λ .

It is clear that the evaluation of the hadronic component of the photon as well as the HO-QCD calculation are difficult and controversial problems. The traditional solution of adding a VDM contribution to the asymptotic structure function seems questionable and consequently the experimental determination of Λ using this procedure, particularly at low and moderate Q^2 , is doubtful. But for both models described before, measurement of F_2 on a large and high Q^2 range would reduce the hadronic effect and could provide a clean test of QCD leading to a reliable Λ determination. Therefore it is interesting to consider what we can expect from the LEP experiments and what progress they provide with respect to PETRA/PEP experiments.

II - FUTURE DETECTOR CAPABILITIES FOR DIS STUDIES

The futur LEP/SLC detectors have quite similar capabilities for measuring the hadronic final state. The ratio $W_{\text{measured}}/W_{\text{true}}$ which was $\sim 60 - 75 \%$ for the PETRA detectors reaches 90% for LEP detectors, allowing less critical unfolding procedures to give results as a function of X_{true} .

The main differences between detectors come from their tagging capabilities :

- measured and identified electrons of 5 GeV or more,
- smallest tagging angle,
- continuous acceptance from this smallest angle to $\pi/2$.

Figure 4 gives the tagging acceptance for the LEP and SLC detectors.

III - COUNTING RATES

Evaluation of the expected counting rates for Structure Function Studies has been done with a Monte-Carlo generator written by D. FOURNIER for the CELLO analysis^[14], using the formulas of section I at the $\gamma\gamma$ interaction level and LUND for the fragmentation of the quarks. Similar to the CELLO analysis, we have taken into account that the target photon could be off the mass shell.

The events were required to meet the following criteria :

- i) one electron with $\theta > \theta_{\min}$ and energy $E' > 7$ GeV,
- ii) at least two charged particles (not including the tagged electron),
- iii) the total visible energy (charged and neutral, but not including the tagged electron) lower than 40 % of the total available energy ($2 E_{\text{beam}}$),
- iv) the visible transverse momentum vector of the hadronic system pointing opposite to the transverse momentum of the tagged electron within ± 0.8 rad,
- v) W^2 and $Q^2 > 1$ GeV².

Detectors are assumed to have perfect detection over 2π in ϕ and for $\theta > \theta_{\min}$.

The following comparisons have been done :

- i) with QCD LO model ($\Lambda = .250$ GeV) :
 - LEP energy ($E_b = 50$ GeV) and LEP detector : $\theta_{\min} = 40$ mrad
(in this case we give also the charm contribution)
 - LEP energy ($E_b = 50$ GeV) and LEP detector : $\theta_{\min} = 30$ mrad
 - PETRA energy ($E_b = 17$ GeV) and LEP detector : $\theta_{\min} = 40$ mrad
 - PETRA energy ($E_b = 17$ GeV) and PETRA detector : $\theta_{\min} = 120$ mrad,
- ii) with QCD HO model^[4] ($\Lambda = .150$ GeV) :
 - LEP energy ($E_b = 50$ GeV) and LEP detector : $\theta_{\min} = 40$ mrad,

Q^2 and W^2 distributions are presented in fig. 5 and fig. 6, showing the large increase of statistics particularly at high Q^2 , W^2 , between PETRA and LEP. Even then the LEP detectors also have a better small Q^2 acceptance ($\theta = 40$ mrad), the mean Q^2 detected values are quite similar (table 1). Table 2 gives the Q^2 , W^2 expected counting rates for an integrated luminosity of 10 pb^{-1} . We can remark that the statistics at high Q^2 reached by previous experiments^[2] : PLUTO (100 events, $\langle Q^2 \rangle = 45 \text{ GeV}^2$) and JADE (50 events, $\langle Q^2 \rangle = 100 \text{ GeV}^2$), will be easily overtaken at LEP.

Moreover, the large number of events at high W^2 will contain a large number of jet events providing a nice test of the point-like behavior of the photon and allowing another reliable test of QCD.

CONCLUSION

Theoretical predictions on the F_2 structure function are actually rather controversial but agree on the necessity of high Q^2 measurements. LEP 1 and still more LEP 2 will provide the unique possibility of reaching high Q^2 and W^2 ranges, with good statistics, allowing reliable QCD tests. From the experimental point of view, the F_2 measurement requires the definition of adapted triggers in the forward region of the detectors, which can be rather difficult due to the important "background" of the other $\gamma\gamma$ reactions.

