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In this case the cells are the grain spacings.

The parameter for scattering, $\langle\alpha^2\rangle$, is determined as follows:

The center of the middle grain will be a measured distance y_4 from the straight line. The variance σ^2 of this distance derived from many observed track-segments will be

$$\sigma^2 = \frac{\langle\psi^2\rangle_\tau}{12} + \frac{3}{2}\langle\delta^2\rangle$$

where $\langle\psi^2\rangle_\tau$ means $\langle\psi^2\rangle$ for a track length τ and is related to $\langle\alpha^2\rangle$ by

$$\langle\psi^2\rangle_\tau = \langle\alpha^2\rangle\langle\tau^3\rangle.$$

The variable ψ was introduced by Barkas.[1] The particle scattering then was shown to be completely determined by it and another random variable χ.

The quantity $\langle\tau_i^3\rangle$ for observed tracks of $(i + 1)$ grains is given by

$$\langle\tau_i^3\rangle = i(i + 1)(i + 2)\langle t\rangle^3,$$

where $\langle t\rangle$ has been defined previously.

In the calculations of coordinates X_i, Y_i for each track, we pick the variables t_i, δ_i, and α_i at random from their respective distributions. For a track of k grains, i will run from 0 to k-1.

The coordinates of a certain point on the particle path where the particle comes closest to the first grain-center are taken to be (0,0), with $\varphi_o = \delta_o = t_o = 0$. Then

$$X_o = 0 \text{ and } Y_o = 0.$$

The formulae (as derived by Barkas[1]) for the coordinates of grain centers

(X_i, Y_i) are:

$$X_{j+1} = X_j + t_{j+1}$$

$$Y_{j+1} = Y_j + t_{j+1}\varphi_j + \psi_{j+1} + \chi_{j+1} + \delta_{j+1} - \delta_j$$

$$\varphi_{j+1} = \varphi_j + \frac{2\psi_{j+1}}{t_{j+1}}$$

$$\psi_{j+1} = t_{j+1}^{3/2}\,\alpha_{j+1}$$

$$\chi_{j+1} = \frac{1}{\sqrt{3}}\, t_{j+1}^{3/2}\,\alpha'_{j+1}$$

j runs from 0 to k-1, where k is the total number of pairs of grain-coordinates. The random numbers α and α' are picked independently from the same distribution of α. This is necessary because ψ and χ are independent random variables which together determine the scattering.

An IBM 7094 program (CARLO) has been written for the purpose of calculating any number of grains for each track. The only input needed for the program are the parameters $\langle t\rangle$, $\langle\delta^2\rangle$, and $\langle\alpha^2\rangle$. The subroutines in the program take care of the proper distributions for t, δ and α.

When t, δ and α do not have the simple distribution functions assumed here, their actual distributions, whatever they are, can be used in the same way. The parameters $\langle\alpha^2\rangle$ and $\langle t\rangle$ also can be permitted to depend on i so as to allow for energy-loss of the particle.

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On the Width of Heavy Ion Tracks in Emulsion

Robert Katz and J. J. Butts
Kansas State University
Manhattan, Kansas

Since the detection of heavy ions in cosmic rays by means of emulsion, the variation of the width of their tracks with range has received considerable attention as a means of identifying Z, and several track width theories have been proposed.

Up to the present work, most attention has been directed to the thindown section of the track, in its last 200 μ. In this region the track is relatively smooth and wedge shaped, and it is relatively easy to make a meaningful width measurement. In the rougher region of the track as few as 25 delta rays in a 10 μ interval may make up the track boundary. Large statistical variations in their number and energy distribution may be expected to show up as an irregular track outline. In the thindown region there is a manyfold increase in the number of delta rays, and a more homogeneous energy distribution because of the clamp in the maximum delta ray energy of 2 $mc^2\beta^2\gamma^2$ imposed by kinematics. Nevertheless our work shows that there are clear advantages to be gained by making width measurements in the rougher regions of the track.

In the present work we have sought to measure the width everywhere along the track length. This we have done by projecting the image of the track onto a screen by means of an Ortholux microscope and a Xenon lamp, and by tracing the outline of 50 μ segments to obtain the area. Isolated delta rays are cut off at their base. Our theoretical formulation is intended to correspond to this experimental requirement. It was our judgement that long isolated delta rays contribute disproportionately to track area measurements, and must be excluded from both theory and experiment.

