

Dark matter, μ problem, and neutrino mass with gauged R symmetryKi-Young Choi,^{1,*} Eung Jin Chun,^{2,‡} and Hyun Min Lee^{3,‡}¹*Department of Physics, Pusan National University, Busan 609-735, Korea*²*Korea Institute for Advanced Study, Hoegiro 87, Dongdaemoon-gu, Seoul 130-722, Korea*³*CERN, Theory division, CH-1211 Geneva 23, Switzerland*

(Received 28 June 2010; published 24 November 2010)

We show that the μ problem and the strong CP problem can be resolved in the context of the gauged $U(1)_R$ symmetry, realizing an automatic Peccei-Quinn symmetry. In this scheme, right-handed neutrinos can be introduced to explain small Majorana or Dirac neutrino mass. The $U(1)_R$ D-term mediated supersymmetry (SUSY) breaking, called the $U(1)_R$ mediation, gives rise to a specific form of the flavor-conserving superpartner masses. For the given solution to the μ problem, electroweak symmetry breaking condition requires the superpartners of the standard model at low energy to be much heavier than the gravitino. Thus, the dark matter candidate can be either gravitino or right-handed sneutrino. In the Majorana neutrino case, only gravitino is a natural dark matter candidate. On the other hand, in the Dirac neutrino case, the right-handed sneutrino can be also a dark matter candidate as it gets mass only from SUSY breaking. We discuss the non-thermal production of our dark matter candidates from the late decay of stau and find that the constraints from the big bang nucleosynthesis can be evaded for a TeV-scale stau mass.

DOI: [10.1103/PhysRevD.82.105028](https://doi.org/10.1103/PhysRevD.82.105028)

PACS numbers: 12.60.Jv, 04.65.+e, 14.60.Pq, 95.35.+d

I. INTRODUCTION

$N = 1$ supersymmetry (SUSY) contains a continuous $U(1)_R$ group transforming different supercharges and thus distinguishing between bosonic and fermionic components of superfields. A discrete subgroup of it, called R -parity, can remain respected after SUSY breaking, and commonly used to explain the stability of proton. In local supersymmetry (supergravity), such an R -symmetry, either discrete or continuous, is gauged and should respect anomaly-free condition [1,2]. A gauged R -symmetry controls all the fields in a theory and thus its anomaly-free condition is very restrictive. Because of this property, R -symmetries can provide a powerful tool for phenomenological applications such as $U(1)_R$ as a family symmetry [3], as a resolution to the μ problem and B/L conservation [4], as an origin of supersymmetry breaking [5] and the D-term inflation [6]. More recently, a $U(1)_R$ mediated supersymmetry breaking model has been constructed based on a six-dimensional flux compactification [7] and its phenomenological application was investigated by some of the authors [8,9]. In 6D compactifications, even if the hidden sector is geometrically separated from the visible sector, the hidden sector SUSY breaking is generically not sequestered [10], as compared to the 5D counterpart. However, it has been shown that the moduli F-term contribution to the soft mass is cancelled by the hidden F-term contribution such that the $U(1)_R$ mediation can be dominant [8,9].

In this paper, extending the previous studies [9], we propose a resolution of the μ problem along the line of Ref. [11], realizing also the axion solution to the strong CP problem [12]. At the same time, the observed neutrino masses and mixing can be explained by introducing three right-handed neutrinos, which can form Majorana neutrinos by the usual seesaw mechanism (at the intermediate axion scale of order 10^{10-12} GeV or at the TeV scale) or Dirac neutrinos with tiny neutrino Yukawa couplings. In this framework, we find that the electroweak symmetry breaking (EWSB) condition requires a peculiar superparticle mass spectrum: all the superparticles in the minimal supersymmetric standard model (MSSM) sector have masses of the TeV scale, whereas the gravitino or right-handed sneutrino masses can be around 100 GeV. This is in contrast to the previous study of the $U(1)_R$ phenomenology [9] where the gaugino masses are assumed to be comparable to the scalar soft masses at the grand unified theory (GUT) scale and the μ and $B\mu$ terms are assumed to be given such that the EWSB condition is satisfied.

As the superpartners in the MSSM turn out to be heavy, either the gravitino or the right-handed (RH) sneutrino can be a natural dark matter candidate. We focus on the non-thermal productions of dark matter depending on the nature of neutrinos and impose the big bang nucleosynthesis (BBN) constraints. When neutrinos are of Majorana type, it is typical that gravitino is the lightest supersymmetric particle (LSP) and stau is the next LSP (NLSP). In this case, the stau decay after the freezeout becomes a dominant source for the nonthermal production of gravitino relic density. On the other hand, when neutrino is of Dirac type, RH sneutrino can be the LSP, and then gravitino is the

*kiyoung.choi@pusan.ac.kr

†ejchun@kias.re.kr

‡hyun.min.lee@cern.ch

NLSP. Because of the tiny Dirac neutrino Yukawa couplings, the thermal production of the RH sneutrino from the decay of heavy superparticles is usually suppressed. Then, the decay of stau can be a dominant source for the RH sneutrino relic density. We point out that in both gravitino and RH sneutrino dark matter scenarios, the BBN problem coming from the late decay of stau can be avoided for the TeV-scale stau mass as required from the EWSB conditions.

The paper is organized as follows. First, we present a 4D effective theory with gauged R -symmetry where the hidden sector SUSY breaking is introduced and the visible sector contains no additional representations under the standard model gauge group other than the MSSM content. In the next section, we show that the soft mass parameters at the GUT scale are determined from the $U(1)_R$ mediation and the μ term and the neutrino masses are dynamically generated with nonzero singlet vacuum expectation values (VEVs). Consequently, deriving the low-energy SUSY spectrum, we discuss the relic density of dark matter and the BBN constraints, depending on the nature of neutrinos. Finally, conclusion is drawn. There are four appendices comprised of the minimization of the singlet scalar potential, the identification of the axion for multiple scalar VEVs, the discussions on the effective R -parity, and Peccei-Quinn symmetry (PQ) symmetry violating terms induced after the R -symmetry breakdown.

II. A SUSY MODEL WITH GAUGED R -SYMMETRY

We first review the consistency of the Fayet-Iliopoulos (FI) term in supergravity and explain the 4D effective gauge supergravity recently derived from a six-dimensional flux compactification. For generation-independent R -charges and renormalizable Yukawa couplings in the MSSM, the gauged $U(1)_R$ is anomaly-free up to a Green-Schwarz counter term. We obtain the μ term and the Majorana/Dirac neutrino mass terms through the superpotential terms with singlets of intermediate-scale VEVs. In both Majorana and Dirac neutrino cases, we present the representative models where negative R -charges are assigned for the MSSM scalar partners except for the Higgs doublets.

A. Fayet-Iliopoulos term in 4D supergravity

It has been recently pointed out that a $U(1)$ gauge theory with constant FI term can be consistent with supergravity, provided that an exact global symmetry is present [13]. However, any global symmetry is believed to be broken in quantum gravity, leading to a conclusion that there is no consistent $U(1)$ with a constant FI term. Here we review the FI term in supergravity focusing on the chiral compensator formalism discussed by Komargodski and Seiberg in Ref. [13] and comment on the consistency of the FI term for the gauged $U(1)_R$.

The general 4D gauged supergravity action in Weyl compensator formalism is

$$S = \int d^4x \left[d^4\theta \mathbf{E} (-3C^\dagger e^{2\xi g_R V_R/3} C e^{-K_0(\Phi_i^\dagger/C, \Phi_i/C)/3}) + \int d^2\theta \mathcal{E} C^3 W(\Phi_i/C) + \text{H.c.} \right] \quad (1)$$

where C is the compensator superfield, which becomes $C = C_0 + \theta^2 F_C$ in super-Weyl gauge, \mathbf{E} and \mathcal{E} are the full and chiral superspace measures, respectively, V_R is the $U(1)_R$ vector superfield, and g_R is the $U(1)_R$ gauge coupling. For the gravitino of R -charge $+1$, the constant Fayet-Iliopoulos (FI) term for the gauged $U(1)_R$ is quantized as $\xi = 2$. The above supergravity action can be made $U(1)_R$ gauge invariant by a super-Weyl transformation [6,8]. A construction of the gauged $U(1)_R$ invariant action in 4D supergravity has been originally done in Ref. [14].

The $U(1)_R$ transformation, which should not be confused with the gauged R -symmetry, is defined such that all the chiral superfields have R -charge $2/3$. We note that the fermionic superpartners have R -charge $-1/3$. The original theory with general superpotential $W(\Phi_i)$ is made R -symmetric by adding an additional chiral superfield C with R -charge $2/3$.

Now we consider a local $U(1)_Q$ without FI term under which the $U(1)$ charges are $Q[\Phi_i] = Q_i$ and $Q[C] = 0$. After introducing a nonzero FI term for the $U(1)_Q$ in the Kähler potential, we need to make charge shifts, obtaining a new local $U(1)_{\bar{Q}}$: $U(1)$ charges are $\bar{Q}[C] = -\xi/3$ and $\bar{Q}[\Phi_i] = Q_i + R_i \xi/2 - \xi/3$, where R_i 's are new R -charges satisfying $\sum_i R_i = 2$ for chiral superfields appearing in each term of the superpotential.

After C gets a nonzero vacuum expectation value, $C = C^\dagger = M_P$, the $U(1)_{\bar{Q}}$ and the $U(1)_R$ is broken down to a gauged R -symmetry, $U(1)_{\bar{Q}} + \frac{\xi}{2} U(1)_R \equiv U(1)_{\bar{R}}$. The R -charges of this gauged R -symmetry are $\bar{R}[C] = 0$, $\bar{R}[\Phi_i] = Q_i + R_i \xi/2 \equiv \bar{R}_i$. The R -charges of the fermionic superpartners of Φ_i are $\bar{R}[\psi_i] = Q_i + (R_i - 1)\xi/2$. Therefore, since $\sum_i \bar{R}_i = \xi$ and $\sum_i R_i = 2$ for chiral superfields appearing in each term of the superpotential, one can draw a conclusion that $\sum_i Q_i = \sum_i \bar{R}_i - (\xi/2)\sum_i R_i = 0$, so there appears a global symmetry for the general superpotential.