E beam (GeV)	Θ_{\min} (mrad)	Model	$\langle Q^2 \rangle$ (GeV ²)	$\langle W^2 \rangle$ (GeV ²)	$\langle X \rangle$
50	40	QCD LO	7.8	25.6 (44)*	0.34 (0.19)*
17	120	QCD LO	8.4	22.1	0.36
50	40	QCD HO	8.6	26.9	0.34

Table 1 : Q^2 , W^2 and $X = Q^2/Q^2 + W^2$ mean values for different hypothesis (* charm contribution)

$Q^2(\text{GeV}^2)$

∞	0 (0)	2 (1)	9 (3)	12 (1)	38 (10)	18 (3)
50	1 (0)	17 (1)	25 (6)	41 (9)	51 (9)	18 (4)
20	15 (1)	44 (2)	53 (9)	69 (16)	62 (15)	24 (5)
10	59 (0)	224 (3)	203 (25)	197 (45)	175 (32)	80 (18)
3	54 (0)	105 (5)	61 (8)	75 (15)	72 (17)	47 (11)
	1	3	10	20	50	200

$W^2(\text{GeV}^2)$

(a) QCD LO -1850 events- $E_b = 50 \text{ GeV}$, $\Theta = 40 \text{ mr}$
(Charm Contribution = 279 events)

$Q^2(\text{GeV}^2)$

∞	0	1	2	5	3	0
50	2	6	8	13	11	0
20	3	19	21	27	11	0
10	25	96	80	87	42	0
3	9	16	15	12	11	0
	1	3	10	20	50	200

$W^2(\text{GeV}^2)$

(b) QCD LO -528 events- $E_b = 17 \text{ GeV}$, $\Theta = 120 \text{ mr}$

Table 2 : Q^2 versus W^2 distribution (see cut in the text)

$Q^2(\text{GeV}^2)$

∞	0	0	14	10	33	19	
50	0	24	19	38	43	10	
20	14	62	53	81	33	29	
10	81	334	239	244	220	129	
3	148	358	253	234	181	143	
	1	3	10	20	50	200	∞

$W^2(\text{GeV}^2)$

(c) QCD LO - 3074 events-

 $E_b = 50 \text{ GeV}, \Theta = 30 \text{ mr}$ $Q^2(\text{GeV}^2)$

∞	0	1	7	12	40	22	
50	1	17	24	44	48	18	
20	9	37	50	61	52	24	
10	53	168	146	149	79	67	
3	33	62	39	48	63	47	
	1	3	10	20	50	200	∞

$W^2(\text{GeV}^2)$

(d) QCD HO -1480 events-

 $E_b = 50 \text{ GeV}, \Theta = 40 \text{ mr}$ **Table 2** : Q^2 versus W^2 distribution (see cut in the text)

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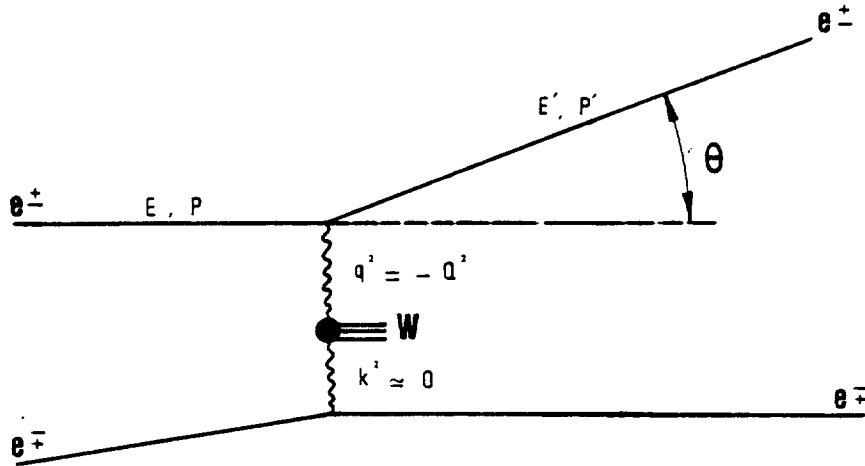


Figure 1 : Schematic diagram describing the lepton-photon deep inelastic scattering

