ON THE THEORY OF PHOTOPRODUCTION OF MESONS ON BOUND NUCLEONS

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Investigation of photoproduction of mesons on bound nucleons affords the only practical possibility of studying the process of photoproduction of mesons on neutrons. Most appropriate for this purpose is the study of the meson photoproduction process on deuterium. As is well known, the importance of data on the elementary process of meson photoproduction on neutrons is due to the fact that the processes of meson photoproduction on neutrons and protons are of different nature. Consequently, unlike the process of meson scattering, the study of meson photoproduction on hydrogen only does not exhaust the problem of elementary interaction even in general features.

From the viewpoint of the theory of wave fields, the production of mesons on complex nuclei, being a more complicated process, is of less interest than photoproduction of mesons on protons or deuterons. It is, however, possible that detailed study of this process may supply valuable information on meson-nucleon interaction. For this purpose it is important, above all, to get a clear idea of the mechanism of meson formation on a group of bound nucleons. What is implied here is whether the existing conceptual models are applicable to this process.

This paper deals with some aspects of the above two problems in the light of the experimental results obtained in the Laboratory of Physics of High Energy Particles, the Physical Institute, USSR Academy of Sciences (by the research group led by Professor V. I. Veksler).

1. *Photoproduction of negative π mesons on deuteron near the threshold*

Investigation of meson photoproduction processes on the deuteron presents considerable difficulties both from the viewpoint of theory and experiment. This results basically from the fact that in dealing with this problem we encounter the problem of three bodies, and the presence of the second nucleon greatly complicates the phenomenon. The prevailing theories of the effect [1,2] have up to now only been tested experimentally to a very slight degree. There are indications [3] that the conventional theory based on impulse approximation is not in agreement with experiment. Apart from this, there are some theoretical arguments pointing to the limited application of impulse approximation. What we mean here are the effects of multiple scattering as estimated by K. A. Brueckner [4] and K. M. Watson [5]. In this connection it is certainly of interest to improve the theory and compare it with the experimental data that are being accumulated. We shall confine ourselves to the region of photons energies $\leqslant$ 200 Mev. In this region the energies of mesons are such that the corresponding phase shifts of meson scattering on nucleons are small, and hence the effects of multiple scattering are of little significance. Therefore, a simpler theoretical approach is justified in the region of near threshold energies of photons and one may expect that the theory is in agreement with experiment to a greater degree. It should also be noted that this region is of particular interest from the viewpoint of checking the results of field theories (see, for example [6]).

The existing theory, based on impulse approximation, should be carefully examined. We shall dwell on the following two points.

First of all, when slow nucleons are formed, the interaction of these nucleons in their final state begins to play an important role. The role of particle interaction effects in their final state is well known from nuclear physics. The influence of the interaction of nucleons in the final state in the reaction of meson production during the collision of nucleons has been dealt with by M.A. Markov (in the case of a square potential well) and by A. B. Migdal (proceeding from the theory of effective length)[7]. A consistent theory of the effect was expounded in K. M. Watson's well-known work [8].

We shall take into account these effects for the reaction :

$$\gamma + d \rightarrow 2p + \pi^- \qquad (1)$$

In our case the above theory cannot be directly applied, since we shall concern ourselves with more detailed characteristics than those it deals with.

Secondly, the usual assumption in the application of impulse approximation was that the transition operator defined by the expression :

$$M = \sum_{\nu} e^{i(\vec{\varkappa}-\mathbf{K})\mathbf{r}_\nu} [(\vec{\sigma}^\nu \mathbf{K}^\nu) + L^\nu] \dagger \qquad (2)$$

† $h = c = \mu = 1$ where μ is the meson mass.

does not depend on the momentum of nucleons. Here $\vec{\sigma}^{\nu}$ stands for Pauli's matrices, and $\vec{\varkappa}$ and **K** are momentum vectors of photon and meson respectively. **K** and L are operators, being the functions of photon and meson momentum vectors, of photon polarization and isotopic spins of mesons and nucleons. The theory usually proceeds from an assumption which disregards the dependence of operators **K** and **L** on nucleon momentum. This assumption is open to question and is not helpful in simplifying calculations. At the same time it is liable to introduce major errors, which can be seen, for instance, when M is computed by means of field theories. Disregard for the effects of nucleon recoil eliminates the substantial difference in the cross-sections of production of π^+ and π^- mesons. From this point of view good results are obtained when terms of the order (v/c) are accounted for in **K** and L (v being the velocity of the nucleon).

$$\mathbf{K}^{\nu} \approx \mathbf{K}^{\nu}_{o} + a\frac{\mathbf{p}^{\nu}}{M} + b\,(\mathbf{K}\mathbf{P}^{\nu})\frac{1}{M}$$

$$L^{\nu} \approx L^{\nu}_{o} + (\mathbf{l}^{\nu}\mathbf{p}^{\nu})\frac{1}{M} \qquad (3)$$

Here $\mathbf{P}^{\nu}$ is the operator of the nucleon momentum, and a, b, $\mathbf{K}^{\nu}_{0}\,L^{\nu}_{0}$, $\mathbf{K}^{\nu}$, $\mathbf{l}^{\nu}$ are operators independent of **p**.

