OPTIMUM MULTIPLE SCATTERING

Douglas E. Greiner

Lawrence Radiation Laboratory
University of California
Berkeley, California
U.S.A.

INTRODUCTION

The purpose of this paper is to study the best utilization of multiple scattering data to determine the scattering rigidity of a particle.

VARIABLES

Consider the projected image of a track as shown in Fig. 1. We define:

y_k = measured ordinates a distance t apart.

λ_i^k = distance between the i^{th} and $i + 1^{st}$ deflections in the k^{th} cell.

ω_i^k = i^{th} angle of deflection in the k^{th} cell

φ_k = angle between projection of track and y axis at k^{th} ordinate

n_k = number of deflections in the k^{th} cell

δ_k = noise associated with the k^{th} ordinate

Assuming δ_k to be a random variable, Barkas has defined two more independent variables χ_k and ψ_k.[1]

$$\chi_k \equiv \frac{\sum_{i=1}^{n_k} \lambda_i^k \sum_{j=1}^{i} \omega_j^k - \sum_{i=1}^{n_k} \omega_i^k \sum_{j=1}^{i} \lambda_{i-1}^k}{2}, \quad \psi_k \equiv \frac{\sum_{i=1}^{n_k} \lambda_i^k \sum_{j=1}^{i} \omega_j^k + \sum_{i=1}^{n_k} \omega_i^k \sum_{j=1}^{i} \lambda_{i-1}^k}{2} \qquad (1)$$

Any order difference ≥ 2 can be expressed in terms of a linear combination of these varables:

$$D_k^r = \sum_{i=1}^{r} (a_i^r \psi_{k+i-1} + b_i^r \chi_{k+i-1}) + \sum_{i=0}^{r} c_i^r \delta_{k+i} \qquad (2)$$

where

$$a_i^r = \frac{(-1)^{r-i}(r-2)!(2i-4-1)}{(r-i)!(i-1)!} \quad b_i^r = \frac{(-1)^{r-i}(r-1)!}{(r-i)!(i-1)!} \quad c_i^r = \frac{(-1)^{r-i}r!}{(r-i)!i!} \tag{3}$$

Hence any difference product average, $\langle D_k^r D_{k+n}^s \rangle$ is a linear combination of $\langle \psi_k^2 \rangle$, $\langle \chi_k^2 \rangle$ and $\langle \delta_k^2 \rangle$.

CALCULATION OF SCATTERING RIGIDITY

If we assume a gaussian distribution for second differences we can relate the mean squared noise-free second difference, $\langle s \rangle$, to the scattering rigidity r.

$$r^2 = \frac{K^2 t^3}{(573)^2} \; \frac{\pi}{2\langle s \rangle} \tag{4}$$

Here one must choose the appropriate scattering "constant" $K.^2$ It is easily shown that :

$$\langle s \rangle = \frac{8}{3} \langle \psi_k^2 \rangle = 8\langle \chi_k^2 \rangle \tag{5}$$

Hence any difference product average is a linear combination of $\langle s \rangle$ and $\langle \delta^2 \rangle$. Solving between two difference product averages we have

$$\begin{aligned} \langle s \rangle &= A(\langle D_k^r \, D_{k+n}^s \rangle + B\langle D_k^t \, D_{k+m}^u \rangle) \\ \langle \delta^2 \rangle &= C(\langle D_k^r \, D_{k+n}^s \rangle + D\langle D_k^t \, D_{k+m}^u \rangle) \end{aligned} \tag{6}$$

Where A, B,C, and D are functions of r, s,t,u,m and n.

CALCULATION OF ERROR

As $\frac{1}{r^2} \propto \langle s \rangle$ we have $\frac{\sigma(r)}{r} = \frac{1}{2} \frac{\sigma(\langle s \rangle)}{\langle s \rangle}$. Using the variables defined above it is possible to define an independent contribution to s from each cell. The calculation is tedious but straightforward. The final result can be put in the form,

$$\frac{\sigma(r)}{r} = \frac{1}{(n)^{1/2}} (a+b\,\lambda+c\lambda^2)^{1/2} \tag{7}$$

where $\lambda = \frac{\langle\delta^2\rangle}{\langle s\rangle}$ and a, b and c depend on the choice of difference product averages used to obtain $\langle s\rangle$ and $\langle\delta^2\rangle$. Figure 2 shows this error as a function of λ for two different combinations of difference products.

We have calculated this error for all possible combinations of order ≤ 3 and found the combinations which yield the smallest error.

