HIGH PRECISION MEASUREMENT OF THE IONIZING POWER OF FAST CHARGED PARTICLES WITH THE HELP OF MULTI-LAYER PROPORTIONAL COUNTERS

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(presented by V. A. Lubimov)

The present report is a survey of a series of investigations carried out during 1952-1956 by a group consisting of A. I. Alikhanov, G. P. Eliseiev, V. A. Lubimov, V. K. Kosmachevski and N. B. Moiseiev; the work was centered on the development of a precise method for measuring the ionization of fast particles, and on this basis, of studying the mass spectrum of cosmic ray particles.

As an illustration of the new technique we include experimental data obtained at sea level and also some experimental results obtained in collaboration with A. V. Khrimian and V. Sh. Kamalian at an altitude of 3250 m., at the Alagyez High Altitude Cosmic Ray Station.

One of the main problems confronting investigators of high energy nuclear stars is the determination of the spectrum of the emitted elementary particles, and especially, measurement of the particle masses. In order to measure the mass of a particle, two quantities must be known and the measurements are usually reduced to the determination of momentum and energy, or momentum and velocity of the particle. The most common procedure for measuring particle masses is the determination of the particle momentum (for example, from magnetic deflection of the trajectories) and residual range in a condensed medium. If the residual range is much smaller than the nuclear interaction path of the investigated particle then the energy losses in the absorber will result only from ionization excitation of the atoms of the medium. The particle energy in this case can be found with minimum error from the range, and the momentum-range method can be applied for mass determination. In thin absorbers, however, momentum-range mass measurements are restricted to a narrow momentum region of the investigated particles.

If one attempts to increase the momentum region by increasing the particle range, errors in the mass determination will inevitably arise due to an appreciable probability of nuclear interaction. The mass measurements may be carried out in a much broader momentum region if range measurements are replaced by energy loss measurements of particles traversing a thin absorbing layer. The ionization caused by the particle in the layer will be proportional to the energy loss and may be found experimentally.

1. Ionization measurements for charged particles in the velocity range $\beta < 0.9$ (left branch of ionization curve)

The ionization produced by charged particles moving with non-relativistic velocities is inversely proportional to the square of the velocity ($I \sim 1/\beta^2$). Thus, for a given particle momentum an error in ionizing power measurements yields a mass measurement error which has a favourable factor of $\frac{1}{2}$

$$\frac{d\mu}{\mu} = \frac{1}{2}\left[1 + \frac{p^2}{\mu^2}\right]\frac{dI}{I}$$

Thus for mass identification, ionization measurements on single elementary particles (π, K, p, d, T, etc.) with velocities $\beta \leqslant 0.9$ must be performed with an accuracy of $\pm$ 10 to 20 per cent. If a gas proportional counter is used in the ionization measurements, the accuracy of the measurement will be $\pm$ 30 to 40 per cent, the fluctuation distribution curve being unsymmetrical with a long "tail" in the region of large ionization losses. It can be shown that this error practically cannot be lowered by changing the filling gas or dimensions of the counter and is inherent in the fluctuational nature of ionization energy losses. The problem of resolving elementary particles in a single ionization measurement can be solved by using a multi-layer (4 or 5 layers) proportional counter in which several independent measurements of the ionizing power of a single particle are carried out. If the distribution of the errors of measurement has an arbitrary form a special study is required.

Fluctuation distribution function for proportional counter ionization measurements (Landau distribution)

In most types of measurements the errors conform to a Gaussian distribution. As is well known, in this case the arithmetic mean value of the separate measurements is accepted as the final value of a set of measurements of given accuracy. The final error will then be $\sqrt{N}$ times smaller (N is the number of measurements) than the error of a single measurement ("arithmetic mean procedure"). If the dispersion of the distribution curve (root mean square deviation) is used to characterize the error of measurement, it follows from a general theorem of probability theory that in the case of an arbitrary distribution application of the "arithmetic mean procedure" also yields a final result which corresponds to a distribution in which dispersion is smaller than that of a single measurement by a factor equal to the square root of the number of measurements. Thus, the "arithmetic mean" method treatment of the measurements can be successively applied not only when the distribution is Gaussian but also for a large class of distributions on the condition that the integral

$$\int_{-\infty}^{+\infty} \varphi(\lambda)\,\lambda^2\,d\lambda \text{ converges.}$$

In ionization measurements we encounter a specific type of distribution. Numerous experiments have shown that the fluctuations in ionization measurements in thin layers of matter (the energy losses in the layer must be much smaller than the particle energy) agree qualitatively with the Landau ionization energy loss distribution [1].

The Landau theoretical distribution (or the experimentally observed distribution of ionization fluctuations), if expressed in terms of the variable $\lambda = (\Delta - \Delta_0)/\xi$, assumes a universal form and can be computed or determined from experiment. Here Δ denotes the energy loss due to ionization (or the ionizing power), Δ_0 is the most probable energy loss (most probable ionization), $\xi = \eta/\beta^2$ and η is a quantity which depends on the thickness of the layer of matter in which the ionization occurs. The Landau distribution $\varphi(\lambda)$ is not symmetrical. It has a high energy tail which decreases as $\sim 1/\lambda^2$. The quantity $A \cdot \xi/\Delta_0$ is a measure of the relative width of this distribution. A is a factor which depends on the shape of the distribution and on the method of determining the width of the distribution (width at half height or "reliability interval", etc.). The velocity dependence of the most probable ionization Δ_0 is the same as that given by the well known Bethe-Bloch formula.

$$\Delta_0 = \xi\left[ln\frac{3\cdot 10^3\eta}{Z^2(1-\beta^2)} - \beta^2\right]$$

It can thus be seen that the width of the Landau distribution which determines the precision of ionization measurements

$$\sigma = \frac{A\xi}{\Delta_0} = \frac{A}{\left[ln\frac{3\cdot 10^3\eta}{Z^2(1-\beta^2)} - \beta^2\right]}$$

depends very weakly (logarithmically) on the gas density or dimensions of the proportional counter. In the non-relativistic case being considered, $\beta < 1$ (left branch of ionization curve), the relative width of the Landau distribution is constant and independent of the magnitude of ionization.

It is easy to see that the dispersion of the Landau distribution tends to infinity since the integral

$$\int_{-\infty}^{+\infty}\varphi(\lambda)\,\lambda^2\,d\lambda \sim \int_{-\infty}^{+\infty}(1/\lambda^2)\,\lambda^2\,d\lambda^2$$

diverges. This means that it would be wrong to apply the "arithmetic mean" method to the results of ionization measurements. As a matter of fact, in regard to fluctuations, the arithmetic mean of N ionization measurements in a proportional counter with a layer thickness η_1

$$\sigma_1 = \frac{A}{ln\frac{3\cdot 10^3\eta_1}{Z^2(1-\beta^2)} - \beta^2}$$

should be equivalent to a single measurement in a proportional counter with a layer N times thicker, $\eta_N = \eta_1 \cdot N$. The distribution in this case will be a Landau curve having the width

$$\sigma_N = \frac{A}{ln\frac{3\cdot 10^3\eta_1 N}{Z^2(1-\beta^2)} - \beta^2} = \frac{A}{ln\frac{3\cdot 10^3\eta_1}{Z^2(1-\beta^2)} - \beta^2 + ln\,N} \cong \sigma_1\left(1 - \frac{ln\,N}{10}\right)$$

Thus, compared to the error of a single measurement for which the distribution function is of the Landau type the arithmetic mean error of N measurements is

$$\sim \frac{1}{1 - \frac{ln N}{10}}$$

times smaller *.

