FIXED FIELD ALTERNATING GRADIENT PARTICLE ACCELERATORS

D. W. KERST, K. R. SYMON, L. J. LASLETT, L. W. JONES, and K. M. TERWILLIGER

Midwestern Universities Research Association *, U.S.A.

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I. General description

Alternating gradient (AG) focusing [1] provides a high degree of stability for both the radial and vertical modes of betatron oscillations in circular particle accelerators. This stability makes possible the construction of many kinds of circular accelerators with magnetic guide fields which are constant in time, called fixed field alternating gradient (hereafter FFAG) accelerators. These machines contain stable equilibrium orbits for all particles from the injection energy to the output energy. These orbits may all be in an annular ring, as in a synchrotron or betraton; the magnetic field must then change rapidly with radius to provide orbits for the different energy particles. If the guide field gradient were made independent of azimuth, one of the modes of betatron oscillation would be clearly unstable. Application of alternating gradient focusing, however, can keep both modes of betatron oscillation stable even with the rapid radial change of magnetic field. It is interesting to note that circular particle accelerators can be classified into four groups according to the type of guide field they use : fixed field constant gradient (conventional cyclotrons, synchro-cyclotrons and microtrons), pulsed field constant gradient (weak focusing synchrotrons and betatrons), pulsed field alternating gradient (AG synchrotrons), and fixed field alternating gradient (FFAG synchrotrons, betatrons, and cyclotrons).

Two types of FFAG design appear the most practical. The radial sector type** achieves AG focusing by having the fields in the successive focusing and defocusing magnets vary in the same way with radius but with alternating signs (or in certain cases alternating magnitudes). Since the orbit in the reverse field magnet bends away from the center, the machine is considerably larger than a conventional AG machine [1] of the same energy having an equal peak magnetic field. This serious disadvantage is largely overcome in the spiral sector type (suggested by D. W. Kerst), in which the magnetic field consists of a radially increasing azimuthally independent field on which is superimposed a radially increasing azimuthally periodic field. The peaks and troughs of the periodic field spiral outward at a small angle to the orbit. The radial separation between peaks is small compared to the radial aperture. The particle, crossing the field ripples at a small angle, experiences alternating gradient focusing. Since the fields need not be anywhere reversed, the size of this machine can be comparable to that of an equivalent conventional AG machine.

FFAG synchrotrons have a number of important advantages over conventional synchrotrons. A major one is beam intensity. Since the magnetic field is time independent in an FFAG synchrotron, the beam pulse rate is determined only by the repetition rate of the radio frequency modulation cycle. In a conventional synchrotron, the beam pulse rate is limited by the time to complete the pulsed magnetic field cycle. It is reasonable to assume that RF cycle repetition rates can be made considerably higher than field recycling rates. In addition, one may consider accelerating several groups of particles simultaneously, so that the interval between times when groups of particles are accepted from the injector may be made much less than the time required to accelerate one group to full energy.

The radio-frequency acceleration may follow a more arbitrary frequency-versus-time program with FFAG synchrotrons since there is no magnetic field tracking requirement as in pulsed-field synchrotrons. This allows the use of a mechanical modulation system with high-Q cavities. With the high-Q realized in unloaded cavities, the required voltage gain per turn could be given the particles by one cavity driven at reasonable pover. Modulation could be accomplished by a moving diaphragm or similar device to tune the cavity capacity. With such a system, model tests indicate a frequency change of a factor of greater than 3:1 is practical. Using 5 Mev injection, a frequency change of 10 : 1 is required to reach relativistic velocities. One might then use one cavity operating as a self-excited oscillator to accelerate particles from injection to about 50 Mev. The voltage on that cavity would then be turned off as voltage on a second cavity is turned on, and acceleration continued with the

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** Suggested by K. R. Symon. This structure was also suggested independently earlier (1953) by T. Ohkawa, University of Tokyo, Tokyo, Japan. (private communication.)

second cavity. The change-over could be triggered by frequency comparison between cavities. The relative phases of the cavities could be controlled by a loose coupling between them. (With the University of Michigan electron synchrotron two cavity RF system, it was observed that is was possible to make the transition from one cavity to another without an observable beam loss.) A third cavity might be added and a second transition made if desired, since it is observed that most of the energy is given the particles after they have reached almost constant velocity, c, and this third cavity could be designed to provide very high voltage over a small frequency range. Fine frequency adjustments would be made with reactance tube loading of the cavities. With this RF system it appears reasonable to accelerate protons to 20 Bev with a repetition rate of two or three per second. While the above system is suggested on the basis of experimental tests already in progress, it is realized that other RF systems might prove more practical. In alternating gradient synchrotrons, phase stability vanishes at a transition energy. It is possible in the radial sector FFAG designs to have k large and negative. In this case there is no transition energy, and high energy orbits lie on the inner radius of the machine. Negative k designs appear o be not practical with spiral sectors.

