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Study of the Multiple Scattering Constant in Emulsion at Great Cell Lengths.

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1. Introduction.

In the present paper the dependence of the scattering constant on cell length is determined from t=0.5 cm up to t=10cm by two different methods.

In the interval 0.5 cm $\leq t \leq$ 3 cm, the "coordinate method" [1] of multiple scattering measurements and for higher cell lengths the "angular dispersion method" [2,3] are used. At t=3 cm K is determined with both methods to check the consistency of the two methods.

Present measurements permit to investigate also the behaviour of the spurious scattering as a function of cell length.

2. Discussion of the different methods.

2.1 The scattering constant can be calculated in the case of the coordinate method from the well known formula:

$$K = \frac{0{,}573 \cdot p\beta \cdot \bar{d}_c}{t^{3/2}} \qquad (1)$$

with the approximation $\bar{d}_{sp} \ll \bar{d}_c$. Here $\bar{d}_c$ and $\bar{d}_{sp}$ are the mean values of the second differences due to multiple Coulomb scattering and spurious scattering respectively. (d, t and $p\beta$ are expressed in μ, 100μ, and MeV/c, respectively). The values of the scattering constant thus obtained are always greater than or at least equal to the true values of K. The less negligible the spurious scattering compared with the Coulomb scattering, the greater this deviation.

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2.2 In the case of the "angular dispersion" method [2], the lateral angular distribution is measured in two or more strips perpendicular to the beam-direction at a distance t given in 100μ units from one another. Then the scattering constant can be determined from

$$K_t = \frac{\left[\overline{|\vartheta_t|}^2 - \overline{|\vartheta_0|}^2\right]^{1/2}}{1{,}225 \cdot t^{1/2}} p\beta \tag{2}$$

where $\vartheta_{0,i}$ and $\vartheta_{t,i}$ are the projected angles in degrees between the i-th measured track and the average direction of the beam in the first strip (t=0), and in another one at a distance t from the first strip, respectively.

The contributions of the various noises (grain, stage, reading noise) to the Coulomb scattering angle are the same in each strip. Thus by this method the effect of any noise is eliminated. However, the effect of small angle diffraction scattering has to be taken into account, Therefore, one has to use in Eq. (2) $\overline{|\vartheta_{t,corr}|}^2 = \overline{|\vartheta_t|}^2 - \langle|\vartheta_{t,diff}|\rangle^2$ instead of $\overline{|\vartheta_t|}^2$, where $\langle|\vartheta_{t,diff}|\rangle^2$ was calculated.

3. Experimental results.

3.1 Measurement with the coordinate method.

3.1.1 Measurements were carried out on tracks of π^--mesons of momentum (17.2 ± 0.2) GeV/c in Ilford G5 emulsion plates of size 14.5 cm x 23.5 cm x 0.06 cm by means of a Koristka R4 microscope using an objective of magnification 55.

Tracks of a total length of about 13 m were measured with a basic cell length t=0.5 cm.

The value of the total noise was found to be (0.150 ± 0.03)μ using 50 μ cell-size.

The distributions of the absolute values of the second differences for cell lengths t=0.5, 1, 2 and 3 cm are cut off at $4\overline{|D_m|}$ and normalised to the total number of second differences. With this cut off 2.7, 1.9, 0.8 and 0.9 % of the total number of second differences were omitted. A χ^2-test analysis showed that the cut off distributions can be well approximated by normal distributions up to the highest cell length measured.

3.1.2 The mean values of second differences corrected for the constant noise (d_m) after cut off are plotted versus cell length in Fig.1 (open circles). The theoretical curve obtained for d_c with values of K(t) calculated by Voyvodić and Pickup [4] having taken into account the finite size of the nucleus, is also shown in the same figure. Because of the effect of spurious scattering one should expect the experimental points to be larger or at least equal to the corresponding d_c values.

3.1.3 For the calculation of the mean second differences due purely either to Coulomb scattering or to spurious scattering the method worked out by Časnikov et al. [5] has been used. This method does not involve the a priori knowledge of the scattering constant and therefore one can investigate the behaviour of the spurious scattering without applying any predicted value of K. By this method the value of $\bar{d}_c$ and $\bar{d}_{sp}$ can be obtained from the measured second ($\bar{d}_m$) and third ($\bar{d}_{m,3}$) differences. Introducing the notations

$$\varrho = \frac{\bar{d}_{m,3}}{\bar{d}_m} \quad , \quad \varrho_c = \frac{\bar{d}_{c,3}}{\bar{d}_c} = \left[\frac{3}{2}\right]^{1/2} \text{and} \quad \varrho_{sp} = \frac{\bar{d}_{sp,3}}{\bar{d}_{sp}}$$

the formulae to be applied are:

$$\bar{d}_c = \bar{d}_m \left[\frac{\varrho^2 - \varrho_{sp}^2}{\varrho_c^2 - \varrho_{sp}^2}\right]^{1/2} \quad , \tag{3}$$

$$\bar{d}_{sp} = \bar{d}_m \left[\frac{\varrho_c^2 - \varrho^2}{\varrho_c^2 - \varrho_{sp}^2}\right]^{1/2} \quad . \tag{4}$$