In ionization measurements a certain degree of cut-off in the high energy tail of the Landau distribution is always inevitable. As a result, besides a decrease of the mean experimental error according to $\sim\sigma\cdot(1 - ln N/10)$ on applying the "arithmetic mean method" for determination of ionization from a set of measurements, there is also a decrease in the error as $\sqrt{\lambda_{max}}/\sqrt{N}$ where the mean square deviation λ_{max} is the cut-off value and is defined by the condition $\int_0^{\lambda_{max}} 1/\lambda^2 \cdot \lambda^2 d\lambda \cong \lambda_{max}$. This is exactly the case encountered in cloud chamber ionization measurements when the cut-off is realized by rejecting blobs due to δ electrons and it is the average number of droplets per unit track length that is determined. Fairly high precision in ionization measurements can be obtained in this manner. Analogous effects were encountered in [2])

* As a matter of fact, in a solid scintillation counter in which the amount of matter in the layer is several thousand times greater than in a gas counter the distribution width decreases by only about two times compared with that for the gas counter.

in which the ionizing power of heavy mesons was determined by the "arithmetic mean" method. Subsequent calculations of the ionization carried out by an exact method confirmed the physical results obtained in this investigation. In the general case when several measurements are performed to determine the ionization, the arithmetic mean procedure is not applicable and for curves of the Landau type a different method of treatment must be used.

A "Universal" method of treatment of results of measurements[3)]

We shall consider the problem in the general form. Assume that N measurements were performed: Δ_1, Δ_2,... Δ_N each of which corresponds to a fluctuation distribution curve $\varphi(\Delta_i, \Delta_0)\,d\Delta$ of arbitrary shape where Δ_0 is the most probable value of the measured quantity. If the function $\varphi(\Delta_i, \Delta_0)$ is known a priori or has been determined experimentally (the form of the distribution function must be known in order to apply any method) and if in the i-th measurement the quantity Δ_i was obtained, then evidently, the probability that the measured value lies between Δ_0 and $\Delta_0 + d\Delta_0$ is given by $\varphi(\Delta_1, \Delta_0)\,d\Delta_0$. For a set of independent measurements Δ_1, Δ_2,... Δ_N the probability that the true value of the measured quantity lies between Δ_0, and $\Delta_0 + d\Delta_0$ is given by $F(\Delta_1, \Delta_2 ... \Delta_N, \Delta_0)\,d\Delta_0 = \varphi(\Delta_1, \Delta_0) ... \varphi(\Delta_N, \Delta_0) d\Delta_0$. The function F is the error curve for the final value determined from the condition $dF/d\Delta_0 = 0$ and the width of the curve F is the statistical error of this value *. Moreover, for a given number of measurements N the shape of the error curve may vary, depending on the concrete values of the fluctuating quantity, Δ_1, Δ_2,... Δ_N and it may serve as a criterion of reliability of the given measurement.

It was found that if ionization measurements were carried out in a proportional counter the "universal" method was very effective. Thus, if four independent measurements of the ionizing power in a proportional counter were carried out with an accuracy of $\pm$ 35% the most probable value of ionization could be determined by the universal method with an accuracy of about $\pm$ 10 to 12%. These circumstances underlie the application of multilayer proportional counters for measurement of the ionization of charged particles for the purpose of mass identification in the velocity region $\beta < 0.9$.

Four and five-layer proportional counters

Four and five-layer proportional counters were used to investigate the mass spectrum of cosmic ray particles in the Alikhanov-Alikhanian mass-spectrometer.

The five-layer proportional counter is a rectangular box, 70 × 60 × 270 mm. The layers are separated by thin aluminium foils 0.1 mm. thick. The common gas mixture is used for all layers of the counter. The counter is made airtight by means of a vacuum "casing" of 0,1 mm. copper and brass foil. The upper wall of the counter is a 0.1 mm. brass foil of the vacuum casing tightly stretched over a frame. Thus, the counter is essentially a thin-wall counter. The reason why the counter had to be thin-walled was that it was placed in the gap of the mass spectrometer magnet, i.e. in the space in which the momentum measurements were carried out. Momentum distortions due to multiple scattering in the proportional counter walls were negligible. The dimensions of a layer were 32 × 70 mm. (fig. 1). Three "blank" Mo wires of 0.1 mm. diameter were stretched over the center of the layer. They were at the same potential as the wall and divided the layer into two compartments. An "operating wire" (Mo 0.07 mm. dia.) was stretched along the center of each compartment; the potential of these wires was ∼ + 4000 V. Each layer essentially consisted of two counters with an optimum cross section which was approximately a square. At the output the operating wires are connected together. This construction was chosen so as to keep to a minimum the dependence of the signal magnitude on the position of the counter in which it is traversed by the particle, which may arise from negative ion formation in the counter. The probability of an electron attachment to electro-negative gas atoms greatly depends on the electric field shape in the counter and is minimum when the counter is square in cross-section.

The counter was filled with propane (C_3H_8) to a pressure of 450 mm. Hg. The gas was first purified by three-fold fractional distillation and by passing it through a melt of potassium and sodium. (8 parts of K and 1 part of Na which is liquid at room temperature.) This purifying procedure ensured complete independence of the proportional counter pulse from the position of the particle trajectory. The high tension applied to the counters was supplied by dry batteries. It was measured by a compen-

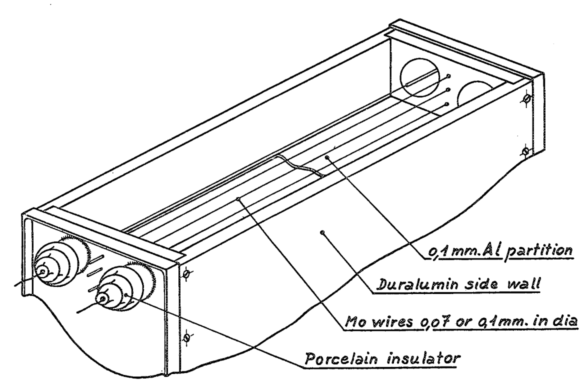

Fig. 1. The view of a layer of a 5-layer counter.

* Here and in the following the distribution width will be defined as comprising 67% of the reliability interval. In other words, the probability that the measured quantity lies within this interval is 0.67. If the distribution is Gaussian this interval will numerically coincide with the mean square error.

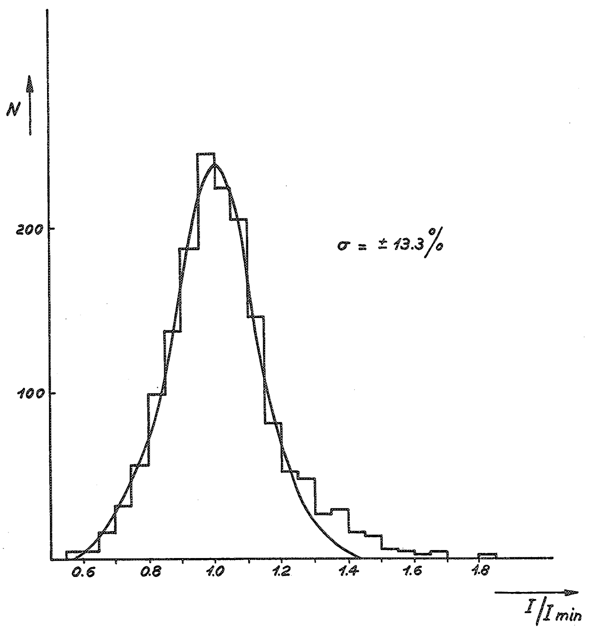

Fig. 2. The histogram of the most probable values of the minimum ionization of μ-mesons measured with a 5-layer proportional counter. The smooth line curve is the ± 13% width Gaussian distribution.

sation method with an accuracy of ± 0.1 V (2.10^{-3} per cent) and was maintained to within ± 1 V.