At the time heavy ions were first observed Freier[1] suggested that the width variation was proportional to ionization, and that thin-down occurs because of electron pickup by the nucleus. In 1953, Lonchamp[2] projected machine accelerated ions into emulsion, and found that track width variations had to be attributed to the spatial distribution of the delta rays rather than to the amount of ionization produced. Electron pickup was found to be of minor importance, occuring perhaps in the last 15 μ of range. The width was only indirectly related to ionization, since in the thindown region the width was decreasing though ionization was still increasing. These basic observations have been substantiated by subsequent observers.

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Since Lonchamp's description of the basic mechanism, all trackwidth theories have had a common base. All use the well-known delta ray distribution formula. All make simplifying assumptions about the angular distribution of the delta rays and attempt to average over scattering of the individual electrons. In a fundamental way the theories differ as to the basic criterion for determining the boundary of a track, and perhaps on the influence of processing on track width.

Lonchamp's model was at best a qualitative one. It assumed that delta rays were projected normally outward from the ion track, and given typical grain diameters for electron sensitive emulsion it proposed that the track edge would be observed as opaque when just 400 delta rays protruded from a cylinder whose diameter was the track width. The delta ray energy at which this occured was converted into width by use of a range energy relationship for electrons. For illustration, we have computed a family of track width curves as a function of range for Z = 5, 10, 15, ... , and have superimposed on these curves data we have obtained from two tracks, as shown in Figure 1a. The experimental track form has no theoretical counterpart.

A significant improvement in the theory was made by Bizzeti and Della Corte^3, who used the energy carried out of a cylinder concentric with the ion path as their width criterion. They assumed that the delta ray distribution was effectively spherically symmetric, and computed an average over emission angles of the energy flux carried away from the track by delta rays. Their principal criterion was that the energy transported through the cylinder defining the track edge was all deposited within a grain diameter of the cylinder, an assumption which is far more reasonable in the thin down region than at high range. The theory leads to widths which are too large at high ion energies. In Figure 1b we show a family of track width curves computed from the Bizzeti-Della Corte theory for Z = 5, 10, 15 ..., and have superimposed on these curves the data from the same two tracks of Figure 1a. Though the agreement is substantially better than in the case of Lonchamp's theory, particularly in the first 100 μ, predicted track widths are greater than any which are observed, and the experimental curves depart from the theory in the region from 1000 to 10,000 μ, even without any Z identification of the two experimental tracks.

We have modified the model of Bizzeti and Della Corte to assert that a photographic grain will be activated if it is in a region where the energy deposited per unit volume by delta rays exceeds a threshold value, E^*. In other words, we use an energy density criterion rather than an energy flux criterion to calculate the radius, x, of the critical cylinder. All grains whose outer edge touches the critical cylinder will grow in development, so that an increment λ_0, the sum of the diameters of a developed and an undeveloped grain, is added to the diameter of the critical cylinder to make up the track width. The track width λ is then given by the sum of the diameter of the critical cylinder and λ_0, or $\lambda = 2x + \lambda_0$.

To calculate x, we start with a computation of $E(x,\beta,Z)$, the energy flux through a cylinder of radius x, due to an ion of effective charge Z moving at speed βc, as derived by Bizzeti and Della Corte. The energy density is then found in a limiting process, to be given by $-(2\pi x)^{-1}\partial E/\partial x$. Since the threshold energy density for grain activation, E^* is unknown beforehand, we adjust this value and solve for x as a function of β and Z. By use of Heckman's formula[4] and a range energy relation for protons, the track width is obtained as a function of range for ions of different Z. In each case the mass chosen for the ion is that of the most abundant isotope.

In obtaining an expression for the energy flux carried by delta rays , $E(x, \beta, Z)$, a range energy relation for low energy electrons in emulsion is required. The empirical formula used by Bizzeti and Della Corte is $r = k\,w^{1.72}$, which gives r in μ when w is in kev and k = 0.021 . This power law gives the practical or the extrapolated range of a group of monoenergetic electrons. We used this equation in the computation of the track width in Figure 1, to obtain width range curves from Lonchamp's theory.

In our track width model we need information about the average energy loss of delta rays as a function of their range. This implies that we must have an expression giving the average range, or the diffusion length, as a function of electron energy rather than an expression giving the extrapolated range. We have chosen to treat the coefficient k of the power law as an adjustable parameter, keeping the exponent at 1.72. We have done this for convenience in the computation, and because experimental data are not available for diffusion lengths of dectrons at the delta ray energies which correspond to observed track radii (less than 15 kev).

