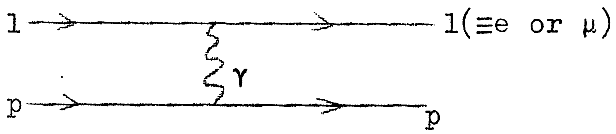
1st and 2nd LECTURES
5th and 15th February 1963

THEORETICAL INTRODUCTION

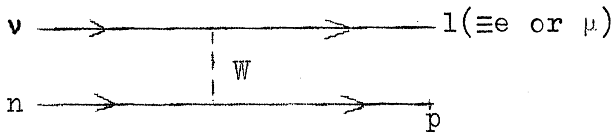
by J. S. Bell

1. The intermediate vector boson.

The idea of current has come to play a large part in weak interaction theory. In particular the idea of the conserved vector current (CVC) has been very successful. In analogy with the interactions of electric currents, it is natural then to suppose that weak interactions are mediated by a vector particle. The process of elastic lepton scattering by a proton, for example,



has for its analogue



representing inverse β decay or μ capture. From the diagram it is clear that W is supposed to be charged; then there are at least two W's, positive and negative. One could try to make theories involving neutral vector particles only [15], but the CVC does not fit naturally into them. Because weak interactions are short ranged, the intermediate vector boson W must be massive. Little is known about the value of the mass. However, the particle mediating K-meson decays must be heavier than the K meson itself, or a fast (semi weak) decay $K \rightarrow W + \gamma$ would be possible. There are some slight quantitative puzzles in μ decay whose

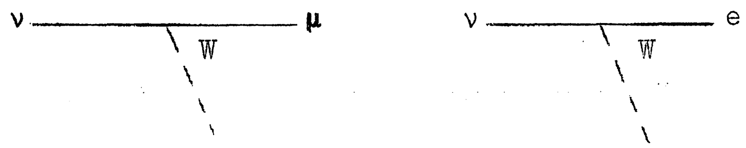
6397/p

resolution would be helped by a W mass not much bigger than that of the Kaon.[1]

If the intermediate boson exists then it should be copiously (i.e. semi-weakly) produced by neutrinos of sufficient energy. We leave the discussion of this for later lectures. Nor do we go here into the complications of the intermediate boson idea required in connection with regularities in the decays of strange particles; but it should be noted that the testing of those regularities at high momentum transfer will eventually be possible and important.[2] Nor will we consider the possibility that interactions involving more than one virtual W (and therefore of higher order in the weak coupling) might be observed. For this, and for general references on weak interactions and neutrino physics, we refer to the excellent reviews of Nilsson and Pais.[3]

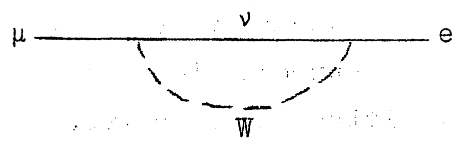
2. <u>How many neutrinos?</u>

If the vertices

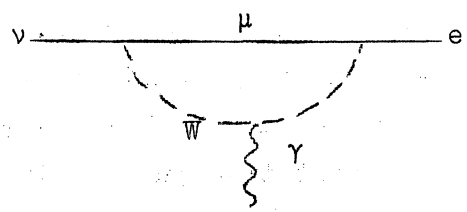


(2.1)

both exist, then so also does the Feynmangraph



This does not conserve energy-momentum, but can be made to do so by including a photon.



This diagram is divergent; in a cut-off calculation the theoretical rate for the process $\mu \to e + \gamma$ comes out as a function of $(\Lambda/M_W)^2$ where Λ is the cut-off momentum. The process has been looked for and not found; the branching ratio to this decay mode of the muon [4] appears to be less than 10^{-7}. This would require $\Lambda \ll M_W$, which seems very unreasonable. The solution of this dilemma is now well known; the neutrinos coupled to the electron and muon in the diagrams (2.1) must be different, ν_e and ν_μ. This is supported by the Brookhaven neutrino experiment in which ν_μ's from pion decay were found to yield muons and not electrons in reactions with nuclei.

