

# SURVEY ON PHOTOPION PRODUCTION BY NUCLEONS

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1. In a first period the interest on photopions was mostly concentrated on their connection with pion-nucleon scattering.

It would have been very surprising if this connection had failed because based on conversation principles; i.e., at large on symmetry properties; the charge independence being applied to the final pion-nucleon states, but, of course, not to the initial ones where a photon is present. The S-matrix formalism plus time reversal has been the most natural way of dealing with this matter. It was worked out by many authors such as Aidzu, Fermi, Watson, Kawaguchi Minami, and others.

To remind the essentials of this analysis let us say that the S-matrix formalism is in some respects a generalized partial waves analysis. Without the use of the isospin as a new degree of freedom, one would proceed as usual; i.e., decomposing the incident (plane wave) photon into parity and angular momenta substates

$$A(\mathbf{r}, t) = \underbrace{(1/\sqrt{2})(\epsilon_x \pm i\epsilon_y)}_{\substack{s=1 \\ m_s = \pm 1}} \underbrace{[e^{i(kz - \omega t)} + \text{complex conj.}]}_{\substack{l_\gamma = 0, 1, 2... \\ m_\gamma = 0}}$$

and getting the series of poles

$J_\gamma$	1	1	2	2
$M_\gamma$	$\pm 1$	$\pm 1$	$\pm 1$	$\pm 1$
Parity	—	+	+	—
	$E_1$	$M_1$	$E_2$	$M_2$ etc.

In the energy intervals up to now, well or fairly well explored; i.e., between 150 and 400 Mev, the average e.m. size  $r$  of the nucleon is according to Hofstadter and co-workers :

$$r \cong 0,7 \hbar/\mu c$$

The corresponding angular momenta of the photon are then

$$\hbar l \cong (E_\gamma/c) r < 2 \hbar$$

However, for the sake of simplicity, only S-P and interference terms of the D-wave were usually considered. This seems somewhat justified by the absence of appreciable D-waves on pion-nucleon scattering and by the attenuating influence of statistical factors.

With these limitations, the cross-section for photo-production

$$d\sigma/d\Omega = A(\nu) + B(\nu) \cos \theta + C(\nu) \cos^2 \theta + \dots \quad (1)$$

(where the  $\nu$  is the photon energy) can be stopped at the third term. The correlated pion-nucleon states and photon pole *amplitudes* are the following (in parentheses are indicated the total angular momenta)

$p^\pi$	$S_{1/2}$	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$
$p^\gamma$	$E_1(1/2)$	$M_1(1/2)$	$M_1(3/2)$ $E_2(3/2)$	$E_1(3/2)$ $M_2(3/2)$
parity	—	+	+	—

For a better understanding of the energy dependence it is sometimes convenient to extract from the A, B and C coefficients the more trivial energy-dependent factors. Then one writes

$$d\sigma/d\Omega = W |T|^2$$

with

$$|T|^2 = A/W + (B/W) \cos \theta + (C/W) \cos^2 \theta = a + b \cos \theta + c \cos^2 \theta \quad (2)$$

and

$$W \cong \frac{\eta \omega}{\left(1 + \frac{\mu \nu}{M}\right)^2} \quad \text{with} \quad \begin{aligned} \eta &= p_\pi/\mu c \\ \omega &= \sqrt{1 + \eta^2} \\ \nu &= E_\gamma/\mu c \end{aligned} \quad (3)$$

W is proportional to the density  $\rho_F$  of the final states divided by the relative velocity  $v_r$  of the particles in the initial state. Then  $T = (\mu/2\pi) H$  where H is the usual matrix element. Frequently instead of the factor W, another factor is used. Precisely  $\chi = W/\nu\omega$ . This is the Møller's relativistically invariant way of writing the cross-section and of course is usually favoured in the theoretical papers.

In the limits of these assumptions, the relationships between the  $|T|^2$  coefficients and the angular momenta are

$$\begin{aligned} a &= a_{os} + a_{op} - \frac{1}{3} a_{SD} = E_1(\frac{1}{2}) \cdot E_1^*(\frac{1}{2}) + a_{op} - \frac{1}{3} a_{SD} \\ b &= -2 \operatorname{Re} [E_1^*(\frac{1}{2}) K] \\ c &= KK^* - a_{op} + a_{SD} \end{aligned} \quad (4)$$

where

$$\begin{aligned} a_{os} &= [\text{S-wave amplitude}]^2 \\ a_{op} &= [\text{P-wave amplitude, non spin-flipping part}]^2 \\ KK^* &= [\text{P-wave, spin flipping part}]^2 \\ a_{SD} &= \text{S-D interference term.} \end{aligned}$$

Now in order to discuss the interference of several amplitudes phases are necessary. But because of the introduction of the isospin as a new degree of freedom, the phases now depend on I as well as J. Precisely the use of S-matrix and time reversal previously mentioned brings us to the conclusion that the complex parts of each multipole are the pion-nucleon scattering phase factors  $e^{i\alpha_{2I, 2J}}$ . We use here the Fermi's notations. I and J are the isospin and the total angular momentum of the final pion-nucleon system.