There are then two unknown parameters which must be adjusted for best fit of the theory to data. We find the best value of k to be 0.006, which gives a diffusion length which is about 1/3 the maximum range of 10 kev electrons, as given by Feldman[5]. The threshold sensitivity which yilds the best fit is $E^* = 6000$ ergs/cm^3 for G-5 emulsion.

The same experimental data shown in Figures 1 are plotted over theoretical width-range curves computed according to our model, in Figure 2. In each case we plot theoretical curves for Z, for $Z + \sqrt{Z}$ and for $Z - \sqrt{Z}$ over the data to indicate the charge discrimination of the theory. The Z assignment made for these cosmic ray tracks is entirely on the basis of overall fit with our theoretical curves. An additional pair of tracks is shown in Figure 3. Thus we have a track width model which yields curves which match tracks found in emulsion to within a grain diameter (about 0.8 μ) over their entire range to a centimeter or two. We do not know Z for these tracks independently.

By way of calibration we have compared our width calculation with published measurements of the width of tracks of machine accelerated ions by Bizzeti and Della Corte, and by Skjeggestad[6], as shown in Figures 4 and 5. The data are in good agreement with the model at $Z = 8$, in the thindown region which is within reach of accelerator energies.

The model predicts that the track width λ is equal to λ_0 at 10 μ range for all Z. This is shown in Figure 6, where $\lambda - \lambda_0$ is plotted against range for $Z = 5, 10, 15 \ldots$. We have verified this experimentally. This finding provides a method for normalizing data for processing variations with depth, by measuring the width at 10 μ of many tracks ending at different depths in the emulsion. Our data have been normalized to a depth of 150 μ where λ_0 was measured to be 0.75 μ. This is consistent with average data given for G-5 emulsion by Barkas[7]. He finds 0.27 μ as the diameter of an undeveloped grain and 0.5 μ as the diameter of a developed grain.

From Figure 6 we see that the theory indicates that there should be crossover in the thindown region where the number of delta rays is very large, and λ is approximately equal to twice the maximum range of the most energetic delta ray, whose energy is determined only by the velocity of the ion, not by its charge. At a range of 50 μ the speed of a $Z = 50$ ion is less than that of a $Z = 20$ ion. The track of the lighter ion is therefore wider than the track of the heavier ion. There would be no crossover on a plot of $\lambda - \lambda_0$ vs. β ; the crossover arises in the translation to range. Generally the curves indicate that the 500 μ range may be the best place to identify Z on the basis of width measurements, the roughness in the track being compensated by the possibility of measuring over longer range intervals to improve statistics. At smaller ranges the width is less sensitive to Z and is even double valued. The position of the maximum width or the thin-down length appears to be relatively independent of Z and of little value in particle identification.

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References

[1] Freier, P., E. J. Lofgren, E. P. Ney, and F. Oppenheimer, Phys. Rev. 74,1818 (1948)

[2] Lonchamp, J. P. , J. Phys. Radium 14,433 (1953)

[3] Bizzeti, P., and M. Della Corte, Nuovo Cimento 11, 317 (1959)

[4] Heckman, H.H., B. L. Perkins, W. G. Simon, F. M. Smith, and W. H. Barkas, Phys. Rev. 117 , 544 (1960)

[5] Feldman, C. , Phys. Rev. 117, 455 (1960)

[6] Skjeggestad, O., Nuovo Cimento 8 , 927 (1958)

[7] Barkas, H, , Nuclear Research Emulsions (Academic Press, Inc., New York, 1963) Vol. 1.

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Captions

Figure 1. Experimental data for two tracks in G-5 emulsion shown as circles and triangles, plotted against a background of theoretical trackwidth-range curves from the theory of (a) Lonchamp (lower curves) and (b) Bizzeti and Della Corte (upper curves). The illustration has been prepared on the output of an IBM-1403 printer.

Figure 2. The two tracks shown in Figure 1 have been identified as Z = 25 (upper graph, previously plotted as triangles) and Z = 10 (lower graph, previously plotted as circles), on the basis of the overall fit of the data with predicitions of the present theory. Curves for the assigned value of Z as well as for a pair of adjacent values are plotted to give some indication of the discrimination of the theory.