This result is due to the assumption that there is a local $U(1)_Q$ in the limit of a vanishing FI term. In order to construct the $U(1)$ theory with nonzero FI term, however, one only has to start with $U(1)_{\bar{Q}}$ symmetry instead of local $U(1)_Q$. Furthermore, when the FI term is quantized as required for charge quantization, there is no limit of a vanishing FI term. It has been shown that a consistent 4D supersymmetric vacuum with gauged R -symmetry can be obtained below the compactification scale in six-dimensional gauged supergravity [7,8]. In this case, the quantization of the FI term is originated from the flux quantization in extra dimensions. For instance, when $\xi = 2$, we only have to start with the $U(1)_{\bar{Q}}$ having charges $\bar{Q}_i = \bar{R}_i - \frac{2}{3}$ in terms of the R -charges of the gauged R -symmetry, while the $U(1)_Q$ symmetry does not need to be

imposed. The charges of the would-be global symmetry $U(1)_Q$ are $Q_i = \bar{R}_i - R_i$, so $U(1)_Q$ would be unbroken only if one can find R_i 's satisfying $\sum_i R_i = 2$ at all orders. However, it is also possible to have $\sum_i \bar{R}_i = 2$ but $\sum_i R_i \neq 2$ for higher dimensional terms in the superpotential.

As will be discussed in later sections, the PQ symmetry appears in our model and it is nothing but a global symmetry with new R -charges given by $\tilde{r}_i = r_i + q_i$ with q_i being the PQ charges and r_i being the R -charges of the gauged $U(1)_R$. Here, $\sum_i \tilde{r}_i = 2$ is guaranteed at low orders in the superpotential by the fact that $\sum_i q_i = 0$ and $\sum_i r_i = 2$. However, at higher orders, even for $\sum_i r_i = 2$, we found that $\sum_i q_i = 0$ or $\sum_i \tilde{r}_i = 2$ is not satisfied any more. For instance, for X, Y singlets considered in our previous paper, the PQ symmetry is an accidental symmetry which holds for the quark/lepton Yukawa couplings and the μ term. However, the PQ symmetry is broken by the other Planck-scale suppressed $U(1)_R$ -invariant interactions.

B. 4D effective supergravity from a 6D flux compactification

In this section, we consider a concrete form of the Kähler potential and the superpotential derived from a flux compactification in six dimensions [8,9]. The bulk theory is based on a 6D chiral gauged supergravity constructed by Nishino and Sezgin [15]. The gauged $U(1)_R$ appears as a partial gauging of the bulk R -symmetry. In this flux compactification, there is a nonvanishing gauge flux along the $U(1)_R$, making the 4D Minkowski space flat, while the 2D extra dimensions are compactified on the sphere with a wedge cut out [7,16]. There are two 3-branes with nonzero equal tension at the poles of the wedged sphere, so visible sector fields are located at one pole and the hidden sector fields are located at the other pole. In order to stabilize the remaining modulus, some bulk dynamics should also be taken into account.

In the 4D effective supergravity, the FI term is given by $\xi = 2$ and part of the Kähler potential without FI term is

$$\begin{aligned} K_0 = & -\ln\left(\frac{1}{2}(S + S^\dagger)\right) - \ln\left(\frac{1}{2}(T + T^\dagger - 8g_R V_R)\right) \\ & - Q_i^\dagger e^{-2r_i g_R V_R} Q_i - Q'^\dagger e^{-4g_R V_R} Q' \\ & - \varphi^\dagger e^{-2r_\varphi g_R V_R} \varphi + M^\dagger e^{-2r_M g_R V_R} M. \end{aligned} \quad (2)$$

Here S, T are the moduli that mix the dilaton and the volume modulus, Q_i are visible brane fields, and $Q', \varphi(M)$ are hidden sector fields living on the hidden brane (in bulk). In our model, the gauged R -symmetry is spontaneously broken by a flux compactification in six dimensions [8,9]. So it is nonlinearly realized by the axion of the bulk T -modulus and the mass of the $U(1)_R$ gauge boson is of order the compactification scale. This is manifest in the above form of the Kähler potential.

The superpotential for the modulus and the hidden sector is

$$W_{\text{moduli}} = W_0 + fQ' + \frac{\lambda}{M^n} e^{-bS} + \lambda' \varphi^p M^2 + \kappa \varphi^q \quad (3)$$

where the R -charges are $r_{Q'} = 2, r_M = -\frac{2}{n}$, and $r_\varphi = \frac{2}{q} = \frac{2(n+2)}{pn}$. Here we introduced the uplifting sector parameterized by f and the bulk sector responsible for the gaugino condensate, in an explicitly $U(1)_R$ invariant fashion. When a Green-Schwarz coupling to the T -modulus is responsible for cancelling the R -symmetry anomalies, the superpotential for the gaugino condensate would get a T -dependent factor with S being replaced by $S + \epsilon T$. But we assume $\epsilon \ll 1$ such that the T -modulus dependence gives a negligible effect on the soft scalar masses.

On the other hand, W_0 stands for a nonzero VEV of the superpotential obtained after a spontaneous breaking of the $U(1)_R$ in the hidden sector. In generalized O'Raifeartaigh model with renormalizable interactions, independent of the R -symmetry breaking, the superpotential VEV is undetermined at tree level as it is proportional to the pseudomoduli [17]. However, it is possible to stabilize the pseudomoduli at a nonzero value from the Coleman-Weinberg potential at one-loop [18]. In this case, the superpotential can get a nonzero VEV, $W_0 \neq 0$. However, the R -symmetry breaking sector giving rise to $W_0 \neq 0$ would necessarily break SUSY and generate a positive vacuum energy, because of the consistency condition, $2W_0 = \sum_i r_i \phi_i \frac{\partial W}{\partial \phi_i}$. For instance, a bulk R -symmetry breaking field Φ with R -charge $+2$ leads to the scalar potential, $V_\Phi = \frac{1}{(\text{Re}S)(\text{Re}T - Q^\dagger Q)} |F_\Phi|^2$. Because of the bound [19] on $|W_0|$: $2|W_0| \leq f_r F$ with $f_r \equiv \sum_i r_i^2 |\phi_i|^2$ and $F^2 \equiv \sum_i |\frac{\partial W}{\partial \phi_i}|^2$, the positive vacuum energy could not be cancelled by the perturbative contribution proportional to $|W_0|^2$ for $f_r \ll M_P$. However, since the R -symmetry anomalies are cancelled by a Green-Schwarz mechanism, a nonperturbative correction may break the R -symmetry dynamically so that we may avoid the no-go theorem based on the perturbative generation of the superpotential. In this case, the order parameter of the R -symmetry breaking is now the superpotential VEV itself, not the VEV of a fundamental scalar field.

The moduli stabilization with the effective superpotential (3) has been discussed in Ref. [9]. The real part of the T modulus was shown to be stabilized at $t \simeq 1$ mainly by the $U(1)_R$ D-term. This is due to the cancellation between the constant Fayet-Iliopoulos (FI) term present in 4D gauged supergravity and the field-dependent FI term coming from the internal gauge flux. On the other hand, the S modulus and M, φ are stabilized by the F -terms at the perturbative regime. Q' is also stabilized radiatively due to the supersymmetric couplings to heavy fields without introducing additional SUSY breaking sources. Then, the resulting F -terms for S, M, φ are negligible while the F -terms for T, Q' are $F^T \simeq 2m_{3/2}, F^{Q'} \simeq \sqrt{2}m_{3/2}$, respectively, and the

$U(1)_R$ D-term is $D_R \simeq -\frac{m_{3/2}^2}{g_R}$. Consequently, because of the cancellation between the moduli and hidden-brane F -terms,¹ scalar soft masses are determined dominantly by the $U(1)_R$ D-term as

$$m_i^2 \simeq r_i g_R D_R \simeq -r_i m_{3/2}^2. \quad (4)$$

The nonzero soft masses for brane scalars proportional to the R -charges can be also derived directly from the 6D action for the nonsupersymmetric flat brane solution with a small warping [20]. The warping induced by unequal 3-brane tensions makes the brane-localized and flux-induced masses uncancelled [8]. Therefore, as a small $U(1)_R$ D-term is generated below the compactification scale (or the 4D GUT scale), it gives rise to a visible effect on the low-energy phenomenology by contributing to the initial SUSY spectrum at the GUT scale, unlike the conclusion of Castano *et al.* in Ref. [2], where it was assumed that the R -symmetry is broken at the Planck scale and the $U(1)_R$ D-term vanishes.

C. The $U(1)_R$ anomalies

Assuming that the renormalizable Yukawa couplings are allowed for quarks and leptons in the MSSM, the anomaly cancellation conditions for the $U(1)_R$ determine the R -charges of sfermions in terms of the squark doublet R -charge \tilde{q} [9] as follows,

$$\begin{aligned} \tilde{l} &= -3\tilde{q} - \frac{16}{3}, & \tilde{e} &= -\frac{3}{7}\tilde{q} - \frac{26}{21}, \\ \tilde{u} &= \frac{17}{7}\tilde{q} + \frac{18}{7}, & \tilde{d} &= -\frac{31}{7}\tilde{q} - \frac{46}{7}, \\ \tilde{h}_d &= \frac{24}{7}\tilde{q} + \frac{60}{7}, & \tilde{h}_u &= -\frac{24}{7}\tilde{q} - \frac{4}{7}. \end{aligned} \quad (5)$$

So, there is one parameter family of solutions to the consistent R -charges. We note that the R -charge of a fermion differs from the one of scalar superpartner by one unit, as $l = \tilde{l} - 1$. We assume that the pure $U(1)_R$ anomalies are cancelled by hidden fermions.² On the other hand, it has been shown that nonzero $U(1)_R$ -SM mixed anomalies, C_a ($a = 1, 2, 3$), can be cancelled by the variation of a Green-Schwarz term [9]:

$$\mathcal{L}_{\text{GS}} = (\text{Im}T) \sum_{a=1}^3 k_a \frac{1}{2} \text{tr}(F^a \tilde{F}_a), \quad (6)$$

¹Because of the absence of sequestering in 6D compactifications [10], the contact term between hidden sector field Q' and the visible sector superfields makes the cancellation happen.

