SOME NEW DATA CONCERNING THE PHOTOPRODUCTION OF CHARGED PIONS IN HYDROGEN AND DEUTERIUM AT LOW ENERGIES

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1. An analysis of the pellicles exposed in the experiments performed at the Betatron Laboratory of the University of Illinois, by A. O. Hanson, E. Goldwasser, R. Reitz and G. Bernardini and already described [1] has been continued and finally completed. Few previous reports already published [2] presenting the early results are up-to-date. A more extensive account including a discussion of the errors, a detailed description of some of the procedures followed in the pellicle analysis, etc., will appear in two forthcoming papers in Nuovo Cimento. The first of them is expected to be published already when these reports on the Cern Symposium are made available.

Hence the present report concerns mostly a summary of the now completed set of experimental investigations. An attempt at a coherent interpretation of the results is also presented here; but again for the details of the rather lengthy and tedious discussion, the reader may refer to the above mentioned forthcoming papers. The data not yet published are those mostly concerning the laboratory angles: 45°, 105° and 150°. In hydrogen they correspond to the c.m. angles: $\sim 60°$, $\sim 120°$, $\sim 160°$. The pellicles which were exposed at 75° (c.m. angle $\sim 90°$) and already partially analysed by Goldwasser and Bernardini have again been analysed for a comparison and also for appreciably increasing the statistic.

In what follows, for the sake of brevity, most of the notations will be the same as those used by Bernardini [3], and their meaning will be repeated from time to time only for the sake of clarity.

2. We first consider the results concerning the reaction

$$\nu + p \rightarrow \pi^+ + n \qquad (1a)$$

The final values of the differential cross-sections at several energies are given in Table I.

The energy-intervals were actually 165-175 corresponding to the average 170; 175-185 corresponding to 180 and so on. These were the 10 Mev (Laboratory) channels used in analysing the pellicles. To visualize better the energy dependence of the corresponding differential cross-section $d\sigma^+/d\Omega$, we consider (see [3]) the expression:

$$|T^+|^2 = \frac{1}{w}\left(\frac{d\sigma^+}{d\Omega}\right)_{c.m.} \; ; \quad w \simeq \eta\omega/[1 + (\mu/M)\,\nu]^2 \qquad (2)$$

As can easily be seen $T = (\mu/2\pi)\,H$, where H is the usual matrix element. $|T^+|^2$ is a function of ν (photon energy,

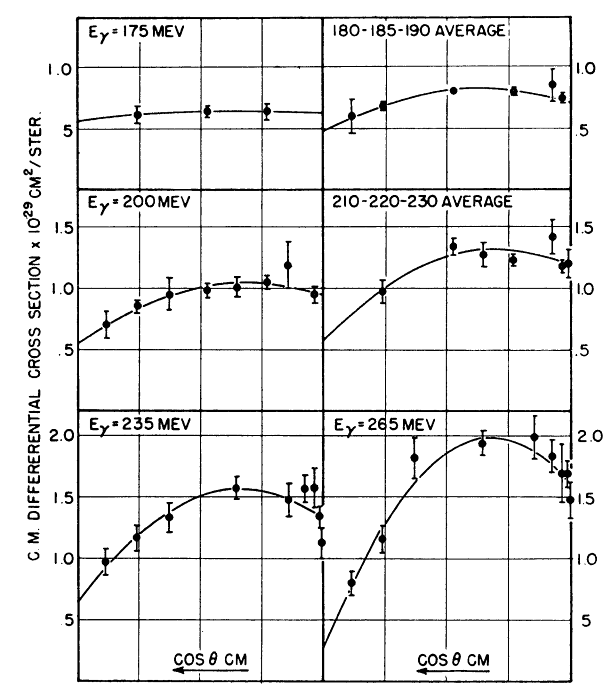

Fig. 1. Angular distribution of charged photo-pions at several energies (see [3]).

c = 1) and ϑ (c.m. angle of projected pion with respect to the incident photon direction). As has already been pointed out [3], in the photon energy interval considered here (170-230 Mev) one may write

$$|T^+|^2 = a_0^+(\nu) + a_1^+(\nu) \cos \vartheta + a_2^+(\nu) \cos^2\vartheta \qquad (3)$$

Fig. 1 where the results of previous experiments are combined with those of Table I, shows the validity of this statement.