It has been suggested [5] that the pairing of neutrinos with charged leptons might be inverted when there is a change of strangeness of the heavy particles involved, 'neutrino flip'. Thus

$$\pi \to \mu + \nu_\mu \quad \text{but} \quad K \to \mu + \nu_e$$

$$e + \nu_e \qquad\qquad e + \nu_\mu$$

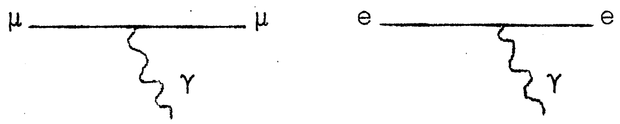
The neutrino beam at Brookhaven contains a substantial contribution from K decay, and so the absence of secondary electrons is evidence against this proposal.

Von Dardel and Ghani [6] suggest that the neutrinos (ν'_e, ν'_μ) associated with strangeness change might be quite different from those associated with no change of strangeness (ν_e, ν_μ). The neutrinos from K decay incident on nuclei would then inevitably produce strange particles in reactions.

It must be said that whereas there was a clear need for the two neutrinos, ν_e ν_μ, there is at present no need whatever for either 'neutrino flip', or 'strange neutrinos'. Moreover either would complicate any intermediate boson scheme; the information about which neutrino is required would have to be transmitted through the boson.

3. <u>Muon and electron conservation</u>.

If the vertices (2.1) above, together with the electromagnetic ones



constitute the only interactions of the leptons, then it is clear that we have two conserved quantities

$$\text{muon number } N_\mu = \begin{pmatrix} \text{number of } \mu^- + \text{number of } \nu_\mu \\ -\text{number of } \mu^+ - \text{number of } \bar{\nu}_\mu \end{pmatrix}$$

$$= \text{muonic number}$$

$$\text{electron number } N_e = \begin{pmatrix} \text{number of } e^- + \text{number of } \nu_e \\ -\text{number of } e^+ - \text{number of } \bar{\nu}_e \end{pmatrix}$$

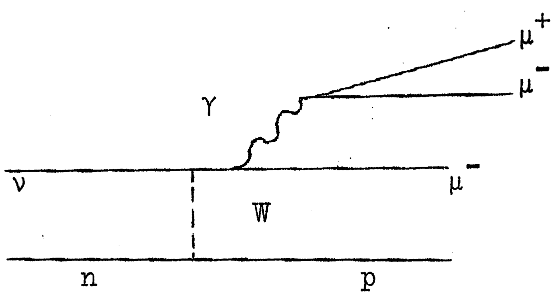
$$= \text{electronic number.}$$

The quantities $N_\mu \pm N_e$ are also conserved, and either may be called 'lepton' number.

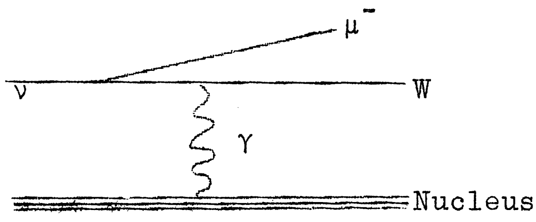
The absence of double beta decay, in which one nucleus transforms into another with emission of two electrons or positrons, is evidence for electron conservation. Electron conservation is also sufficient to forbid unobserved decays such as $\mu \to e + \gamma$. Therefore one might look particularly for reactions forbidden only by muon conservation, such as

$$\nu_\mu + \text{nucleus} \to \mu^+ + \text{heavy particles.}$$

However it should be noted that such events might be simulated by others if additional final leptons are not identified. For example



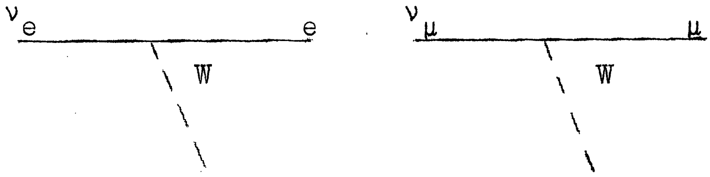
This would be expected to be small because of the additional electromagnetic coupling. More important is



with subsequent decay of W into $(\mu^+ + \nu)$. Indeed the observation of fast leptons of the wrong kind in high energy neutrino experiments would most plausibly be attributed to this mechanism, even if the other leptons could not be reliably identified. One would try to confirm the mechanism by observing the characteristic rapid rise of cross section with energy.

4. <u>μ-e universality</u>.