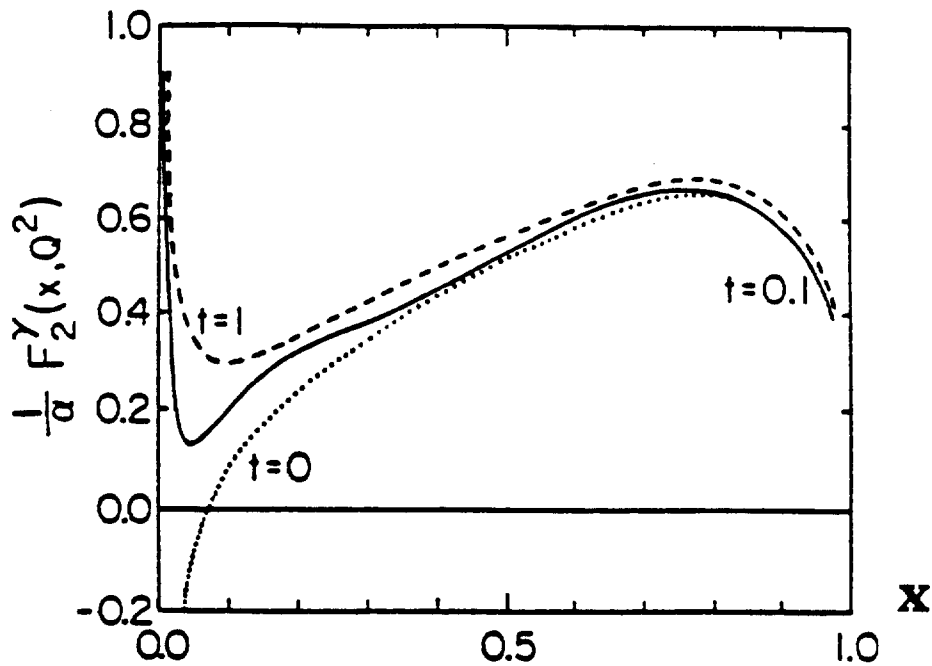


Figure 2 : The result of Antoniadis and Grunberg^[12] for the unregularized case ($t=0$, dotted line) and two values ($t=0.1$, solid line and $t=1.0$, dashed line) of the regularization parameter.

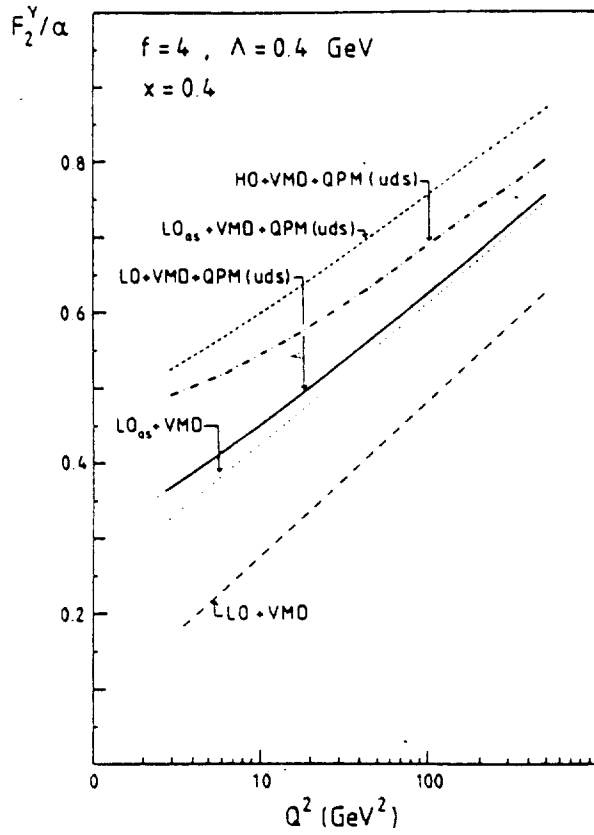


Figure 3 : Comparison of the predicted Q^2 dependences of various LO and HO calculations at a fixed $x=0.4$ ^[13]:
 - VMD + QPM corresponds to an input $q_{\text{QPM}}^Y(x, Q_o^2 = 5.9 \text{ GeV}^2)$,
 - VMD corresponds to an input at $Q_o^2 = 1 \text{ GeV}^2$,
 - LO and HO are calculated in ref. 13

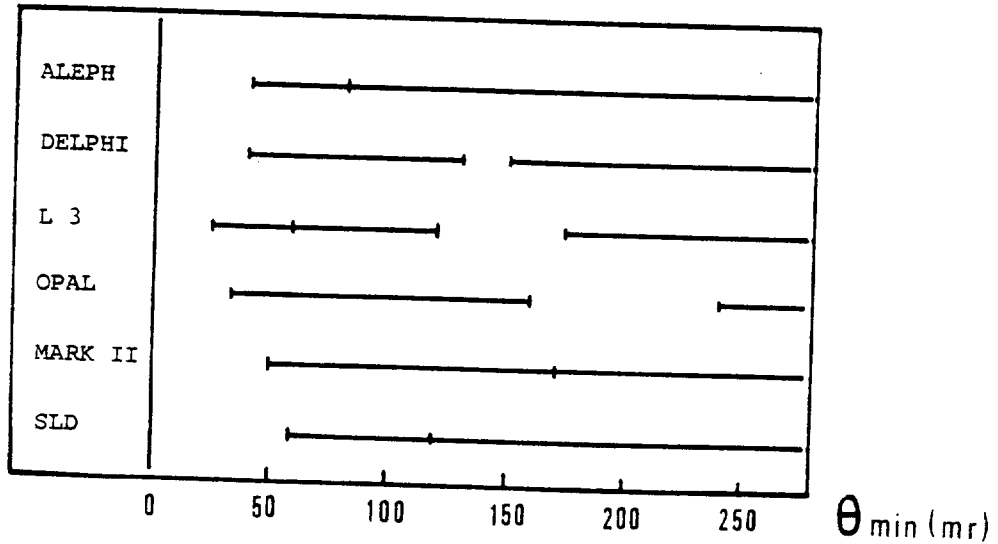


Figure 4 : Tagging acceptance for LEP and SLC detectors.