TABLE I

c.m. differential cross-sections in units 10^{-30} cm^2/ster

E_γ (Mev., lab.)	59°	93°	123°	159°
170	5.2±.4	5.4±.4	—	—
180	6.5±.5	7.5±.4	7.6±.5	6.6±.9
190	7.2±.5	9.1±.5	8.7±.6	7.5±.6
200	8.0±.6	9.8±.6	10.5±.6	9.5±.7
210	10.0±.7	12.2±.7	10.7±.7	10.7±.8
220	9.7±.9	13.4±.7	12.1±.8	12.1±.8
230	—	—	15.4±.9	13.0±1.0

Table II gives the corresponding values of the a^+ coefficients. The interval of energy 175-195 has been split to make better use of other experiments.

As already mentioned [3], when terms higher than $\cos^2 \vartheta$ are neglected the relationships between the coefficients of the a's of (3) and the angular momenta possibly involved are the following:

$$\begin{aligned} a_0^+ &= |S^+|^2 + a_{op}^+ - {}^1/_3\, a_{SD} \\ a_1^+ &= -2K^+S^+ \\ a_2^+ &= |K^+|^2 - a_{op}^+ + a_{SD} \end{aligned} \qquad (4)$$

It should be remembered that S^+ is the amplitude of the S wave, K^+ is the amplitude of the spin-flip P wave, a_{op}^+ is the remainder of the P wave, and a_{SD} the SD interference term.

The form in which the Eq. (4) are written implies that the different amplitudes are real; this is a good approximation because the complex parts of such amplitudes are of the type $e^{i\alpha}$ where α is the corresponding phase shift of the pion nucleon scattering. Up to ≈ 230 Mev all α's are lower than 10°.

It must be pointed out that, within the limits of experimental errors, a_0^+, as a function of energy, can very well be represented by *a constant*. Precisely

$$a_0^+ = (1.48 \pm .02) \times 10^{-29}\ cm^2.$$

This must also be the value of $|S^+|^2$ at the threshold, because at the threshold a_{op} and a_{SD} must vanish.

As has been already mentioned, a discussion of set (4) has been done by the authors and may be found in the above mentioned forthcoming paper.

The conclusions of this discussion are the following:

Taking into account the two experimental facts:

i) a_0^+ is constant between rather small error limits;
ii) the value of a_1 is quite small and hence K^+ is very small; one is compelled to abandon the presumption *of giving a consistent interpretation* of the experimental

TABLE II

(units 10^{-30} cm^2/ster)

E_γ (Mev, lab.)	W	a_0	a_1	a_2	σ total × × 10^{-30} cm^2
170	.372	14.6±1.1			
175	.428	15.0±0.9	−0.7±2.1	−1.1±5.4	78±7
182.5	.509	14.4±0.6	−2.0±0.9	−2.4±1.6	87±3
192.5	.609	14.8±0.5	−2.7±0.9	−4.5±1.5	101±4
200	.679	14.8±0.6	−3.0±0.8	−3.6±1.6	116±3
210	.770	15.3±0.7	−2.2±1.2	−4.0±2.1	136±5
220	.861	14.6±0.6	−3.6±1.2	−4.3±1.7	142±6
235	.995	14.9±0.8	−3.4±0.6	−5.1±1.3	167±5
265	1.255	14.8±0.7	−5.4±0.6	−7.3±1.1	195±6

results based on the idea that a_{SD} is negligible and a_0^+ *is simply the gauge-invariance term* $G_{os}/\nu\omega$ already discussed in [3]. But in the same discussion it was also stated that we are forced to give up the idea that an appreciable a_{SD} amplitude can be used to re-establish the consistency of the equations (4). Actually the most plausible introduction of higher order waves into (4) can be obtained by using the direct interaction of the photon with the pion current i.e. (see[3]) the term

$$H \propto \frac{\vec{\sigma}\cdot(\vec{\eta}-\vec{\nu})\ \vec{\varepsilon}\cdot\vec{\eta}}{\nu^2\,(1-v\cos\vartheta)}$$

which includes all orbital angular momenta. But in conclusion one finds that (still on the assumption that $a_{os} = G_{os}/\nu\omega$) up to terms in v^2 (v = pion velocity) the introduction of H does not affect the magnitude and energy dependence of the coefficients K^+ and a^+_{op} to such an extent as to explain the above-mentioned lack of consistency.

