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ON THE MEASUREMENT OF COULOMB SCATTERING IN THE PRESENCE OF TRACK DISTORTION AND SPURIOUS SCATTERING

by

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I. The present work is another attempt to obtain an estimate for the Coulomb signal on tracks scattered in nuclear emulsions in the presence of spurious scattering (ss) and/ or C-shaped track curvature due to emulsion distortion and/ or to magnetic deflection.

It has been recognised that the-most obvious-direct elimination procedure, viz. subtraction of the spurious signal measured on calibration tracks, is cumbersome and inefficient. Previous attempts [1-4] to gain information as to the spurious signal from the measured track itself have made use of either the cell-length dependence of second differences or of the ratio of third to second differences, with various assumptions as to the parameters describing s.s. These assumptions are open to criticism on the following grounds :

a) The absolute mean second differences D are subject to track curvature; Fowler's simple correction uses too little information (only the first and the last cell!).

b) In the frequently used ratio ρ of third to second differences, only the numerator is free from curvature.

c) Moreover the corresponding ratio ρ_{ss} due to s.s. only is known to be strongly dip-dependent.

We suggest the following way of handling scattering data: readings are taken on a basic cell-length t on which the Coulomb and the s.s. contributions are of comparable amplitude. For this and the double cell-length, r.m.s. deviations $s(t)$ and $s(2t)$ of the mean second differences x_i, as well as absolute mean third differences $T(t)$ and $T(2t)$ are estimated. Assuming that ρ_{ss} does not depend on cell-length (as shown in sec. IV below) an estimate for the true Coulomb signal is given by eq. (19). Its proof is given in sec. II and the experimental checks in sec. III.

II. Provided an adequate cut-off is applied, the x_i are normally distributed with zero mean and variance

$$\sigma \sim (p\beta)^{-1} \quad ; \tag{1}$$

in the presence of C-shaped curvature, the mean is shifted to a value

$$A \simeq \frac{t^2}{\mathcal{R}} \tag{2}$$

where $\mathcal{R}$ is the radius of curvature; now provided the x_i are statistically independent, the quantity

$$s^2 \equiv \frac{1}{n-1} \sum (x_i - a)^2 \tag{3}$$

where

$$a \equiv \frac{1}{n} \sum x_i \tag{4}$$

and n is the number of second differences, is a sufficient estimate for σ, independent of A (hence of curvature). The quantity ns^2/σ^2 is distributed like χ^2 with n-1 degrees of freedom; this gives the confidence limits.

We proceed now to show that, to all practical purposes, the x_i <u>are</u> statistically independent. Indeed, let R be the correlation coefficient of x-values from adjacent cells. Since

$$x(2t) = x_i(t) + 2x_{i+1}(t) + x_{i+2}(t) \tag{5}$$

and

$$\langle x_i x_{i+2} \rangle = 0 \tag{6}$$

it is easily shown that

$$\mu^2 \equiv \frac{\sigma^2(2t)}{\sigma^2(t)} = 6 + 8R \tag{7}$$

For pure Coulomb scattering

$$\mu = 2^{3/2} \tag{8}$$

whence

$$R_c = +1/4 \tag{9}$$

and, as is easily seen,

$$\rho_c = \sqrt{3/2} \tag{10}$$

Now, let σ_c, σ_ν and σ_{ss} be respectively the components of σ due to Coulomb scattering, noise and s.s. With

$$q_c \equiv \frac{\sigma_c}{\sigma} \quad \text{, etc.} \tag{11}$$

we have

$$R = q_c^2 R_c + q_\nu^2 R_\nu + q_{ss}^2 R_{ss} \tag{12}$$

Since

$$\rho_\nu = \sqrt{10/3} \tag{13}$$

we have, from eq. (7),

$$R_\nu = -\frac{2}{3} \tag{14}$$

hence, noise weakens the correlation.