²See the $U(1)_R$ anomaly coefficients in the presence of hidden fermions with nonzero R -charges in Ref. [9]. There may be an additional R -symmetry breaking in the process of giving hidden fermions masses. If this occurs only in the hidden sector, there is no problem with interactions of the additional R -breaking fields and the MSSM fields. There might appear also light fermions and one of them could be the LSP.

where the $U(1)_R$ gauge transform of $\text{Im}T$ is $\delta_R(\text{Im}T) = 4g_R \Lambda_R$, and k_a are related to the anomaly coefficients as $k_a = C_a/16\pi^2 g_R$ with $C_1 = -15$ and $C_2 = C_3 = -9$. Consequently, after a supersymmetric completion of the Green-Schwarz term, the gauge kinetic functions for the brane-localized SM gauge fields are modified to

$$f_a = 1/g_{a,0}^2 + k_a T, \quad (7)$$

where $g_{a,0}$ are the tree-level SM gauge couplings. For unified tree-level gauge couplings with $g_{3,0}^2 = g_{2,0}^2$ and $g_{1,0}^2 = \frac{3}{5}g_{2,0}^2$ at the compactification scale, $k_a = \frac{5}{3}k_2$ is consistent with the favorable choice of $\sin^2\theta_W = \frac{3}{8}$ at the compactification scale, as there is no exotics charged under the SM between the unification scale and the electroweak scale. However, given that the R -charges of the matter fields are nonuniversal in the same GUT multiplet, the matter multiplets should appear as split multiplets below the compactification scale as in orbifold GUT models.

D. The μ term

In the presence of the gauged $U(1)_R$, the μ term is forbidden at tree level by the consistent R -charge assignment in (5). Therefore, for the resolution of the μ problem, we introduce higher dimensional interactions with two singlets, Y, X , in the superpotential³:

$$W_\mu = \frac{h}{M_P} Y^2 H_u H_d + \frac{\kappa}{M_P} Y X^3. \quad (8)$$

Here we note that there exists an automatic Peccei-Quinn (PQ) symmetry⁴ which provides the axion solution of the strong CP problem [12]. As shown in Appendix A, in the presence of weak-scale soft mass terms, the dimension-5 interaction⁵ between singlets gives rise to intermediate-scale singlet VEVs so that we can get a weak-scale μ term. Note that this realizes the idea of Ref. [11] by imposing a fundamental $U(1)_R$ symmetry. In the same appendix, we have shown the mass spectrum of the singlet sector after minimizing the singlet potential. It turns out that the masses of axino and saxion partners in X, Y singlets are heavier than the gravitino mass. The details for the axion property are explained in Appendix B. Even with an explicit PQ-breaking term and after the R -symmetry breakdown for nonzero singlet VEVs, the PQ symmetry breaking is small enough for maintaining the axion solution to the strong CP problem. The details on this aspect are shown in Appendix C.

³We note that other dimension-5 singlet operators, $Y^2 X^2$ and $Y^3 X$, are problematic because there is no minimum with nonzero singlet VEV.

⁴It is not possible to write the self-interaction for the singlet in the superpotential to break PQ symmetry explicitly.

⁵If the μ term comes from a renormalizable singlet interaction, a necessary small singlet VEV would lead to a dangerous axion due to low PQ symmetry breaking scale.

E. Neutrino masses

In this section, we introduce right-handed neutrinos which can have Majorana or Dirac masses. It further constrains the allowed R -charges of the MSSM sector, consequently determining the scalar soft masses via the $U(1)_R$ mediation. We first consider the Majorana neutrino case with intermediate or TeV-scale Majorana mass for the RH neutrino. Then, we go on to discuss the Dirac neutrino case with vanishing Majorana mass for the RH neutrino.

1. Majorana neutrino case

We consider the neutrino mass term in the superpotential with right-handed neutrino N as follows,

$$W_\nu = \lambda_\nu LH_u N + \frac{\lambda_N}{2M_P^{n-1}} X^n NN. \quad (9)$$

The standard high-scale seesaw mechanism is applied for $n = 1$ while the TeV-scale seesaw mechanism must be used for $n = 2$. Then, for the R -charge of the Y singlet, $r_Y = -3$, we obtain the R -charges of the other singlets as $r_X = \frac{5}{3}$ and $r_N = 1 - \frac{5n}{6}$. Then, the R -charge of the squark doublet, that determines all the other R -charges through Eq. (5), is determined to be

$$\tilde{q} = -\frac{29}{27} - \frac{7n}{54}. \quad (10)$$

Thus, we obtain the doublet squark R -charge to be $\tilde{q} = -\frac{65}{54}$ for $n = 1$ and $\tilde{q} = -\frac{4}{3}$ for $n = 2$. Note that the PQ -charges can be assigned in an appropriate way and thus the axion solution to the strong CP problem persists even after introducing right-handed neutrinos. For $n = 2$, the R -charges and PQ -charges are shown in Table I.

The PQ symmetry is nothing but a global R -symmetry with new R -charges \tilde{r}_i given by the shifted ones from the local R -charges, $\tilde{r}_i = r_i + q_i$ with q_i being the PQ charges. As shown in the Appendix C, the PQ symmetry is broken explicitly by higher order $U(1)_R$ -invariant terms in the superpotential. The same is true of the Dirac neutrino case.

2. Dirac neutrino case

We note that in the absence of the Majorana neutrino mass term, it is possible in our framework to realize the

TABLE I. R -charges and PQ -charges for the Majorana neutrino case with $n = 2$.

	Q	U	D	L	E	H_u	H_d	Y	X	N
$U(1)_R$	$-\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{2}{3}$	4	4	-3	$\frac{5}{3}$	$-\frac{2}{3}$
$U(1)_{PQ}$	-3	0	0	-2	-1	3	3	-3	1	-1

tiny Dirac neutrino Yukawa coupling for neutrino masses in the superpotential.

One possibility (Type I) is to take the following Dirac neutrino Yukawa coupling,

$$W_\nu = \frac{\lambda_\nu}{M_P} XY LH_u N. \quad (11)$$

Since the R -charge of the right-handed (RH) sneutrino becomes

$$r_N = \frac{45}{7}\tilde{q} + \frac{194}{21}, \quad (12)$$

the R -charges of all fields except X, Y are determined in terms of the doublet squark R -charge. We note that the doublet squark R -charge is not determined unlike the Majorana neutrino case. For this type of Dirac neutrino mass, taking into account the condition on \tilde{q} for getting positive soft squared masses of squarks, sleptons and RH sneutrino, we give an example with rational R -charges in Table II.

The other possibility (Type II) is to take the following superpotential;

$$W_\nu = \frac{\lambda_\nu}{M_P^2} X^2 LH_u N. \quad (13)$$

Then, the R -charge of the RH sneutrino is

$$r_N = \frac{45}{7}\tilde{q} + \frac{32}{7}. \quad (14)$$

We note that the $Y^2 LH_u N$ coupling would lead to the R -charge of the RH sneutrino as $r_N = \frac{45}{7}\tilde{q} + \frac{292}{21}$, giving rise to a tachyonic RH sneutrino for the allowed range of \tilde{q} .

In either case without tachyonic RH sneutrino, a necessary tiny Dirac Yukawa coupling can be generated when the singlets get intermediate-scale VEVs as shown in Appendix A. For instance, in the former case, we obtain the neutrino mass as

$$m_\nu = y_\nu v \sin\beta \simeq 0.01 \text{ eV}, \quad (15)$$

where $y_\nu \equiv \frac{\lambda_\nu}{M_P^2} \langle XY \rangle$. Thus, plugging the singlet VEVs (A7) in the above, we require $\frac{\lambda_\nu}{\kappa} \sim 10^4$ for $\frac{m_{3/2}}{M_P} \sim 10^{-16}$. In Type II case, we would need $\frac{\lambda_\nu}{\kappa} \sim 10^3$ for the same gravitino mass.

TABLE II. R -charges and PQ -charges for the Dirac neutrino case.

	Q	U	D	L	E	H_u	H_d	Y	X	N
$U(1)_R$	$-\frac{13}{9}$	$-\frac{59}{63}$	$-\frac{11}{63}$	-1	$-\frac{13}{21}$	$\frac{92}{21}$	$\frac{76}{21}$	-3	$\frac{5}{3}$	$-\frac{1}{21}$
$U(1)_{PQ}$	-3	0	0	-2	-1	3	3	-3	1	1

III. INITIAL SOFT MASS PARAMETERS

Let us first summarize the $U(1)_R$ -mediated SUSY breaking in the MSSM sector. Then, we also determine the soft mass parameters for the singlet sector, that is responsible for generating the μ term as well as the neutrino masses.

For a scalar field with R -charge r_i , the $U(1)_R$ D-term determines the scalar soft mass [9] as $m_i^2 = -r_i m_{3/2}^2$. Thus, from the R -charges of the MSSM fields in Eq. (5), the scalar soft masses are given by

$$\begin{aligned} m_{\tilde{q}}^2 &= -\tilde{q} m_{3/2}^2, & m_{\tilde{l}}^2 &= \left(3\tilde{q} + \frac{16}{3}\right) m_{3/2}^2, \\ m_{\tilde{e}}^2 &= \left(\frac{3}{7}\tilde{q} + \frac{26}{21}\right) m_{3/2}^2, & m_{\tilde{u}}^2 &= -\left(\frac{17}{7}\tilde{q} + \frac{18}{7}\right) m_{3/2}^2, \\ m_{\tilde{d}}^2 &= \left(\frac{31}{7}\tilde{q} + \frac{46}{7}\right) m_{3/2}^2, & m_{\tilde{h}_d}^2 &= -\left(\frac{24}{7}\tilde{q} + \frac{60}{7}\right) m_{3/2}^2, \\ & & m_{\tilde{h}_u}^2 &= \left(\frac{24}{7}\tilde{q} + \frac{4}{7}\right) m_{3/2}^2. \end{aligned} \quad (16)$$

Note that all squarks and leptons squared masses are positive when the doublet squark R -charge lies in the range $-\frac{46}{31} < \tilde{q} < -\frac{18}{17}$. In this R -charge range, the soft mass squareds of the scalar Higgs doublets are negative. The corresponding trilinear soft terms for Yukawa couplings are universal as

$$A_{ijk} = -2m_{3/2} \quad (17)$$

for all i, j, k .