OVERLAPPING CELLS

In order to use a cell length longer than the measurement cell length one can ignore the intermediate points or form differences of the form below for each cell.

$$D^2_{k,n} = Y_{k+2n} - 2Y_{k+n} + Y_k \tag{8}$$

It can be shown that these differences are related in the same manner to the signal and noise as the differences for unit cell length. The two equations;

$$D^2_{k,n} = \sum_{\ell=0}^{n-2} (\ell-1)\, D^2_{k+\ell} + \sum_{\ell=n-1}^{2n-2} (2n-\ell-1)\, D^2_{k+\ell} \tag{9}$$

$$D^r_{k,n} = D^{r-1}_{k+nr,n} - D^{r-1}_{k,n} \tag{10}$$

can be used to relate the signal to the variables χ, ψ and δ.

The error calculation proceeds exactly as earlier.

Figure 3 shows fractional error as a function of number of overlaps for several initial noise to signal ratios.

ELIMINATION OF SPURIOUS SCATTERING

The assumption that δ is an independent variable is violated by the presence of spurious scattering which is correlated in some manner to cell length.

In a region of cell lengths where $\langle s \rangle \propto t^3$ the spurious scattering contribution is small and the signal is increasing as required by our assumptions.

This suggests the following method for determing scattering rigidity.

1. Measure ordinates of the track at a short cell length.

2. Calculate $\langle s \rangle$ at several multiples of the measurement cell length.

3. When in the region where $\langle s \rangle \propto t^3$, calculate rigidity and error.

4. Test other cell lengths in this region for smaller error.

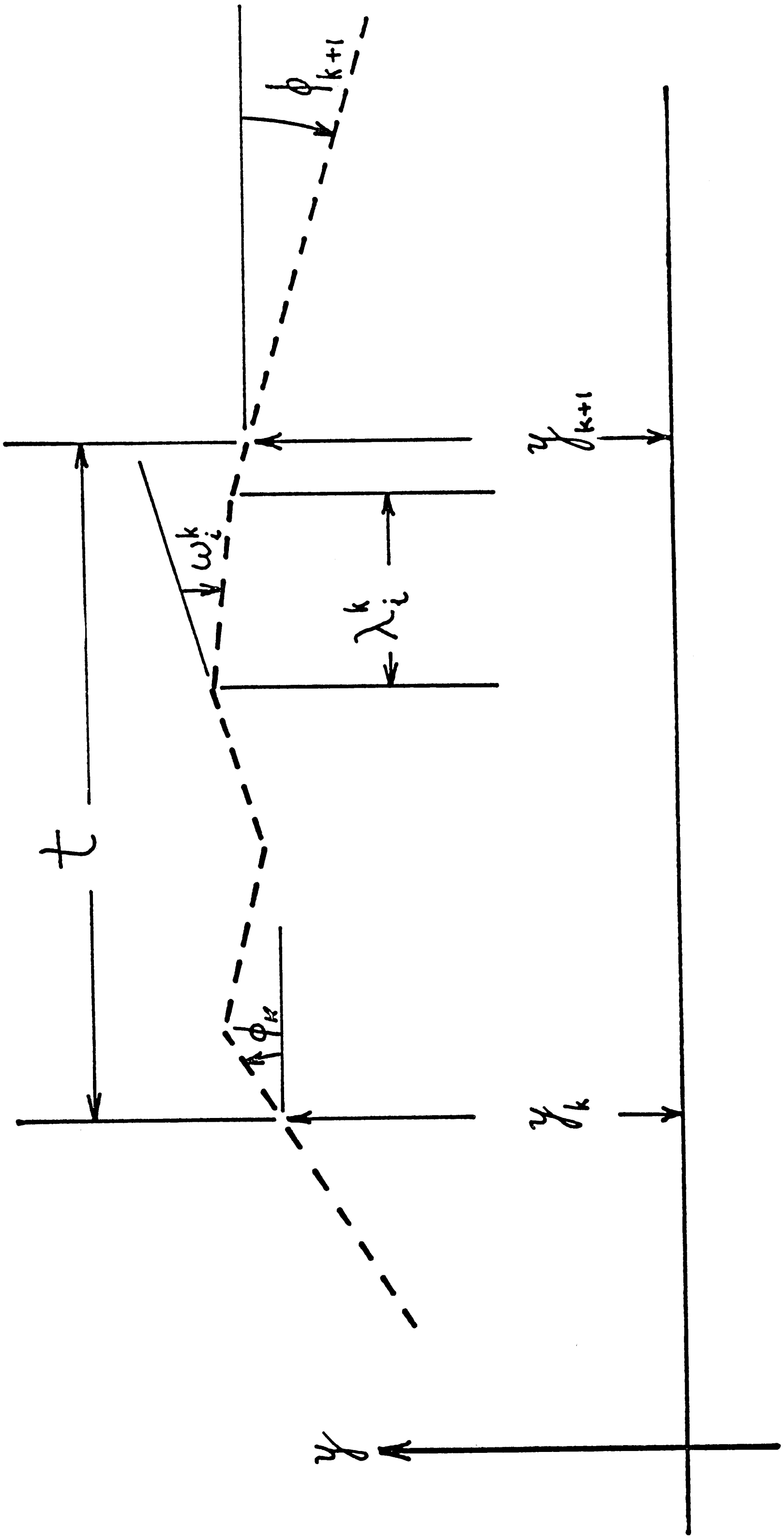
We have applied this method to tracks of known momenta and found good agreement up to several BeV/c.