On a semilogarithmic scale the dependence of the proportional counter pulse on the voltage is a straight line whose slope corresponds to a change of the pulse magnitude by 0.8 per cent when the voltage varies by one volt.

A relativistic particle traversing a single layer produces about 270 ion pairs. The gas amplification was approximately 50.000. The pulses from each layer of the proportional counter were preamplified and fed into an analyzing arrangement which recorded each particle traversing the mass spectrometer *. Thus a 5 layer proportional counter made five independent measurements of the ionizing power of each particle and by using the "universal" method and special tables the most probable ionization could be computed.

A histogram of the most probable values of the minimum ionization of μ mesons measured with a 5 layer proportional counter is shown in fig. 2. The width of the histogram is ± 13.3 per cent. This means that, on the average, the error in each ionization measurement performed with a 5 layer proportional counter on a minimum ionizing particles is about ± 13.3 per cent. This accuracy is sufficient to carry out momentum-ionization mass determinations on particles moving with $\beta < 0.9$.

Magnetic mass spectrum investigation of the mass spectrum of cosmic ray particles with the aid of 4 or 5-layer proportional counters

For the sake of illustration we reproduce in fig. 3 some mass spectra of cosmic ray particles obtained with the magnetic mass spectrometer at sea level under 5 cm. of lead, for $\beta < 0.9$ [4]. The particles were stopped in a graphite trapping device (7 to 12 cm. C). The ionization measurements were carried out with 4-layer proportional counters.

The particle distribution obtained by range-momentum mass determination is shown in the upper part of the fig. 3. The mass distribution of the same particles obtained by momentum-ionization measurements is reproduced in the lower part of the fig. 3. Both methods of mass measurement yield μ meson mass distributions which are approximately Gaussian. It can be seen from fig. 3 that range-momentum measurement of π meson masses yields values which in some cases are too high; this is due to π-meson stopping by other processes than ionization. π mesons thus imitate particles with larger mass. On the other hand, the masses of the same π mesons determined from momentum-ionization measurements closely group around a mean value. The mean error in mass of a π meson arising in momentum-ionization measurements is about the same as that arising in measurement of μ meson masses.

Each of the mass determination methods yields a distribution possessing a characteristic shape and the difference is especially marked in the case of protons. In range-momentum measurements the distribution is skew with a tail at large mass values due to particles stopping by other processes than ionization. The latter also make it practically impossible to separate protons and deuterons by momentum range measurements whereas they are easily resolved in momentum-ionization measurements. In each measurement the average error in momentum-ionization determination of proton mass was about ± 14.2 per cent.

The possibility of mass resolution in a narrow interval of range values is not the only advantage of the momentum-ionization mass determination method with respect to the momentum-range method. As a matter of fact ionization measurements make it possible to extend the investigation of the particle spectrum to particles stopping by other processes than ionization. The nature of the stopping mechanism in the particle absorbing arrangement permits one to determine the interaction cross-section for particles identified by momentum and ionization. With a knowledge of the interaction cross-section it is possible to construct complete momentum spectra of nuclear-active particles in a very broad momentum region. The most interesting problem of this type is the investigation of the π meson spectrum.

A π meson spectrum obtained by momentum-ionization identification in a mass spectrometer experiment under

* The electronic circuits used in conjunction with the 4 and 5-layer counters were designed by V. K. Kosmachevski and B. N. Moiseiev.

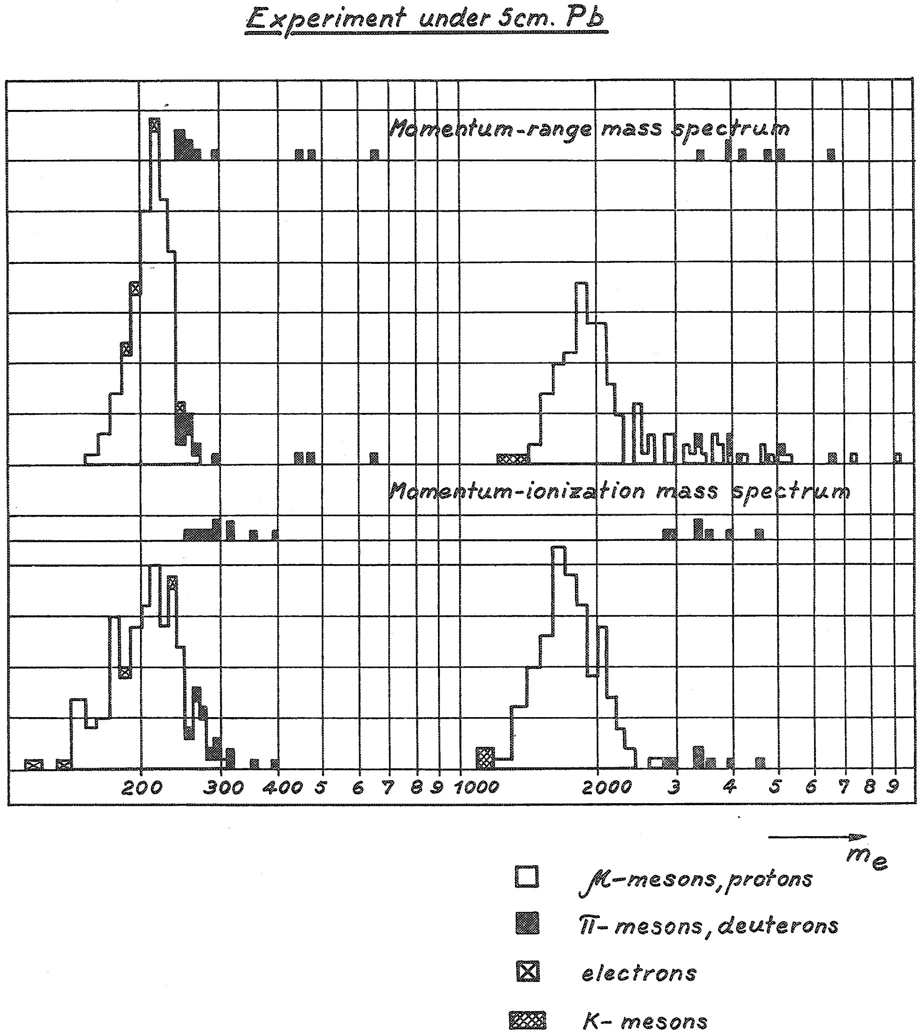

Fig. 3. The mass spectra of cosmic ray particles obtained with the magnetic mass-spectrometer at sea level under 5 cm. of lead, for $\beta < 0.9$. The range interval — 7 to 12 cm. C.

a— Momentum - range mass spectrum.
b— Momentum - ionization mass spectrum.

