ABSOLUTE YIELDS AND ENERGIES OF MESONIC X-RAYS

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(presented by M. Stearns)

Introduction

The capture process of negative mesons in condensed matter has been discussed by several authors [1]. The meson is first bound to a particular nucleus in a high quantum state ($n \simeq 16$) and then proceeds to cascade inwards toward the nucleus, transferring its energy by radiative and Auger transitions. At each intermediate level the meson can (1) be absorbed directly from the intermediate level by the nucleus, (2) make a radiative transition to a lower state and emit a quantum, (3) make a radiationless transition to a lower state, transferring the energy difference to an atomic electron (mesonic Auger effect). If the energy difference is sufficiently large it might, alternatively, create an electron pair or excite the nucleus, but such events are highly unlikely in the light mesonic atoms discussed here.

The work reported here is on some recent direct measurements of process No. 2. We shall first discuss measurements of the absolute radiative yields of the K and L X-ray lines from both μ and π mesonic atoms as a function of Z. In the last section we shall report on our most recent measurements of the 2p → 1s (the K_α line) energies of π mesonic atoms.

Absolute radiative yields

The absolute radiative yield of a given transition (for example, 2p → 1s) is defined as the number of X-rays of the given transition per captured meson. It is given by

$$Y_R = \frac{P\,T_R}{T_R + T_C + T_A} \qquad (1)$$

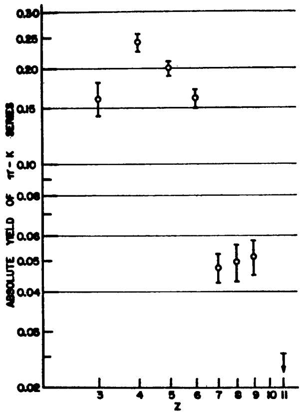
where P is the population of the initial state (2p in our example) and T_R, T_C and T_A are the radiative, capture, and Auger transition probabilities respectively. Since $T_R \sim Z^4$, $T_A \approx$ constant, and T_C is assumed to go as Z^n where $n > 4$, the Z dependence of the radiative yield is given very nearly by

$$Y_R \simeq \frac{P(Z)\,Z^4}{Z^4 + a\,Z^n + b} \qquad (2)$$

P(Z) is an unknown but slowly varying function of Z and depends on the cascade process following the capture of the meson in a very high orbit.

No attempt has been made at this time to separate the components of a given series. By the K yield we mean the integrated yield of all the K lines, (2p → 1s, 3p → 1s, 4p → 1s, etc.) and the same is true for the L yield. In all cases the sum of the higher transitions, i.e. $\beta + \gamma + \delta + \ldots$ constitutes about 20-30% of the total yield.

Fig. 1 shows the absolute radiative yield of the π-K series. Note that the yield flattens out around the magic nucleus, oxygen, and becomes practically zero at Na (Z = 11). Direct nuclear capture from the P states is strongly predominant here. The Li yield is lower than its neighbour, Be, probably due to "Auger" competition. Fig. 2 shows the corresponding absolute radiative yield of the π-L series. Again there appears a magic number effect around Z = 28. The "Auger" competition shows up very clearly here at smaller Z while nuclear capture dominates at higher Z.



Fig, 1. Absolute radiative yield of π-mesonic K-series as a function of Z.

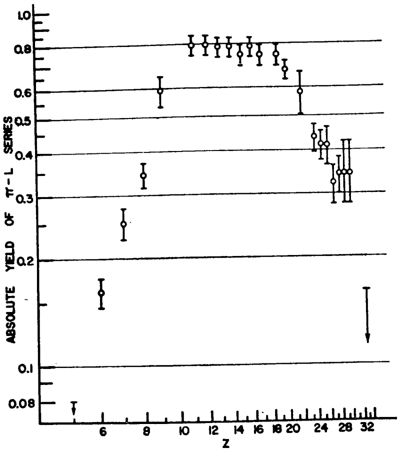


Fig. 2. Absolute radiative yield of π mesonic L-series as a function of Z.

Figs. 3 and 4 show the μ-K and μ-L radiative yields respectively. The absolute values are not as accurate as the corresponding pion yields because the smaller intensity muon beam was contaminated with an electron component. There is no evidence of nuclear capture at higher Z, which is to be expected since the muon-nucleus interaction is very weak. The crosses in fig. 3 give the theoretical values calculated from the Auger transition probabilities as given by Burbridge and de Borde [2] and assuming a $2l + 1$ population in the $n = 14$ level. It appears that the theoretical Auger transition probabilities are too small by a couple of orders of magnitude to explain the reduced yields at smaller Z. This discrepancy between theory and experiment has not been resolved, nor is it easy to invent a process to account for it. Almost all of the obvious and reasonable explanations, such as a depletion of atomic electrons, would enhance the discrepancy rather than reduce it. Nor can the discrepancy be removed by an appropriate choice of P(Z). We have found no P(Z) that will both satisfy the general results of all the yield curves and yet give the necessary large attenuation of yield at small Z.