For instance, it is found that

$$\begin{aligned} E_1(\frac{1}{2}) &= \sqrt{2} E_3 e^{i\alpha_3} + (1/\sqrt{2}) E_1 e^{i\alpha_1} \\ M_1(\frac{1}{2}) &= \sqrt{2} M_{31} e^{i\alpha_{31}} + (1/\sqrt{2}) M_{11} e^{i\alpha_{11}} \\ M(\frac{3}{2}) &= \sqrt{2} M_{33} e^{i\alpha_{33}} + (1/\sqrt{2}) M_{13} e^{i\alpha_{13}} \\ &\text{etc.} \end{aligned} \quad (5)$$

The numerical coefficients are the Clebsch-Gordan ones and  $M_{33}$  etc. are real quantities depending upon pion and photon energy (momentum). The relationships (5) are the net result of the S-matrix formalism. However, this formalism is not and could not be adequate for determining the magnitude and energy dependence of the coefficients  $E_1$   $M_{31}$  etc. For this purpose a theory or some models going into the detailed e.m. structure of the physical nucleon are required. This may be obvious because, as it has been said, the analysis is based only on conservative principles. The simple aspect of the poles indicated by (5) reveals only the independence of the isospin variables from the electro-magnetics ones. But of the possible transitions some will be emphasized others depressed according to the particular electro-magnetic and mesonic structure of the nucleon states involved.

2. Theoretically, the first very significant step can be taken by using the Kroll and Ruderman theorem. Briefly, this theorem states: In the limit in which the total meson energy  $\omega$  goes to zero, the renormalized PSPS matrix  $H_r$  for photoproduction coincides with that  $H_\pi$  obtained in the same limit by the second order perturbation approximation (Born approximation).

That is

$$\lim_{\omega \rightarrow 0} H_r = \lim_{\omega \rightarrow 0} H_\pi = \frac{e g_r}{\sqrt{2M}} \frac{\vec{\sigma} \cdot \vec{\varepsilon}}{\sqrt{\nu\omega}} \quad (6)$$

where  $\omega$  is the total energy of the outgoing real pion and  $e = 1/\sqrt{137}$ ;  $g_r$  is the renormalized symmetrical PSPS coupling constant;  $\vec{\sigma}$  = nucleon spin,  $\vec{\varepsilon}$  = photon vector,  $M$  = nucleon mass,  $\nu$  = photon energy-momentum.

In the real case  $\mu \neq 0$ , as a consequence of the theorem itself one finds that *at threshold* for charged pions

$$\begin{aligned} \langle n^+ | H | p^\gamma \rangle^2 &= (1/\nu\omega) [G_{os} + \omega/M G_1 + H_1(\omega/M)] \\ \langle p^- | H | n^\gamma \rangle^2 &= (1/\nu\omega) [G_{os} + \omega/M G_2 + H_2(\omega/M)] \end{aligned} \quad (7)$$

where  $G_1$  and  $G_2$  for charged pions are independent from  $\omega/M$  and equal and opposite in sign. Obviously  $G_1$ ,  $H_1$  etc... are nucleon-recoiling terms, all vanishing when  $M \rightarrow \infty$ . The first term  $G_{os}$  is a S term only and coincides with that of the non-relativistic limit when  $M \rightarrow \infty$ .

In other words, according to the equivalence theorem of Dyson

$$G_{os}/\nu\omega = \langle n^+ | i\alpha \vec{\sigma} \cdot \vec{\varepsilon} | p^\gamma \rangle^2 \quad (8)$$

where

$$\alpha = \frac{ef}{\mu} \sqrt{\frac{2}{\nu\omega}}, \quad f = g_r \left( \frac{\mu}{2M} \right) \quad (9)$$

$f$  being the PSPV renormalized coupling constant.

For the neutral pions, the first term of the matrix element similar to (7) is zero. Then, at threshold the neutral pion—S-wave, is due to nucleons recoils only. Thus the Kroll and Ruderman theorem gives us the possibility of knowing, at least near threshold, the essential part of the electric dipole amplitude  $E_{1/2}$  in (5). Precisely neglecting nucleon recoils, for charged pions

$$E_{1/2} \cong \sqrt{G_{os}} / \sqrt{\nu\omega}$$

and for the neutral ones  $E_{1/2}^0 = 0$ . The actual behaviour of the S-waves near threshold will be discussed in the report by Beneventano et al.<sup>1)</sup>

3. Assuming that in this way, the origin of the main part of the S amplitude is known, as a second step, one may try to explain the origin of the higher angular momentum waves; that is in our limitations, essentially P-waves. Obviously beside the difficulties of the evaluations one is immediately tempted to establish the most possible and most close connection with the successful theories of pion-nucleon scattering.