In these formulae we put $\varrho_{sp}^2 = 10/3$ x/. The mean values of second

x/ This assumption is supported by the fact, that experimental values very close to $\varrho_{sp}^2 \approx \varrho_{u}^2 = 10/3$ were obtained by Časnikov et al. with the formula given in ref.[5]:

$$\varrho_{sp}^2 = \left[\left(\frac{t_2}{t_1}\right)^3 d_{m,3}^2(t_1) - d_{m,3}^2(t_2)\right] \Big/ \left[\left(\frac{t_2}{t_1}\right)^3 d_m^2(t_1) - d_m^2(t_2)\right]$$

where t_1 and t_2 represent two different cell lengths.

differences due purely to Coulomb scattering calculated from Eq.(3) are also plotted in Fig.1 (crossed points). It can be seen, that all the experimental points for $t \geqslant 1$ cm lie under the theoretical curve, i.e. the deviation is more pronounced than in the first case (see 3.1.2) despite of the greater statistical error of the individual values.

The mean values of second differences due to spurious scattering obtained from Eq.(4) for different cell lengths are plotted in Fig.2. One can see that the trend of the spurious scattering does not show with increasing cell length any saturation or break down tendency.

In the present measurements the spurious scattering obeys the power law

$$\bar{d}_{sp} = a\,t^{n} \qquad (5)$$

with $n=1.22 \pm 0.17$ and $a=(6.84 \pm 1.3)\times 10^{-3}$ obtained using the method of least squares.

The average of the ϱ -values in our measurements is 1.45 ± 0.02 significantly greater than $\varrho_c = 1{,}225$ predicted theoretically for multiple Coulomb scattering.

The values of the scattering constant calculated with the assumption $\bar{d}_{sp} \ll \bar{d}_c$ and using $\bar{d}_c$ values obtained from Eq.(3) are listed in Table I, respectively.

Table I.

t (100μ)	K if $\bar{d}_{sp} \ll \bar{d}_c$	K_{corr}
50	31.8 ± 1.7 x/	27.5 ± 2.0
100	30.4 ± 2.0	27.1 ± 2.2
150	30.1 ± 1.1	26.3 ± 2.7
200	30.0 ± 1.2	27.8 ± 4.4
250	29.5 ± 1.7	27.2 ± 2.0
300	28.7 ± 1.7	

x/ This value is higher than expected because the assumption $\bar{d}_{sp} \ll \bar{d}_c$ does not hold at this cell length.

3.2 Measurement with the angular dispersion method.

3.2.1 Measurements were carried out in the same region of the emulsion plates as in the former case. The plate was placed on the stage in such a way that the angle between the direction of stage motion and the average direction of the pion-beam was about zero. In order to include only primary particles, tracks having projected angles to the average direction of the beam smaller than 1^{o} were measured.

In three stripes at distances 1.8, 4.8 and 11.8 cm from the edge of the plate (the second and the third stripes being at t=3 and t=10 cm from the first one) about 1000 tracks were measured.

In order to eliminate the possible background due to single Coulomb scattering, knock-on electrons and secondaries from inelastic nuclear interactions a cut off was applied at $4|\vartheta|$. In this way the same background events are eliminated as in the case of the coordinate method. In the first, second and third stripes 0.6, 1.5 and 0.6 % of the total number of tracks were omitted, respectively.