Usually $q_\nu^2 \ll 1$ and hence the correlation is determined essentially by q_c^2/q_{ss}^2. Again from eq. (7) we have

$$\rho_{ss}^2 = 2(1 - R_{ss})\,; \tag{15}$$

if, as usual, the spurious signal is assumed to obey a power law in t, say

$$D_{ss} \sim t^{-\delta} \tag{16}$$

we have

$$2^{2\delta} = 14 - 4\rho_{ss}^2 \,. \tag{17}$$

Eq. (17) shows that:

a) The information content of any corrected signal obtained from D-estimates on multiple cells is essentially the same as that of the corrected signal obtained from higher order differences.

b) For the most frequently quoted values, $\delta \simeq 1$ or, equivalently, $\rho_{ss} \simeq 1{,}6$

$$R_{ss} \simeq -1/4 \qquad (18)$$

which, used in eq. (12), leads practically to statistical independence of adjacent cells, provided $\rho_c^2 \sim \rho_{ss}^2$.

In a more detailed forthcoming paper based on a world survey of scattering data and on our own results, it will be shown that ρ_{ss} is neither dip-independent nor does it assume a universal value ; however, no significant cell-length dependence could be detected. With the last assumption it is easily shown that

$$D_c^* = \sqrt{\frac{2}{\pi}}\, s(2t) \left\{ \frac{\rho''^2(t) - \rho''^2(2t)}{8{,}56\left[\rho''^2(t) - \rho_c^2\right] - \frac{s^2(2t)}{s^2(t)}\left[\rho''^2(2t) - \rho_c^2\right]} \right\} \qquad (19)$$

where D_c^* is the estimate for the true Coulomb sagitta and $\rho'' = \sqrt{\frac{\pi}{2}}\frac{I}{s}$.

III. $\sim$ 5000 readings were taken on 7m of track in Ilford G-5 emulsions exposed to 17 GeV π^- and NIKFI-R emulsion exposed to 7 GeV π^- and 10GeV p (the latter in a 12 kgauss magnetic field). We shall quote here briefly only the main results.

a) The r.m.s. estimates s for σ were found free from curvature and obey χ^2 -distributions, as expected from the (checked) normal distribution of the second differences and their practical statistical independence.

b) Use of eq. (19) yields consistent estimates for the Coulomb signal in the presence of s.s. increasing $\sim t^1$ and magnetic track curvature (see fig. 1a and b for the distribution of estimates D_c^* from 51 tracks at 7 GeV, and 55 tracks at 10 GeV, respectively).

c) Assumption of a "universal" ρ_{ss} -value [4] yields also consistent estimates on the flat tracks measured here although their spread is somewhat wider as that of D_c^* [eq. (19)] and imaginary values occur too. In view of its independence on dip, the estimate given by eq. (19) appears more reliable.

d) A statistical analysis of the sequence of positive and negative sagittae based on the test of runs has yielded quantitative evidence in favour of Aditya's [5] interpretation of s.s. as a superposition of local macroscopic C-shaped distortions. Moreover, a spurious component has been detected among the large single scatters rejected by the usual cut-off criteria. This implies a possibly important contribution of such sharp breaks to the accepted sagittae too, and/or to shadow scattering.

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FIGURE CAPTION

Fig. 1 Distribution of D_c^* on a) 51 7 GeV π^- tracks and b) 55 10 GeV proton tracks. Lower arrows : expected Coulomb sagitta; upper arrows : sampling mean of D_c^*.