This can be achieved in many ways, more or less complicated, more or less rewarding. They have a common

pattern. The interaction  $H_{ef}$  contains of course both the coupling constants  $e$  and  $f$ . As far as  $e$  is involved a (first order) perturbation theory may be applied. That is one calculates the matrix element starting with an initial state which is a real nucleon, and treating it, as far as is the e.m. interaction (and this alone !) in a first order approach. The final state consists of a real nucleon and a real meson. The way of connecting through the interaction these two states is the main job of any theory or model. All of them bring us to the almost inevitable conclusion that all multipoles amplitudes can be split into the product of two parts. The first expresses the electro-magnetic interaction of the photon with the nucleon (including the pion-current); the second represents the pion-nucleon interaction. Thus, on the basis of general arguments, one may write for each pole (see for notations formula (5).

$$\left. \begin{array}{l} M_{2I,2J} \\ E_{2I,2J} \end{array} \right\} = \left. \begin{array}{l} m_{2I,2J} \\ e_{2I,2J} \end{array} \right\} \gamma^I f(\nu) F(\eta) \quad (10)$$

A central-type force model of the pion-nucleon system has been discussed by Fermi and Gell-Mann<sup>2)</sup>. According to this simple model the matrix element associated with a definite value of  $l_\pi$  can be written :

$$\langle \psi_f | H | \psi_i \rangle = \psi_f(\bar{r}) \int Q(r) r^2 dr \quad (11)$$

where  $\psi_f$  is the final pion-nucleon state; i.e. the pion-nucleon scattering wave function. The radial factor  $\psi_f(\bar{r})$  is then the phase-shifted radial solution of the pion-nucleon system, i.e. the linear combination of the regular and irregular Bessel functions :

$$\psi_f(r) = [j_l(\eta r) \cos \alpha_{2I,2J} + \eta_l(\eta r) \sin \alpha_{2I,2J}] \quad (12)$$

$\psi_f(\bar{r})$  is its value for the averaged distance  $\bar{r}$ . Obviously  $\bar{r}$  may be considered as the "effective distance" at which the pion was created.

The integrals :  $\int Q(r) r^2 dr$  do not depend upon  $\alpha_{2I,2J}$  and  $\eta$ , but may well depend upon  $I, J$  and the particular e.m. pole. These integrals are essentially the unknown functions  $f(\nu)$  and they represent the e.m. properties of the nucleon pion-system. Through these functions the photopion processes have some interesting relationships with the nucleon structure different from that of the scattering experiments. With this very simple model by comparison with (10), one has, for instance, for the magnetic dipole and the electric quadrupole matrices of the "enhanced" pion nucleon state  $I = 3/2, J = 3/2$ , the following expressions :

$$M_{33} = m_{33} f(\nu) \sin \alpha_{33}/\eta^2; \quad E_{33} = e_{33} f(\nu) \sin \alpha_{33}/\eta^2$$

Near resonance, this energy dependence for the enhanced state, derives immediately from (12) but<sup>3)</sup> is expected for any short range strongly interacting two-particles system and will be found as well in the more elaborated model of Chew and Low. Near threshold the Fermi's model (11)

and (12) says that both the functions in (10) :  $f(\nu)$  and  $F(\eta)$  are  $\cong \text{const.}$

The so-called "general enhanced model" of Watson and others follows these lines of reasoning. On this model of the P phases only  $\alpha_{33}$  is considered different from zero and its behaviour directly taken from the pion-proton scattering. The amplitudes (5) then assume the simplified forms  $\sqrt{2} M_{33} e^{i\alpha_{33}} + (1/\sqrt{2}) M_{13}$  etc. All waves, including the S-ones previously discussed and the contributions due to the nucleon recoils are considered, but the squared partial elements due to the non-enhanced P-waves are considered very small and dropped. The products as  $m_{33} f(\nu)$  etc. in (10) are left undetermined and deduced from the experimental values of the coefficients in (1). To some extent one may say that the analysis is a method for presenting the essential of the experimental data. Via the known phases of the pion-scattering the values of e.m. parameters of the pion-nucleon system are given as experimentally determined functions of incident photon energy.

Thus we may say that this "enhanced" model represents more a very useful guide on the analysis of the experiments than a theory. In this respect the success of this model has been emphasized many times.