From the modified gauge kinetic term, Eq. (7), in the presence of the nonzero F -term of the T -modulus, the gaugino masses for the SM gauge group are given by the $U(1)_R$ -SM mixed anomalies and are universal at the GUT scale:

$$M_a = k_a g_a^2 F^T \simeq -\frac{9}{16\pi^2 g_R} m_{3/2}, \quad a = 1, 2, 3, \quad (18)$$

where the used relations are $16\pi^2 g_R k_a g_a^2 = -9g_{\text{GUT}}^2 \simeq -\frac{9}{2}$ and $F^T \simeq 2m_{3/2}$. The gaugino masses can be larger or smaller compared to the gravitino mass depending on the $U(1)_R$ gauge coupling (e.g. $|M_a| \gtrsim m_{3/2}$ for $g_R \lesssim \frac{9}{16\pi^2}$). In particular, for a small $U(1)_R$ gauge coupling as required for a large gaugino mass, the Green-Schwarz terms with large k_a give a large negative contribution to the SM gauge kinetic terms. In this case, to get the unified value of the gauge couplings, $g_{\text{GUT}}^2 \simeq \frac{1}{2}$, we need to cancel the large contribution of the Green-Schwarz term by considering small tree-level SM gauge couplings. Henceforth we treat the universal gaugino mass $M_{1/2}$ to be a free parameter. We will see that $M_{1/2} \gg m_{3/2}$ is required for a proper electro-weak symmetry breaking.

A. Majorana neutrino case

For the Majorana neutrino case, the soft mass terms for singlets are

$$\begin{aligned} \mathcal{L}_{\text{soft}} \supset & -m_X^2 |X|^2 - m_Y^2 |Y|^2 - m_N^2 |N|^2 - \frac{h}{M_P} A_h Y^2 H_u H_d \\ & - \frac{\kappa}{M_P} A_\kappa Y X^3 - \lambda_\nu A_\nu L H_u N - \frac{\lambda_N}{2M_P^{n-1}} A_N X^n N N \\ & + \text{c.c.} \end{aligned} \quad (19)$$

The soft mass parameters for the singlet sector are determined in the $U(1)_R$ mediation as follows:

$$m_X^2 = -\frac{5}{3} m_{3/2}^2, \quad m_Y^2 = 3m_{3/2}^2, \quad m_N^2 = \left(-1 + \frac{5n}{6}\right) m_{3/2}^2. \quad (20)$$

The A terms in the neutrino sector also follow the relation $A \simeq -2m_{3/2}$ as shown below;

$$\begin{aligned} A_h &= -F^I \partial_I \ln \left(\frac{h}{C Y_Y^2 Y_{H_u} Y_{H_d}} \right) = \frac{F^C}{C_0} + 4 \left(\frac{F^S}{6s} - \frac{F^T}{3t} \right) \\ &\simeq -2m_{3/2}, \\ A_\kappa &= -F^I \partial_I \ln \left(\frac{\kappa}{C Y_Y Y_X^3} \right) \simeq -2m_{3/2}, \\ A_\nu &= -F^I \partial_I \ln \left(\frac{\lambda_\nu}{Y_L Y_{H_u} Y_N} \right) = 3 \left(\frac{F^S}{6s} - \frac{F^T}{3t} \right) \simeq -2m_{3/2}, \\ A_N &= -F^I \partial_I \ln \left(\frac{\lambda_N}{C^{n-1} Y_X^n Y_N^2} \right) \\ &= (n-1) \frac{F^C}{C_0} + (n+2) \left(\frac{F^S}{6s} - \frac{F^T}{3t} \right) \simeq -2m_{3/2} \end{aligned}$$

where $\frac{F^C}{C_0} \simeq \frac{2}{3} m_{3/2}$ and $F^S \ll F^T \simeq 2m_{3/2}$. Here, Y_i 's are defined from the expansion of the superconformal factor, $\Omega = -3e^{-K/3}$; $\Omega \simeq -3e^{-K_0/3} + Y_i Q_i^\dagger Q_i$ where Q_i are all the brane-localized chiral superfields, K_0 is independent of the brane fields, and $Y_i = (\frac{1}{2}(S + S^\dagger))^{1/3} (\frac{1}{2}(T + T^\dagger) - Q_i^\dagger Q_i - \varphi^\dagger \varphi)^{-2/3}$. In the presence of nonzero singlet VEVs, we obtain μ , $B\mu$ terms as follows,

$$\mu = \frac{h}{M_P} \langle Y^2 \rangle, \quad (21)$$

$$B\mu = A_h \mu + \frac{2h\kappa^*}{M_P^2} \langle Y X^3 \rangle = \mu \left(A_h + \frac{2\kappa^*}{M_P} \langle Y^{-1} X^3 \rangle \right). \quad (22)$$

After the singlet VEVs (A7) are inserted in the above, we find that

$$\mu \simeq 0.0272 \frac{h}{\kappa} m_{3/2}, \quad (23)$$

$$B \simeq 7.49 m_{3/2}. \quad (24)$$

Moreover, the RH neutrino masses are also determined as follows,

$$M_N = \frac{\lambda_N}{M_P^{n-1}} \langle X^n \rangle, \quad (25)$$

$$\begin{aligned}
B_N M_N &= A_N M_N + \frac{3n\lambda_N \kappa^*}{M_P^n} \langle Y^* X^{*2} X^{n-1} \rangle \\
&= M_N \left(A_N + \frac{3n\kappa^*}{M_P} \langle Y^* X^{*2} X^{-1} \rangle \right). \quad (26)
\end{aligned}$$

In the $n = 2$ case, from Eq. (16) with R -charges given in Table I, the MSSM scalar soft masses at the GUT scale are determined as follows,

$$\begin{aligned}
m_{\tilde{q}}^2 &= m_{\tilde{l}}^2 = \frac{4}{3} m_{3/2}^2, \\
m_{\tilde{u}}^2 &= m_{\tilde{d}}^2 = m_{\tilde{e}}^2 = \frac{2}{3} m_{3/2}^2, \\
m_{H_u}^2 &= m_{H_d}^2 = -4m_{3/2}^2.
\end{aligned} \quad (27)$$

In this case, from Eqs. (25) and (26), the RH neutrino masses are

$$M_N \simeq 0.849 \frac{\lambda_N}{\kappa} m_{3/2}, \quad (28)$$

$$B_N \simeq -1.09 m_{3/2}. \quad (29)$$

Then, the mass eigenvalues of the RH sneutrino are

$$\begin{aligned}
m_{\tilde{N}_{\pm}}^2 &= M_N^2 + m_{\tilde{N}}^2 \pm |B_N| M_N \simeq m_{3/2}^2 \left[\left(0.849 \frac{\lambda_N}{\kappa} \right)^2 \right. \\
&\quad \left. + \frac{2}{3} \pm \left(0.849 \frac{\lambda_N}{\kappa} \right) \right] \geq \frac{5}{12} m_{3/2}^2. \quad (30)
\end{aligned}$$

B. Dirac neutrino case

In the Dirac neutrino case with $XYLH_u N$, the soft mass terms for singlets are

$$\begin{aligned}
\mathcal{L}_{\text{soft}} \supset & -m_X^2 |X|^2 - m_Y^2 |Y|^2 - m_N^2 |N|^2 - \frac{h}{M_P} A_h Y^2 H_u H_d \\
& - \frac{\kappa}{M_P} A_\kappa Y X^3 - \frac{\lambda_\nu}{M_P} A_\nu XYLH_u N + \text{c.c.} \quad (31)
\end{aligned}$$

The scalar soft masses for the X , Y singlets and the trilinear couplings corresponding to the μ term and the singlet interaction are the same as in the Majorana neutrino case, so, after X , Y singlets get VEVs, the μ and $B\mu$ terms are given by Eqs. (8) and (24), respectively.

As the R -charges of all fields are determined in terms of the squark R -charge, so are the scalar soft masses in the $U(1)_R$ mediation. Then, the RH sneutrino scalar soft mass is given by $m_N^2 = -(\frac{45}{7} \tilde{q} + \frac{194}{21}) m_{3/2}^2$. For the doublet squark R -charge, $-\frac{46}{31} < \tilde{q} < -\frac{194}{135}$, not only all the squarks and sleptons but also the RH sneutrino have positive scalar squared soft masses⁶ as

$$0 < m_N^2 < 0.301 m_{3/2}^2. \quad (32)$$

We note that, in this region of $|\tilde{q}|$, the relic density coming from neutralino as the LSP tends to be too large [9]. So, it is

⁶For the Dirac Yukawa coupling $X^2 L H_u N$, the RH sneutrino mass is given by $2.24 m_{3/2}^2 < m_N^2 < 4.97 m_{3/2}^2$.

natural to take the RH sneutrino or gravitino as a dark matter candidate. The trilinear coupling for the Dirac neutrino Yukawa coupling is given by

$$\begin{aligned}
A_\nu &= -F^I \partial_I \ln \left(\frac{\lambda_\nu}{C^2 Y_X Y_Y Y_L Y_{H_u} Y_N} \right) \\
&= 2 \frac{F^C}{C_0} + 5 \left(\frac{F^S}{6s} - \frac{F^T}{3t} \right) \simeq -2m_{3/2}
\end{aligned}$$

following again the relation $A_{ijk} \simeq -2m_{3/2}$.

IV. LOW ENERGY SPECTRUM AND DARK MATTER

In this section, we consider the constraints on the SUSY spectrum coming from the EWSB conditions. Even after the R -symmetry breakdown, the R -parity is a good symmetry at the perturbative level as the R -parity violating terms appear at sufficiently higher orders as shown in Appendix D. Depending on the nature of neutrino masses, we take either gravitino or RH sneutrino to be a dark matter candidate. We discuss the dark matter relic density and the BBN constraints on a late decaying NLSP in either dark matter scenario.

A. The EWSB condition and the SUSY spectrum

The Higgs mass terms contributing to the Higgs potential are given by

$$\begin{aligned}
V_{h,\text{mass}} &= (|\mu|^2 + m_{H_u}^2) |H_u|^2 + (|\mu|^2 + m_{H_d}^2) |H_d|^2 \\
&\quad + (B\mu H_u H_d + \text{c.c.}) \quad (33)
\end{aligned}$$

In order to achieve electroweak symmetry breaking, the following conditions at the weak scale must be fulfilled:

$$|B\mu|^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2), \quad (34)$$

$$2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 - 2|B\mu| > 0. \quad (35)$$

Then, the minimization conditions for the Higgs potential impose the following conditions;

$$\sin(2\beta) = \frac{2|B\mu|}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2}, \quad (36)$$

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}. \quad (37)$$

The above EWSB conditions require that the μ -term and Higgs scalar soft masses must be large at the EWSB scale to be compatible with the large B -term as compared to gravitino or scalar soft masses at the GUT scale in Eq. (24). Therefore, the necessary large loop corrections to the Higgs scalar soft masses can be obtained for the gaugino mass which is much larger than the gravitino mass.