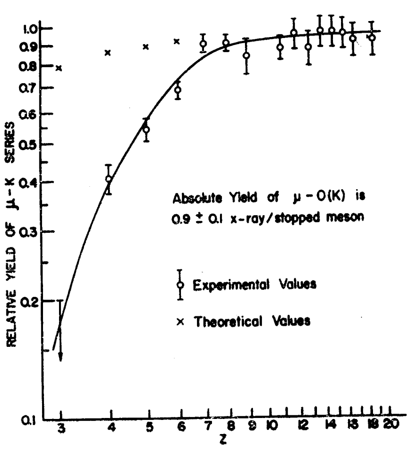


Fig. 3. Relative radiative yield of μ mesonic K-series as a function of Z.

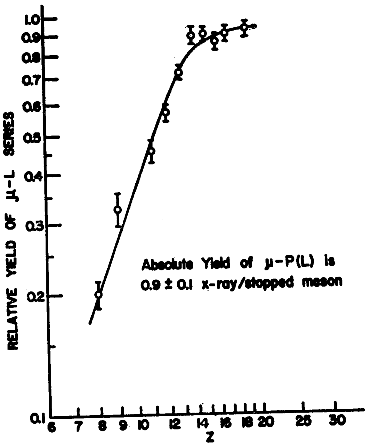


Fig. 4. Relative radiative yield of μ mesonic L-series as a function of Z.

Energies of the 2p → ls π mesonic X-rays

The energies of the 2p → 1s π mesonic X-rays were measured for the elements Li to F inclusive. The X-rays from elements greater than F could not be measured since nuclear capture directly from the 2p state is practically complete at Na ($Z = 11$). In particular, we have measured the 2p → 1s lines from both the B^{10} and B^{11} isotopes.

For the purpose of comparison with the experimental results the energies of the lines were computed with the Klein-Gordon equation (for $m_{\pi^-} = 272.8\ m_e$) using a potential due to a point charge and corrected for reduced mass, finite nuclear size, vacuum polarization, fine structure, electronic screening, nuclear polarization, etc. For the light elements used here all these corrections were negligible except for the reduced mass, vacuum polarization, and the finite extent of the nucleus. The last two are of opposite sign and therefore tend to cancel each other to some extent. Table II gives the calculated energies and corrections in tabulated form.

The energies were measured by two different methods [3]. The first was that of the critical absorption technique familiar from X-ray work. For this work a sequence of absorbers were interposed, one at a time, between the meson target and the NaI detector, and the transmission of the mesonic line was measured as a function of the absorber Z. The presence of an absorption discontinuity brackets the energy of the line between the two neighbouring K-edges which, in principle, can be known very accurately. In two cases we were singularly fortunate. (1) The $_{59}$Pr K-edge split the Be(2p → 1s) line thus giving an accurate measurement of its energy. (2) The $_{72}$Hf K-edge lay within the natural width of the $B^{10}(2p \rightarrow 1s)$ line. Combining this information with an independent (pulse height) measurement of the B^{10} energy, described below, we obtained a rough measure of the half width of the B^{10} 1s level, namely 1.1 ± 1.4 Kev. The results of the critical absorption measurements are given in Table 1.*

TABLE I

Kα energies obtained from critical absorber measurements

Element	Calculated Energy (Kev)	Measured Energy (Kev)
$_3$Li	24.61	$23.22 < E < 24.35$
$_4$Be^9	43.95	close to 41.98
$_5$B^{10}	68.76	close to 65.2
$_5$B^{11}	68.83	$61.31 < E < 65.32$
$_6$C^{12}	99.10	$90.53 < E$

The second method was to determine the energy of the Kα line by a measurement of pulse height. The experimental procedure was to bracket each Kα line under investigation with two calibration lines of similar energy and these were run alternatively several times. The stability of the apparatus allowed a determination of the Kα energy to 1% or better for the elements investigated. π-Lα lines (and occasionally π-Mα lines) were used for calibration. Fig. 5 shows, as an example, the B^{10} Kα line bracketed by the Na and Mg Lα lines. A summary of our results of the pulse height measurements, as well as the calculated energies, is given in Table II. The percent energy shift listed in the Table is defined by $(E_{calc} - E_{meas})/E_{calc} \times 100$.