Another reason for high beam intensity is the large injection aperture possible in the FFAG designs. Whereas injection from a 50 Mev proton linear accelerator is planned for 25 Bev pulsed-field accelerators, a 5 Mev Van de Graaff electrostatic generator might be used to inject into FFAG synchrotrons for the following reasons. Eddy current effects on the magnetic fields are absent in FFAG synchrotrons and the effects of remanent magnetic fields can be reduced by properly distributed currents (or by a demagnetizing procedure at the end of an operating day), so that injection into weaker magnetic fields appears practical. By enlarging the injection aperture space charge and gas scattering effects may be reduced, allowing the lower injection energy. Conventional synchrotrons must inject into a region where the magnetic field will later be pulsed to its maximum value, so that an increase in injection aperture would require an increase in peak magnet power and stored energy. The use of electrostatic generator injection with FFAG synchrotrons would have the advantages of higher pulse currents, greater simplicity, lower cost, and better beam energy and size resolution than are at present realized with proton linear accelerators. Although one-turn injection using a pulsed inflector with a pulsed current of milliamperes is the most obvious injection system, many-turn injection might be used to give greater beam currents if methods of circumventing the space charge limit are found.

Other advantages of the FFAG synchrotron are engineering and maintenance simplifications. The direct current magnet power supply is simpler and cheaper than a pulsed supply to construct and to maintain. The magnets do not have to be laminated, and field trimming is all time independent. Disadvantages of the FFAG synchrotron are the large increase in circumference for the radial sector type (at least a factor of three) and the increase in complexity of the magnetic fields, particularly for the spiral sector machine.

Fixed field betatrons have potentially a much higher intensity than conventional betatrons.* Beam can be injected for a considerable fraction of a cycle, if extra accelerating flux is available, rather than the few tenths of a microsecond presently possible. The only beam current limitation appears to be space charge at injection, and this may be decreased by such techniques as high voltage injection. An FFAG betatron has no problems of tracking a pulsed guide field with the accelerating flux, and has also other engineering simplifications mentioned in the synchrotron case.

Application of the FFAG principle to a cyclotron allows the radial dependence of the magnetic field to be such as to keep the particle revolution rate constant, independent of energy even in the relativistic region. Present high energy cyclotrons must be frequency modulated to compensate for the relativistic increase of mass. A constant frequency cyclotron should increase the beam output about two orders of magnitude. A radial sector cyclotron, in which the field alternates between high and low values, was first suggested by Thomas [2]. The spiral sector design seems even more advantageous for application to the cyclotron.

II. Types of FFAG design

1. *Radial sector type*

Circular particle accelerators with radial sectors can be built with the high energy orbits at the outer edge of the machine and the injection orbits at the inside edge, or vice versa. This discussion assumes the highest energy orbits are at the outside edge. (We will refer specifically to FFAG synchrotrons, but most of our comments will apply also to betatrons and cyclotrons.) In the radial sector design the magnet structure consists of N identical sectors, each composed of a focusing magnet and a defocusing magnet. The magnet which is focusing for radial oscillations is of course defocusing for vertical oscillations and vice versa. The azimuthal boundaries of the magnets are on radii from the machine center (hence the name). The magnetic field direction in one magnet of a sector is opposite to that of the other, while the radial dependence of the field is the same in both. The field in the median plane at any azimuth is

$$H = H_0 \left(\frac{r}{r_0}\right)^k \tag{1.1}$$

where r is the distance from the machine center to the equilibrium orbit and k is a constant for the machine. This field shape requires that orbits for different energy particles are similar, i.e. photographic images of each other. Ideally,

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* This has been pointed out independently by Miyamoto, Tokyo University, Tokyo, Japan, at a symposium on nuclear physics of the Physical Society of Japan in October, 1953. (private communication.)

the field along a closed equilibrium orbit is constant through each magnet, and the path is composed of arcs of circles. This situation is perturbed by the impossibility of a sharp field boundary. If we assume the ideal situation, a particularly simple case occurs when the fields for a given energy orbit have the same magnitude in the positive and negative field magnets.