For instance it is well known that it predicted the determinant role of the  $\alpha_{33}$  state on the behaviour of the total cross-section  $\sigma_\gamma^0$  of the photo-production of  $\pi^0$ ; particularly the maximum of this cross-section due to the "33 resonance" around the expected photon energy  $E_\gamma = 320$  Mev (Lab.).

It predicts also quite a few more refined facts. For instance it is true that for  $\pi^0$  the first coefficient of (1) :  $A_0 = (d\sigma/d\Omega)_0$  at  $90^\circ$  c.m. around resonance is about twice  $A_+ = W_{\text{opp}}$  as it is expected immediately from charge independence. But far from resonance for instance for  $E_\gamma \cong 250$  Mev,  $A_0$  differs from  $2 A_+$  mainly for an extra-term due to the interference of the enhanced and non-enhanced P amplitudes. This term is roughly proportional to  $\cos \alpha_{33}$  and thus at energies far from resonance  $A_0$  is appreciably smaller (or beyond resonance, larger) than twice the corresponding coefficient for charged pion i.e.  $2 A_+$ .

Another example which may be useful in discussing some recent experimental results by Corson and by Osborne et al.<sup>4)</sup> is the comparison between the two  $\cos \theta$ -coefficients for charged and neutral photopions. With reference to the Eq. (1) (which can be written as well for charged (+) as for neutral (0) pions) one finds<sup>5)</sup> that  $B_+$  depends mainly on the expression  $\cos(\alpha_{33} - \alpha_3) + 2 \cos(\alpha_{33} - \alpha_1)$  while  $B_0$  depends more upon the product  $r[\cos(\alpha_{33} - \alpha_3) + \frac{1}{2} \cos(\alpha_{33} - \alpha_1)]$ . Here  $r$  is the ratio between the changing sign S amplitude due to the nucleon recoil and the total S amplitude. It can be determined by the ratio  $\sigma_s^-/\sigma_s^+$  of the S-wave cross-sections for negative and positive pions. Taking into account the Orear's values of  $\alpha_3$  and  $\alpha_1$ , the coefficients  $B_+$  and  $B_0$  are not supposed to be both vanishing (as it was considered some times) near resonance. Instead  $B_0$  is likely vanishing at an energy appreciably lower than resonance ( $\cong 270$  Mev according

to some evaluations made by L. J. Koester) and certainly lower than the energy at which  $B_+$  is vanishing. The recent results of Corson <sup>9)</sup> seem to give indication of this fact.

Finally the analysis of the coefficients A, B and C for charged and neutral pions allows the derivation of some conclusions about the particular behaviour of some of the multipoles involved. An interesting case is presented by the electric quadrupole  $E_2(3/2)$ . It is a typical "dynamical" e. m. pole in the sense that  $E_2(3/2) = 0$  for  $\nu \rightarrow 0$ . In other words its presence reveals an appreciable "excitation" of the pion cloud. However, at low energies ( $\nu \lesssim 2\mu$ ) it is supposed to be rather small; as well as all "dynamical" terms. At least this is expected on the base of the Chew and Low theory. The well known Caltech results <sup>7,8)</sup> gave originally  $E_2(3/2) \cong M_1(3/2)$  as a consequence of the fact that experimentally it was found  $A_0/C_0 \cong 1.25$ . However, adding to the early Cal Tech data the new ones obtained by Corson <sup>4)</sup>, the ratio  $A_0/C_0$  raises up and more nearly approaches the value  $5/3$  required by a negligible  $E_2(3/2)$ . The situation is shown in fig. 1 where the early Cal Tech data and the new ones due to Corson are reported.

It is worthwhile to mention that the effective quadrupole amplitude coming in this ratio is the half of  $E_2(3/2)$ . This half is the amplitude considered by Chew and Low. Perhaps one may notice that this conclusion about the smallness of  $E_2(3/2)$  seems to be in contradiction with the rather well determined value of  $B_+$  below resonance. This coefficient at least at low energies is rather small <sup>1)</sup>. This fact has as a consequence (see equation 4) that  $|K|^2$  is smaller than  $a_{0p}$  at least for a factor 10. But as it may easily be seen <sup>9)</sup>  $K = M_1(3/2) - [\frac{1}{2}E_2(3/2) + M_1(1/2)]$ . It would follow that either  $E_2(3/2)$  or  $M_1(1/2)$  or both were not