3. As an alternative (and probably as the more plausible one) way of interpreting the experimental data the introduction of nucleon recoils, fairly large also at very low energies, has been considered. To do this one takes into account also the reaction

$$\nu + N \rightarrow P + \pi^-$$

Then considering both pion-signs the set (4) can be written

$$\begin{cases} a_0^{\pm} = (V \mp R)^2 + a_{op}^{\pm} \\ a_1^{\pm} = -2\,(V \mp R)\,K^{\pm} \\ a_2^{\pm} = |\,K^{\pm}\,|^2 - a_{op}^{\pm} \end{cases} \tag{5}$$

Here R is the part of the nucleon recoil $\left(\lim_{M\to\infty} R = 0\right)$ due to the term in the Hamiltonian which is invariant (scalar) with respect to rotations in isospin-space. V is the S amplitude which transforms as a third component in isospin-space, and

$$\lim_{\mu/M\to 0} V = \sqrt{\frac{G_{os}}{\nu\omega}} \tag{6}$$

Let us write :

$$V = \frac{g_{os}}{\sqrt{\nu\omega}} + \rho \tag{7}$$

where ρ is the part of the nucleon recoil which changes sign with g_{os} *.

Obviously V and R could be experimentally determined if also the cross-section σ^- (and hence the amplitude $S^- = V + R$) for photoproduction of π^- from *free* neutrons at several photon energies, were known. But σ^- is up to now known only through the experiments in deuterium. Precisely (because of the competing process $\nu + D \rightarrow N + P$ and of the indefiniteness of the kinematics due to the internal nucleon motion) only the ratio π^-/π^+ between the rates of production for negative and positive pions in the $\nu + D \begin{smallmatrix} \nearrow (PP) + \pi^- \\ \searrow (NN) + \pi^+ \end{smallmatrix}$ reactions are fairly well known.

At low energies the results obtained with a target of liquid D_2 are those given in Table III.

TABLE III

The π^-/π^+ ratio as function of E_γ and ϑ

E_γ (In Mev) \ ϑ	59°	93°	123°	159°
170	1.67±.19	1.50±.15	—	—
180	1.35±.12	1.41±.10	1.47±.11	—
190	1.19±.10	1.41±.09	1.23±.16	1.38±.13
200	1.25±.12	1.28±.09	1.14±.15	1.46±.12
210	1.16±.12	1.05±.08	1.62±.21	1.37±.12
220	1.10±.12	1.26±.09	1.34±.16	1.31±.10
230	.96±.21	1.01±.08	1.32±.10	1.67±.13

As has been pointed out many times, this π^-/π^+ ratio in D_2 deviates from that of free nucleons mainly because of the influence of the Coulomb interaction of the final states. The amount of this deviation depends upon the photon energy, or more properly from the relative velocities of the two final nucleons and from the velocity of the outgoing pion.

The value of this deviation has been theoretically estimated many times but the results depend very much upon the several possible assumptions and seem to be rather vague and contradictory.

Considering the uncertainties of the evaluations mentioned above it was considered more correct and safe to neglect completely at these energies any correction for the influence of the final states.

We are supported in this attitude by the conclusions of Baldin's work [16], (kindly communicated by Prof. Veksler and by the author) which seems to be the most detailed discussion on this matter. Fortunately at the photon-energies considered here (from ~ 170 up to ~ 230 Mev) the expected corrections are always less than ~ 10%; while the experimental errors are of the same magnitude or even larger. Further, the work of

* Frequently and more properly the ± sign instead of being applied to R is displaced to V, but for what follows, it was found more convenient to apply the sign to the scalar part R because it is directly correlated with the pion-sign.

Adamovich and others[17]), shows directly that within our limits of error significant deviations from the scheme of free-nucleons are not expected for photon energies $\geqslant$ 170 Mev.