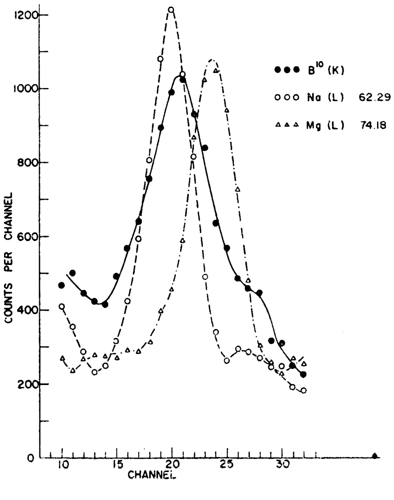


Fig. 5. Bracketing of the B^{10} π-K-line.

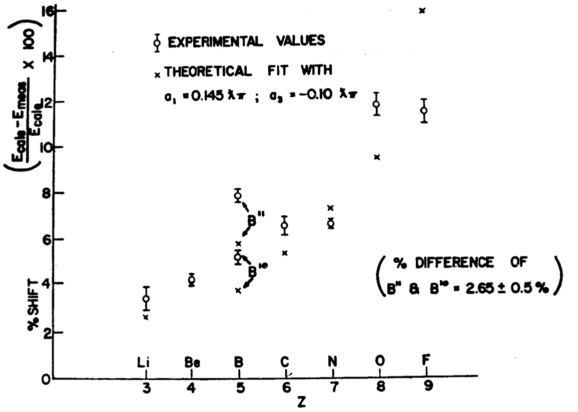


Fig. 6. Energy shifts of 2p-1s transition as a function of Z, theoretical and experimental.

In each case the measured Kα energy is lower than its calculated electromagnetic energy indicating a net repulsive interaction between the meson in the 1s state and the nucleus.

Deser et al.[4], have related the energy shift of the mesonic 1s level to the meson-nucleon scattering phase shifts at zero energy. They assume that the effects of the individual nucleons are simply additive. Their expression for the fractional shift of the π–Kα line is

$$\frac{\delta E}{E_{K\alpha}} = -\frac{4}{3}\frac{\mu}{\bar{\mu}}\frac{4Z}{r_B}\left[\frac{2}{3} Z a_1 + \frac{(3N+Z)}{3} a_3\right] \qquad (3)$$

where δE is the energy shift in the 1s level, r_B is the mesonic Bohr radius (1.94×10^{-11} cm.), a_1 and a_3 are the s scattering lengths for the isotopic spin states $^1/_2$ and $^3/_2$, and μ and $\bar{\mu}$ are the reduced meson masses with respect to nucleus and nucleon respectively.

Using our experimentally observed energy shifts for the pure isotopes, Be through F, we have made a least squares determination of the best values of a_1 and a_3. The values so obtained are $a_1 = 0.14_5 \bar{\lambda}_\pi$ and $a_3 = -0.10 \bar{\lambda}_\pi$ where $\bar{\lambda}_\pi$ is the π meson Compton wave-length. The theoretical percentage shifts, plotted in Fig. 6, were calculated with these values. An independent determination of a_3 can be derived from Eq. 3 using the measured energy difference between B^{10} and B^{11}. This gives $a_3 = -0.12 \pm 0.02 \bar{\lambda}_\pi$. These scattering lengths can be compared with Orear's values [5] obtained from an analysis of low energy π p scattering: $a_1 = 0.16 \bar{\lambda}_\pi$ and $a_3 = -0.11 \bar{\lambda}_\pi$. As can be seen, there is good agreement between the two results. This may be fortuitous. The 1s level shift predicted by Deser et al.[4] should vary as Z^2 for $Z = N$. This appears to be too strong a Z dependence for the measured elements B^{10},

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* We are very grateful to Professor J. W. M. DuMond and Dr. R. L. Shacklett for measuring the K-edge profiles of many of the absorbers used in our experiments.