It is tempting to believe that the vertices



are of identical strength. Then we have μ-e universality; the pairs (ν_e, e) and (ν_μ, μ) are interchangeable. There is evidence for this in pion decay and to some extent in the comparison of β decay with μ capture [7]. To test it at high momentum transfer would require a source of high energy ν_e's, which means a beam of high energy particles which decay into electrons and neutrinos with appreciable probability. The only plausible candidates as ν_e parents seem to be the W's, which should have among other decay modes

$$W^- \rightarrow \mu^- + \nu_\mu \qquad W^- \rightarrow e^- + \nu_e$$

$$W^+ \rightarrow \mu^+ + \bar{\nu}_\mu \qquad W^+ \rightarrow e^+ + \bar{\nu}_e$$

The rate for any of these would be expected to be very large (semi-weak). Dimensionally

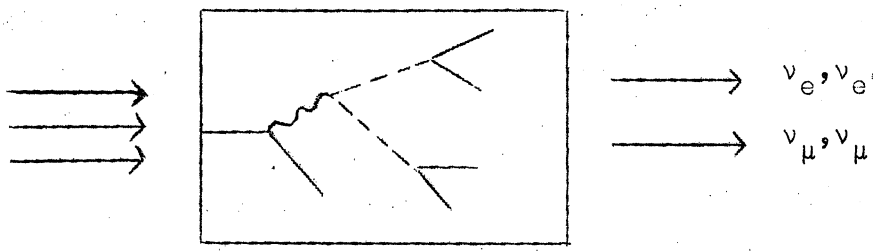
$$\Gamma \approx g^2 M_W$$

where g is the boson-lepton coupling constant. From the muon decay rate, for example,

$$g^2/M_W^2 \approx \frac{10^{-5}}{\sqrt{2}\, M_p^2}$$

whence $$\Gamma \approx 10^{-5} \frac{M_W}{M_p} \times 10^{24} \sec^{-1}$$

Since $M_W > M_k$, $\Gamma \gtrsim 10^{18} \sec^{-1}$ (a more careful calculation gives $\gtrsim 10^{17}$). The high energy W's could be produced electromagnetically; Schwarz has proposed that the beam of the future Stanford electron linac enter a block of material sufficient to contain all secondaries other than neutrinos. The high energy neutrinos emerging would come from the W's, since other particles yielding decay neutrinos would be slowed before decay. Reactions induced by these neutrinos should involve secondary μ, μ^+, e, and e^+, 's in essentially equal numbers if the μ, e universality is correct.



5. <u>Neutral currents</u>

If there are charged intermediate bosons, why not neutrals? We could then have an isobaric triplet W^+ W^o W^-. If such a neutral boson exists, and is coupled to leptons, it must not be coupled to strangeness changing currents. Otherwise it would give rise to unobserved decays such as

$$K \rightarrow \pi + (\mu^+ + \mu^-)$$
$$\pi + (e^+ + e^-)$$
$$\pi + (\nu + \bar{\nu})$$

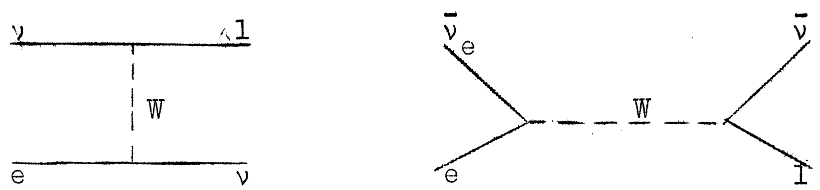
However there is no evidence either for or against the existence of W^o coupled to leptons and non-strange currents; decays in which strangeness and charge of the heavy particles are unaltered can go in any case with the intervention of virtual photons. The only practical way to test for W^o apart from producing them seems to be with fast neutrinos, looking for such reactions as

$$\nu + \text{nucleon at rest} \rightarrow \nu + \text{recoil nucleon}$$

From the Brookhaven experiment[8] it was concluded that such a cross section could not be large compared with the usual charge exchange process; but one would not have expected it to be so.

6. Electron as target.

The simplest reactions theoretically are those involving only leptons:



The second process has a threshold at lab. energy $[(m_l{}^2/2m_e)-\frac{1}{2}m_e]$; suppose we are well above this. For centre of mass energies small compared with the boson mass, (lab energy $\ll M_W{}^2/2m_e$) and with the conventional $(1 + \gamma_5)$ couplings [9].

$$\sigma(\nu + e \rightarrow l + \nu_e) \approx 3\sigma(\bar{\nu}_e + e \rightarrow \bar{\nu} + l) =$$
$$= \frac{1}{\pi} G^2 \text{ (centre of mass energy)}^2$$
$$\approx 10^{-41} \text{ (lab. energy in GeV) cm}^2$$

where $G = \sqrt{2}\, g^2/M_W^{\ 2} \approx 10^{-5}/M_p^{\ 2}$ from muon decay. Because of centre of mass motion these cross sections are down on those with nucleons by a factor of about 10^3 in the GeV range. The fast final lepton is confined to angles [$\approx$ (electron mass/lab energy)$^{\frac{1}{2}}$] of about a few degrees; within that cone the differential cross section is comparable with that for nucleon targets.