Consequently the product $\sigma^+ \times (\pi^-/\pi^+)$ is supposed to be a fairly good value of σ^-, i.e. of the cross-section of the reaction (8) $\nu + n \rightarrow \pi^- + p$. The values of σ^- for several energy intervals have been evaluated and the corresponding a^- coefficients have been estimated as in the case of the a^+s, but of course with appreciable errors. For instance averaging in the energy interval 180-220 Mev one finds

$$a_1^-/a_1^+ = 2.1 \pm 0.2 \quad a^-_2/a^+_2 = 0.9 \pm 0.4$$

and for a^-_0 the following values

E_γ	170-190	200-220
a_0^-	2.12 ± 0.12	1.76 ± 0.10

In the same energy interval, i.e. around 200 Mev one finds

$$R = \tfrac{1}{2}(S^- - S^+) \approx (0.3 \pm 0.1) \times 10^{-15} \quad \text{and}$$

$$\tfrac{1}{2}\sqrt{\nu\omega}\,(S^- + S^+) = \sqrt{\nu\omega}\; V = (4.21 \pm 0.18) \times 10^{-15}$$

Now as a direct consequence of the Kroll and Ruderman theorem [18]) it is found that *at the threshold*, ρ in (7) has to be of an order in (μ/M) smaller than R. If one considers this fact as a good argument for supporting the assumption that near the threshold ρ is always much smaller than R and practically (within our limits of error) negligible, then

$$g_{os} \simeq \sqrt{\nu\omega}\, V \simeq \text{constant} = (4.21 \pm 0.18) \times 10^{-15}.$$

It is worth mentioning that this has been found to be correct within the limits of error, also considering separately the several energy intervals of Tables I and III.

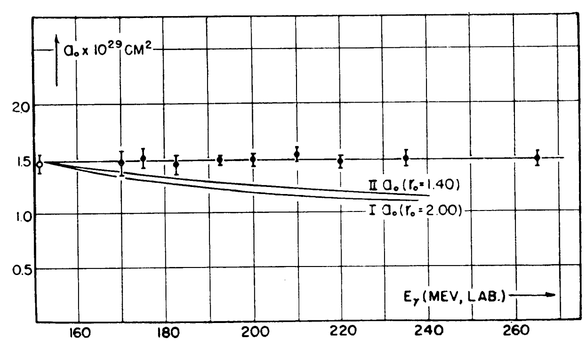

Fig. 2. Plot of the coefficient a_0 (see text) showing in which error limits it has to be considered constant with respect to the photon-energy.

Simultaneously the value of R at the threshold ($\omega = 1$, $\nu = \nu_t = 0.935$) would turn out to be

$$R(\omega = 1) = V(\omega = 1) - \sqrt{a_0} \simeq 0.7 \pm 0.2 \qquad (10)$$

In (10) the value of a_0 is the extrapolated value $a_0 = (14.8 \pm .02) \times 10^{-30}$ **.

As was seen in Table II the lowest energy reached in our measurement is $\sim$ 170 Mev. Our measurements do not exclude an increase of a_0 in the last 20 Mev over threshold. However, the experiment performed by Leiss, Penner and Robinson [15]) in the energy interval 150-170 Mev gave a value of a_0 at the threshold in excellent agreement with the value mentioned above. The situation is well represented in fig. 2 where the results of the two experiments are plotted. The straight line corresponds to the least square-fit of the data of Table II only. Hence at present * it seems quite reasonable to consider (and certainly within the limits of error) $a_0 = 14.8 \times 10^{-30}$ over the entire range between the threshold and 250 Mev.

If this is correct, and actually $\rho \ll R$ then the value of $R(\omega = 1)$ with respect to the average value of R found for $E_\gamma \simeq 200$ Mev seems to deviate from the expected energy dependence.

With a slightly different approach one may reach some more definite conclusion in the same direction. The assumptions, certainly correct within the limits of error are:

i) $K^+ \neq K^-$ but still $|K^-|^2 \ll |a_2^-|$ and then

ii) $-a_2^+ \simeq a_{op}^+ \simeq a_{op}^- \simeq -a_2^-$.