When $M_{1/2} \gg m_{3/2}$, after renormalization group equation (RGE) running from GUT scale to EWSB scale, the soft terms at the EWSB scale becomes of the order of the gaugino mass at GUT scale while B and μ -terms do not change much. For $\tan\beta \gtrsim 1$, considering $|\mu|^2 \simeq -m_{H_u}^2 \sim M_{1/2}^2$ from Eq. (37), we find roughly

$$B \simeq 7.5m_{3/2} \sim \frac{|\mu|}{\tan\beta} \sim \frac{M_{1/2}}{\tan\beta}. \quad (38)$$

at the EWSB scale. This is the common feature of this $U(1)_R$ gauged model. For the correct magnitude for μ -term we need a large ratio between h and κ as $h/\kappa \sim 10^3$ from Eq. (8).

Since gaugino mass is much larger than scalar soft masses at GUT scale, at low energy, all the masses of the SUSY particles are of the order of the gaugino mass and the lightest ordinary supersymmetric particle becomes lighter stau $\tilde{\tau}_1$. Therefore the possible dark matter (DM) candidate must be gravitino or sneutrino which is outside of the MSSM sector. We note that the axino and saxion partners of the X, Y singlets are heavier than gravitino or sneutrino so they cannot be LSP. In the following sections, we consider the corresponding DM candidate for each model introduced in the previous section.

B. Majorana neutrino case

First, we consider the Majorana neutrino with $n = 1$ in Eq. (9), the mass of the scalar RH neutrino mass is $M_N \simeq \lambda_N(X) \sim 10^{10-12}$ GeV. Thus, gravitino, as the LSP, is the only dark matter candidate while stau is the NLSP. In this case, there are two sources for the relic density of gravitino DM: nonthermal production from the decay of stau NLSP and thermal production from the thermal scattering after reheating. We will not consider the thermal production which depends on the reheating temperature.

When NLSP decays after freezeout, the nonthermal production of LSP dark matter is determined by

$$\Omega_{\text{DM}} h^2 = \frac{m_{\text{DM}}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}} h^2. \quad (39)$$

The abundance of stau from the thermal freeze-out is [21]

$$\Omega_{\tilde{\tau}_1} h^2 \simeq 0.2 \left(\frac{m_{\tilde{\tau}_1}}{1 \text{ TeV}} \right)^2. \quad (40)$$

For gravitino LSP and stau NLSP which decay via $\tilde{\tau}_1 \rightarrow \tau + \tilde{G}$, from Eqs. (39) and (40), the nonthermal production of gravitino is

$$\Omega_{\tilde{G}} h^2 \simeq 0.02 \left(\frac{m_{3/2}}{100 \text{ GeV}} \right) \left(\frac{m_{\tilde{\tau}_1}}{1 \text{ TeV}} \right). \quad (41)$$

For a correct relic density, we need a stau mass of TeV-scale which is consistent with the EWSB conditions in our scenario.

However the decay products of a long-lived decaying particle can be problematic with respect to the standard BBN. When a long-lived particle is negatively charged, it can make a bound state with nuclei and even worse, the situation catalyzed big bang nucleosynthesis (CBBN) [22]. To avoid the CBBN constraint, the lifetime of stau must be less than 5×10^3 sec or $Y_{\tilde{\tau}_1} \equiv n_{\tilde{\tau}_1}/s \lesssim 10^{-15}$ for longer lifetime [23–27]. Such a small abundance of stau requires unusual situations [28–31]. As stau decays dominantly to gravitino and tau lepton, the lifetime of stau is given by

$$\begin{aligned} \tau(\tilde{\tau}_1 \rightarrow \tilde{G} + \tau) &\simeq 48\pi M_p^2 \frac{m_{3/2}^2}{m_{\tilde{\tau}_1}^5} \\ &\simeq 1.8 \times 10^3 \text{ sec} \left(\frac{m_{3/2}}{100 \text{ GeV}} \right)^2 \left(\frac{1 \text{ TeV}}{m_{\tilde{\tau}_1}} \right)^5. \end{aligned} \quad (42)$$

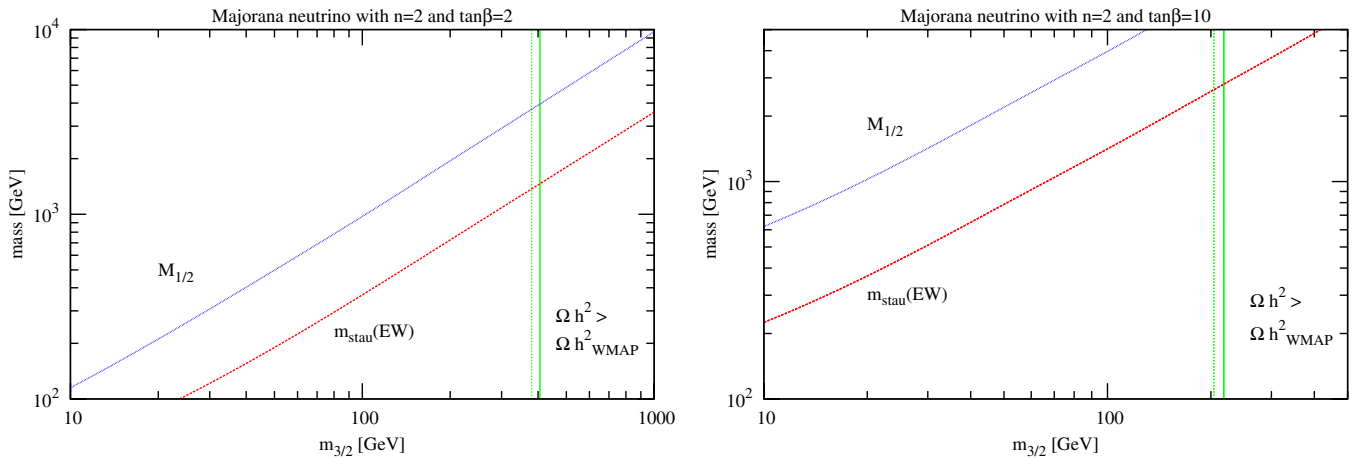


FIG. 1 (color online). Plot of gaugino mass at GUT scale (upper blue line) and stau NLSP at EWSB (lower red line) for a given gravitino mass with correct EWSB condition in the case of Majorana neutrino with $n = 2$. Between the vertical green lines, the nonthermally produced gravitino from stau decay satisfies the correct dark matter abundance by WMAP.

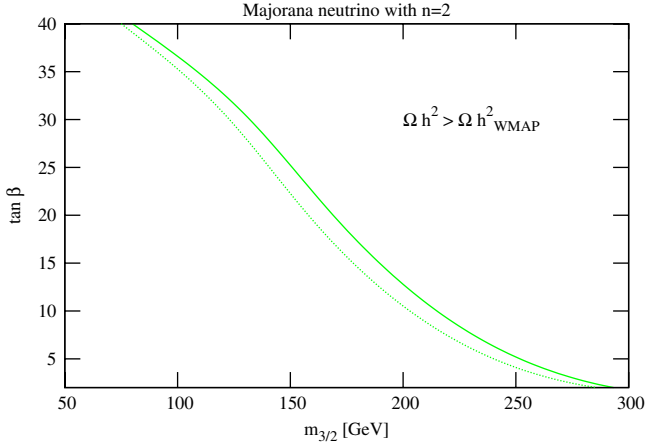


FIG. 2 (color online). The contour plot correct relic density of gravitino from nonthermal production in the plane of $\tan\beta$ and gravitino mass in the case of Majorana neutrino with $n = 2$.

The CBBN constraint is automatically satisfied for $m_{\tilde{\tau}_1} \gtrsim 2$ TeV in the region of producing the right relic density of gravitino from stau decay according to Eq. (42). For example, for $\tan\beta = 10$, our model with correct EWSB predicts $m_{3/2} \simeq 200$ GeV and $m_{\tilde{\tau}_1} = 2.6$ TeV, which leads to the lifetime of stau around 100 sec.

In the case of Majorana neutrino with $n = 2$ in Eq. (9), the mass of RH sneutrino is determined by Eq. (31). If we consider the small magnitude of $\kappa \sim \mathcal{O}(10^{-3})$ with $\lambda_N \sim \mathcal{O}(1)$, then $m_{\tilde{N}_\pm} \simeq (0.85\lambda_N/\kappa)m_{3/2} \sim 10^3 m_{3/2}$ and becomes much heavier than the other SUSY particles as well as gravitino. This gives gravitino DM as in the $n = 1$ case. For $\lambda_N \sim \kappa$, we may obtain $m_{\tilde{N}_\pm} \sim m_{3/2}$ so two scenarios are possible: RH sneutrino is LSP and gravitino is NLSP and vice versa. In both cases, however, the thermal production of the RH sneutrino LSP would highly overclose the Universe [21], and thus it is excluded.

In Fig. 1, we show the gaugino mass $M_{1/2}$ at the GUT scale and the stau mass at the EWSB scale vs the gravitino mass after taking into account the correct EWSB. The green

lines show the region where the gravitino nonthermal production from stau decay is within the range of cold dark matter from WMAP 7-year data [32], $0.105 < \Omega_{\text{DM}} h^2 < 0.119$. The region right to the solid green line, where $\Omega_{\text{DM}} h^2 > 0.119$, is excluded. In Fig. 2, we show the region of correct relic density in the plane of $\tan\beta$ and $m_{3/2}$.

Gravitino can be produced thermally, depending on the reheating temperature after inflation. In the region left to the green lines, the thermal production of gravitino is required for the correct relic density of gravitino.

C. Dirac neutrino case

First, we consider Dirac neutrino Type I with the superpotential Eq. (11). Since the renormalization group evolution of the (Dirac) RH sneutrino mass $m_{\tilde{N}}$ is negligible due to the smallness of Yukawa coupling, the RH sneutrino mass at EWSB is smaller than the gravitino mass from Eq. (32). Therefore the RH sneutrino is the LSP dark matter, and gravitino is the NLSP.