5 cm. of Pb at sea level is shown in fig. 4 [4]. An anomalous increase in the number of particles is observed in the momentum range from 700 to 1000 Mev/c. In this momentum region K and π mesons cannot be resolved in an individual momentum-ionization measurement. However, the ionizing power of the particles comprising the anomaly in the π meson spectrum deduced from all the particle measurements (14 in all) was found to be ($1.14 \pm 0.04\ I_{min}$) ($I^+ = 1.15 \pm 0.06\ I_{min}$; $I^- = 1.13 \pm 0.06\ I_{min}$ which definitely differs from the π meson ionization value which in this momentum region should be $1.03\ I_{min}$. These results thus indicate that there was an intensive generation of K mesons in the 700-1000 Mev/c momentum range.

All K mesons were stopped in the absorbers of the trapping arrangement whose total thickness (0.2 to 6 cm. Pb) was several times smaller than the ionization range of these particles; this is an experimental proof of the strong nuclear interaction between K mesons and matter.

Automatic treatment of ionization measurements obtained with 4 or 5-layer proportional counters. The "cut-off" method [5]

An essential feature of the ionization measurement method in which utilization is made of 4 or 5 layer proportional counters is that each of the 4 or 5 ionization readings is independently recorded by an electronic circuit and after reduction by the "universal" method the most probable value can be deduced. In cosmic ray experiments this feature is not a shortcoming since treatment of the ionization data takes less time than the experiment as a whole. However, in accelerator work a lengthly treatment of ionization data becomes a major weakness. On the other hand, reduction of the data by the universal method cannot be carried out automatically by simple means. For this purpose it is necessary to find a simple but effective method which would make it possible to

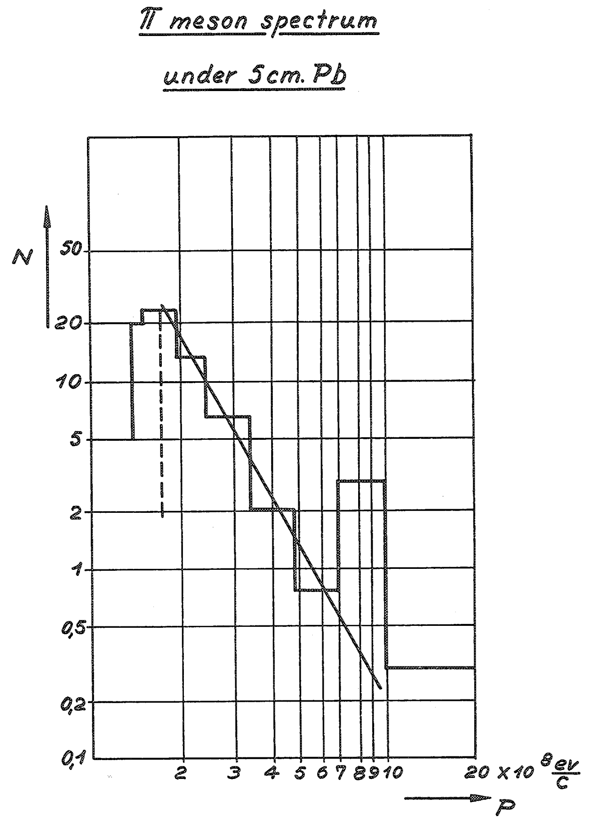

Fig. 4. π meson spectra under 5 cm. Pb, at sea level.

compute the most probable ionization with the aid of electronic circuits.

Let us consider qualitatively the shape of the Landau distribution curve for ionization energy loss fluctuations. On passage through matter a charged particle suffers a large number of collisions with atomic electrons of the medium and energy losses take place in small portions ("long range" collisions). In this case the fluctuations can be described by a Gaussian distribution which forms the central part of the Landau distribution. "Head-on" collisions in which energetic δ-electrons are formed and the ionizing particles lose an appreciable part of their energy are much more rare *. The tail in the Landau distribution is due just to these "short range" collisions. If it is possible to divide the collisions into "long" and "short" range and then consider only the long range collisions (this is equivalent to the introduction of an energy transfer cut-off) then the ionization fluctuations would be of a Gaussian type and the most probable value of the ionization could be obtained as the arithmetic mean of several measurements. This type of division automatically takes place in photographic plates and, as we have already mentioned, it can be accomplished in cloud chamber measurements and be utilized in ionization determinations. In a proportional counter, however, this separation cannot be realized. If a multilayer proportional counter is used, the picture changes considerably.

Suppose the maximum amount of energy that can be transferred in a single collision is W_0, this value being chosen in such a manner that the average energy loss equals the most probable, i.e.

$$\overline{\Delta} = \int_0^{\Delta_0 + W_0} \Delta\, \varphi \cdot \frac{1}{\xi}\, d\Delta = \Delta_{\text{prob}}$$

where Δ_{prob} is obtained from the condition $(\varphi(1/\xi))'_{\Delta} = 0$ where $\varphi\,(1/\xi)$ is the Landau distribution. This means that the tail of the Landau distribution has been cut off in such a way that the centre of gravity of the curve obtained coincides with the position of the maximum. Then, of N measured ionization values $\Delta_1, \Delta_2, \ldots \Delta_n$ the $(1\text{-}\alpha)N$ largest values should be rejected and the most probable ionization can thus be determined as the mean of the remaining values :

$$\Delta_{\text{prob}} = \frac{\Delta_i + \Delta_k + \ldots}{\alpha N}$$

where
$$\alpha = \frac{\int_0^{\Delta_0 + W_0} \varphi \frac{1}{\xi}\, d\Delta}{\int_0^{\Delta_0 + W_{\max}} \varphi \frac{1}{\xi}\, d\Delta}$$

is the ratio of the area of the cut-off distribution to that of the whole curve. The error in the value found in this way will be $\delta\Delta_{\text{prob}} = \sigma/\sqrt{\alpha N}$, whree σ is the half-width of the Gaussian equivalent of the "cut-off" distribution.

In our case this corresponds to rejecting 35 per cent of the largest ionization values.

Numerous checks have shown that the efficiency of this method of treatment of the experimental material is as satisfactory as the "universal" method, irrespective of the number of ionization measurements (see fig. 5).

Utilizing the simplicity of this method some simple electronic apparatus was constructed, which "rejected"

* Although the ejection of an energetic δ-electron is a rare event, in the total balance of energy losses the role of "close" collisions is nevertheless significant (the integral $\int_{-\infty}^{+\infty} \lambda\, \varphi\,(\lambda)\, d\lambda \cong \int_{a}^{+\infty} \lambda \cdot 1/\lambda^2 \cdot d\lambda$ diverges logarithmically).