When the cm energy is not small compared with the boson mass the cross section for $(\nu + e \rightarrow 1 + \nu_e)$ acquires an additional factor

$$M_W^{\ 2}/(M_W^{\ 2} + E_{cm}^{\ 2})$$

which reduces it to a constant value at large energy. On the other hand the cross section for $(\bar{\nu}_e + e \rightarrow \bar{\nu} + 1)$ is <u>increased</u> by [9]

$$M_W^4/[(M_W^{\ 2} - E^2_{\ cm})^2 + M_W^{\ 2}\Gamma^2]$$

where Γ is the decay width of W. The average cross section, between say $E^2_{\ cm} \approx \frac{1}{2}M_W^2$ and $E^2_{\ cm} \approx 1{,}5 m_W^{\ 2}$ is increased very roughly by a factor

$$(M_W/\Gamma\ ) \approx g^{-2} \approx 10^5\ (M_p/M_W)^2$$

and is then of order

$$(10^5/3\pi)\ \ G^2 M_p^{\ 2} \approx 10^{-34}\mathrm{cm}^2.$$

However to obtain this large cross section needs very large neutrino energies

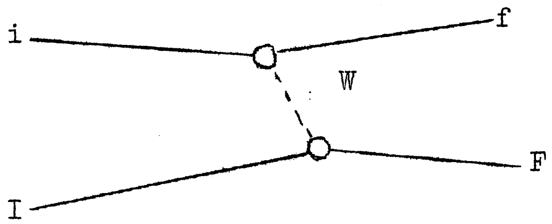
$$\text{lab. energy} \approx M_W^{\ 2}/2m_e \gtrsim M_K^{\ 2}/2m_e \gtrsim 250 \text{ GeV}$$

At present one would have to look for such energies in cosmic rays [9], but there most of the neutrinos are ν_μ (from pion decay) which should not annihilate with electrons.

7. Vector particle exchange

The hypothesis that weak interactions are mediated by a particle of spin one imposes definite restrictions on the energy dependence of cross sections [10]. These should be tested in future neutrino experiments.

Consider the collision of particles i, I with 4-momenta k, K, resulting in particles f, F with 4-momenta k', K'.



If the interaction Lagrangian involving the vector particle W is

$$g(W_\mu j_\mu + W_\mu J_\mu) + \text{herm conj.}$$

Then apart from trivial phase space factors the probability for the process is given by the square of the covariant matrix element

$$g^2 (f|\bar{j}_\mu|i) \left\{ \frac{\delta_{\mu\nu} - q_\mu q_\nu/M_W^2}{M_W^2 + q^2} \right\} (F|J_\nu|I)$$

$$= \frac{g^2}{M_W^2} (f|\bar{j}_\mu|i)(F|J'_\mu|I)$$

where $q = k' - k = K - K'$.

$$J_\mu' = (J_\mu + q_\mu q_\nu J_\nu/M_W^2)/(1 + q^2/M_W^2)$$

If we square this, average over initial and sum over final spin, the result is an invariant quantity.

$$P = m_{\mu\nu} \quad M_{\mu\nu}$$

$$m_{\mu\nu} = g^2 \sum_f \bar{\sum_i} (i|j_\mu|f)(f|\bar{j}_\nu|i)$$

$$M_{\mu\nu} = \sum_f \bar{\sum_i} (I|\bar{J}'_\mu|F)(F|J'_\nu|I)$$

where bar denotes complex conjugation in general with an additional change of sign when μ or ν is 4.