The use of this kind of operators permits one essentially to take into account with sufficient accuracy all the effects related to the nucleon recoil. This is particularly well-illustrated if operators **K** and L are obtained according to the field theories (2).

The calculations made in (2) result in the following equation for the cross-section of charged meson photoproduction :

$$d\sigma = \frac{16}{(2\pi)^2}\Big\{|\mathbf{K}_o|^2\Big[\frac{2}{3}|I_a|^2 + \frac{1}{3}|I_s|^2\Big] + |L_o|^2|\mathbf{I}_a|^2$$
$$+ 2\,\mathrm{Re}\,\frac{1}{M}\Big[\Big((a\mathbf{K}^*_o + (\mathbf{K}_o\,\mathbf{b})\,\mathbf{K}_1)\,,\,(\frac{2}{3}I^*_a\,\mathbf{I}_a - \frac{1}{3}I^*_s\,\mathbf{I}_s)\Big)$$
$$+ L^*_o\cdot(\mathbf{l},\,I^*_a\,\mathbf{I}_a)\Big]\Big\}\,d\mathbf{p}\,d\mathbf{q}\,\delta\,(E_o - E_f) \qquad (4)$$

where $\mathbf{p} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{2}$ is the relative momentum of two nucleons and $\mathbf{q} = \frac{\mathbf{P}_1 + \mathbf{P}_2}{2}$:

$$I_a = \int \varphi_{fa}\,e^{i\mathbf{qr}}\,\varphi_d\,d\mathbf{r} \qquad I_s = \int \varphi^*_{fs}\,e^{i\mathbf{qr}}\,\varphi_d\,d\mathbf{r}$$

$$\mathbf{I}_a = \int \varphi_{fa}\,e^{i\mathbf{qr}}\,\mathbf{p}\,\varphi_d\,d\mathbf{r} \qquad \mathbf{I}_s = \int \varphi^*_{fs}\,e^{i\mathbf{qr}}\,\mathbf{p}\,\varphi_d\,d\mathbf{r}$$

Indexes "a" and "s" mean that the coordinate part of the wave function defining the relative momentum of two nucleons is respectively antisymmetrical or symmetrical.

It is not difficult to estimate the integral cross-section by using closure approximation. When integrating terms containing I_a and $\mathbf{I}_a$ we use the equation :

$$\int \varphi_d\,e^{2i\mathbf{qr}}\,\mathbf{p}\varphi_d = -\,\mathbf{q}I(2q)$$
$$\text{where} \qquad I(2q) = \int|\varphi_d|^2\,e^{2i\mathbf{qr}}\,d\mathbf{r} \qquad (5)$$

Simple calculations result in the following equation :

$$d\sigma_{\pi^+} = \frac{1}{(2\pi)^2}\frac{K^3 d\Omega_K}{\Big[\frac{K^2}{E} + \frac{K^2 - (\vec{\varkappa}\,\mathbf{K})}{2\,M}\Big]}\Big\{|\mathbf{K}_o|^2 + |L_o|^2$$
$$+ \Big[2\mathrm{Re}\,(\mathbf{q}_1\,\mathbf{x}) - \frac{1}{3}|\mathbf{K}_o|^2 + |L_o|^2\Big]\,I\,(2q)\Big\}$$

$$\text{where} \quad \mathbf{x} = \frac{1}{M}\Big[\frac{1}{3}\,a^*\mathbf{K}_o + (\mathbf{K}^*_o\,\mathbf{b})\,\mathbf{K}_, + L^*_o\,\mathbf{l}\Big] \qquad (6)$$

It should be noted that the results of the last calculations are in general suitable for the determination of the total cross-section. This is confirmed by comparing the total cross-section obtained by the equation (6) with the experimental one.

A more detailed comparison with experiment (6) is not required for the reason that when closure approximation is used, the term p^2/M in the conservation of energy principle is disregarded. Corresponding calculations demonstrate that the role of the region of variables is important when $p \sim q$. As pointed out above, we take into account both the nuclear and Coulomb interaction of protons. To begin with, we confine ourselves to the region of small p :

$$p^2 \ll q^2 \qquad (7)$$

This case I conforms with "forks" observed in experiments, i.e. such events when the resultant momentum of protons greatly exceeds the relative momentum. This differs from analysis of the case : $p \sim q$ which will be made separately. The effects of interaction are taken into account through the selection of the wave function which describes the relative motion of two protons and which forms part of the following integrals :

$$\int \psi_f(\mathbf{r})\,e^{i\mathbf{qr}}\,\varphi_d(\mathbf{r})\,d\mathbf{r} \qquad (8)$$

Due to our condition (7) only the lower values of orbital moments and of the final state of the two nucleons will play a major role in the integrals (8). As seen from (8), the succeeding *l* will contribute, as compared with the preceding, a term of the order of $(p/q)^2$. In other words, when calculating I we shall require the wave function for the S state of the two protons, and that for the P- state when calculating $\mathbf{I}_a$.