TABLE II

Calculated K_α energies and experimental values from pulse height measurements

Transition	Energy from Klein-Gordon equation *	Finite size correction (Kev)	Vac. polar. correction (Kev)	Calculated energy (Kev)	Measured energy ** (Kev)	Energy shift (Kev)	% Shift
Li(K_α)	24.546	-0.033	0.096	24.61	23.77 ± 0.12	0.84	3.4 ± 0.5
N(L_α)	25.004		0.064	25.07			
O(L_α)	32.708		0.094	32.80			
F(L_α)	41.464		0.136	41.60			
Be(K_α)	43.869	-0.123	0.202	43.95	42.09 ± 0.10	1.86	4.2 ± 0.25
S(M_α)	46.022		0.126	46.15			
Na(L_α)	62.056		0.230	62.29			
$B^{10}(K_\alpha)$	68.708	-0.324	0.376	68.76	65.2 ± 0.2 ***	3.5_4	5.15 ± 0.3
$B^{11}(K_\alpha)$	68.802	-0.347	0.376	68.83	63.5 ± 0.2	5.35	7.8 ± 0.3
Mg(L_α)	73.894		0.288	74.18			
Al(L_α)	86.801		0.365	87.17			
C(K_α)	99.284	-0.765	0.576	99.10	92.6 ± 0.4	6.5	6.5 ± 0.4
Si(L_α)	100.72		0.430	101.15			
P(L_α)	115.73		0.53	116.26			
N(K_α)	135.52	-1.57	0.84	134.79	126.0 ± 0.3	8.8	6.55 ± 0.2
S(L_α)	131.74		0.63	132.37			
Cl(L_α)	148.84		0.74	149.58			
O(K_α)	177.46	-2.93	1.20	175.73	155.2 ± 0.8	20.5	11.7 ± 0.5
Zn(M)				159.45±0.25			
K(L_α)	186.16		0.97	187.13			
F(K_α)	225.25	-5.28	1.60	221.57	196.3 ± 1.0	25.3	11.4 ± 0.5
Ca(L_α)	206.38		1.10	207.48			

* $m_\pi = 272.8\ m_e$.

** The errors quoted are r.m.s. values; see text for discussion of possible systematic errors.

*** $E_{B^{10}} - E_{B^{11}} = 1.8 \pm 0.3$ Kev; % difference in energy shift = (2.65 ± 0.5)%.

N, C, and O. Also the experimental fluctuations between neighbouring elements do not agree very well with those predicted by Eq. 3. This disagreement with the theory may indicate that the simple assumption of additivity of the effects of individual nucleons must be modified. Brueckner 6) has made an estimate of the effect of absorption and reemission of the meson by pairs of nucleons and has found that it contributes a shift of about the same magnitude and sign as that due to scattering from single nucleons. Thus, the total shift, as given by Brueckner, would be about twice as large as that experimentally observed—assuming reasonable values of the scattering length. (This theory is sensitive to the meson absorption rate and the internal momentum distribution of the nucleons, and a modification of these parameters would change the shift.) * In addition to the shift the Brueckner theory gives an estimate of the line width, $Z^2E_z/2150$ for the half width of the 1s level. In general this is too small to be observed with our resolution. However, as described earlier, we have been able to get a rough measure of the width of the $B^{10}K_\alpha$ line by means of critical absorbers. The half-width obtained is 1.1 ± 1.4 Kev. This is the same order of magnitude as that given by Brueckner, ≈ 1.1 Kev.

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* Karplus, in extending Brueckner's theory, has shown that certain choices of nucleon momentum distributions will even change the sign of the shift. (Sixth Rochester Conference.)

LIST OF REFERENCES

1. (*a*) Fermi, E. and Teller, E. The capture of negative mesotrons. Phys. Rev., *72*, p. 399-408, 1947.
   (*b*) Marshak, R. E. Meson physics. Chapter 5, New York, 1952.
   (*c*) Wheeler, J. A. Some consequences of the electromagnetic interaction between μ-mesons and nuclei. Rev. mod. Phys., *21*, p. 133-43, 1949.
2. (*a*) Burbidge, G. R. and de Borde, A. H. The mesonic Auger effect. Phys. Rev., *89*, p. 189-93, 1953.
   (*b*) de Borde, A. H. The Auger effect and negative meson capture. Proc. phys. Soc., Lond. A, *67*, p. 57-67, 1954.
3. Stearns, M. and Stearns, M. B. Phys. Rev. (to be published Sept. 1956.)
4. Deser, S., Goldberger, M. L., Baumann, K. and Thirring, W. Energy level displacements in pi-mesonic atoms. Phys. Rev., *96*, p. 774-6, 1954.
5. Orear, J. Low energy behaviour of the phase shifts in pion-proton scattering. Phys. Rev., *96*, p. 176-9, 1954.
6. Brueckner, K. A. Energy levels of π mesonic atoms. Phys. Rev., *98*, p. 769-71, 1955.

DISCUSSION

P. T. Matthews stressed that the Deser theory does not give a level width, although such a width has to be expected due to the nuclear capture and has actually been observed. It would therefore be desirable to know, whether the Brueckner theory, which gives the width, can be brought into quantitative agreement with the experiments. In its original version it contained some errors, but those were corrected by Thouless in Cambridge and by Karplus (reported in Rochester).