The tensor M can depend only on the momenta K and K'; the most general form is

$$A\delta_{\mu\nu} + B\, K_\mu K_\nu + C\, K'_\mu K'_\nu + D\, K_\mu K'_\nu + E\, K'_\mu K_\nu + F\varepsilon_{\mu\nu\rho\sigma} K_\rho K'_\sigma$$

where the quantities A - F are the functions of the scalars K^2 and K'^2 (the negatives of the masses of particles I and F) and $q^2 = (K'-K)^2$. The tensor m must be a similar expression involving k and k'. When m and M are multiplied together the resulting P is a bilinear combination of the new invariants

$$K_\mu k_\mu, \; K'_\mu k_\mu, \; K_\mu k'_\mu, \; K'_\mu k'_\mu$$

with coefficients which are functions of k^2, K^2, k'^2, K'^2, q^2. Now each of the new invariants can be expressed as a linear combination of the old invariants and the (centre of mass energy)^2 s:

$$s = -(k + K)^2 = -(k' + K')^2$$

For example

$$\begin{aligned} k\cdot K' &= k\cdot K + k\cdot(K' - K) \\ &= k\cdot K + k\cdot(k - k') \\ &= k\cdot K - k\cdot k' + k^2 \\ &= \tfrac{1}{2}(k + K)^2 - \tfrac{1}{2}k^2 - \tfrac{1}{2}K^2 \\ &\quad + \tfrac{1}{2}(k - k')^2 - \tfrac{1}{2}k^2 - \tfrac{1}{2}k'^2 + k^2 \\ &= -\tfrac{1}{2}s + \tfrac{1}{2}q^2 - \tfrac{1}{2}K^2 - \tfrac{1}{2}k'^2 \end{aligned}$$

Clearly the highest power of s that can occur is the second, and so we arrive at the general form

$$P = A' + B's + C's^2$$

where the coefficients are functions of the masses

$$-k^2, \; -k'^2, \; -K^2, \; -K'^2$$

and the invariant momentum transfer q^2. So for given masses and momentum transfer q^2, P varies at most quadratically with the square of the centre of mass energy.

Another form is for some purposes more convenient. Defining s, t, and u by

$$\begin{aligned} s &= -(k + K)^2 = -(k' + K')^2 \\ t &= -q^2 = -(k - k')^2 = -(K - K')^2 \\ u &= -(k' - K)^2 = -(K' - k)^2 \end{aligned}$$

we recall that

$$s + t + u = m_i^2 + m_f^2 + M_I^2 + M_F^2$$

whence

$$2s = (s - u) - t + m_i^2 + m_f^2 + M_I^2 + M_F^2$$

The result can then be rewritten

$$P = A + B(s - u) + C(s - u)^2 \qquad (7.1)$$

where the coefficients are combinations of the old coefficients, and again depend only on q^2 and masses.

Consider now the 'elastic' neutrino reactions

$$\nu + n \rightarrow l + p \tag{7.2}$$

$$\bar{\nu} + p \rightarrow \bar{l} + n \tag{7.3}$$

One can show that the <u>same</u> quadratic in $(s - u)$, apart from the sign of the linear term, governs these processes. We ignore here the n-p mass difference but do not need any other special symmetries. Assuming the interactions to be hermitian, so also (in the one W-exchange approximation) is the matrix element. Then the conjugate reactions

$$\nu + n \rightarrow l + p \tag{7.4}$$

$$l + p \rightarrow \nu + n \tag{7.5}$$

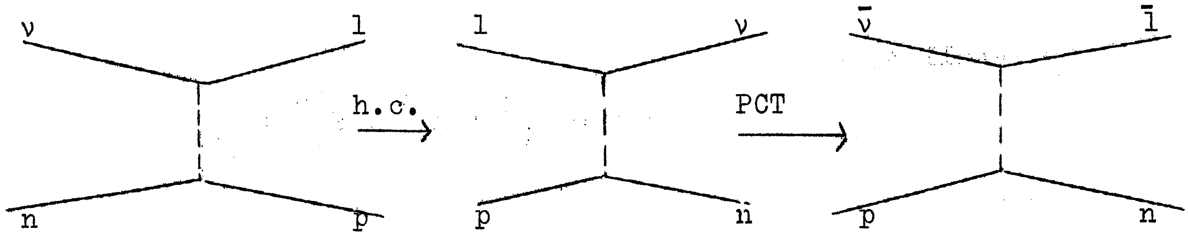
have the same squared matrix elements. In going from (7.5) to

$$\bar{\nu} + p \rightarrow \bar{l} + n \tag{7.6}$$

we have only to replace

$$(\nu|\bar{j}_\mu|l) \quad \text{by} \quad (\bar{l}|\bar{j}_\mu|\bar{\nu})$$

But these are equal by PCT symmetry (with suitable spin adjustments that do not matter in the sum over spins).