As is known from the p-p scattering theory, the interaction of protons in the p state in the region of lower energies becomes practically reduced to the Coulomb interaction. Consideration of the weak repulsion forces would have only increased the effect discussed below. Therefore, the expansion of the combination of Whittaker functions for $l = 1$ representing the incoming wave at infinity should be taken as ψ_{fa}

$$\psi_{fa} = -\frac{1}{(2\pi)^{3/2}}\, C\sqrt{1+\eta^2}\, i\,(\mathbf{p}\,\mathbf{r})\, e^{i\beta} \tag{9}$$

where $e^{i\beta}$ is an irrelevant phase factor, $\eta = \frac{e^2}{v}$

$$C^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$$

v– is the relative velocity of the two nucleons.

A similar expression for ψ_{fs} appears like this [9]:

$$\psi_{fs} = \frac{1}{(2\pi)^{3/2}}\left\{C\cos\delta + \frac{\sin\delta}{pr}\right.$$

$$\left.\times\left[\frac{1}{C} + \frac{r}{RC}\left(h(\eta) + 2\gamma - 1 + ln\frac{r}{R}\right)\right]\right\} e^{i\beta}, \tag{10}$$

where $h(\eta) = -\,ln\,\eta + \mathrm{Re}\,\frac{\Gamma^1(-i\eta)}{\Gamma(-i\eta)}$;

$$\mathrm{ctg}\,\delta = \frac{2\eta R}{C^2}\left[-\frac{1}{a} - \frac{h(\eta)}{R} + \frac{1}{2}r_0p^2\right] \tag{11}$$

In practical calculations it is more convenient to use not formula (11), but to substitute δ obtained in experiments on proton-proton scattering [9].

Calculations lead to the following values Ia Is :

$$Ia = \frac{1}{\sqrt{2\pi}}\left(\frac{7\alpha}{9\pi}\right)^{\frac{1}{2}} C\sqrt{1+\eta^2}\, e^{i\beta}\, 4\,(\mathbf{p}\mathbf{q})$$

$$\times\left[\frac{1}{(\alpha^2+q^2)^2} - \frac{1}{(49\alpha^2+q^2)^2}\right]$$

$$Is = \frac{2}{\sqrt{2\pi}}\left(\frac{7\alpha}{9\pi}\right)^{\frac{1}{2}}\left\{\left[C\cos\delta + \frac{1}{cpR}\right.\right.$$

$$\left.\times\left(h(\eta) + 2\gamma - 1 + ln\frac{1}{qR}\right)\right]\left(\frac{1}{\alpha^2+q^2} - \frac{1}{49\alpha^2+q^2}\right)$$

$$\left. + \frac{\sin\delta}{cp}\frac{1}{q}\left(\mathrm{arctg}\frac{7\alpha}{q} - \mathrm{arctg}\frac{\alpha}{q}\right)\right\} \tag{12}$$

The function $\varphi_d = \left(\frac{7\alpha}{9\pi}\right)^{\frac{1}{2}}\frac{e^{-\alpha r} - e^{-7\alpha r}}{r}$ is taken as the deuteron wave function, $\alpha = \sqrt{M\varepsilon} = 0{,}31$ ε is the binding energy of deuteron.

The substitution of formulae (9) and (12) in (4) provides a general expression for the cross-section which should be compared with the experiment so as to ascertain the applicability of impulse approximation and determine the values of parameters involved in this expression. The statistical data available on the reaction under discussion is as yet so insufficient that it is hardly possible to compare the differential cross-section. For this reason it becomes necessary to integrate the above cross-section with respect to some of the variables and to obtain the integral cross-sections with sufficiently pronounced characteristics. It follows from expressions (4), (9) and (12) that the relative value K_0 and L_0 substantially defines both the total cross-section of forming "forks" and the dependence of the cross-section on p and q. Now let us find these relationships.

In order to integrate, we have to make some assumptions regarding K_0, L_0, a, b, **l**. As the experimentally measured total cross-section is approximately proportional to the first power of the meson momentum, this means, as shown by (6), that K and L may be considered constants with sufficient accuracy. This is all the more justified since we are to confine ourselves to a consideration of a rather narrow interval of photon energies ranging from 150 to 200 Mev. Apart from this, in order to simplify the calculation, we shall disregard the terms $\sim$ v/c, this may lead to errors of $\sim 10\%$. The experiment is somewhat less accurate than the above figure. As the absolute measurements of the cross-sections involves a number of non-controlled errors, it is advisable to analyse the relationship between the differential cross-section and the total one. In addition, we shall take into consideration the non-monochromatic nature of the photon beam, by introducing the factor $d\varkappa/\varkappa$ and integrating with respect