Let us note that the thermal production of the RH sneutrino can be obtained from the decay of supersymmetric particles but it is suppressed by the small Dirac neutrino Yukawa coupling $y_\nu \sim 10^{-13}$. If there is no enhancement factor due to a small mass difference between left-handed and RH sneutrinos or degenerate neutrino masses (requiring large y_ν), one typically gets the relic density of the RH sneutrino from the thermal production as $\Omega_{\tilde{N}_\pm} h^2 < \mathcal{O}(10^{-3})$ [21]. Thus, the main contribution to the RH sneutrino DM can come from the nonthermal production due to the decay of the NLSP. Using Eqs. (39) and (40) for RH sneutrino LSP, the relic density of the RH sneutrino produced nonthermally from stau decay is

$$\Omega_{\tilde{N}_\pm} h^2 = \frac{m_{\tilde{N}_\pm}}{m_{\tilde{\tau}_1}} \Omega_{\tilde{\tau}_1} h^2 \simeq 0.02 \left(\frac{m_{\tilde{N}_\pm}}{100 \text{ GeV}} \right) \left(\frac{m_{\tilde{\tau}_1}}{\text{TeV}} \right), \quad (43)$$

which includes the RH sneutrino produced from stau decay to gravitino and gravitino decay to RH sneutrino. The region which gives the correct relic density for DM is shown in the

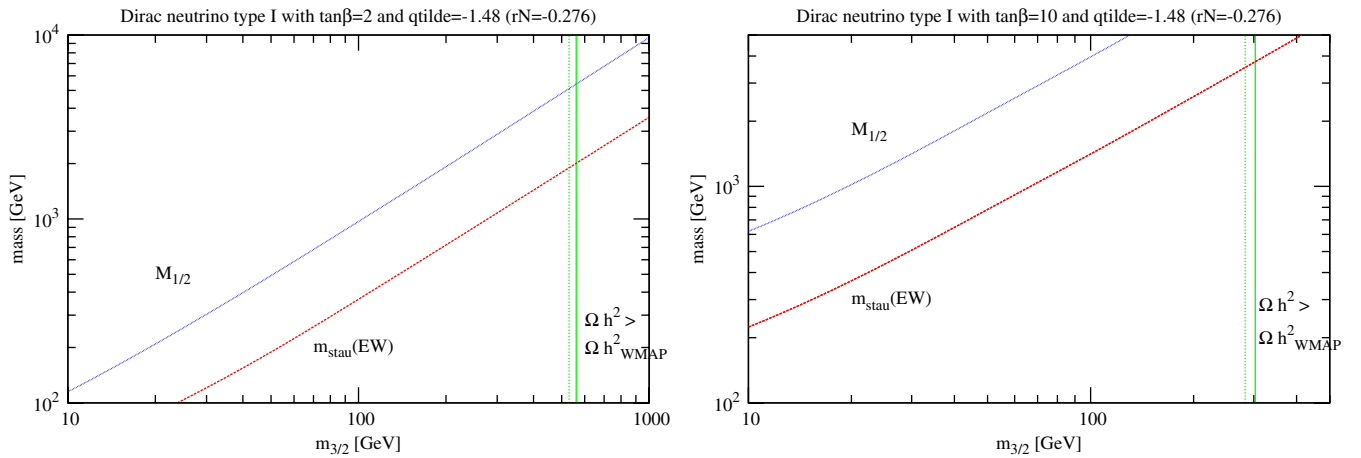


FIG. 3 (color online). The same as Fig. 1 but in the case of Dirac neutrino type I.

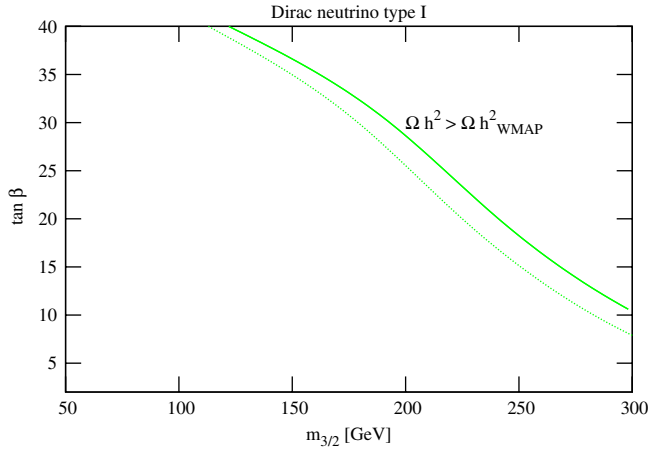


FIG. 4 (color online). The same as Fig. 2 but in the case of Dirac neutrino type I.

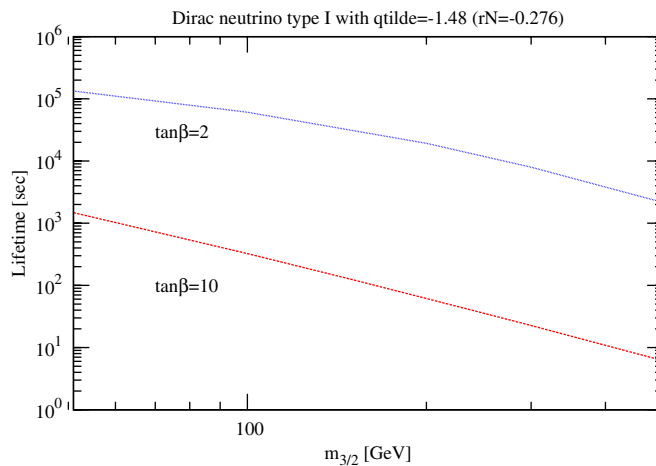
Fig. 3 for fixed $\tan\beta = 2, 10$ and in the Fig. 4 on the $\tan\beta$ and $m_{3/2}$ plane. Here we used $r_N = -0.276$ so that the mass of RH sneutrino is $m_{\tilde{N}_\pm} = 0.52m_{3/2}$. With smaller r_N , the green lines in the Figures move to the right direction correspondingly.

In the case of Dirac neutrino, the decay rate of stau to RH sneutrino and W gauge boson can be comparable to the one of stau to gravitino and tau lepton, causing a BBN problem. The latter decay rate is given in Eq. (44) and the former one is given by [21,33]

$$\Gamma(\tilde{\tau}_1 \rightarrow W^- + \tilde{N}_\pm) \approx \frac{\sin^2\theta_{\tilde{\tau}}}{32\pi} \left(\frac{m_{\tilde{\tau}_1}^2}{m_{\tilde{\nu}_L}^2 - m_{\tilde{N}_\pm}^2} \right)^2$$

$$\frac{|\mu \cot\beta - A_\nu^*|^2 m_\nu^2}{m_{\tilde{\tau}_1} v^2} \approx 3.3 \times 10^{-26} \text{ GeV} \sin^2\theta_{\tilde{\tau}}$$

$$\left(\frac{1 \text{ TeV}}{m_{\tilde{\nu}_L}} \right)^4 \left(\frac{m_{\tilde{\tau}_1}}{1 \text{ TeV}} \right)^3 \left(\frac{|\mu \cot\beta - A_\nu^*|}{1 \text{ TeV}} \right)^2 \left(\frac{m_\nu}{0.01 \text{ eV}} \right)^2, \quad (44)$$



where use is made of $v \simeq 174 \text{ GeV}$, m_ν is the neutrino mass, and $\theta_{\tilde{\tau}}$ is the left-right mixing angle of stau, i.e. $\tilde{\tau}_1 = \tilde{\tau}_R \cos\theta_{\tilde{\tau}} + \tilde{\tau}_L \sin\theta_{\tilde{\tau}}$. Here we used Eq. (15) and $A_\nu \simeq -2m_{3/2} - 0.59M_{1/2}$. Using this we show the plot of the lifetime of stau and the branching ratio of stau decay to RH sneutrino and W boson in Fig. 5 for $\tan\beta = 2, 10$, respectively.

As can be seen from Fig. 5, for $\tan\beta = 10$, one finds that the decay rate of stau to gravitino and tau lepton (44) is much larger than the one for stau to RH sneutrino and W gauge boson. Then, the lifetime of stau is determined by the decay rate to gravitino and tau lepton so it is less than about 100 sec. Therefore, in this case, the hadronic particles produced from W boson decay do not have a BBN problem [21,33]. For $\tan\beta$ smaller than 10, the stau decay rate to RH sneutrino and W boson becomes sizable, and thus it would cause the BBN problem.

On the other hand, the late decay of gravitino to RH sneutrino and neutrino could cause a BBN problem. But, the BBN constraints on the late decaying gravitino may be avoided if the mass difference between gravitino and sneutrino is less than about 100 GeV [34].

For the Dirac neutrino Type II with the superpotential Eq. (13), the mass range of RH sneutrino is

$$1.49m_{3/2} \lesssim m_{\tilde{N}} < 2.23m_{3/2}. \quad (45)$$

In this case, RH sneutrino is NLSP and gravitino is LSP as a DM candidate. The correct relic density of gravitino can be obtained from the stau decay in some region of heavy stau mass while the BBN constraint can be avoided in the same way as Dirac neutrino Type I model discussed above.

In both Dirac neutrino cases, in the region where the nonthermal production is not enough, the thermal production of gravitino may give rise to the correct DM of gravitino (or RH sneutrino DM from gravitino decay) with appropriate reheating temperature around $10^9\text{--}10^{10} \text{ GeV}$ (or scaled by $m_{3/2}/m_{\tilde{N}}$).

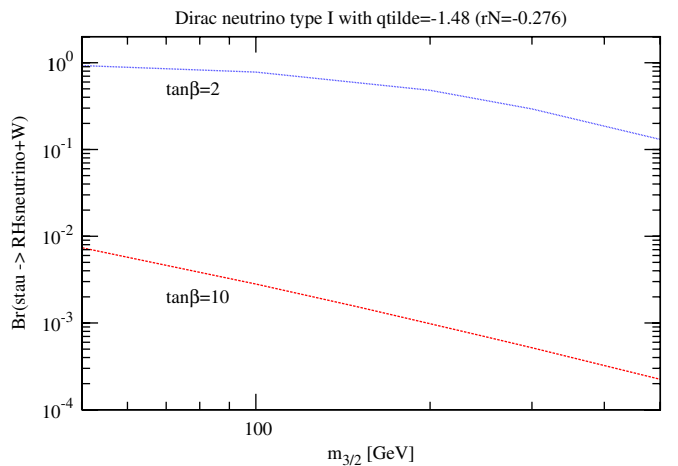


FIG. 5 (color online). The lifetime (left) and the branching ratio of stau decay to RH sneutrino and W boson (right) for $\tan\beta = 2, 10$ in the case of Dirac neutrino type I.

V. CONCLUSION

We have shown that the gauged $U(1)_R$ symmetry naturally realizes the solution to the μ problem and accommodates the axion solution to the strong CP problem. The interplay between the higher dimensional interaction for singlets and the singlet soft masses coming from the $U(1)_R$ mediation gives rise to the stabilization of the singlets at an intermediate scale, consequently generating the small μ term from a higher dimensional interaction.