Thus (7.6) is governed by the same matrix element as (7.5) and (7.4). But in this comparison the label K remains attached to the neutron and K' to the proton.

Returning to our original convention that K refers to the initial nucleon and K' to the final involves the replacement in $M_{\mu\nu}$ of K by -K' and K' by -K. The change of sign does not effect $M_{\mu\nu}$ (which is bilinear) and is made to preserve the relation $k + K = k' + K'$. In the final expression P, (s - u) simply changes sign

$$\begin{aligned} s - u &= -(k + K)^2 + (k - K')^2 \\ &= - 2k\,(K' + K) \end{aligned}$$

So (7.1) and (7.2) are determined respectively by

$$P_{\nu,\bar{\nu}} = A \pm B(s-u) + C(s-u)^2 \quad (7.7)$$

Because P is a proper scalar the pseudotensors ($\epsilon_{\mu\nu\rho\sigma}$ terms) in $m_{\mu\nu}$ and $M_{\mu\nu}$ can contribute only in combination with one another. Because they are antisymmetric respectively in (k,k') and (K,K'), they cannot contribute to the $(s-u)^2$ term. With certain assumptions they can be shown to contribute only to the (s-u) term and to be entirely responsible for it. The existence of such a linear term then implies mixed parity in both currents. Suppose for example that the heavy particle current has definite G-parity, i.e. is unaltered apart from a sign by the operation $n \to \bar{p}$, $p \to -\bar{n}$. Then (with appropriate spins and signs)

$$\begin{aligned} (pK'|J|nK) &= (\bar{n}K|J|\bar{p}K') \quad \text{by PCT} \\ &= (pK|J|nK') \quad \text{by G} \end{aligned}$$

It would follow that $M_{\mu\nu}$ is symmetric under the exchange $K \rightleftharpoons K'$, or, since it is quadratic, under $K \to -K'$, $K' \to -K$. This would mean that P is symmetric under $s \rightleftharpoons u$ together with $K^2 \rightleftharpoons K'^2$. Ignoring the nucleon mass difference the coefficient of (s-u) would have to vanish. However it is customary to assume opposite G-parities for the axial and vector currents; the (s-u) term can then arise from, and only from, vector-axial interference. Even without G-symmetry for nucleon currents, this same result is obtained if the lepton mass-difference is ignored and an explicit form

$$i\bar{\Psi}_1(a\gamma_\mu + b\gamma_\mu\gamma_5)\Psi_\nu W_\mu$$

assumed for the W-lepton vertex. The two parts of this interaction have a leptonic analogue of G-symmetry (with $\nu \to \bar{l}$, $l \to -\bar{\nu}$) and then a parallel argument goes through, for the exchange $k \to -k'$, $k' \to -k$ again changes the sign of (s-u).

Similar considerations can be made for electron and positron scattering. In the case of elastic coulomb scattering, in the one photon exchange approximation, the term linear in (s-u) can be ruled out in several ways; for example it would change sign between l and $\bar{l}$ scattering and so conflict with charge conjugacy in the leptonic vertex. Ignoring the lepton mass, the expression

$$A(q^2) + B(q^2)\ (s-u)^2 \tag{7.8}$$

can then be cast into the familiar form

$$A'(q^2) + B'(q^2)\ \cot^2 \theta/2 \tag{7.9}$$

because

$$s - u = 2M_I(l + l')$$

(where l and l' are lab. lepton energies)

$$= 2M_I \left\{ \left(\frac{q^2}{2M}\right)^2 + q^2\left(1 + \cot^2 \frac{\theta}{2}\right)\right\}^{\frac{1}{2}}$$

The simpler expression, 7.8 or 7.9, applies also to the <u>sum</u> of the differential cross sections for $\nu + n \to l + p$ and $\bar{\nu} + p \to \bar{l} + n$. Measurement of both differential sections will clearly facilitate the determination of the structure factors although one of them is in principle sufficient.

8. <u>Form factors in $\nu + n \rightarrow l + p$ and $\bar{\nu} + p \rightarrow \bar{l} + n$</u>.