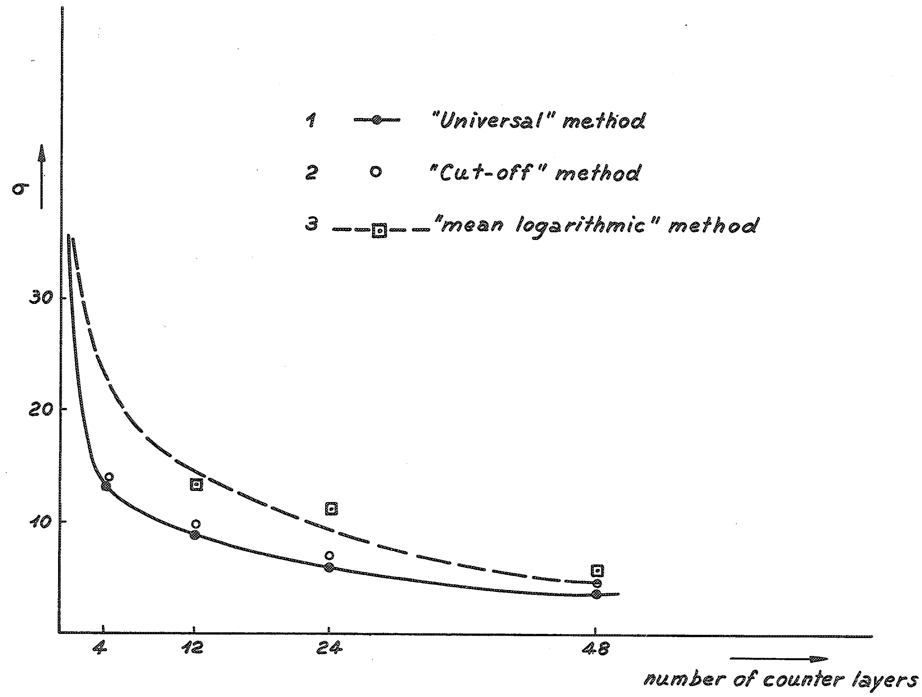

Fig. 5. The dependence of the accuracy of ionization measurements on the number of proportional counter layers.

1. "Universal" method.
2. "Cut-off" method.
3. "Mean Logarithmic" method.

the maximum value when four pulses were recorded or the two largest values when five pulses were recorded, and added together the remaining values *.

A histogram of the most probable ionization values for 277 minimum ionizing μ mesons is given in fig. 6; the measurements were made with a 4 layer proportional counter and an arrangement which automatically computed the ionization by the "cut-off" method. The average error which this method yields in measurements on the ionization of a single μ meson is $\pm$ 12 per cent and the precision is therefore as good as that which the alternative method yields, this being exactly what one should expect. In this form the experimental arrangement can be successfully used in accelerator work.

2. Measurement of ionization on the logarithmic rise of the ionization curve for velocities $\beta \rightarrow 1$

(Right hand part of ionization curve)

As is well known, theory predicts [6] and experiment confirms [7] that at a velocity of about $\beta = 0{,}96$ the ionizing power of a particle is minimum and with further increase of the velocity it increases logarithmically until a constant ("plateau") value, which is due to polarization effects, is attained. In a gas under normal conditions the logarithmic ionization increase continues up to a value $p/\mu = 100$ and the "plateau" ionization is about 1,35 times greater than the minimum value.

It is highly tempting to use this relation for momentum-ionization particle identification in the ultra-relativistic velocity region **. (Mesons, for example, can be studied

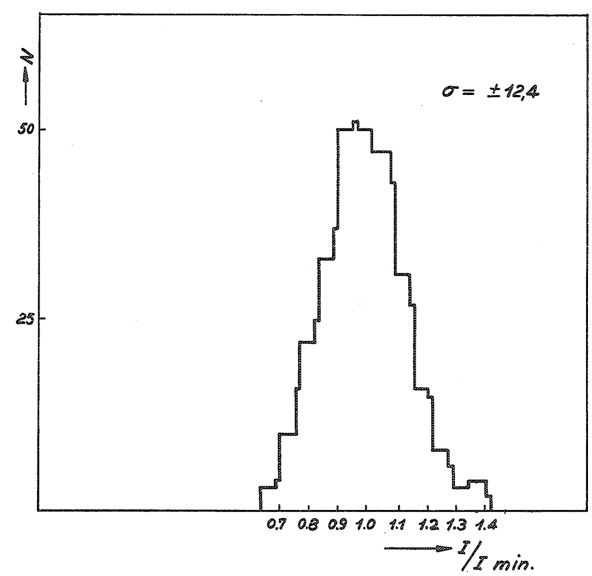

Fig. 6. The histogram of the most probable ionization values for a μ mesons. The measurements were made with a 4-layer proportional counter and an arrangement which automatically computed the ionization by the "Cut-off" method.

* The electronic circuits were designed by B. N. Moiseiev and A. Talizin.

** If it is desirable to know whether the measured ionization value belongs to the right or left branch of the ionization curve one can use an auxiliary Čerenkov counter or a set of scintillation proportional counters for which the Fermi-plateau approximately corresponds to the minimum ionization.

by this method up to energies $\sim 10^{10}$ ev and protons up to $\sim 10^{11}$ ev.) However, in the relativistic region the dependence of the ionization on p/μ is very weak and therefore to obtain the same precision in momentum-ionization mass measurements, as that attainable in the non-relativistic region, the precision of ionization measurements must be 10 times higher. An accuracy of $\pm$ 1-2 per cent in ionization measurements is apparently beyond the possibilities of modern experimental techniques.

We shall consider some possible ways of approaching this limit in ionization measurements. In order to make full use of the ionization increase, the ionization should be measured in a rarefied medium. It is natural to use for this purpose a gas counter. One possible method of attack is further development of the multilayer proportional counter method.

Reduction of the error in multilayer proportional counter ionization measurements with increase of number of layers (when the most probable ionization is computed by the "universal" method)

After a treatment of the measurements by the "universal" method an error curve (probability distribution for the measured quantity) is obtained which is characteristic of the precision of an individual value obtained for a given detail set of values; the shape of this curve may be different for different sets consisting of the same number of measurements depending on the concrete values of the individual measurements. Thus, the error curve completely determines the measurement problem. However, in order to characterize the efficiency of the method of treatment with respect to the number of independent measurements it is necessary to know the average accuracy which a definite number of measurements yields. For this purpose one must know the error curve for a given number of measurements averaged over all possible combinations of readings for a prescribed number of measurements *. Let us assume that a large number of measurements corresponding to the distribution $\varphi(\Delta_1, \Delta_0)$ has been performed (the deduction is valid for a large number of measurements) and that the results of the measurements are represented by a histogram, each division of which corresponds to $\delta\Delta$. When the number of measurements is sufficiently large the histogram can be satisfactorily described by the distribution irself– $\varphi(\Delta_1, \Delta_0)$. Suppose that n_k measurements on the histogram correspond to a quantity Δ_k. n_k can then be represented as follows:

$$n_k = N \cdot \varphi(\Delta_k, \Delta_0) \cdot \delta\Delta$$

where N is the total number of measurements and $\varphi(\Delta_k, \Delta_0)$ is a normalized function. In agreement with the "universal" method these measurements yield an error curve for the unknown quantity Δ_0 which has the form

$$\Phi(\Delta_1, \Delta_2, \dots \Delta_N; \Delta_0) = \varphi^{n_1}(\Delta_1, \Delta_0)\, \varphi^{n_2}(\Delta_2, \Delta_0) \dots$$
$$\dots \varphi^{n_N}(\Delta_N, \Delta_0) = \prod_i^N [\varphi(\Delta_i, \Delta_0)]^{N\varphi(\Delta_i, \Delta_0)\delta\Delta};$$

taking the logarithms

$$ln\, \Phi(\Delta_1, \Delta_2, \dots \Delta_N; \Delta_0) = N\, \delta\, \Delta \sum_i^N \varphi_i ln \varphi_i$$

and passing to the limit

$$ln\, \Phi(\Delta_{prob}, \Delta_0) = \lim N\delta\Delta \sum_i^N \varphi_i\, ln\varphi_i =$$
$$N \int_{-\infty}^{+\infty} \varphi(\Delta, \Delta_0) ln\, \varphi\, (\Delta_1, \Delta_0)\, d\Delta.$$