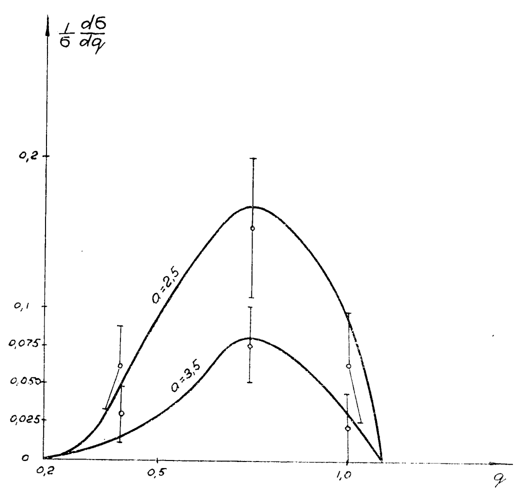

Fig. 1. Solid curve shows theoretical distribution of "forks" with q for two values of a. Experimentally observed points are also shown.

to $d\varkappa$. By using formula (6), we are able to calculate the following expression for the total cross-section :

$$\sigma = [5.3|K|^2 + 3.8|L|^2] \cdot 10^{-2}$$

The integration of (4) with respect to solid angles $d\Omega_p$ and $d\Omega_q$, account being taken of (9) and (12), results in the following expression for the differential cross-section :

$$d\sigma = 3.36\alpha \left\{4C^2(1+\eta^2)p^2q^2\left(\frac{2}{3}|K|^2+|L|^2\right)\right.$$
$$\times\left[\frac{1}{(\alpha^2+q^2)^2}-\frac{1}{(49\alpha^2+q^2)^2}\right]^2+\frac{1}{3}|K|^2\left[\left(C\cos\delta+\frac{1}{cpR}\right.\right.$$
$$\left.\times\left[h(\eta)+2\gamma-1+ln\frac{1}{qR}\right]\sin\delta\right)\left(\frac{1}{\alpha^2+q^2}-\frac{1}{49\alpha^2+q^2}\right)$$
$$\left.\left.+\frac{\sin\delta}{cp}\cdot\frac{1}{q}\left(\operatorname{arctg}\frac{7\alpha}{q}-\operatorname{arctg}\frac{\alpha}{q}\right)\right]^2\right\}$$
$$\times\left[\frac{q-\dfrac{q^2}{M}}{1+4q^2}-\frac{1+4q^2}{16\varkappa^2_{max}\,q}\right]\cdot p^2\,dp\,q^2\,dq \tag{13}$$

Let us define "forks" as events in which $Ap_1 \leqslant q$. Our theory is applicable for $A > 2.5 - 3$. To compare with the experiment, we shall take two values of A : 2.5 and 3.5.

Analysis of the case $L \neq 0$, $K = 0$ leads to the following value for the relationship between the total cross-section of "forks" formation and the total cross-section of the effect $\gamma + d \rightarrow 2p + \pi^-$

$$\sigma_f/\sigma < 1/A^5 \tag{14}$$

In other words, the probability of recording of even one event with $A \sim 5\text{-}10$ is extremely small when observing some 500 cases of the reaction (1).

This sharply contradicts the experimental data available. Hence the assumption $K = 0$ is not valid.

Let us consider the case $K \neq 0$. The assumptions concerning the value L are of little significance in determining the value of differential cross-section as it is clear from the above analysis that the factor at L is negligibly small.

Successive integrations of differential cross-section, of which the last are made numerically, give the following values of the relationship σ_β/σ :

$$\frac{\sigma_\beta}{\sigma} = \frac{0.19}{5.3 + 3.8\dfrac{|L|^2}{|K|^2}} \quad \text{When } A = 3.5$$
$$\frac{\sigma_\beta}{\sigma} = \frac{0.46}{5.3 + 3.8\dfrac{|L|^2}{|K|^2}} \quad \text{When } A = 2.5 \tag{15}$$

If $|L|^2 / |K|^2$ is small, these relationships prove to be respectively :

$$\sigma_\beta/\sigma = 0.035 \quad \text{and} \quad \sigma_\beta/\sigma = 0.086,$$

which agrees very well with the experimentally obtained values 0.039 ± 0.012 and 0.085 ± 0.018.

Proceeding from these experimental values and formulae (15), $|L|^2 / |K|^2$ can be determined. It then follows that $|L|^2$ does not exceed 10% of $|K|^2$.

As is seen from (15), the number of "forks", with the total number of events of the reaction (1) being approximately equal to 500, is such, that an attempt can be made to compare differential cross-sections with the experiment, too. Figs. 1 and 2 illustrate the distribution of "forks" against p and q when A = 2.5 and 3.5. The experimental values are likewise plotted there. As is seen from the figures, there appears to be satisfactory agreement, with the possible exception of the region of very low p.

The results so obtained have shown that impulse approximation appears to be justified in the region of energies under discussion and that some knowledge of the operators **K**,**L**, **k**,**a**,**b**,**l** may be obtained by means of the above formulae, when more statistical data is available. In other words, data can be obtained on meson photoproduction on neutrons. The available experimental material leads to the assumption that it is the part of the transition matrix depending on the spin operator that plays a major role in meson photoproduction on neutrons.