The gauged $U(1)_R$ symmetry restricts the generated $B\mu$ term to be larger than the other scalar soft masses of order the gravitino mass, resulting in $M_{1/2} \gg m_{3/2}$. Thus, we found that superpartner masses at the EWSB scale in the MSSM sector are much larger than the mass of gravitino or RH sneutrino. Therefore, only gravitino or RH sneutrino can be a natural dark matter candidate.

Depending on whether the Majorana mass term for the RH neutrino exists, we considered a different candidate for dark matter: gravitino for the Majorana neutrino case and RH sneutrino for the Dirac neutrino case. In both dark matter scenarios, the NLSP in the MSSM sector is stau. For the stau decaying after the freezeout, we showed that the correct relic density of dark matter can be generated by the nonthermal production mechanism through the stau decay. At the same time, the BBN constraints on such a long-lived charged particle can be evaded for the TeV-scale stau mass. Gravitino and RH sneutrino LSP could be also produced thermally, by the reheating after inflation and by the decay of the heavier superparticles in thermal bath, respectively. However, we have not considered the thermal production because the former case depends on the reheating temperature and the latter case depends on the (fine-tuned) enhancement factor of the decay rate.

ACKNOWLEDGMENTS

K. Y. Choi acknowledges support from the Korea Research Foundation funded by the Korean government Grant No. (KRF-2008-341-C00008) and the second stage of Brain Korea 21 Project. E. J. C. acknowledges support from the Korea Neutrino Research Center through National Research Foundation of Korea Grant No. (2009-0083526).

APPENDIX A: MINIMIZATION OF THE SCALAR POTENTIAL FOR SINGLETS

In this section, in order to get nonzero singlet VEVs, we focus on the minimization of the scalar potential for the singlets, X and Y .

The F -term potential for singlets X, Y is

$$V_F = \left| \frac{2h}{M_P} Y H_u H_d + \frac{\kappa}{M_P} X^3 \right|^2 + \frac{9\kappa^2}{M_P^2} |Y|^2 |X|^4. \quad (\text{A1})$$

Since the Higgs doublet gets small VEVs compared to the singlet VEVs, we ignore the singlet coupling to the

Higgs doublets in the F -term potential. As far as singlet F -terms are negligible and $|X|, |Y| \ll M_P$, the $U(1)_R$ D-term contributes to the singlet scalar potential only through the soft mass terms, which are determined after the moduli stabilization. Then, adding the singlet soft mass terms to the F -term potential, we obtain the scalar potential for singlets as

$$\begin{aligned} V(X, Y) = V_F + m_X^2 |X|^2 + m_Y^2 |Y|^2 \simeq & \frac{\kappa^2}{M_P^2} |X|^6 \\ & + \frac{9\kappa^2}{M_P^2} |Y|^2 |X|^4 + m_X^2 |X|^2 + m_Y^2 |Y|^2 \\ & + \frac{\kappa}{M_P} A_\kappa Y X^3 + \text{c.c.} \end{aligned} \quad (\text{A2})$$

Writing $X = |X|e^{i\theta_X}$, $Y = |Y|e^{i\theta_Y}$, we find that for $A_\kappa \simeq -2m_{3/2} < 0$ and $\kappa > 0$, the trilinear terms stabilize one of linear combinations of angles at $3\theta_X + \theta_Y = 2n\pi$ with integer n . Thus, the other combination of angles becomes a massless axion. After plugging the minimization condition for the angles into Eq. (A2), the scalar potential becomes

$$\begin{aligned} V(X, Y) = & \frac{\kappa^2}{M_P^2} |X|^6 + \frac{9\kappa^2}{M_P^2} |Y|^2 |X|^4 + m_X^2 |X|^2 + m_Y^2 |Y|^2 \\ & - \frac{2\kappa|A_\kappa|}{M_P} |Y||X|^3. \end{aligned} \quad (\text{A3})$$

For the soft mass parameters given in the Majorana neutrino case with $n = 2$, redefining the singlet fields as $x^2 = \frac{\kappa|X|^2}{m_{3/2}M_P}$ and $y^2 = \frac{\kappa|Y|^2}{m_{3/2}M_P}$, we rewrite the scalar potential as

$$V(x, y) = \frac{m_{3/2}^3 M_P}{\kappa} \left(x^6 + 9y^2 x^4 - \frac{5}{3} x^2 + 3y^2 - 4yx^3 \right). \quad (\text{A4})$$

The extremum conditions for x and y are

$$0 = 6x^5 + 36y^2 x^3 - \frac{10}{3} x - 12yx^2, \quad (\text{A5})$$

$$0 = 18yx^4 + 6y - 4x^3. \quad (\text{A6})$$

Consequently, we find a minimum at $x \simeq 0.921$ and $y \simeq 0.165$ while $x = y = 0$ is a saddle point. Then, the singlet VEVs are

$$|X| \simeq 0.921 \sqrt{\frac{m_{3/2} M_P}{\kappa}}, \quad |Y| \simeq 0.165 \sqrt{\frac{m_{3/2} M_P}{\kappa}}. \quad (\text{A7})$$

Expanding the singlets, X and Y , around the background VEVs, as $X = \langle X \rangle + \frac{1}{\sqrt{2}}(h_1 + i\varphi_2)$ and $Y = \langle Y \rangle + \frac{1}{\sqrt{2}} \times (h_2 + i\varphi_2)$, we obtain the nonzero mass eigenvalues for singlets: for real bosons,

$$M_{h_\pm}^2 = \frac{1}{2} (a + b \pm \sqrt{(a-b)^2 + 4c^2}) \quad (\text{A8})$$

with

$$a = \frac{15\kappa^2}{M_P^2}\langle X \rangle^4 + \frac{54\kappa^2}{M_P^2}\langle Y \rangle^2\langle X \rangle^2 - \frac{5}{3}m_{3/2}^2 + \frac{6\kappa}{M_P}A_\kappa\langle Y \rangle\langle X \rangle, \quad (\text{A9})$$

$$b = \frac{9\kappa^2}{M_P^2}\langle X \rangle^4 + 3m_{3/2}^2, \quad (\text{A10})$$

$$c = \frac{36\kappa^2}{M_P^2}\langle Y \rangle\langle X \rangle^3 + \frac{3\kappa}{M_P}A_\kappa\langle X \rangle^2, \quad (\text{A11})$$

and

$$M_{\varphi_+}^2 = \frac{12\kappa^2}{M_P^2}\langle X \rangle^4 + \frac{18\kappa^2}{M_P^2}\langle Y \rangle^2\langle X \rangle^2 + \frac{4}{3}m_{3/2}^2 - \frac{6\kappa}{M_P}A_\kappa\langle Y \rangle\langle X \rangle; \quad (\text{A12})$$

for Weyl fermions,

$$M_{f_\pm}^2 = \frac{1}{2}(a' + b' \pm \sqrt{(a' - b')^2 + 4c'^2}) \quad (\text{A13})$$

with

$$a' = \frac{36\kappa^2}{M_P^2}\langle Y \rangle^2\langle X \rangle^2 + \frac{9\kappa^2}{M_P^2}\langle X \rangle^4, \quad (\text{A14})$$

$$b' = \frac{9\kappa^2}{M_P^2}\langle X \rangle^4, \quad (\text{A15})$$

$$c' = \frac{18\kappa^2}{M_P^2}\langle Y \rangle\langle X \rangle^3. \quad (\text{A16})$$

Another combination of the imaginary part is massless and it appears as a Goldstone boson for breaking the PQ symmetry. For the obtained singlet VEVs (A7), we can determine the mass eigenvalues: for the radial modes, which are almost mass eigenstates due to a small mixing, $M_{h_+}^2 = 9.67m_{3/2}^2$ and $M_{h_-}^2 = 8.39m_{3/2}^2$; for the massive angular mode, $M_{\varphi_+}^2 = 12.2m_{3/2}^2$; for Weyl fermions, $M_{f_+}^2 = 9.26m_{3/2}^2$ and $M_{f_-}^2 = 4.54m_{3/2}^2$. Here we note that the radial modes, h_\pm are almost mass eigenstates $h_{2,1}$ due to a small mixing.

APPENDIX B: AXION FOR MULTIPLE SCALAR FIELD VEVs

We identify the axion when multiple scalar fields participate in PQ symmetry breaking.

As shown in Appendix A, a linear combination of angles of singlet scalar fields, X and Y , in our model, i.e. $3\theta_X + \theta_Y$, is stabilized by the A -term for YX^3 term in the superpotential. From the PQ -charges of X and Y , this combination of angles does not transform under the $U(1)_{PQ}$. So, the

orthogonal combination of angles plays a role for the QCD axion.

From the kinetic term for X, Y , $-\langle X \rangle^2(\partial_\mu \theta_X)^2 - \langle Y \rangle^2(\partial_\mu \theta_Y)^2$, we find the canonical axion field as follows,

$$a = \frac{1}{M}(\langle X \rangle a_X - 3\langle Y \rangle a_Y) \quad (\text{B1})$$

where $a_X = \frac{\theta_X}{\langle X \rangle}$, $a_Y = \frac{\theta_Y}{\langle Y \rangle}$ and $M = \sqrt{9\langle Y \rangle^2 + \langle X \rangle^2}$. In the presence of multiple scalars with VEV v_i and PQ -charge q^i , the axion field is generalized [35] to $a = \frac{1}{M}\sum_i a_i q^i v_i$ with $M = \sqrt{\sum_i (q^i v_i)^2}$.

Then, the axion coupling to the gluon field is given by the following effective Lagrangian,

$$\mathcal{L}_{agg} = \frac{a}{f_a} \frac{g^2}{32\pi^2} \text{tr}(G_{\mu\nu} \tilde{G}^{\mu\nu}) \quad (\text{B2})$$

where $f_a = \frac{M}{\mathcal{A}}$ with $\mathcal{A} = \sum_i q^i l_i$ being $U(1)_{PQ} - SU(3)_C - SU(3)_C$ anomaly and l_i being the $SU(3)_C$ quadratic index of a fermion with PQ -charge q^i . In our case, we obtain the anomaly as $\mathcal{A} = -3$. Thus, the axion decay constant is given by $|f_a| = M/|\mathcal{A}| = \frac{1}{3}\sqrt{9\langle Y \rangle^2 + \langle X \rangle^2}$.

APPENDIX C: PQ SYMMETRY BREAKING TERMS AND AXION SOLUTION TO STRONG CP PROBLEM

We must also check other higher dimensional operators which are not PQ symmetric and thus can potentially spoil the property of the PQ symmetry for solving the strong CP problem.