Figure 3 An additional pair of tracks identified as Z = 18 and Z = 8 plotted against the theoretical curves.

Figure 4. Width of the tracks of machine accelerated ions against the present theory. Data obtained by Bizzeti and Della Corte.

Figure 5. As in Figure 4. Data published by Skeggestad.

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Figure 6. Width minus λ_o , in microns, versus range, in microns, for the tracks of ions for Z = 5, 10, 15 ... 50 in G-5 emulsion, according to the present calculation. Here, as in Figure 1, the width increases regularly with Z at high range.

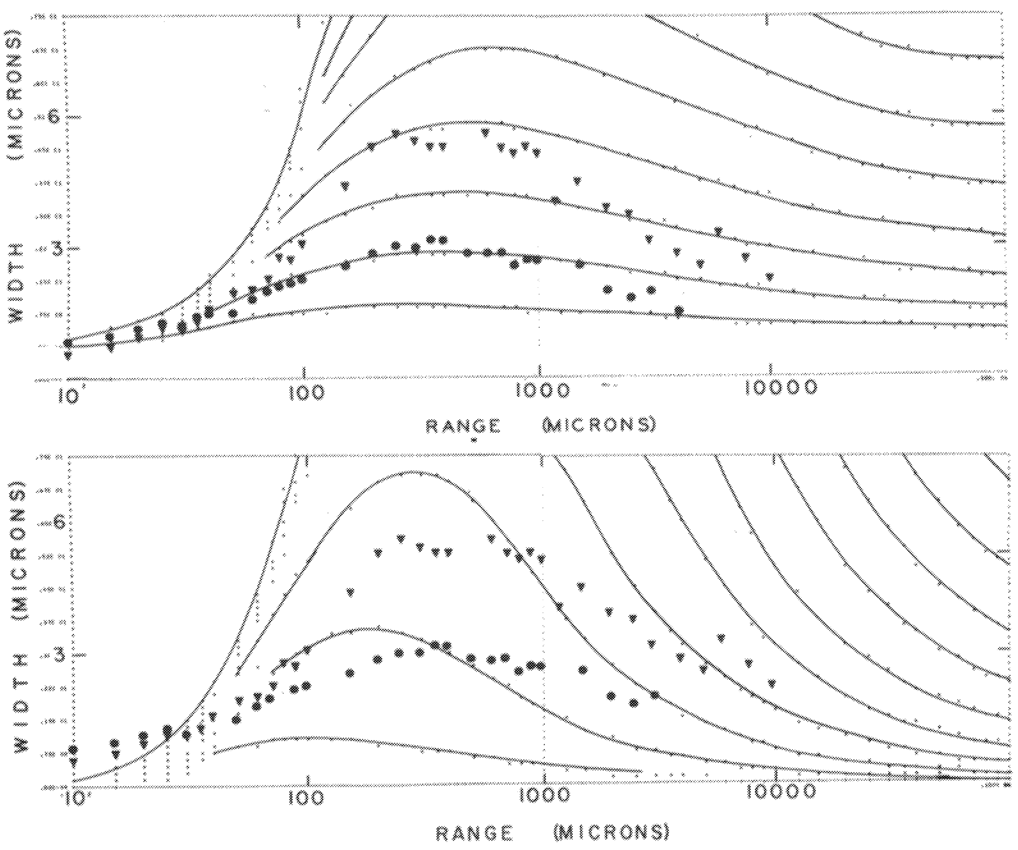

Figure 1

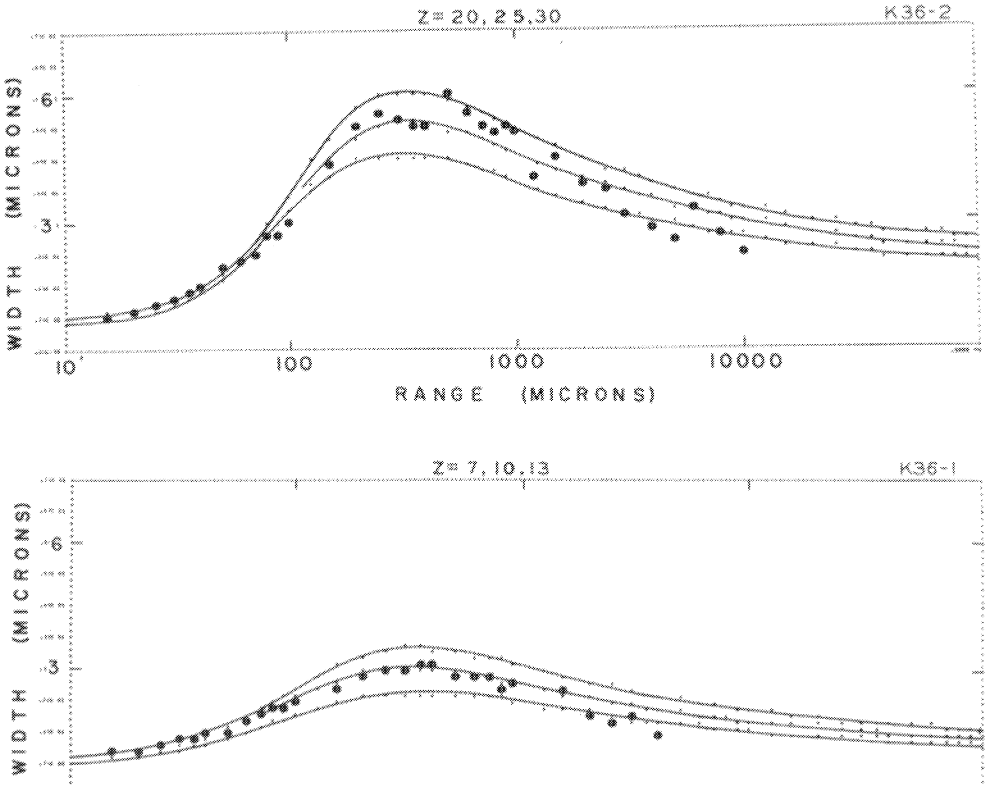