Assume now the usual form for the lepton vertex

$$\bar{l}\gamma_\mu (1 + \gamma_5)\nu$$

The matrix element of the heavy particle current is restricted by Lorentz invariance to the form

$$(K'|J|K) = \bar{P} \left\{ \begin{array}{l} f \quad i\gamma_\alpha - f_2\sigma_{\alpha\beta} iq_\beta - h_V\, q_\alpha \\ +f_A \quad i\gamma_\alpha\gamma_5 + f_P q_\alpha\, \gamma_5 + h_A(K_\alpha + K'_\alpha)\gamma_5 \end{array} \right\} n$$

where n and p are initial and final state spinors for neutron and proton, and the f's are functions of q^2, where $q = (K - K')$. In general there are six complex form factors f, but only three real structure factors A, B, C. It is clear therefore that in experiments where polarizations are not used all of the form factors cannot be determined. Various theoretical restrictions might be imposed to reduce the number of unknowns. Time reversibility would imply the reality of the f's. G-symmetry would imply $h_V = h_A = 0$. The CVC hypothesis (with μ-e universality) makes $h_V = 0$ and f_1 and f_2 identical with the isovector electromagnetic form factors.

If time reversibility and G-symmetry are assumed, and a very large boson mass, the structure factors are

$$\begin{aligned} A = {} & 4m^2(f_A^{\,2} - f_1^{\,2}) + m^4\{ 4\, f_A\, f_p - (f_1 + 2f_2)^2 - f_A^{\,2} \} \\ & +q^2 \{4f_A^{\,2} - 4f_1^{\,2} - 4m\, f_A\, f_p + 4m^2 f_1 f_2 + m^4 f_P^{\,2} \} \\ & +q^4 \{f_1^{\,2} + 4f_2^{\,2} + 8f_1 f_2 + f_A^{\,2} + m^2(f_P^{\,2} + f_2^{\,2}) \} \\ & -q^6 f_2^{\,2} \end{aligned}$$

$$B = 4q^2(f_1 + 2f_2)\, f_A$$

$$C = f_1^{\,2} + f_A^{\,2} + q^2 f_2^{\,2}$$

The proton mass has been taken as unity, and m is the lepton mass. In units of (proton compton wavelength/$2\pi)^2$ the differential cross section is

$$\frac{d\sigma}{dq^2} = \frac{G^2}{32\pi} \frac{1}{|\underline{k}|^2} \{A \pm B(s-u) + C(s-u)^2\}$$

with the upper sign for $\nu + n \rightarrow p + l$ and the lower for $\bar{\nu} + p \rightarrow n + \bar{l}$; $\underline{k}$ is the laboratory incident momentum. Finally, to allow for finite boson mass, J has to be replaced by J' and therefore the f's by f''s:

$$\left\{ f_1', f_2', f_A', h_A' \right\} = \left(1 + \frac{q^2}{M_W^2}\right)^{-1} \left\{ f_1, f_2, f_A, h_A \right\}$$

$$f_P' = f_P + \left(1 + \frac{q^2}{M_W^2}\right)^{-1} 2f_A \qquad h_V' = h_V$$

If time reversibility, G-symmetry, and CVC are all assumed, determination of B and C gives f_A and the boson mass M_W; A then gives f_P.

If the form factors f' fall off fast enough with increasing q^2, it is clear that the C term dominates the total cross sections at high energy. Since then $(s-u) \propto |\underline{k}|$, the two cross sections become constant and equal. Several authors [12)] have estimated this asymptotic value to be $\approx 7.10^{-39}$ cm^2. They used all the possible theoretical restrictions mentioned above, and in addition arbitrarily supposed f_A to be similar to the proton electromagnetic form factors and $M_W \approx \infty$. The estimate is uncertain for these reasons. Only in the forward direction ($q^2 \approx 0$) can confident predictions be made, using values of $f_1(0)$ and $f_A(0)$ from beta decay and muon capture.

The above discussion applies immediately only to nucleon targets. Current experiments are made with nuclei, and their interpretation may be complicated by final state interactions and other nuclear effects. In particular one has to consider the Pauli principle [13], which inhibits reactions with small momentum transfer, and possibly the Fermi motion.