This is equivalent to including any nucleon recoil effect of the p-wave in its flipping part. Hence from (5) independently of any structure of S and considering only the a-coefficients corresponding to the hydrogen experiment one has:

$$R = \tfrac{1}{2}\sqrt{a_0}\,\{\sqrt{r - a_{op}/a_0} - \sqrt{1 - a_{op}/a_0}\,\} \qquad (11)$$

where $r = a_0^-/a_0^+$ i.e. the ratio $\sigma^-(\theta)/\sigma^+(\theta)$ at 90° c.m. In Table IV are listed some values of R corresponding to four different energies.

From (11), R turns out to be a definitely decreasing function of the energy because experimentally r is just decreasing while a_{op} is increasing. It should be noted that this conclusion is based on two experimental results, i.e. the constancy of a_0, and the behaviour of σ^-/σ^+ at 90° where better data are available, and where Cal Tech results are in good agreement with ours.

* One may consider the Leiss, Penner and Robinson experiment to be somewhat indirect. In fig. 2 are also plotted the expected energy dependence of a_0 for the extreme values of the π^-/π^+ ratio at the threshold. See later.

** Note added in proof. In this extrapolation the influence of the direct interaction term at low energies has not been taken into account. In accordance with Chew and Low [Phys. Rev. *101*, 1579 (1956)] this term introduces a minimum at 10-15 Mev above the threshold in the square of the matrix element. With this term the value of the coupling constant needed to fit the experimental results is $f^2 \simeq 0.073$.

On the other hand

$$V = (g_{os}/\sqrt{\nu\omega}) + \rho = (\sqrt{a_0}/2)\{\sqrt{r - a_{op}/a_0} + \sqrt{1 - a_{op}/a_0}\} \quad (12)$$

At present the accuracy in the measurements of a_0, a_2 and r consents only a very crude treatment of the data. Accordingly, we assume consistently with the constancy of $\sqrt{\nu\omega}\,V$ that ρ is negligible with respect to R. With this approximation the σ^-/σ^+ ratio at 90° c.m. can be expressed as follows :

$$r = (1/a_0)\{\sqrt{a_0\nu_t/\nu\omega}\,(1 + \sqrt{r_0}) - \sqrt{a_0 + a_2}\}^2 - a_2/a_0 \quad (13)$$

where r_0 is the value of σ^-/σ^+ at threshold.

This formula is merely an *empirical* formula from which the influence of the pure P-wave term a_2 is properly subtracted. No particular assumptions are given as to the dependence of r or R on ω except the oversimplifications indicated previously. The purpose of using (13) is merely to find the upper energy limits for which the data can be used for the evaluation of $R(\omega = 1)$. In this sense, a first check of (13) is to find out up to what energy it gives, within the limits of error, a constant value of r_0. The r values obtained by the deuterium experiment * are given in Table V together with the corresponding values of r_0.

TABLE IV

(Units 10^{-15} cm.)

E_γ (Mev, lab.)	R	V	$g_{os}/\sqrt{\nu\omega}$
170	.46±.12	4.07±.15	4.10±.15
185	.39±.06	3.86±.14	3.82±.14
205	.16±.06	3.46±.19	3.51±.19
225	.14±.06	3.31±.23	3.25±.23

TABLE V

E_γ (Mev,lab.)	170	185	205	225
r	1.50±.15	1.41±.07	1.15±.06	1.12±.06
r_0	1.82±.22	1.93±.22	1.78±.33	2.00±.47

From this Table one has $r_0 \equiv 1.87 \pm .13$.

Knowing r_0 one is able to estimate R and g_{os}. With $r_0 = 1.87 \pm .13$ we find

$$R(\omega = 1) = \tfrac{1}{2}\sqrt{a_0\nu_t}\,(\sqrt{r_0} - 1) = (.68 \pm .09) \times 10^{-15}\text{ cm.}$$

$$g_{os} = \tfrac{1}{2}\sqrt{a_0\nu_t}\,(\sqrt{r_0} + 1) = (4.39 \pm .09) \times 10^{-15}\text{ cm.}$$

In Table IV, column 4, are given the values of V obtained directly from the g_{os} calculated above. It is seen that the V values of the third and fourth columns are practically coincident. This means that *within the limits of error ρ is negligible.* This means also that *the curious energy dependence indicated by* (11) *is explained not by ρ but by the neglected higher order terms of R.*

From the above value of g_{os} the interaction constant in the (PS, PV) coupling turns out to be

$$f^2 = g_r^2(\mu/2M)^2 = .067 \pm .003$$

where g_r is the renormalized PSPS interaction constant.