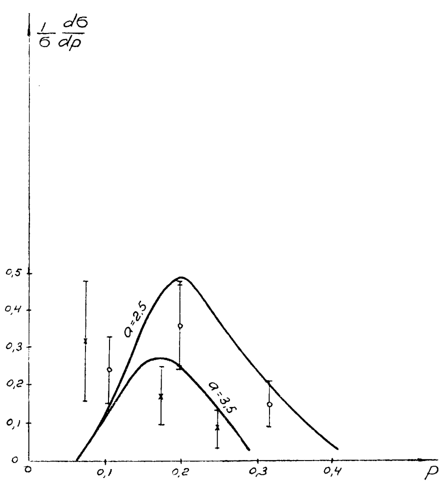

Fig. 2. Solid curve shows theoretical distribution of "forks" with p for two values of a. Experimentally observed points are also shown.

As is well known [10], the laws of conservation of angular momentum and parity and the assumption that mesons are produced in S-state in the region of near-threshold energies lead to the following expression for the transition operator: $D\,(\vec{\sigma}\,\vec{\lambda})$, which is precisely of the type obtained above. Moreover, isotropic angular distribution of mesons has been obtained in the experiments under discussion as well as linear dependence of the cross-section on the meson momentum. All these facts quite definitely indicate that π^- mesons are produced near the threshold in a S-state as a result of electrical dipole transition.

Let us consider the other limiting case II

$$p^2 \sim q^2 \gg \alpha^2 \qquad (7^{II})$$

As follows from the calculations the effects of the final state interaction are negligible in this case. Neglecting those terms in eq (4) which yield a contribution of the order of 10%, and integrating with respect to $d\Omega_p$, $d\Omega_q$ and $d\varkappa$ we find:

$$d\sigma = \frac{14}{9\pi^2}\, ln\, \frac{1.4\,q}{0.25 + q^2} \cdot \frac{q^2 + 2q\varepsilon}{\varepsilon^2 + \alpha^2}\, |K^0|^2 dq\, d\varepsilon \qquad (13^{II})$$

here we have introduced a new variable

$$\varepsilon = p - q$$

As may be seen from (13^{II}) the dependence of $d\sigma$ on ε is very great. This dependence may be used in order to check the impulse approximation.

From an absolute measurement of the cross sections of the reaction (1) we can obtain the magnitude of the photoproduction cross section on a neutron in the center-of-mass system:

$$\frac{d\sigma^-}{d\Omega} = |T^-|^2 \frac{kE}{\left(1 + \frac{\varkappa^2}{2M}\right)^2}$$

$|T^-|^2 = (1/(2\pi)^2)\,|K|^2$ may be obtained by three independent ways: from case I, from case II, and from the total cross-section, deduced in the "closure" approximation (the last method is somewhat doubtful).

The data are tabulated in the Table I.

TABLE I

	from case I	from case II	from the total cross-section
$\lvert T^- \rvert^2$	$(2.9 \pm 0.9) \cdot 10^{-29}$ cm^2	$(2.6 \pm 0.3) \cdot 10^{-29}$ cm^2	$3.1 \pm 0.3) \cdot 10^{-29}$ cm^2
σ^-/σ^+	1.9 ± 0.6	1.7 ± 0.15	2.1 ± 0.15

We used data of Bernardini and Goldwasser [11] in order to obtain the last line.

More detailed data concerning photoproduction on a neutron will be obtained when data with greater statistical accuracy is available.

2. *Photoproduction of slow mesons on nuclei* †

M. Lax and H. Feshbach suggested in 1951 [12] a simple and rather natural single-nucleon model of meson photoproduction on nuclei. The checking of this model appears to us to be of considerable interest. M. Lax and H. Feshbach proposed that their model should be checked by comparison of the calculated energy distribution of mesons with experiment. It was then that they discovered a peculiar discontinuity in the slope of this distribution. However, similar calculations made by the author jointly with V. V. Mikhailov [13] have proved that the integration of the momentum distribution of nucleons was made incorrectly in the work carried out by M. Lax and H. Feshbach and that there was no discontinuity in the slope referred to by the investigators. The energy spectrum of the mesons proved to be a smooth function which is non-sensitive to the details of the mechanism. The above-mentioned paper proved at the same time that angular distributions of mesons were of some interest. It appeared that the angular distributions of mesons produced on nuclei should only slightly differ from those produced on free nucleons if the single-nucleon model was appropriate. A comparison of the data obtained by Jakobson, Shultz, and White [14] on measuring the angular distribution of mesons produced on helium with identical data regarding hydrogen has proved that they are similar. Both angular distributions have a maximum in the region of the meson emergence angle $\theta = 90°$. Values of this type are not, however, numerous and contain large errors. From our standpoint, an experimental study of the stars accompanying the slow mesons and observed on a photographic emulsion [15] was the most convincing confirmation of the single-nucleon model. Fast protons of an energy up to 70 Mev were detected among the star prongs accompanying the π^- mesons. These protons possessed a pronounced anisotropic angular distribution. An explanation of this effect should evidently be sought in the single-nucleon model. These protons appear to be recoil protons arising during the production of π^- mesons on nuclear neutrons.