3.2.2 Having applied the cut off the angular distributions can be well approximated by normal ones.

For the calculation of the scattering constant one has to use Eq.(2) which can be written in the form

$$\overline{|\vartheta_{t,corr}|}^2 = \overline{|\vartheta_o|}^2 + \alpha t \tag{6}$$

where

$$\alpha = \left[K \frac{1,225}{p\beta}\right]^2.$$

In Fig.3 $\overline{|\vartheta_{t,corr}|}^2$ is plotted vs. cell length. The experimental points yield a straight line, indicating the constancy of K in this interval. Using the method of the least squares, the mean value of K for the cell length interval $3 \leq t \leq 10$ cm turned out to be

$$\overline{K} = 30.2 \pm 2.5$$

4. Discussion and conclusions.

4.1 It can be seen from Table I that the interval $0.5\ \text{cm} \leq t \leq 3$ cm the K values obtained by the "coordinate method" show a small but significant dependence on cell length, and can be well approximated by a constant $\overline{K} = 30.2 \pm 0.6$. Applying the "angular dispersion method" we obtained in the cell length interval 3cm t 10 cm $\overline{K} = 30.2 \pm 2.5$ which is in agreement with the above value. After having corrected for spurious scattering one finds that the independency of the K-values of cell length becomes even more pronounced in the same interval.(see Table I), and the mean value of K is 27.1 ± 1.2.

We have collected from 20 published measurements the K-values which were measured at different energies in the $0.1\ \mathrm{cm} \leq t \leq 10$ cm cell length interval. Table II. We have divided the data into two parts with respect to the cell lengths used: $0.1\ \mathrm{cm} \leq t \leq 1$ cm and $1\ \mathrm{cm} \leq t \leq 10$ cm. The weighted averages of the K-values over the first, second and the total cell length intervals are 28.9 ± 0.2, 29.3 ± 0.3, and 29.0 ± 0.2, respectively.

In conclusion we can say that for high energy particles
a) K is constant at cell lengths $t \geq 0.1$ cm and
b) smaller than expected from the theory of Voyvodič and Pickup, even if the influence of the spurious scattering is not taken into account.

x x x

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Figure Captions.

Fig.1 Variation of $\overline{d}_m$ with t . The crossed points correspond to second differences corrected for spurious scattering.

Fig.2 Dependence of spurious scattering on cell length t.

Fig.3 Variation of $\overline{|d_{t,corr}|}^2$ vs. t. The smooth line was calculated using the method of least squares.

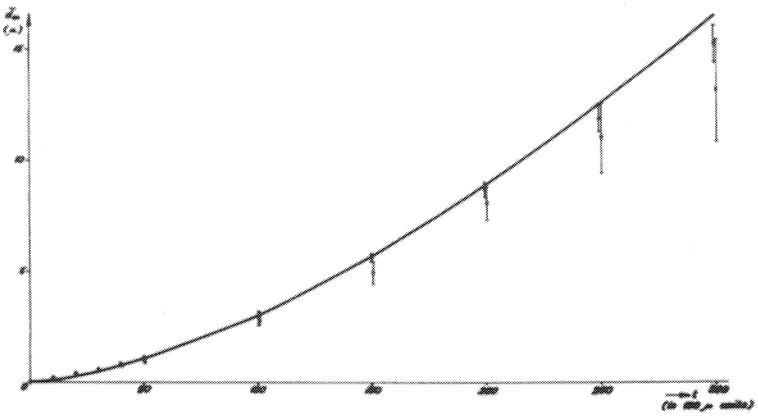
TABLE I.