Figure 2

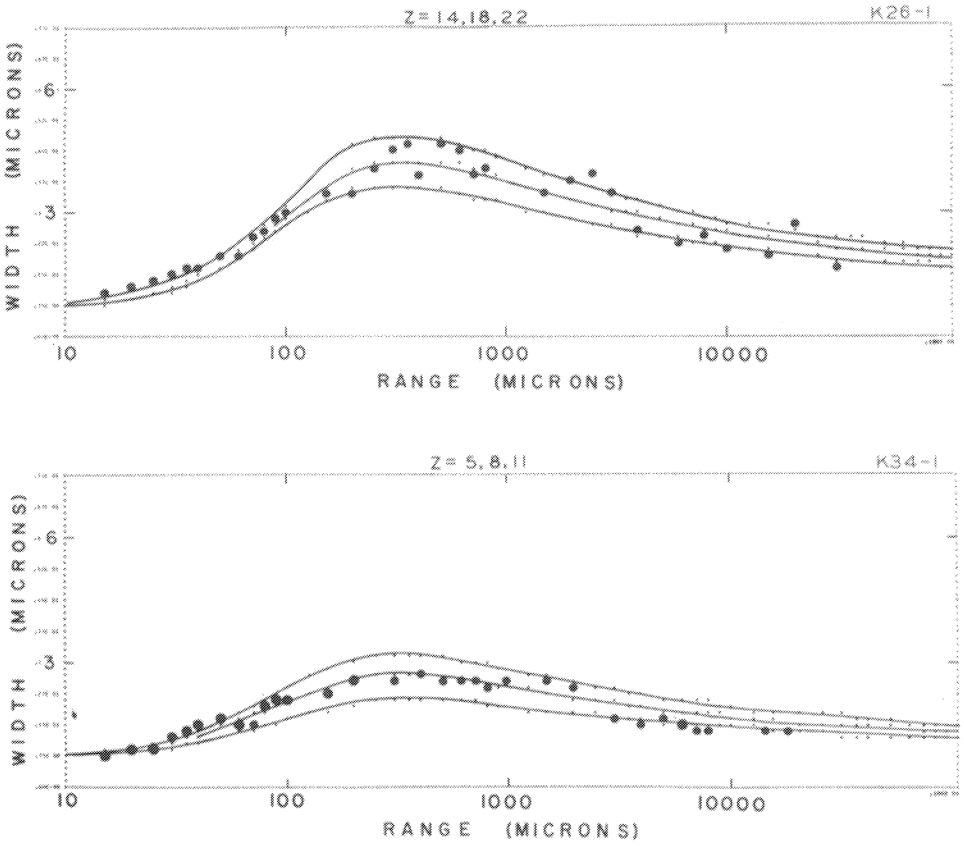

Figure 3

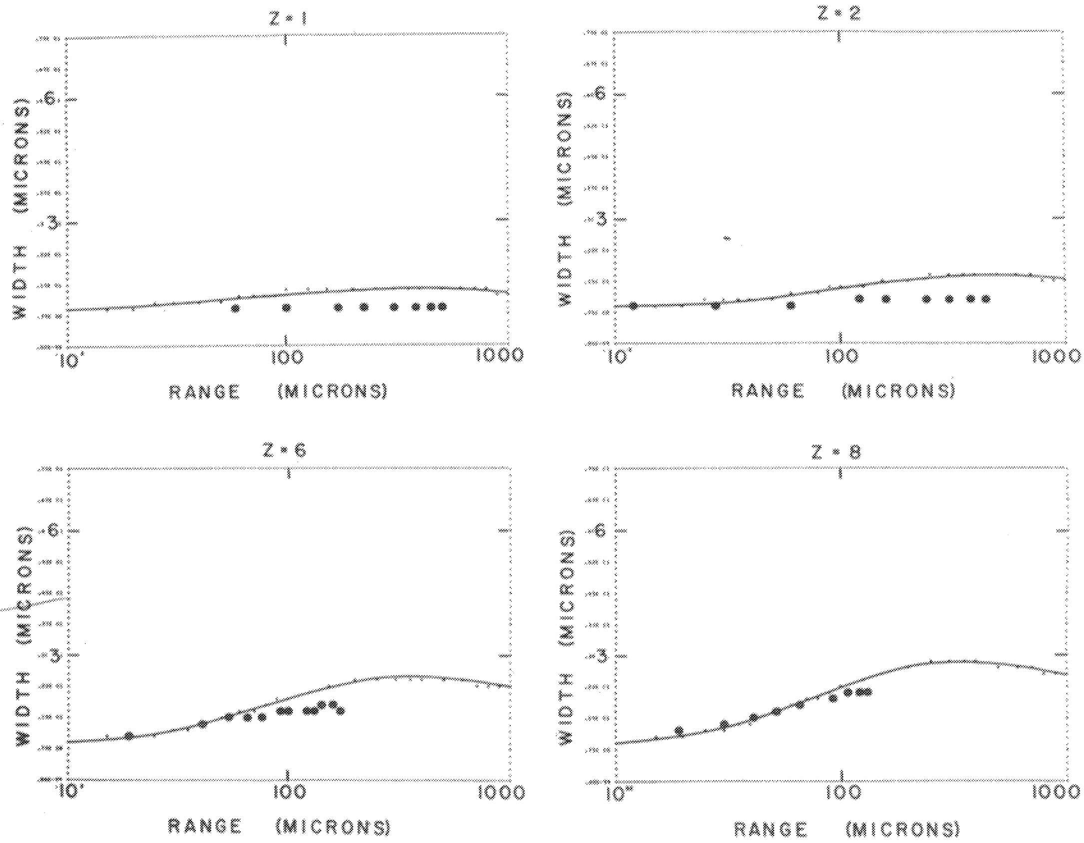

Figure 4

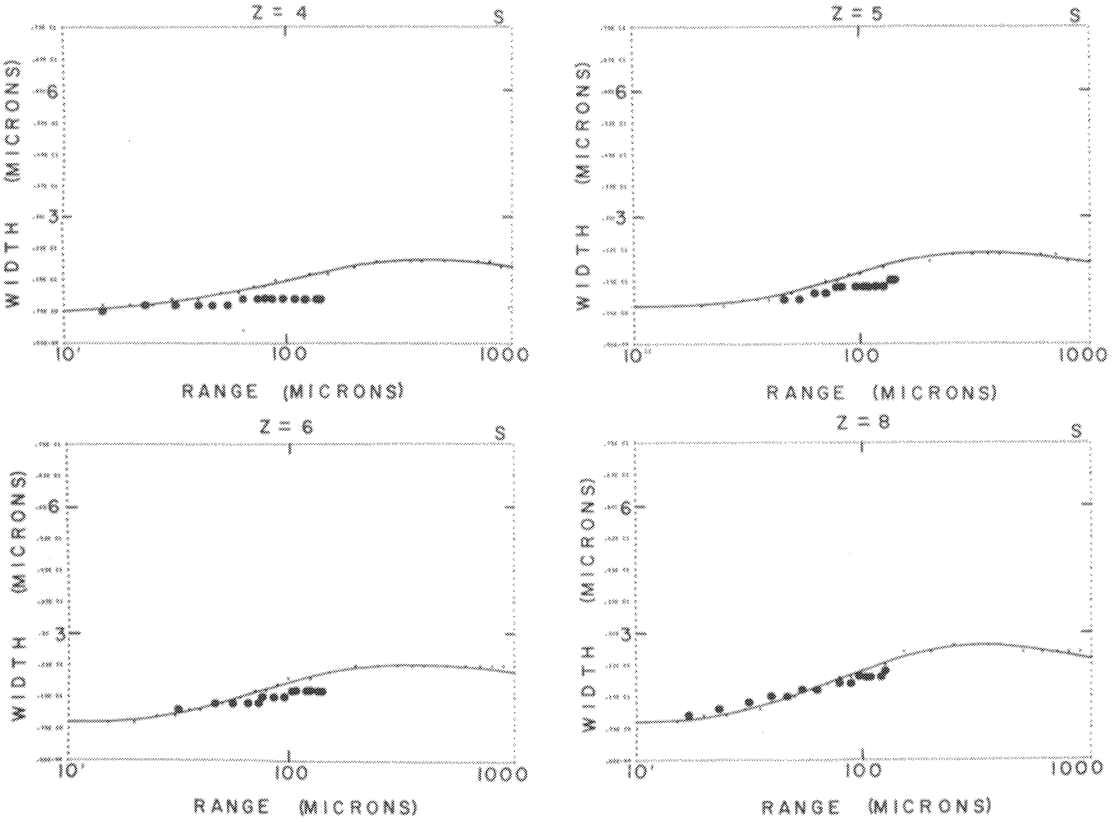

Figure 5

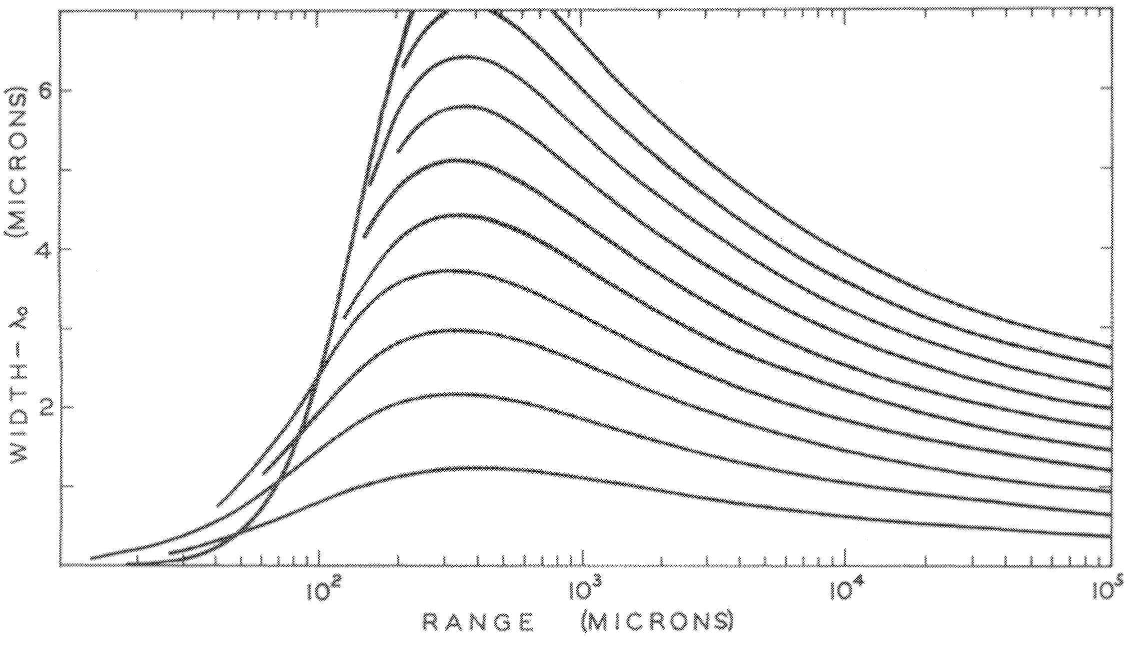

Figure 6