The principle features in the recoil proton distributions resulting from the single-nucleon model are considered below. By "fast" protons we imply protons of an energy exceeding 20 Mev, and by "slow" mesons those of an energy less than 10 Mev.

First of all, it is worth mentioning that the cross-section of the formation process of slow mesons accompanied by the emergence of fast protons greatly depends on the momentum distribution of nucleons in the nucleus. For

† The author wishes to acknowledge the considerable help he received from A. N. Lebedev in making the calculations for this part of the paper.

example, if the formation of mesons proceeds on a nucleon at rest, the energy of the recoil nucleon (according to conservation laws) should be within :

$$\frac{(E-k)^2}{2M} \leqslant \frac{P^2}{2M} \leqslant \frac{(E+K)^2}{2M} \quad (16)$$

the limiting energy of the meson being $E = \sqrt{1+K^2}$. From this it follows that if the mesons possess energy less than 7 Mev, no recoil nucleons of an energy exceeding 20 Mev can appear at all, irrespective of the photon energy.

The cross-section of π meson photoproduction on nuclei, using the model of Lax and Feshbach, appears as follows :

$$d\sigma = N(2\pi)^{-2}(|K|^2 + |L|^2)\,\mathbf{dk}\,[\int \rho(\mathbf{p})\,\mathbf{dp}\,\delta(\mathbf{p}_f + \mathbf{k} - \vec{\varkappa} - \mathbf{p}) \times \delta(E_f + E - \varkappa + \varepsilon)]\cdot \mathbf{dp}_f\,\frac{d\varkappa}{\varkappa} \quad (17)$$

where N is the number of neutrons in the nucleus, $\rho(\mathbf{p})$ is the momentum distribution of the nucleons in the nucleus, E_f, P_f are the energy and the fast proton momentum, and ε equals the mass of the nucleus-residue plus the mass of the proton minus the mass of the initial nucleus. The other notations are the same as above. In order to obtain the angular distribution, the expression (17) must be integrated by all the variables with the exception of the angle x between the vectors $\vec{\varkappa}$ and $\mathbf{P}_f$. The amount of change in the variables is predetermined by the

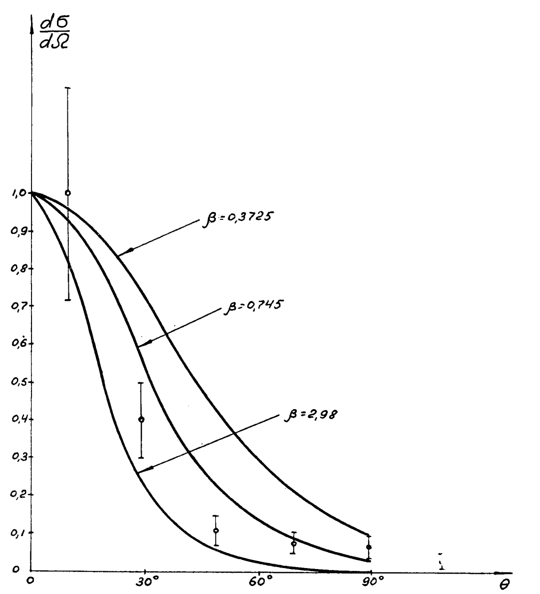

Fig. 3. Theoretical angular distribution of fast protons accompanying slow mesons for different values of β, compared with the experimental points.

functions of the conservation laws and the conditions $\varkappa \leqslant \varkappa_{max}$; $E \leqslant 1.07\ P_f^2/2M > 0.14$ where $\varkappa_{max}$ is the upper limit of bremsstrahlung spectrum, $E = 1.07$ corresponds to the kinetic energy of the meson of approximately 10 Mev, and $P_f^2/2M = 0.14$ corresponds to the nucleon energy of 20 Mev.

For $\rho(\mathbf{p})$ we took the distribution

$$\rho(\mathbf{p}) = (\beta/\pi)^{3/2}\, e^{-\beta\rho^2}$$

A number of successive integrations leads to a rather cumbersome expression. The last two integrals must be taken numerically. The results of these calculations are graphically shown in fig. 3. The figure illustrates the angular distributions of fast protons accompanying slow mesons, the parameter β being of a different value. The experimental values obtained by E. Gorzhevskaia and N. Panova are plotted on the same graph. It shows that the agreement with experiment is attained at $1/2\beta M = 0.1$ (14 Mev) i.e. at the same value which is obtained in the analysis of other experiments.

Since the measurement of total cross-sections involves errors, it is worth while determining the relationship between the cross-section and the total cross-section of slow meson photoproduction when the slow meson is accompanied by the emergence of a fast proton. (In the experiment this relationship proved to be equal to 0.25 ± 0.06.) For this purpose expression (17) was integrated with respect to all the variables with the additional condition $P_f^2/2M > 0.14$ and without it. The ratio between the cross-sections proved to be equal to 0.6. It should, however, be considered that in the case of nucleons of an energy in the range of 20-70 Mev the nucleus is not quite transparent.