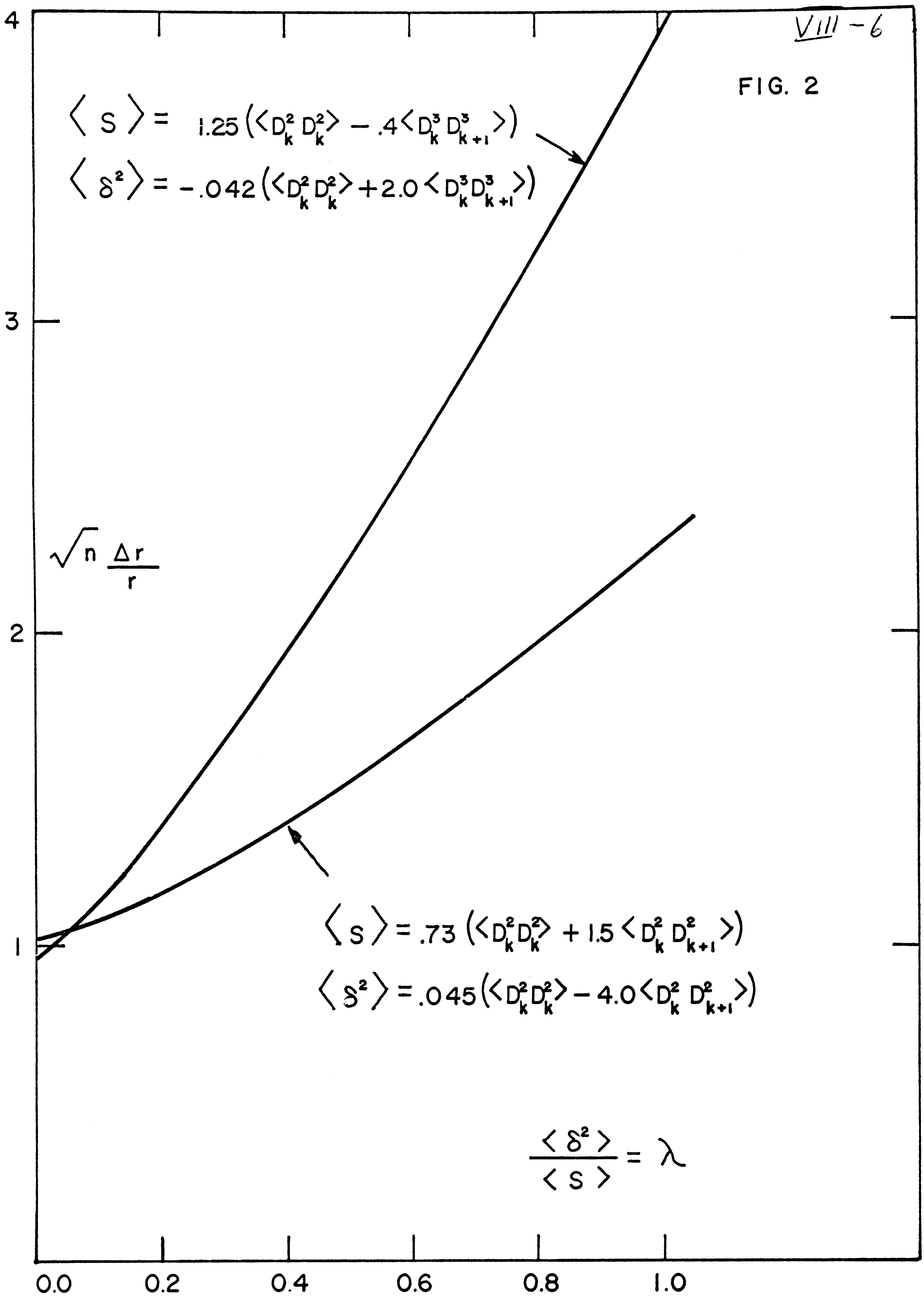
REFERENCE

1. W. H. Barkas. Nuclear Research Emuslion, Chapter 8, Academic Press, New York, 1963.

2. W. H. Barkas, op cit, page 299-301.

FIG. 1





⟨ S ⟩ = 1.25 (⟨D_k^2 D_k^2⟩ − .4 ⟨D_k^3 D_{k+1}^3⟩)
⟨ δ^2 ⟩ = −.042 (⟨D_k^2 D_k^2⟩ + 2.0 ⟨D_k^3 D_{k+1}^3⟩)
⟨ S ⟩ = .73 (⟨D_k^2 D_k^2⟩ + 1.5 ⟨D_k^2 D_{k+1}^2⟩)
⟨ δ^2 ⟩ = .045 (⟨D_k^2 D_k^2⟩ − 4.0 ⟨D_k^2 D_{k+1}^2⟩)
√n Δr/r
⟨ δ^2 ⟩ / ⟨ S ⟩ = λ
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0