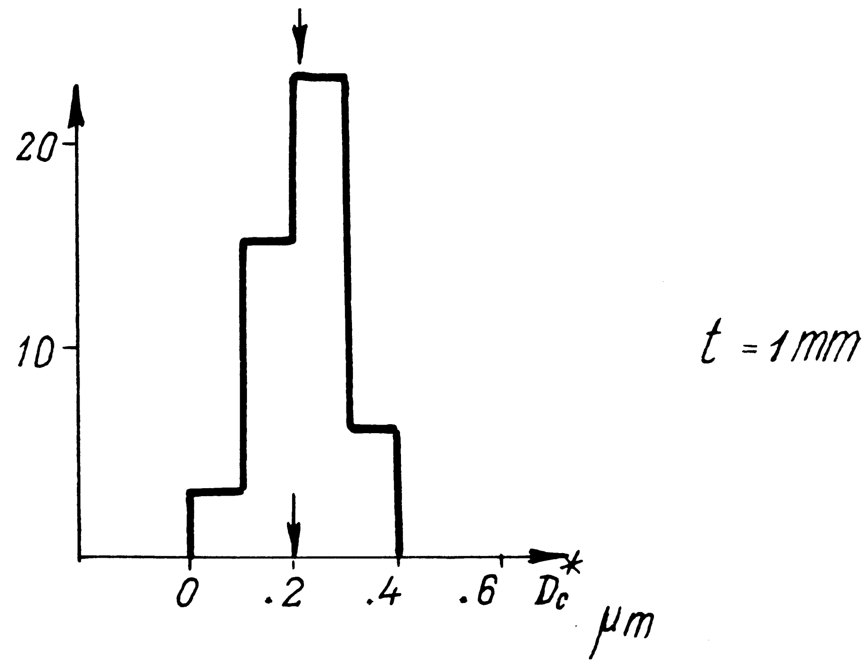

Fig. 1a.

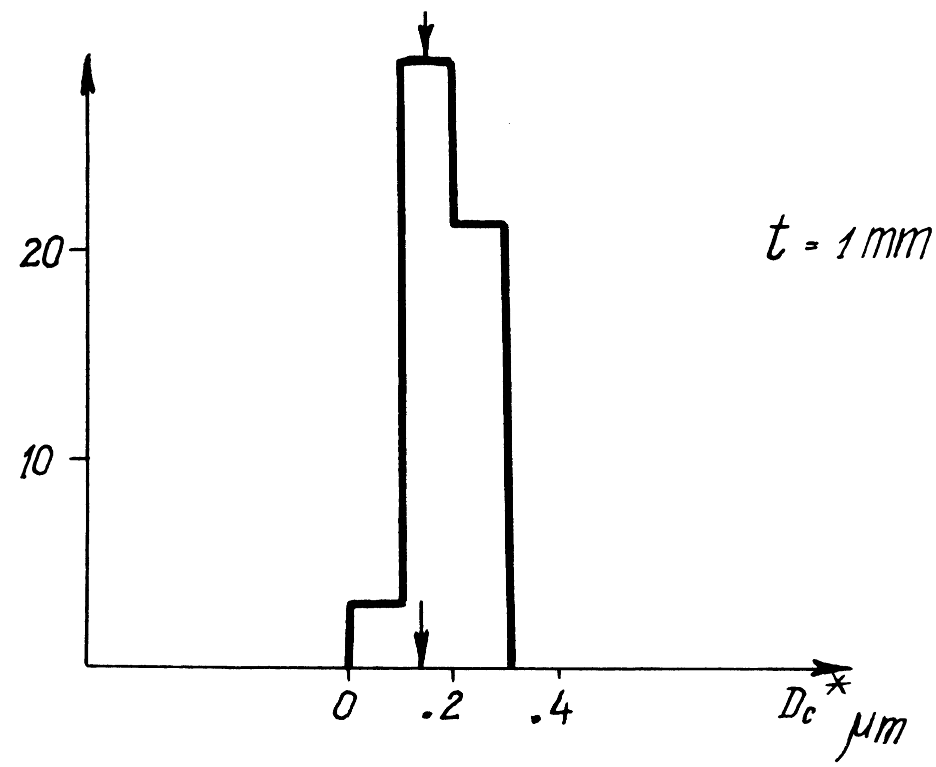

Fig. 1 b.