Hence, for N measurements the error curve has the form

$$\Phi_N(\Delta_{prob}, \Delta_0) = f^N(\Delta_{prob}, \Delta_0),$$

where

$$f(\Delta_{prob}, \Delta_0 = \exp\left[\int_{-\infty}^{+\infty} \varphi ln \varphi d\Delta\right]$$

Thus for an arbitrary distribution the averaged error curve in the "universal" method can be represented by a function raised to a power equal to the number of measurements. This function (which may be termed universal) does not depend on the number of measurements but only on the form of the initial distribution. A result which seemed unexpected at first glance was obtained after numerical integration when an empirical ionization distribution function was accepted as the initial distribution: it was found that the universal function f $(\Delta_{prob}, \Delta_0)$ itself is the initial distribution function. Thus, for a given number of measurements, the average error curve is

$$\Phi_N(\Delta_{prob}, \Delta_0) = \varphi^N(\Delta_{prob}, \Delta_0)$$

where $\varphi(\Delta_{prob}, \Delta_0)$ is the initial distribution. It may be noted that for a single measurement a simple result is obtained:

$$\varphi_1(\Delta_{prob}, \Delta_0) = \varphi(\Delta_{prob}, \Delta_0)$$

On the other hand the relation under discussion also leads to a familiar result if the distribution is Gaussian

$$\varphi_N(\Delta_{prob}, \Delta_0) = \varphi^N = Ae^{-\frac{(\Delta_{prob}-\Delta_0)^2 N}{2\sigma^2}} \doteq Ae^{-\frac{(\Delta_{prob}-\Delta_0)^2}{2\sigma_N^2}}$$

where $\sigma_N = \sigma/\sqrt{N}$ that is, one obtains the well known rule that the error decreases as $\sqrt{N}$ increases.

* Properly speaking, the histograms of the most probable ionization values obtained with 4 or 5-layer proportional counters and characterizing the average precision of a single ionization measurement are averaged experimental error curves of this type.

The dependence of the accuracy of ionization measurements on the number of proportional counter layers was deduced by applying the relations given above. This dependence is in very good agreement with direct experimental determination of the accuracy of ionization measurements for various numbers of measurements and for small as well as large number of layers (fig. 5).

These data show that in order to obtain an accuracy of 2 per cent in ionization measurements about 200 layers would be required in the proportional counter.

A counter with 200 layers would be a very complex device, indeed. However, it is quite feasible to construct a 50 layer counter. It was mentioned above that when 4 or 5 layer counters were used each counter pulse was separately electronically measured. If the same method were used in measurements with a 50 layer counter, several thousand radio tubes would be required to construct the electronic circuits. Thus the electronic difficulties would be practically insurmountable in this case. New means of carrying out the measurements with a 50 layer proportional counter must be found. A possible solution is an ionization calculation method which may be very simple and effective for a large number of ionization readings (that is, for a large number of counter layers) and which may be realized with the aid of electronic circuits, only one pulse analyzer being necessary in this case.

"Logarithmic mean" method

Assume that the function $\chi = \chi(\Delta)$ transforms the coordinates in such a manner that the shape of the Landau distribution function $\varphi(\Delta, \Delta_0)$ (ionization fluctuation distribution for a single layer) becomes almost Gaussian.

$$\varphi(\Delta, \Delta_0)\, d\Delta = \varphi(\chi, \chi_0)\,(\partial\Delta/\partial\chi)\, d\chi = \psi(\chi, \chi_0)\, d\chi$$

where $\chi = \chi(\Delta)$ is the coordinate transformation function and $\psi(\chi, \chi_0)$ is a distribution similar to a Gaussian one.

The best method of treatment of the data in this case would to be find the arithmetic mean of all the measurements :

$$\overline{\chi} = \frac{\chi(\Delta_1) + \chi(\Delta_2) + \ldots \chi(\Delta_N)}{N}$$

The inverse transformation then yields

$$\Delta = \chi^{-1}[\overline{\chi}(\Delta_1, \Delta_2, \ldots \Delta_N)]$$

It happens that a very satisfactory function is a logarithmic one. Indeed, if the Landau distribution function is plotted on a logarithmic scale the resulting distribution will be found to be very similar to a Gaussian one. If the number of measurements is not large the distribution on the logarithmic scale will become only slightly more narrow (naturally in accordance with the $1/\sqrt{N}$ law) and on the inverse transformation to a linear scale the asymmetry of the error curve will still be considerable. If, however, the number of measurements is large and the narrowing is therefore appreciable, then over a small interval occupied by the distribution the logarithmic scale will be almost linear. Thus, on inverse transformation the error curve will assume a symmetrical, Gaussian form on a linear scale. This indicates that for a large number of readings the efficiency of the "logarithmic mean" method is about the same as that of the "universal" method of treatment. Indeed, it can be seen from fig. 5 that when the number of layers is approximately 50 the accuracy of the ionization measurements computed by the "logarithmic mean" method is about the same as that computed by the "universal" method. The final value of the ionization obtained by this method is then found to be

$$\Delta = \exp\left[\frac{1}{N} \sum_i ln\, \Delta i\right] = \sqrt[N]{\Delta_1 \cdot \Delta_2 \cdot \cdots \Delta_N}$$

It is very easy to carry out this type of calculation electronically by making use of the logarithmic dependence of the current in a diode on the voltage. It is convenient to measure the sum of the proportional counter pulses logarithms since in this case the relative accuracy in measurement of the ionization remains constant with varying magnitude of the ionization and the range of ionization measurements can be increased. If a 50-layer counter were used, only 250 radio tubes would be required. A system of this type has been built and operated in conjunction with a 4 layer proportional counter **.

A histogram of the ionization values of 277 singly ionizing μ mesons measured with a 4 layer proportional counter and the apparatus above is shown in fig. 7. On a logarithmic scale the histogram can excellently be described by a Gaussian curve. From these data it can be deduced that if a 50 layer proportional counter and the above mentioned apparatus were used the precision of an ionization measurement would be $\pm$ 4 per cent and on a linear scale the error curve would be Gaussian.

50-layer proportional counter

The 50-layer proportional counter is a rectangular box of internal dimensions 156 $\times$ 600 $\times$ 600 mm. The counter consists of 50 identical layers divided by an aluminium foil 0.1 mm. thick. Each layer consists of a rectangular frame, the end wall of which is made of an insulator–fluoroplast–4. Twentyfive Mo wires 0.1 mm. in dia. are stretched over the frame; 13 of the wires are "operating" ones and 12 are "blanks". The potential of the blank wires is the same as that of the counter body. The wires divide the layer into 13 counters, each with a square cross-section 12 $\times$ 12 mm. (fig. 8). The "ope-

* The logarithmic mean method was suggested by B. N. Moiseiev.