If there is a Planck-scale induced non- PQ symmetric term in the potential [36]:

$$V = \frac{1}{M_P^{2n}} \phi^{2n+3} (\alpha \phi + \alpha^* \phi^*), \quad (\text{C1})$$

it gives additional contribution to the axion mass $m^2 = |\alpha| |\phi|^{2n+4} \cos \delta / (M_P^{2n} f_a^2)$ with δ being the phase of α . In order not to perturb the axion potential term from QCD instanton effect, this mass must be smaller than about 10^{-5} times the usual axion mass (m_a): $m^2 < 10^{-9} m_a^2$.

First, we note that the PQ symmetry is an approximate global symmetry because it is broken explicitly by the Planck-scale suppressed $U(1)_R$ -invariant higher dimensional interactions, e.g. $W = \frac{1}{M_P^{15}} Y^6 X^{12}$. But, the leading term breaking the PQ symmetry while preserving the R symmetry is given by $W = \frac{\alpha}{M_P^{17}} (W_0)^4 Y^2$. Then, from the F -term potential for Y , we obtain the additional term for the axion potential as

$$\Delta V(a) = \frac{2\kappa\alpha}{M_P^{12}} (W_0)^4 X^{*3} Y + \text{c.c.} \quad (\text{C2})$$

Thus, by expanding the above potential around the axion minimum, we get the correction to the axion potential as

$$\Delta V(a) \simeq m_*^2 a^2 - \frac{3}{2} f_a m_*^2 (\tan \delta) a \quad (\text{C3})$$

where $m_*^2 = -\frac{32\kappa|\alpha|}{9f_a^2 M_p^{12}} (W_0)^4 |X|^3 |Y| \cos \delta$ and $e^{-i\delta} = \frac{\alpha}{|\alpha|}$. Then, from the bound $\langle \bar{\theta} \rangle = \frac{\langle a \rangle}{f_a} < 10^{-9}$, we obtain

$$\frac{3}{4} \left| \frac{m_*^2 \tan \delta}{m_a^2 + m_*^2} \right| < 10^{-9} \quad (\text{C4})$$

where $m_a^2 = \frac{\Lambda_{\text{QCD}}^4}{f_a^2}$. Therefore, for $|m_*^2| \ll m_a^2$, this bound becomes $\frac{3}{4} \frac{|m_*^2|}{m_a^2} \lesssim 10^{-9}$. For nonzero singlet VEVs given in Eq. (A7), the axion mass correction is $|m_*^2| \sim 0.3 \times |\alpha| \left(\frac{m_{3/2}}{M_p} \right)^3 m_{3/2}^2 \sim 10^{-25} \text{ eV}^2$ for $m_{3/2} \sim 100 \text{ GeV}$. Compared to the axion mass bound, $m_a = (0.6 \times 10^7 \text{ GeV}/f_a) \text{ eV} \gtrsim 0.6 \times 10^{-5} \text{ eV}$ for $f_a < 10^{12} \text{ GeV}$, the axion mass correction is negligible.

APPENDIX D: R-PARITY VIOLATING TERMS

In this appendix, we consider the R -parity violating terms induced after the R -symmetry breakdown. We focus on the Majorana neutrino case with $n = 2$.

The effective R -parity violating terms are generated by the following higher dimensional interactions:

$$\begin{aligned} \frac{1}{M_p^6} YX^5 LQD, & \quad \frac{1}{M_p^6} YX^5 LLE, \\ \frac{1}{M_p^6} (W_0)^2 UDD, & \quad \frac{1}{M_p^7} W_0 Y^2 X^2 LH_u. \end{aligned}$$

However, after the superpotential and the singlets develop a nonzero VEV, the induced R -parity violating couplings are negligible. Therefore, it is possible to have a stable LSP.

Moreover, we also note that the following terms are allowed:

$$\frac{1}{M_p^9} (W_0)^2 X^2 QQQ, \quad \frac{1}{M_p^{12}} (W_0)^3 YXUUE.$$

Thus, the baryon/lepton (B/L) violating dim-5 operators are negligible, so the proton stability is also justified.

-
- [1] L. E. Ibanez and G. G. Ross, *Nucl. Phys. B* **368**, 3 (1992).
[2] A. H. Chamseddine and H. K. Dreiner, *Nucl. Phys.* **B458**, 65 (1996); D. J. Castano, D. Z. Freedman, and C. Manuel, *Nucl. Phys.* **B461**, 50 (1996).
[3] E. J. Chun, *Phys. Lett. B* **367**, 226 (1996); I. Jack and D. R. T. Jones, *Phys. Lett. B* **491**, 151 (2000).
[4] H. P. Nilles and N. Polonsky, *Nucl. Phys.* **B484**, 33 (1997); K. Choi, E. J. Chun, and H. D. Kim, *Phys. Rev. D* **55**, 7010 (1997).
[5] N. Kitazawa, N. Maru, and N. Okada, *Nucl. Phys.* **B586**, 261 (2000); *Phys. Rev. D* **62**, 077701 (2000); **63**, 015005 (2000).
[6] P. Binetruy, G. Dvali, R. Kallosh, and A. Van Proeyen, *Classical Quantum Gravity* **21**, 3137 (2004).
[7] H. M. Lee and A. Papazoglou, *J. High Energy Phys.* 01 (2008) 008.
[8] H. M. Lee, *J. High Energy Phys.* 05 (2008) 028.
[9] K. Y. Choi and H. M. Lee, *J. High Energy Phys.* 03 (2009) 132.
[10] A. Falkowski, H. M. Lee, and C. Ludeling, *J. High Energy Phys.* 10 (2005) 090.
[11] J. E. Kim and H. P. Nilles, *Phys. Lett. B* **138**, 150 (1984); E. J. Chun, J. E. Kim, and H. P. Nilles, *Nucl. Phys.* **B370**, 105 (1992); E. J. Chun and A. Lukas, *Phys. Lett. B* **297**, 298 (1992).
[12] For a review, see, J. E. Kim, *Phys. Rep.* **150**, 1 (1987).
[13] Z. Komargodski and N. Seiberg, *J. High Energy Phys.* 06 (2009) 007; Z. Komargodski and N. Seiberg, *J. High Energy Phys.* 07 (2010) 17; K. R. Dienes and B. Thomas, *Phys. Rev. D* **81**, 065023 (2010).
[14] D. Z. Freedman, *Phys. Rev. D* **15**, 1173 (1977); R. Barbieri, S. Ferrara, D. V. Nanopoulos, and K. S. Stelle, *Phys. Lett. B* **113**, 219 (1982); S. Ferrara, L. Girardello, T. Kugo, and A. Van Proeyen, *Nucl. Phys.* **B223**, 191 (1983).
[15] H. Nishino and E. Sezgin, *Phys. Lett. B* **144**, 187 (1984).
[16] A. Salam and E. Sezgin, *Phys. Lett. B* **147**, 47 (1984); Y. Aghababaie, C. P. Burgess, S. L. Parameswaran, and F. Quevedo, *Nucl. Phys.* **B680**, 389 (2004).
[17] Z. Komargodski and D. Shih, *J. High Energy Phys.* 04 (2009) 093.
[18] E. Witten, *Phys. Lett. B* **105**, 267 (1981); S. Dimopoulos, G. R. Dvali, R. Rattazzi, and G. F. Giudice, *Nucl. Phys.* **B510**, 12 (1998); D. Shih, *J. High Energy Phys.* 02 (2008) 091.
[19] M. Dine, G. Festuccia, and Z. Komargodski, *J. High Energy Phys.* 03 (2010) 11.
[20] G. W. Gibbons, R. Gueven, and C. N. Pope, *Phys. Lett. B* **595**, 498 (2004); Y. Aghababaie *et al.*, *J. High Energy Phys.* 09 (2003) 037; H. M. Lee and C. Ludeling, *J. High Energy Phys.* 01 (2006) 062.
[21] T. Asaka, K. Ishiwata, and T. Moroi, *Phys. Rev. D* **73**, 051301 (2006); **75**, 065001 (2007).
[22] M. Pospelov, *Phys. Rev. Lett.* **98**, 231301 (2007); K. Kohri and F. Takayama, *Phys. Rev. D* **76**, 063507 (2007); M. Kaplinghat and A. Rajaraman, *Phys. Rev. D* **74**, 103004 (2006).
[23] K. Hamaguchi, T. Hatsuda, M. Kamimura, Y. Kino, and T. T. Yanagida, *Phys. Lett. B* **650**, 268 (2007).
[24] M. Kawasaki, K. Kohri, and T. Moroi, *Phys. Lett. B* **649**, 436 (2007).

- [25] J. Pradler and F.D. Steffen, *Phys. Lett. B* **666**, 181 (2008).
- [26] K. Jedamzik, *J. Cosmol. Astropart. Phys.* 03 (2008) 008.
- [27] K. Jedamzik and M. Pospelov, *New J. Phys.* **11**, 105028 (2009).
- [28] M. Ratz, K. Schmidt-Hoberg, and M.W. Winkler, *J. Cosmol. Astropart. Phys.* 10 (2008) 026.
- [29] J. Pradler and F.D. Steffen, *Nucl. Phys.* **B809**, 318 (2009).
- [30] S. Bailly, K. Y. Choi, K. Jedamzik, and L. Roszkowski, *J. High Energy Phys.* 05 (2009) 103.
- [31] L. Boubekeur, K. Y. Choi, R. R. de Austri, and O. Vives, *J. Cosmol. Astropart. Phys.* 04 (2010) 005.
- [32] E. Komatsu *et al.*, [arXiv:1001.4538](https://arxiv.org/abs/1001.4538).
- [33] K. Ishiwata, M. Kawasaki, K. Kohri, and T. Moroi, *Phys. Lett. B* **689**, 163 (2010).
- [34] J. L. Feng, S. f. Su, and F. Takayama, *Phys. Rev. D* **70**, 063514 (2004).
- [35] K. S. Choi, H. P. Nilles, S. Ramos-Sanchez, and P. K. S. Vaudrevange, *Phys. Lett. B* **675**, 381 (2009).
- [36] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins, and L. M. Widrow, *Phys. Lett. B* **282**, 132 (1992); M. Kamionkowski and J. March-Russell, *Phys. Lett. B* **282**, 137 (1992); S. M. Barr and D. Seckel, *Phys. Rev. D* **46**, 539 (1992).