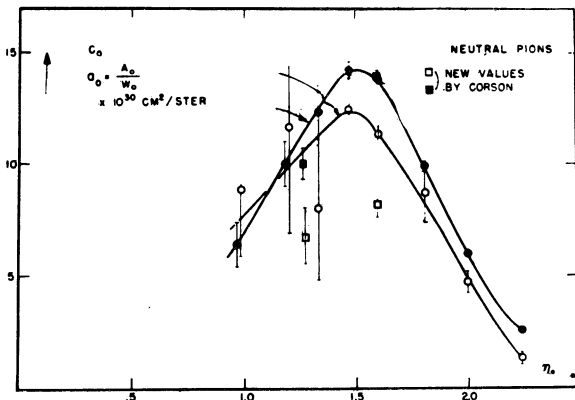


Fig. 1. The values of the coefficients  $A_0$  and  $C_0$  (see text) according to the old Cal Tech data and the new ones due to Corson.

negligible. Quite likely it is  $M_1(1/2)$  which near the threshold is not so negligible; while it is so in comparison with  $M_1(3/2)$  near the resonance. This sketch of a discussion concerning the electric quadrupole is just an example of the other similar cases. This type of a discussion points out in what direction new experiments and theoretical evaluations may be pursued for a better understanding of the nucleon structure.

At present the most uncertain informations are those concerning the neutral pions. Much more accurate measurements of  $[d\sigma(\nu,0)/d\Omega]_0$  will be of considerable help in solving some puzzling questions concerning: the behaviour of the S-phases  $\alpha_1$  and  $\alpha_3$ ; the contribution of the nucleon recoils on the S and P amplitudes; the importance of the "dynamical" part of the e.m. poles of the nucleon; etc.

4. Of all recent theories and models the most successful has been that originally suggested by Chew <sup>10)</sup> and then extensively developed by Chew and Low <sup>11)</sup>. Unfortunately as far as it concerns the photomeson production it does not share the essential simplicity of the other fundamental paper by the same authors on the "p-wave pion-nucleon interaction" <sup>12)</sup>. Of course this statement may reflect only the difficulties encountered by the speaker in dealing with this matter.

The starting point is as usual, the matrix element

$$H_k(q) = \langle \psi_q^-, \int d\nu (-\mathbf{j} \cdot \mathbf{A}_k) \psi_0 \rangle \quad (13)$$

where  $\mathbf{j}$  is the *total* current density operator for pion-nucleon system. Here the index  $k$  refers to the photon (which momentum is  $\mathbf{k}$  and polarization  $\vec{\epsilon}$ ) while the index  $q$  summarizes the momentum coordinates and the isospin of the pion.  $\psi_0$  is the single-nucleon state and  $\psi_q$  the one-pion nucleon state. They are both real or physical pion-nucleon states, i.e. solutions of the Schrödinger equation.

$$(H-E) \psi^\pm = 0 \quad (14)$$

where  $H$  is the *complete* Hamiltonian for a fixed nucleon. That means the sum of free pion-field plus the PSPV pion-nucleon interaction <sup>13)</sup>. The ( $\pm$ ) signs indicate respectively the presence of a "scattered wave" containing *no* ingoing/outgoing pions at very large distance. It may be emphasized that here (as in the scattering theory) one of the essential features of the Chew and Low's approach consists of the use of a representation the basic set of which is the set of eigenstates  $\psi_n^+$  (or  $\psi_n^-$ ) belonging to the *complete* pion-nucleon Hamiltonian  $H^*$ . The advantage of using the set  $\psi_n^-$  lays on the fact <sup>13)</sup> that the S-matrix is directly related to the coefficients of  $\psi_n^-$  in the expansion of a scattered wave.\*\*

\* In (13)  $\psi_q^-$  represents the final one-pion state, but the *complete* set of eigenstates  $\psi_n$  would include, beside the (four degenerate)  $\psi_0$ , also states with *several* { ingoing } pions and possibly bound states.

\*\* And for  $t \rightarrow +\infty$  any expansion in  $\psi_n^-$  can be evaluated replacing  $\psi_n^-$  by its asymptotic plane wave part  $\Phi_n$ ; i.e. by the eigenfunctions of the free-field Hamiltonian  $H_0$ .

Actually it is found that the elements of S are :

$$\langle q | S | p \rangle = \delta_{qp} - 2\pi i \delta(E_p - E_q) T_p(q) \quad (15)$$

where

$$T_p(q) = (\psi_q^-, V_p \psi_0) \quad (16)$$

being

$$V_p = (4\pi)^{1/2} (f_0/\mu) \tau_p i \mathbf{p} \cdot \vec{\sigma} v(p)/\sqrt{2\omega_p} \quad (17)$$

the essential part of the interaction. In (17)  $f_0$  is the *unrenormalized* coupling constant and  $v(p)$  the cut-off function for the pion momenta. The pion type is characterized by the index  $p$  and  $\tau_p$  is the component of the nucleon isospin corresponding to the pion  $p$ . It operates as a yes/no changing operator on the charge of the nucleon according to the types of pions absorbed or emitted.