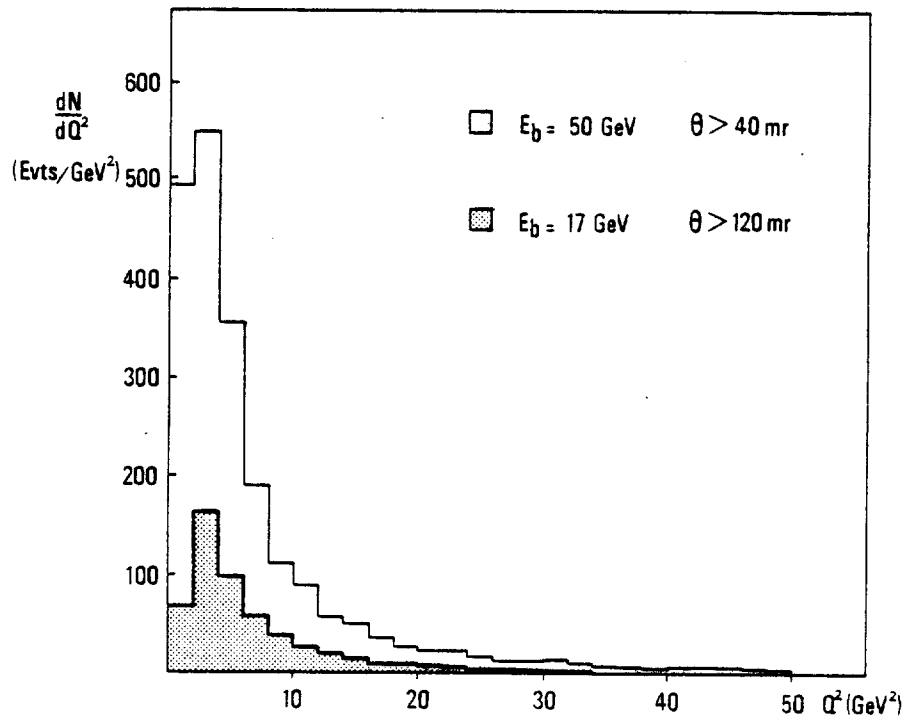


Figure 5 : Q^2 distribution for LEP and PETRA detector

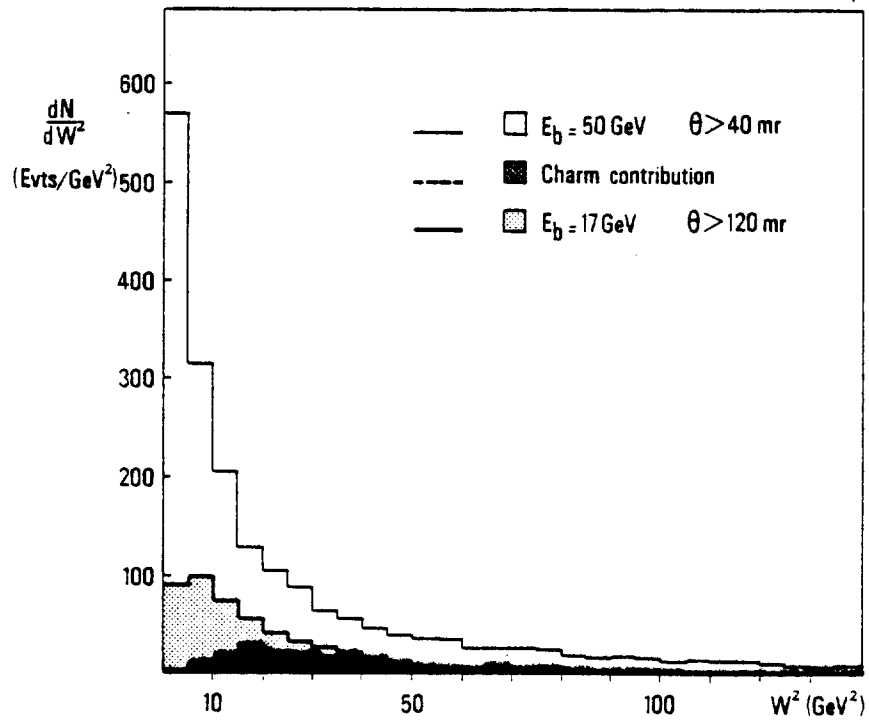


Figure 6 : W^2 distribution for LEP and PETRA detectors