9. Inelastic reactions. [14]

The most likely interactions, other than those in which W's are produced, should be those in which pions appear:

$$(\nu \text{ or } \bar{\nu}) + (n \text{ or } p) \rightarrow (n \text{ or } p) + \pi\text{'s}$$

It is not possible to make definite predictions, but present guesses are that the total cross section for such processes will be comparable with the elastic cross section in the GeV region and increase rather slowly with energy. Reactions in which strange particles appear are expected to be more uncommon, as with pion beams. In particular such simple reactions as

$$\nu + n \rightarrow l + \Sigma^+$$
$$\bar{\nu} + n \rightarrow \bar{l} + (\Sigma^- \text{ or } \Xi^-)$$
$$\bar{\nu} + p \rightarrow \bar{l} + (\Sigma^0 \text{ or } \Lambda \text{ or } \Xi^0)$$

are related to the surprisingly slow leptonic decays of hyperons. Even well above the relevant thresholds therefore one might expect them to be down on the elastic processes, by an order of magnitude for Λ, about two orders of magnitude for Σ, and still more where Ξ appears.

The general form (7.1) for a squared matrix element applies also to inelastic reactions. The final state F may be either a single particle or a group of particles. In the latter case internal momenta as well as spins are summed over to obtain the stated result. The mass of F, on which the coefficients (A, B, C) depend, is then a continuous variable; it may be regarded as a function of q^2 and $T(= l - l')$ the loss of lab. energy by the leptons. The differential cross section for the production of any specified type of final state then has the form

$$\frac{d^2\sigma}{dq^2 dT} = \frac{1}{|\underline{k}|^2} \left\{ A + B(s-u) + C(s-u)^2 \right\}$$

where A, B, C depend on q^2 and T but not on (s-u), which is essentially the mean lepton energy!

$$s-u = -(k + K)^2 + (k' - K)^2 =$$
$$= -m^2 + 2M(l + l').$$

In the inelastic case there seems to be no simple relation between ν and $\bar{\nu}$ reactions. However if the lepton current satisfies the analogue of G symmetry and the lepton mass difference is neglected, then once again the B-term arises only from parity mixture in both currents.

REFERENCES.

1. T. D. Lee, Phys. Rev. 128, 899 (1963).
R. A. Shaffer, Phys. Rev. 128, 1452 (1963).

2. For an account of this, and references, see Sirlin's lecture at the 1962 Bergen Spring School, and Bell, Meyer and Prentki, Phys. Letters 2, 349 (1963).

3. Reviews on weak interactions and especially on neutrino reactions. S. M. Berman, in Theoretical Aspects of High Energy Phenomena, Cern 1961.
J. Nilsson, Chalmers University of Technology, Report No 258, 1962.
A. Pais, lecture at Trieste, August 1962.
Pais, Lee and Feinberg in Particle Physics, Cern Seminars 1961.

4. See for example the report of Wolfenstein to the 1963 High Energy Conference.

5. Feinberg, Gursey and Pais, Phys. Rev. Letters 7, 208 (1961).
See also S. Bludman UCRL 9667.

6. Ghani and Dardel, Cern NP 62 - 9. (1962).

7. Pais, Report on Weak Interactions, 12th Solvay Congress 1961.

8. Danby et al., Phys. Rev. Letters 9, 36 (1962).
Schwarz, in Proc. of 1962 Int. High Energy Conf. at Cern.

9. Glashow, Phys. Rev. 118, 316 (1960).

10. The application of such ideas to neutrino reactions was considered first by Lee and Yang, Phys. Rev. Letters 4, 307 (1960). Pais extended the result (7.1) to the case of finite lepton mass (Phys. Rev. Letters 9, 117 (1962)). The true generality of (7.1) and in particular the irrelevance of local action of lepton current was established by Martin and Gourdin (Cern preprint). The general analysis of neutrino reactions without summing over spins has been given by Yang and Lee, Phys. Rev. 126, 2239 (1962).

11. I am indebted for these formulae to Dr. Løvseth. Other, presumably equivalent, versions appear in the literature.

12. Cabibbo and Gatto, Nuovo Cimento 15, 304 (1960).
Yamaguchi, Prog. Theor. Physics, 23, 1117 (1960) and CERN 61-2 (1961).
Lee and Yang, Phys. Rev. Letters 4, 307 (1960).

13. S. M. Berman, in Theor. Aspects of Very High Energy Phenomena, CERN 1961.

14. Bell and Berman, Nuovo Cimento 25, 404 (1962).
Dombey, Phys. Rev. 127, 653 (1962).
Dennery, Phys. Rev. 127, 664 (1962).
Cabibbo and Da Prato, Nuovo Cimento 25, 611 (1962).
Fujii, Frascatti Not. Int. No. 163 (1962).

15. Kinoshita, Phys. Rev. Letters 4, 37 (1960).