In dealing with (13) it has certainly been a very shrewd idea to treat the operator  $\mathbf{j}$  in a way which exploits at the maximum the  $\psi_n$  representation. Precisely Chew and Low decompose  $\mathbf{j}$  in two parts  $\mathbf{j} = \mathbf{j}_N + \mathbf{j}_\pi$  where by definition,  $\mathbf{j}_N$  commutes with the pion-absorption,—creation operators  $a_q, a_q^*$ , but have the same matrix element as the total  $\mathbf{j}$  between the *single-physical* nucleon states. Thus

$$\langle \psi_0 | \mathbf{j}_N \psi_0 \rangle = \langle \psi_0 | \mathbf{j} \psi_0 \rangle; \quad \langle \psi_0 | \mathbf{j}_\pi \psi_0 \rangle = 0 \quad (18)$$

This decomposition emphasizes the distinction between the ground state  $\psi_0$  of the physical ("clothed") nucleon and the ("excited") states of the pion-nucleon system built by a nucleon and one (or more) real pions.

This distinction turns out to be essential for the simplicity of the results. Actually it is found that up to fairly high energies ( $\nu \ll 2\mu$ ) the absorption of the incident photon involves mainly the e.m. poles of the single physical nucleon i.e. (because the nucleon is fixed) the static magnetic moments of proton and neutron.

A partial interpretation of these results might be found in two facts: i) the appreciable contribution of the Dirac magnetic moment; ii) the large coherence and stability of the pion cloud excitation.

Thus at low energies the photon absorption which originates the dominant ( $^{3/2} \text{ } ^{3/2}$ ) P-wave (the S-wave being large only for the charged pions, as it has been already mentioned in Section 2) is in a large extent, simply due to an operator of the type :

$$H_m = -\vec{\mu} \cdot \mathbf{H} = -\frac{1}{2} [\mu_N (1 - \tau_3) + \mu_P (1 + \tau_3)] \vec{\sigma} \cdot \vec{\nabla} \times \mathbf{A} \quad (19)$$

$$= -\frac{1}{2} \left[ \left( \frac{\mu_N + \mu_P}{2} \right) + \left( \frac{\mu_P - \mu_N}{2} \right) \tau_3 \right] \vec{\sigma} \cdot \vec{\nabla} \times \mathbf{A}$$

where  $\mathbf{H}$  is the magnetic field and  $\mu_N$  and  $\mu_P$  the neutron and proton static magnetic moments.

As it is suggested by (19) the actual evaluation is pursued splitting  $\mathbf{j}_N$  into its isospin components; i.e. writing

$$\mathbf{j}_N = \mathbf{j}_V + \mathbf{j}_S \quad (20)$$

where the subscript S and V indicate the parts of  $\mathbf{j}_N$  behaving respectively as a scalar and as the 3rd component of a vector in the isospin-space.\*

Obviously the transitions induced by the S-part obey the selection rule  $\Delta I = 0$ . Hence this term cannot produce as a final state the "enhanced" one ( $^{3/2} \text{ } ^{3/2}$ ). Furthermore taking into account the values of the proton and neutron magnetic moments the magnitude of this S term is five times smaller than the V-one. It may be seen that it corresponds to a part of the Watson's nucleon recoil effect. The V term is instead the dominant one in producing the photopion P-wave. It is found<sup>11)</sup> that the corresponding matrix element may be written as follows :

$$H_k^V(q) \cong \left\langle \psi_q^-, -\frac{f_0}{f} \left( \frac{\mu_P + \mu_N}{2} \right) \tau_3 i \frac{\vec{\sigma} \cdot \mathbf{k} \times \vec{\varepsilon}}{\sqrt{2k}} \psi_0 \right\rangle \quad (21)$$

The close connection with (19) is evident and it derives from (13) (18) and (20) when one writes

$$\int \mathbf{j}_V A_k(\mathbf{r}) d\mathbf{v} = \int \mathbf{j}_V \cdot \frac{\vec{\varepsilon}}{\sqrt{2k}} e^{i \cdot \mathbf{k} \cdot \mathbf{r}} d\mathbf{v} \\ = \text{const } i \tau_3 \vec{\sigma} \times \mathbf{k} \cdot \vec{\varepsilon} F(k^2) \quad (22)$$

The last step being motivated by the argument that the integral must be a polar vector proportional to  $\tau_3$  and to the vectors  $\mathbf{k}$  and  $\vec{\sigma}$ . These are the only ones vectors available in a frame *at rest* with the nucleon.