FIG. 2

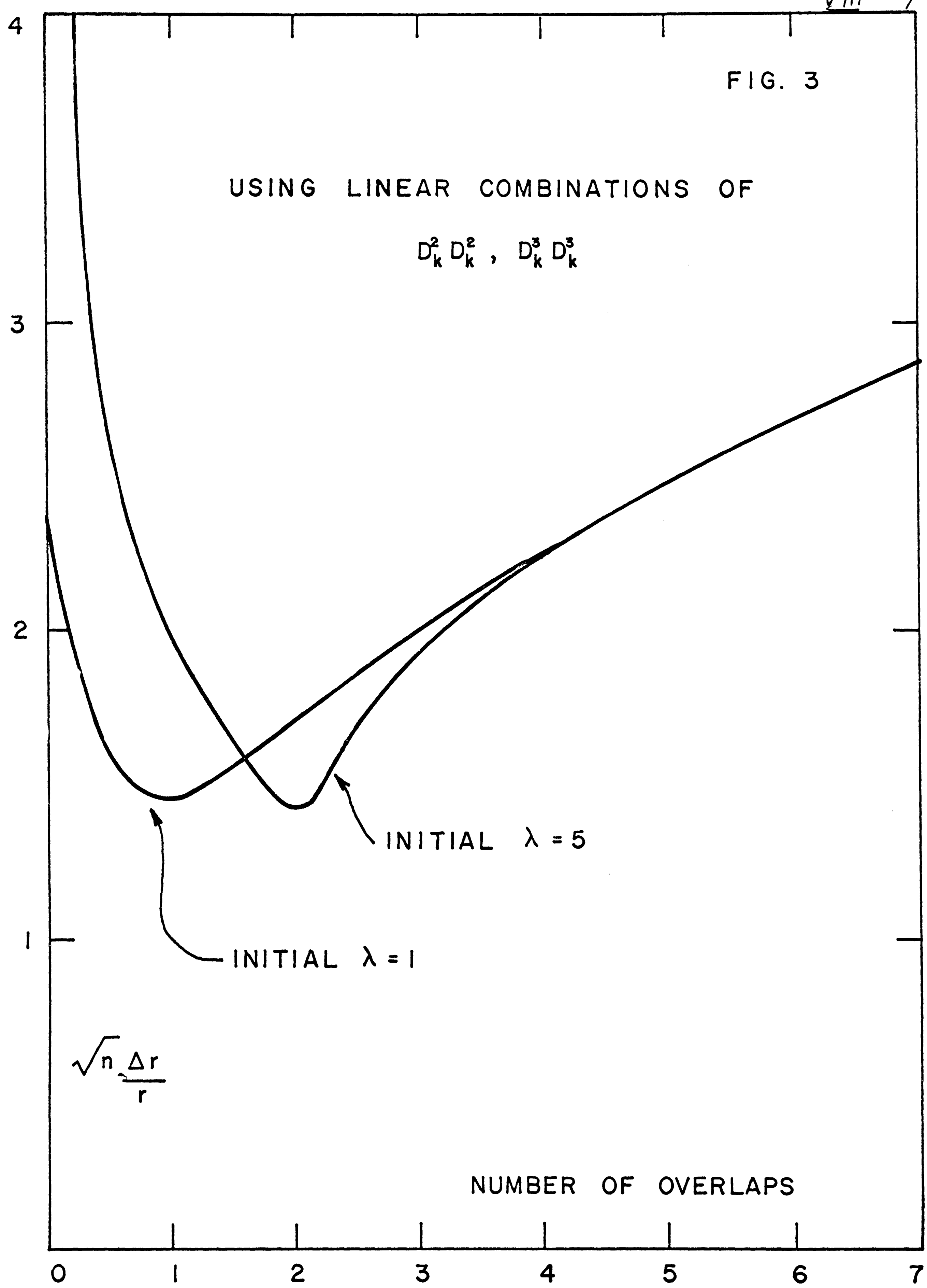


FIG. 3
USING LINEAR COMBINATIONS OF
$D_k^2 D_k^2$, $D_k^3 D_k^3$
INITIAL λ = 5
INITIAL λ = 1
$\sqrt{n}\,\frac{\Delta r}{r}$
NUMBER OF OVERLAPS
4
3
2
1
0
1
2
3
4
5
6
7