** The electronic apparatus was designed by V. A. Lubimov and V. K. Verzunov.

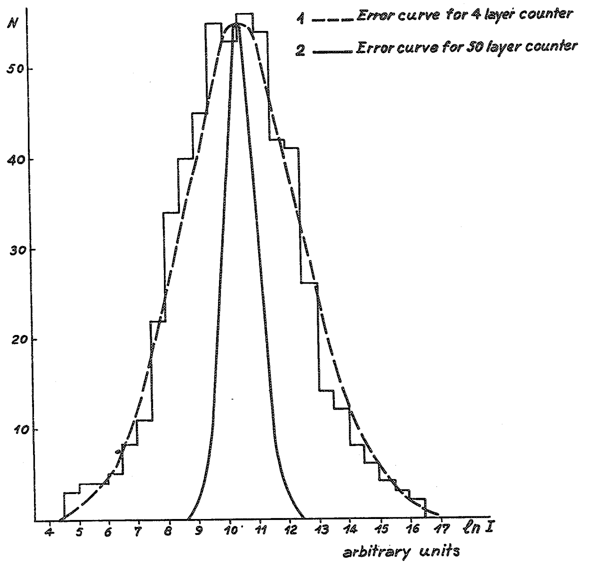

Fig. 7. The histogram of the most probable ionization values for μ mesons. The measurements were made with a 4-layer proportional counter and an arrangement which automatically computed the ionization by the "mean logarithmic" method.

1. Error curve for 4-layer counter.
2. Error curve for 50-layer counter.

rating" wires are connected together to a single output passing through a porcelain insulator. The counter is kept air-tight by means of a "vacuum casing" made of 0.1 mm. thick brass foil. The brass foils of the "casing" are stretched on a frame and serve as the upper and lower walls of the counter. The total thickness of the counter for a particle passing through it is $\sim$ 1.5 gm/cm^2. The errors in momentum measurements due to this amount of matter are negligeable. Small pulses can be measured with high precision outside the space occupied by the counter. The counter is supplied with a metallic sodium "trap" which was included to capture electro-negative atoms of gases which may be released from the counter walls during operation of the counter.

Each layer of the 50 layer proportional counter was tested with the aid of cosmic ray particles. The counter was filled with propane to a pressure of 550 mm. Hg. The high voltage was supplied to the "operating" wires. Experiments showed that there was no dependence of the counter pulse magnitude on the position of the paths of the charged particles. The ionization fluctuation distribution function is essentially the same as that for the usual type of gas proportional counter. On a semi-logarithmic scale the magnitude of the counter pulse is a linear function of the voltage applied to the counter wires up to gas

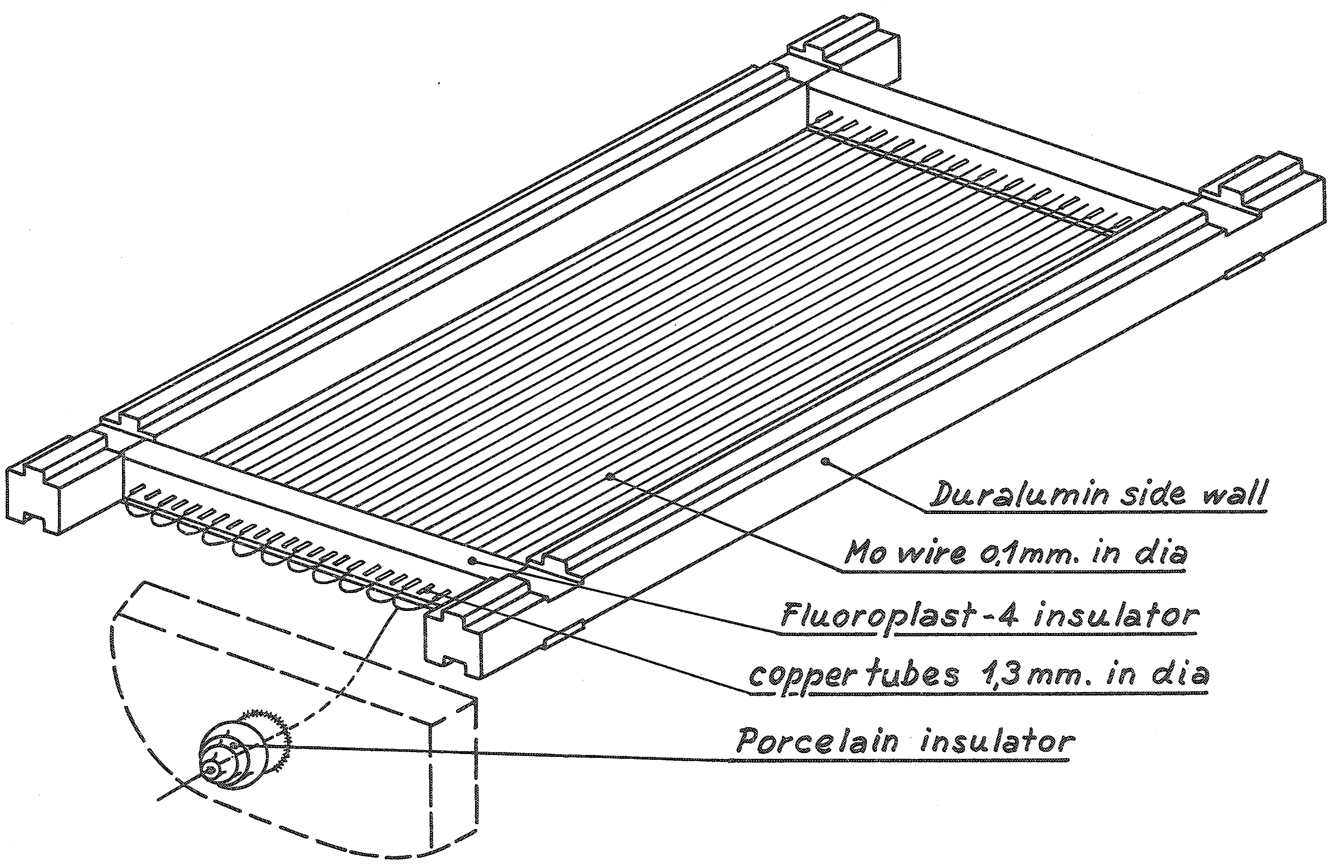

Fig. 8. The view of a layer of 50 layer counter.

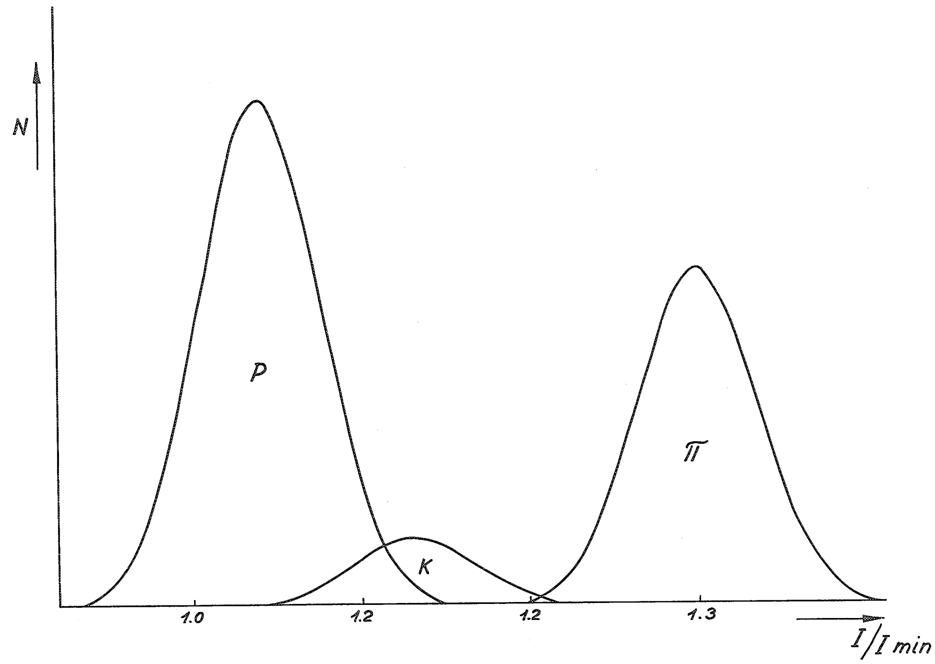

Fig. 9. The distribution of ionization fluctuations due to π and K mesons and to protons with an energy of 7 Bev as measured with a 50-layer proportional counter.