The form factor  $F(k^2)$  would be the only one really involving the nucleon structure; but the electron-scattering experiments by Hofstadter and McAllister<sup>14)</sup> indicate that in the energy range here considered  $F(k^2) \cong F(0)$  and this can be normalized to unity.

The multiplying constant in (22) may be determined remembering that according to (18) and (20),  $\mathbf{j}_V$  should have the correct *one-physical* nucleon expectation value. Hence

$$\text{const} = (f_0/f) (\mu_P - \mu_N)/2 \quad (23)$$

where  $f$  is the "renormalized" pion-nucleon coupling constant. The  $f_0/f$  ratio is motivated by the pion-nucleon interaction effectively included in (21). Actually the momentum  $\mathbf{k} \times \vec{\varepsilon}$  may be considered as the momentum absorbed by the nucleon and transferred by it to a virtual pion of index  $p$ . In a crude intuitive way one may think that the nucleon absorbs the photon through its magnetic moment and hence its spin vibrates. Being  $f_0 \gg e$  the interaction (17) corresponding to  $\tau_3$  takes over the reemission of a photon, and a neutral

\* For the pion-current the charge is  $Q = e I_3$  and then it contains only a term of the V-type<sup>3)</sup>.

pion  $\mathbf{p}$  corresponding to the momentum  $\mathbf{k} \times \vec{\varepsilon}$ , and isotopic variable 3 is created.

Consequently guided by (17) we write (21) as follows :

$$H_k^v(\mathbf{q}) = -\frac{1}{f} \left( \frac{\mu_p - \mu_n}{2} \right) \left( \frac{\omega_p}{k} \right)^{1/2} \frac{1}{v(\mathbf{p})} (\psi_{q^-}, V_p \psi_0) \quad (24)$$

This formula says that effectively  $\mathbf{j}_v$  induces the creation of a neutral pion which (virtually scattered by the nucleon) brings the pion-nucleon system into the final (charged or neutral pion) state  $\psi_{q^-}$ .

It would now remain to consider the contribution of  $\mathbf{j}_\pi$ . This operator is rather obscure. Actually being  $\varphi_1, \varphi_2, \varphi_3$  the components of the pion-field,

$$\mathbf{j}_\pi \neq -e (\varphi_1 \nabla \varphi_2 - \varphi_2 \nabla \varphi_1) = \mathbf{j}_T \quad (25)$$

because  $\mathbf{j}_T$  is the *total* pion-current density around the "bare" nucleon and putting  $\mathbf{j}_\pi = \mathbf{j}_T$  would be equivalent to neglect the difference between "bare" and physical "clothed" nucleon. In other words the basic functions  $\psi_n$  of the complete Hamiltonian (14) depend on the nucleon and pion variables in a nonfactorable way. The only clear property enjoyed by  $\mathbf{j}_\pi$  is (18) i.e. its matrix elements between *single*-nucleon states shall vanish.

To surpass these difficulties Chew and Low take as a "guide" the relativistic theory (where there are not ambiguities) and then examine the corresponding matrix element in the zero-energy limit. It coincides with the approach followed by Kroll and Ruderman in dealing with the S-wave of charged pions. It is found that the corresponding matrix element is

$$H_k^\pi(\mathbf{q}) \cong -\frac{i e f}{\sqrt{4\omega_q k}} \left( \frac{\tau_3 \tau_q - \tau_q \tau_3}{2} \right) \times \left[ \frac{\vec{\sigma} \cdot \vec{\varepsilon}}{\sigma \cdot \varepsilon} - 2 \frac{[\vec{\sigma} \cdot (\mathbf{q} - \mathbf{k})][\mathbf{q} \cdot \vec{\varepsilon}]}{(\mathbf{k} - \mathbf{q})^2 + \mu^2} \right] \quad (26)$$

plus a quite complicated additional term which was not written. This term is found to be always small but near resonance. However, also near resonance the corresponding amplitude is never more than 20% of  $H_k^v(\mathbf{q})$ . Of course one is then tempted of noticing that the net result of this rather crude approximation is simply the sum of the S-wave already discussed in Section 2 plus the direct pion-current interaction as it would be derived starting from the conventional operator  $\mathbf{j}_T$  of (25). In other words reminding the Kroll and Ruderman theorem  $H_k^\pi(\mathbf{q})$  in (26) is simply the usual renormalized Born approximation.

In comparing the theory with the experiments Chew and Low neglected also the term  $H_k^s(\mathbf{q})$  due to the scalar component of (20). As already mentioned  $H_k^s(\mathbf{q})$  is

responsible for transitions to the  $1/2$  pion-nucleon isospin state only. To neglect it represents an approximation which is very crude near threshold but much better around resonance. In conclusion Chew and Low keep only the terms (24) and (26). We may say that at present this theory would be in principle a quite complete treatment of photo-pion production by an infinitely heavy nucleon. However, practically to avoid very complicated and difficult evaluations, it ends in a fairly accurate approximation only for the neutral pions. For this kind of pion only the term (24) survives, and it is really the dominant and the more simple. For the charged pions the precision already reached by the experiments usually surpasses the accuracy of the theoretical predictions.