The net result of the preceding analysis is merely the following : *the observed angular distributions for photopions produced by protons can be explained by the use of an appropriate value of r_0. This value was not picked up* ad hoc *but is the result of measurements on deuterium at energies where the influence of binding and Coulomb forces is supposed to be within the limits of the experimental errors small and properly estimated. Further, the extrapolation to zero energy was based on the hydrogen experiment only.*

This value of r_0 is very high. Particularly it is much higher than that predicted by the perturbation theory. However, a large value of r_0 has the great advantage of making the analysis of the s-wave phase shifts more consistent. In fact, one may apply again the Anderson-Fermi argument. The difference lies now in the fact that (as indicated by fig. 2) one may consider the extrapolated value of $(1/W)\,\sigma_T(p_\gamma \,|\, n^+)$ for $\eta \to 0$ which is now quite well determined.

The most recent value of the s-wave phase shifts is

$$(\alpha_1 - \alpha_3)/\eta = 0.27 \pm .03.$$

The value of the Panofsky branching ratio suggested by recent experiments is **

$$\frac{\langle p^- | n^0 \rangle^2}{\langle p^- | n^\gamma \rangle^2} = 1.25 \pm .10\,.$$

The corresponding value of r_0 could be $r_0 = 2.11$ with an estimated error of about 20%. It is then consistent with the high value of r_0 previously mentioned.

* The values given here are averages in the intervals 170-190 Mev, 200-210 Mev, and 220-230 Mev.

** We took here the average of the following values :

.95 ± .20	Panofsky et al.
1.46 ± .20	Cassels et al.
1.38 ± .20	Merrison et al.
1.10 ± .50	Lederman et al.

It may be worthwhile to notice that independently of the value of r_0, whether large or not, the approximation $R = const.$* seems to be almost excluded by the fact that a_0 is constant up to 265 Mev. This is shown in fig. 2 where curve II represents the expected trend of a_0 when it is assumed $r_0 = 1.40$, while curve I is calculated taking $r_0 = 2.00$. Obviously, the introduction of p-wave recoils does not change this conclusion because, as long as $|K|^2$ is negligible or small, the p-wave included in a_0 is given directly by a_2.

Beside this point, one should admit that the quite large value of σ^-/σ^+ seems rather unusual. At present it seems very difficult to find any plausible explanation of this fact.

However, it may be pointed out that several determinations of the interaction constant based on pion-nucleon scattering systematically give a value of f^2 around 0.08 or larger. Instead, by using the value of r_0 required by the perturbation theory, i.e., $r_0 = 1.35$ the value of f^2 determined from the threshold cross-section of charged photopions drops to the very low value $f^2 = 0.056 \pm 0.005$.

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DISCUSSION

V. I. Veksler : In this report there are no remarks about taking into account nuclear interaction of the nucleons in the final state, but this interaction, as was pointed out by our investigation concerning the photodisintegration of deuteron, is very important.

M. Beneventano : In the hypothesis of "impulse approximation" the π^-/π^+ ratio on D_2 is :

$$\frac{\pi^-}{\pi^+} = \frac{\int \{ |K^-|^2 + |L^-|^2 \} I_1 N_\gamma dv + \ldots}{\int \{ |K^+|^2 + |L^+|^2 \} I_1 N_\gamma dv + \ldots}$$

where N_γ is the differential spectrum of the γ-rays, $|K|^2$ and $|L|^2$ are the contributions of the spin-flipping and not spin-flipping part of the interaction and I_1 is the same integral quoted by M. Lax and H. Feshbach. The pure nuclear interaction contributes with the same factor in the numerator and denominator.

The Coulomb interaction is present only in the numerator and has been roughly evaluated. In the energy region of our experiment, this contribution seems to be not more than 7% with a sign which was not well known. Now this sign seems to be well established by A. M. Baldin.

* Such an assumption was also followed recently by Klein.