amplification factors ~ 10^6; the slope is 0.9 per cent signal magnitude per volt applied to the counter wires. The counter operates satisfactorily in the proportional region up to an amplification of ~ 10^6 without changing its properties.

The 50 layer proportional counter was designed primarily for cosmic ray work. Its comparatively complicated design was due to the necessity of subtending a sufficiently large solid angle.

In accelerator work the counter design may be much more simple.

Particle identification with a 50-layer proportional counter in the relativistic velocity region (on the increasing part of this ionization curve)

The tests of the counter layers and measuring apparatus described above were apparently sufficient to permit one to assume that the real precision in measurement of ionization with a 50 layer proportional counter was ± 4 per cent.

The distribution of ionization fluctuations due to π and K mesons and to protons with an energy of 7 Bev as measured with a 50 layer proportional counter is illustrated in fig. 9.

At an energy of 7 Bev the ionizing power of protons, K mesons and π mesons is respectively 1.03 I_{min} 1.13 I_{min} and 1.30 I_{min}. It can be seen from the figure that when negative particles of this energy are studied the π and K mesons can be distinctly separated in ionization measurements for any ratio between the intensities of the particles.

When positive particles are studies, only protons and π mesons can be resolved with the 50 layer proportional

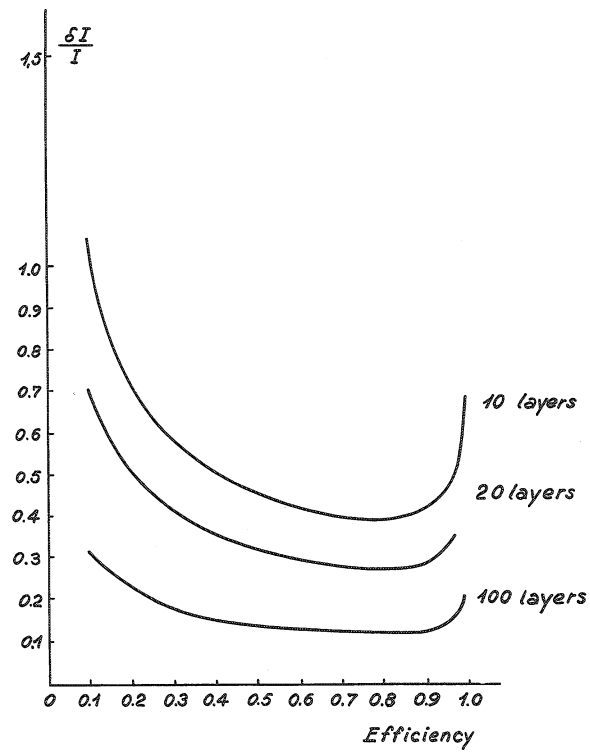

Fig. 10. Dependence of the relative error δI/I in the determination of ionization with a low-efficiency counter, on the efficiency of a layer q and on the number of counter layers.

counter. This separation of protons and π mesons can be carried out up to particle momenta of 20-30 Bev/C.

Another possible way of precise and sufficiently rapid measurement of ionization with gas counters is the use of multilayer low-efficiency counters. In a low-efficiency multilayer counter the ionization of a particle can be expressed as follows:

$$I \sim \ln N - \ln K$$

where N is the total number of layers and K is the number of untriggered layers.

The mean square root error in determination of I is then

$$\frac{\delta I}{I} = \pm \frac{\sqrt{Nq\,(1-q)}}{N\,(1-q)\,\ln(1-q)}$$

where q is the average efficiency of a layer with respect to a particle with a given I. The dependence of $\delta I/I$ on the number of layers and on the efficiency q is shown in fig. 10.

A twenty layer low-efficiency counter was successfully used to determine the dependence of the primary ionization of mesons on energy [8].

Very simple electronic apparatus is required in low efficiency counter measurements, but another difficulty then arises in order to attain a high accuracy the number of layers must be large.

In order to attain an accuracy of 2 per cent in a single measurement about 200 layers would be required in a proportional counter whereas $\sim$ 4,000 would be required in a low efficiency counter. Thus the main difficulty in high precision ionization measurements with multilayer proportional counters is the necessity of using very complex electronic apparatus; if low efficiency counters are utilized the main difficulty is in the design of the counter itself.

LIST OF REFERENCES

1. Landau, L. On the energy loss of fast particles by ionization. J. Phys. USSR, *8*, p. 201-5, 1944.
2. Alikhanian, A. I. and Kharitonov, V. (Particles with mass intermediate between masses of meson and proton. Doklady Akad. Nauk SSSR, *85*, p. 295-8, 1952.
3. Eliseiev, G. P., Kosmoachevski, V. K. and Lubimov, V. A. (Dependence of primary and complete ionization of μ mesons on energy.) Doklady Akad. Nauk SSSR, *90*, p. 995-8, 1953.
4. Lubimov, V. A., Eliseiev, G. P. and Kosmachevski, V. K. Izv. Akad. Nauk SSSR, Ser. Fiz., *19*, p. 720, 1955.
5. Lubimov, V. A., Eliseiev, G. P. and Kosmachevski, V. K. Izv. Akad. Nauk SSSR, Ser. Fiz., *19*, p. 753, 1955.
6. (Problems of modern physics.) Ser. 5, *7*, 1953.
7. (*a*) Ghosh, S. K., Jones, G. M. D. B. and Wilson, J. G. Ionization in oxygen by μ mesons. Proc. phys. Soc., Lond., A, *65*, p. 68-9, 1952.
   (*b*) Parry, J. K., Rathgeber, H. D. and Rouse, J. L. Ionization of cosmic ray mesons in argon. Proc. phys. Soc., Lond., A, *66*, p. 541-8, 1953.
   (*c*) Kharitonov, V. and Barski, A. Doklady Akad. Nauk SSSR, *93*, p. 645- , 1953.
   (*d*) Lubimov, V. A., Eliseiev, G. P., Kosmachevski, V. K. and Kovda, A. V. (Probable ionization by μ mesons in gas in the momentum interval 2×10^8 to 1.2×10^{11} ev/c.) Doklady Akad. Nauk SSSR, *100*, p. 883-6, 1955.
8. Eliseiev, G. P., Lubimov, V. A. and Frolov, A. M. (Primary ionization spectrum of fast μ mesons.) Doklady Akad. Nauk SSSR, *107*, p. 233-5, 1956.