An estimation of the probability of emergence of the nucleon out of the nucleus without collisions was made on the basis of simple geometrical considerations and an optical model. The composition of the emulsion and the role of individual components were also taken into account. As a result, the above probability proved to be equal to $0.3 \div 0.4$. Thus the final value of the relationship between the cross-sections discussed proved to be equal to $0.18 \div 0.24$, which agrees reasonably with the experimental data.

Hence, the experimental data obtained at the Physical Institute of the USSR Academy of Sciences confirm the value of the single-nucleon model.

Another important problem relating to the mechanism of meson formation on complex nuclei is that of the interaction between the produced meson and the residual nucleus. This problem was dealt with in a number of papers [2,16] on the basis of an optical model. It was then shown that the observed dependence of the photoproduction cross-section on the atomic weight $A^{2/3}$ could be explained on the basis of a self-absorption model.

In the region of photon energies near the meson photoproduction threshold, when the length of the meson wave is great in comparison with the nucleus dimensions, the meson interacts, with a large number of nucleons.

It might be expected that in this case the value of the cross-section is influenced not so much by the effect of meson self-absorption as by the effect of usual interaction, which can be described by some real potential acting between the meson and the nucleus.

It is known [2], for instance, what influence is exercised by the Coulomb interaction on the value $\sigma_{\pi^-}/\sigma_{\pi^+}$ for mesons of the energy $\sim$ 10 Mev.

If the effect so recorded does exist, it should affect, for example, the dependance of σ on Z.†

By a careful analysis of the stars in a photographic emulsion, accompanying the production of slow mesons, E. Gorzhevskaia and N. Panova succeeded in determining the relationship of cross-sections of meson production on light (effective $Z_l = 7.3$) and on heavy ($Z_h = 41$) nuclei of the emulsion. Possible errors are related only to an overestimation of the cross-section of π meson production on heavy nuclei, and to an underestimation of the cross-section of production on light nuclei.

For mesons of an energy in the 1.5–2.5 Mev interval the following equation was obtained:

$$\sigma_l/\sigma_h = 0.26 \pm 0.06$$

Had there been no above-mentioned effect of the specifically nuclear interaction between the meson and the nucleus, and the nucleus had been a charge point the relationship would have been approximately:

$$\sigma_l/\sigma_h \approx C_l^2/C_h^2 \cdot \sigma_l^0/\sigma_h^0.$$

where C_l^2 and C_h^2 are usual Coulomb factors, and σ_l^0 and σ_h^0 are cross-sections, no account being taken of the Coulomb interaction between the produced meson and the residual nucleus. Even assuming that $\sigma^0 \sim$ Const $A^{2/3}$ this relationship should be

$$\sigma_l/\sigma_h = 0.2\,(A_l/A_h)^{2/3} = 0.06$$

And if in the case of very slow mesons $\sigma^0 \sim$ Const A, the relationship would have been still smaller.

Now we shall try to explain this effect by the specifically nuclear interaction meson-nucleus. We shall analyse the meson photoproduction when the length of the meson wave considerably exceeds the dimensions of the nucleus. Instead of C_l and C_h we shall introduce a factor defining the distortion of the wave function near the surface of the nucleus:

$$D = \frac{|\psi(r_1)|^2}{|\psi_0(r_1)|^2} = \frac{1}{(Kr_1)^2}\,|F_0(r_1)\cos\delta + G_0(r_1)\sin\delta|^2$$

Where ψ_0 is the wave function of free motion, and ψ is the wave function with interaction being taken into consideration.

F_0 and G_0 are regular and irregular functions describing two charged particles in an S-state under electrostatic attractive interaction only. They were obtained from the corresponding functions for repulsive interaction [9] and expressed in terms of sums of Bessel functions of integral order with the real argument $2\sqrt{r/R}$:

$$F_0 \cong G\,kr\left[\left(\frac{r}{R}\right)^{-\frac{1}{2}} J_1 - \frac{(kr)^2}{3}\,\frac{R}{r}\,J_2\right]$$

$$G_0 \cong \frac{1}{c}\left\{-\left(\frac{r}{R}\right)^{\frac{1}{2}} Y_1 + J_1\,\frac{(kr)^2}{3} + \frac{(kr)^2}{3}\,Y_2 - h(\eta)\,\frac{1}{CkR}\,F_0(r)\right\}$$

where

$$R = \frac{1}{2Ze^2}\,;\ \eta = \frac{Ze^2}{v}\,;\ h(\eta) = \mathrm{Re}\,\frac{\Gamma'(i\eta)}{\Gamma(i\eta)} - \ln\eta;$$

Conclusions concerning nuclear interaction, characterised by the phase shift δ may be drawn from an analysis of the dependence σ on Z. These conclusions might well supplement the results obtained from a study of the spectrum of mesic atoms [17]. As there are indications that repulsive forces act between the meson and the nucleus