5. We want more to say briefly something about the comparison of the Chew and Low theory with the experimental data.

A quite detailed comparison of this type has been done quite recently by Koester for neutral pions and by Goldwasser for the charged ones. Likely a full report concerning the work done by these authors will be published in the next few months.

Let us start with the neutral pions. In this case as pointed out by Koester<sup>15)</sup> the Chew and Low theory leads (in the limits of the approximations already discussed) to an extremely simple relation between the total cross-section  $\sigma_\gamma^0$  for photoproduction and that  $\sigma_0^0$  for  $\pi^0$  nucleon scattering.

This follows immediately from (24) where

$$(\psi_{q^-}, V_p \psi_0) = T_p(\mathbf{q}) \quad (27)$$

is nothing else than the matrix element for pion-scattering from the state  $p$  to the state  $q$ .

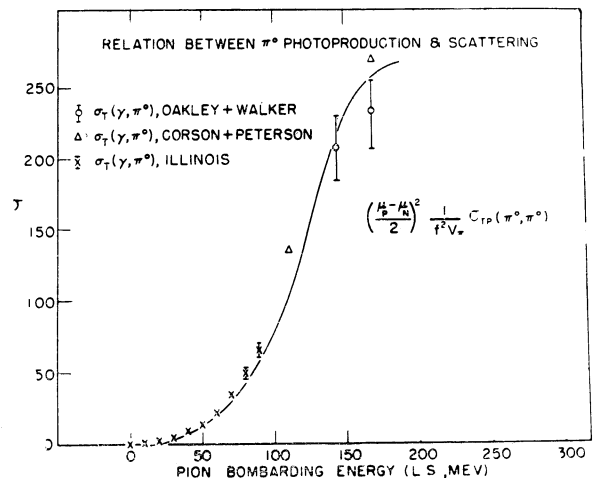


Fig. 2. The total cross-section  $\sigma_\gamma^0$  for neutral pion photoproduction. The solid line (see L. J. Koester<sup>15)</sup>) gives the behaviour of  $\gamma^0$  as deduced from the  $\sigma_{\pi^0 \rightarrow \pi^0}$  nucleon scattering.

Squaring and taking into account the appropriate statistical factors one finds

$$\sigma_{\gamma^0} \cong \left( \frac{\mu_P - \mu_N}{2} \right)^2 \frac{1}{f^2 v_{\pi}} \sigma_0^0 \quad (28)$$

where  $v_{\pi} = q_0/\omega_0$  is the velocity of the incident neutral pion corresponding to the incident photon, i.e. such that  $v \cong \omega_{q_0}$ . With  $f^2 = 0.08$  one has  $\sigma_{\gamma^0} = 0.0026 (\sigma_0^0/v_{\pi})$ . Fig. 2 (due to Koester<sup>15</sup>), shows how the evaluated  $\sigma_{\gamma^0}$  (solid line) fits the experimental data on  $\sigma_{\gamma^0}$ . Of course  $\sigma_0^0$  has been deduced by subtraction from scattering experiments, i.e.

$$\sigma_0^0 = \frac{1}{2} (\sigma_{+^+} + \sigma_{-^-} + \sigma_{-^0})$$

and implies rather large errors.

As far as concerns the charged pions at low energies a rather detailed discussion can be found in the Beneventano's and Osborne's reports.

It seems quite remarkable that the M.I.T. group had been able of giving, with a nice technique, the first experimental evidence of the second term of (26). Taking into account

the rather large errors this evidence is merely qualitative but even so it is very nice to know that this term exists and has the right order of magnitude. It is the first direct evidence of the pion-cloud current. In the near future similar experiments for charged pions and at very forward angles will be able of offering some quite interesting informations about the dynamical behaviour of the pion-cloud.

The picture will be completed by accurate measurements on photoproductions of neutral pions near the threshold. The Koester and Mills analysis already points clearly out a quite appreciably neutral-pion S-wave around 200 Mev and warns us about the importance of the nucleon recoil. This importance is also very much emphasized in the Beneventano's report. Nucleon recoils and the dynamical terms neglected in (26) are the main unavoidable limitations of the Chew and Low theory.

However, the success of this theory is such that also for the charged pions the lack of agreement with the experimental data is confined to the details. It becomes more evident only on the coefficients B and C of (1) where the interferences among the several amplitudes emphasize the importance of the so-called "small" terms.

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