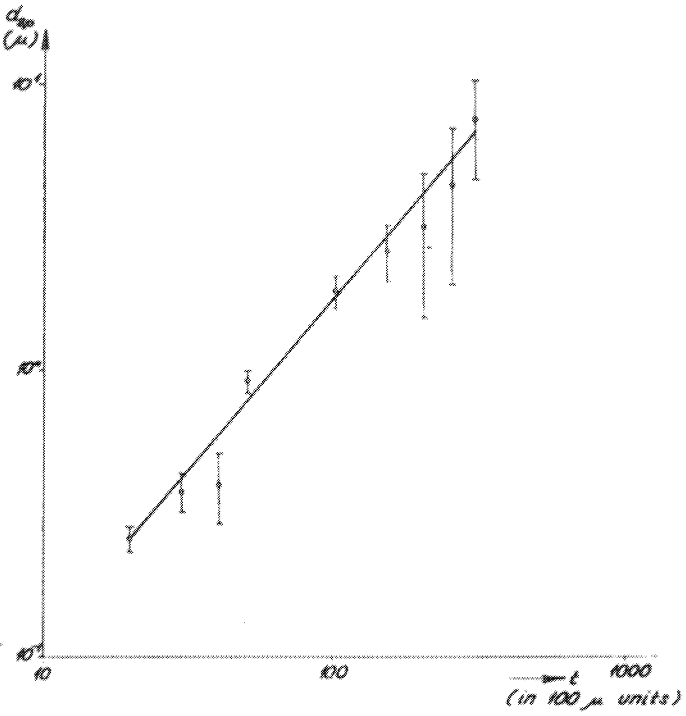
Cell length t (cm)	Expected values	VALUES OF THE SCATTERING CONSTANT Measured by																				
		[11]	[8]	[9]	[10]	[10]	present authors	authors x/	[12]	[9] xxx/	[10]	[10]	[10]	[13] xxx/	[8]	[2]	present authors xx/	[2]	[3] xx/	[6]	[6]	[6]
0.1	27.3	28.2±1.0							30.6±0.9													
0.15	27.7	27.9±1.4																				
0.2	28.3	28.6±1.7			28.8±0.4				30.1±1.2	29.9±0.9												
0.3	28.6								30.9±1.7													
0.4	28.9				27.8±0.5	29.0±1.8			31.0±2.1	30.2±1.5	30.6±1.2	31.8±1.9	28.0±2.2	30.4±1.0								
0.5	29.2						31.8±1.7	27.5±2.0	31.3±2.3													
0.6	29.4				28.1±0.7				32.2±2.9													
0.8	29.8			30.5±1.7	27.8±0.7				29.8±3.4	29.0±2.6												
1.0	30.1			26.9±1.7	28.9±1.0		30.4±2.0	27.1±2.2														
1.5	30.6						30.1±1.1	26.3±2.7							27.5±2.5					31.2±2.7	30.0±1.7	
2.0	30.9		26.6±1.2	23.2±2.6	29.3±0.8		30.0±1.2	27.8±4.4														
2.5	31.2						29.5±1.7	27.2±4.0												28.6±2.5	27.6±1.6	
3.0	31.2		25.4±1.8		27.2±1.1		28.7±1.7									30.3±0.7				27.1±2.4	27.3±1.6	
4.0	31.2				27.6±1.6															28.1±2.5		
5.0	31.2																	30.4±0.9		28.3±2.6		
6.0	31.2																			26.5±2.5		
7.0	31.2																		29.4±2.0			
7.2	31.2																			27.4±2.7		
8.0	31.2																			28.5±2.8		
9.5	31.2																					
10.0	31.2																					
Mean value of K		28.2±0.7	26.3±1.0	27.7±1.1	28.3±0.2		30.1±0.6	27.1±1.2	30.6±0.6	29.9±0.8							30.2±2.5			28.2±0.9	28.2±0.9	31.5±1.7
Method applied		Coordinate														Angular dispersion				y dispersion		
		cut off with replacement	without cut off	with cut off										without cut off		with cut off						
Measurements were made		on single tracks				by rel.sc.	on single tracks		by rel. sc.						on single tracks	on single tracks				on single tracks		
Particle Type		π								p					μ	π		p		p		μ
Particle Energy (GeV)		4.5	16.2	17					6.2	27					8	0.3	17	6	9	27		8

x/ corrected for d_{sp}

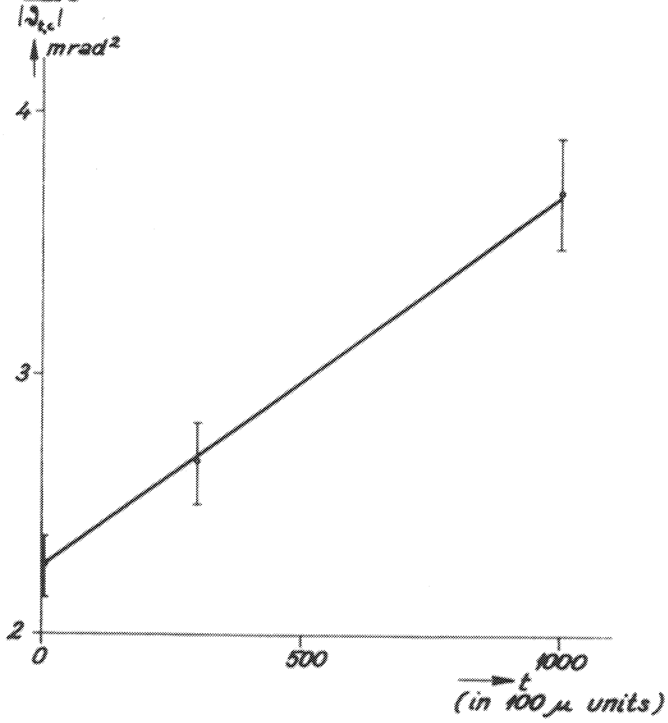
xx/ corrected for small angle diffraction scattering

xxx/ This values have to be multiplied by a factor of 27/24. (see.ref. [6])





d_{sp}
(μ)
10^1
10^0
10^{-1}
10
100
1000
t
(in 100 μ units)



mrad²
4
3
2
0
500
1000
t
(in 100 μ units)