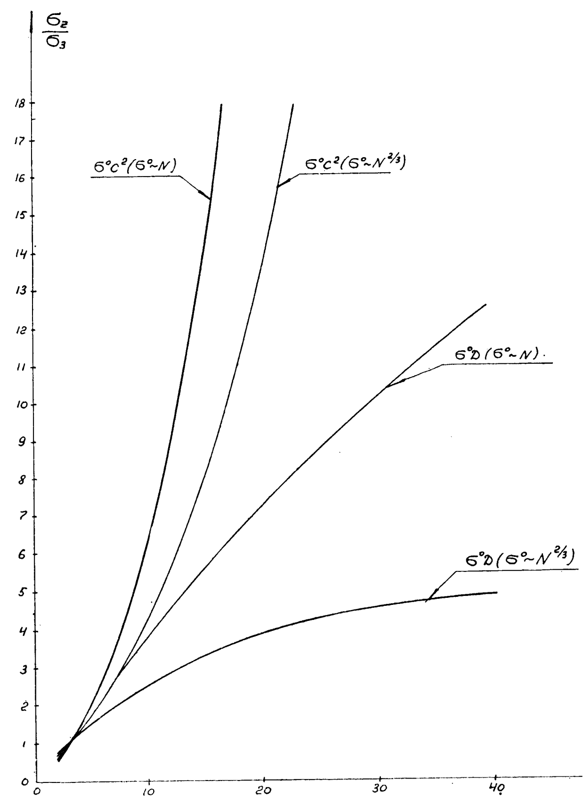

Fig. 4. Theoretically predicted dependence of the cross-section with atomic number for the light nuclei.

† Corresponding experiments have been performed, and a comparison of our results with the experiment is discussed in the report of Belousov et al. (see p. 288).

in an S-state,[14] we shall attempt to choose a repulsive potential of a height independent of Z and of width approximately equalling the nucleus radius.

The usual condition of continuity of a logarithmic derivative of wave functions inside and outside the nucleus shows :

$$\frac{|\psi(r_1)|^2}{|\psi_0(r_1)|^2} = \frac{1}{(\varkappa r_1 \mathrm{cth}\, \varkappa r_1 F_0 - F_0' r_1)^2 + (r_1 G_0^1 - \varkappa r_1 \mathrm{cth}\, \varkappa r_1)^2}$$

where $\varkappa = \sqrt{2v-k^2}$

By simple calculations the following value is obtained for the height of the potential $V \approx 2 \div 7$ Mev, $\sigma_l/\sigma_h = 0.3$. Fig. 4 shows the predicted dependence σ on Z for light nuclei at $V = 4.8$ Mev.†

Naturally enough, these last considerations are preliminary. We should like, however, to stress that the study of meson production on nuclei, when the wave length exceeds the dimensions of the nucleus, may supply valuable information on the interaction between the meson field and the nucleon group. Incidentally, this is true not only for meson production by gamma-rays but for meson production under the influence of any agent as well.

The author is indebted to the members of the experimental group led by V. I. Veksler, M. I. Adamovich, S. P. Kharlamov, V. G. Larionoua, G. V. Kuzmicheva, E. Gorzhevskaia and N. Panova for informing him of their experimental results and numerous discussions.

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DISCUSSION

G. Bernardini remarked that Koerster and Goldwasser had made measurements of the photoelastic production of mesons on He. This appears to be the best case to test Baldin's theory as all assumption he makes appear to be fulfilled.

A. Baldin pointed out that He is a more closed system.

G. Bernardini said that in He there are simultaneously present

1. a strong elastic production;
2. an incoherent pion production (with low yield because of the Pauli principle); and
3. the photodisintegration. The effects found in

† This value of V was obtained from data for mesic atoms [17].

heavy nuclei (surface effect, etc.) are all found in He. But He is so small and simple that the new approach of Baldin can perfectly well be applied to this experiment of Koerster and Goldwasser.

A. Baldin said that indeed the theory should apply to He.

G. Bernardini emphasized the other part of the paper of Baldin where the production of π on D is studied extensively. Orear made a calculation 3 or 4 years ago, neglecting the interaction of the pions with the parent nucleon, which was criticised. Now the paper of Baldin shows this assumption to be good down to low energies.

M. Petermann: "The total S-wave contribution, up to the μ/M terms, to the π^-/π^+ ratio is not contained in the result $(1 + \mu/M)(1 - \mu/M)^{-1}$ given by A. Klein. Such a S-wave contribution to the μ/M terms is the electric dipole absorption.

$$\varepsilon \, \Phi \, \gamma_5 \, (\sigma \, F)$$

where Φ stands for any linear combination of $(\vec{\tau}, \vec{\varphi})$, $(\vec{\tau} \times \vec{\varphi})_3$ and φ_3; F is the electromagnetic field, ε a numerical constant which is not known. Then the theoretical value of about 1.4 for the $\pi^-/\pi+$ ratio may well be changed if this term contributes effectively.

A. Baldin remarked that the values of π^+/π^- ratio contained in his paper include results from both the Veksler group and Bernardini's group.

G. Bernardini pointed out that the calibrations of the beams in the two laboratories should be good as the deuterium results were in good agreement.