It is evident that particles deviating from the equilibrium orbit experience AG focusing. The numbers of radial and vertical betatron oscillations around the machine, ν_x and ν_z, are determined by k and the magnet lengths. Both ν_x and ν_z are constant for all energies.

It is desirable to make the negative field magnets as short as possible, to keep the radius of the machine small; the minimum length of the negative field magnet is of course determined by the necessity for preserving stability of the vertical betatron oscillations. Some vertical focusing and radial defocusing occur because the orbits are scalloped and do not cross the magnet edges at right angles. In machines in which the number of sectors is large and the effects of orbit scalloping small, the negative field magnet can be made no shorter than about $^2/_3$ of the positive field magnet if we wish to preserve vertical stability. This means that, neglecting straight sections, the circumference of the machine is five times that necessary if there were no negative field magnets. The ratio (in this case, five) between the actual orbit circumference of a circle whose radius is the minimum radius of curvature at any point along the orbit, we call the circumference factor. The fixed magnetic field in an FFAG machine can be made considerably larger than the pulsed field of a conventional accelerator, so a machine of the radial sector type might actually be about three times the size of a pulsed field AG accelerator of the same energy. It is also desirable to make the radial extent of the magnets as small as possible, which requires a high field gradient. The allowable gradient is determined by the effect of magnet misalignments. Reasonable values indicate a minimum radial aperture of about 2% of the radius of the machine.

2. *Spiral sector type*

The spiral sector design of FFAG accelerator has the high energy orbits at the outside edge of the machine. It is not practical to have the high energy orbits on the inside and inject at the outside edge, because stability of the radial oscillations becomes virtually impossible to achieve.

The guide field on the median plane, if there are no straight sections, is given by

$$H = H_0 (r/r_0)^k \{1 + f \cos [N \theta - N \tan \zeta \ln (r/r_0)]\} \tag{2.1}$$

where r is again the distance from the center of the machine; k, the mean field index; θ, the azimuthal angle, also measured from the center of the machine; f, the flutter factor (the fraction of field variation); N, the number of sectors (periods of the field variation) around the machine; and ζ is the spiral angle between the field maximum and the radius. The equilibrium orbits are all similar figures, whose linear dimensions are proportional to the radius, but their positions rotate with radius due to the spiraling periodic field. A particle going around the machine experiences a gradient first of one sign then the opposite as it crosses the periodic field peaks and troughs at a small angle, so there is AG focusing of the betatron oscillations. The negative gradient is less than the positive gradient, due to the radial increase of field. This is somewhat compensated by the scalloping of the orbits, which causes the particle to experience a longer path in the negative gradient and a shorter path in the positive gradient than if it moved on a circle. The strength of betatron focusing depends on the rate of radial increase of the field, the spiral angle, and the number of sectors. The minimum size of radial aperture is limited primarily by the difficulty of achieving strong AG focusing with a periodic field while requiring a given vertical aperture. A flutter factor of about $^1/_4$ gives the largest vertical gap for a fixed strength of focusing when iron magnet poles are used without distributed backwindings and forward windings. This small flutter factor means the machine has a circumference factor (in this case, $1 + f$), close to unity, so the radius of an FFAG spiral sector synchrotron is about the same as that of an equivalent energy conventional synchrotron. By using a field variation in the median plane which is more rectangular than sinusoidal, some increase in vertical aperture and also in the maximum stable amplitude of vertical oscillations is achieved at some sacrifice of circumference factor. The minimum radial aperture for reasonable parameters is about 3% of the radius.

3. *Other FFAG types*

Both the radial sector and spiral sector designs discussed above have equilibrium orbits of constant shape scaled in proportion to the orbit radius. There are many modifications of these designs. Some differ only in that the fields are not the square wave type used in the radial sector design described or the sinusoidal shape used in the spiral sector design. There are other variations of these designs which preserve betatron oscillation stability, hold ν_x and ν_z constant, but do not retain the property of similarity of equilibrium orbits. The magnet edges of focusing and defocusing sectors can be made non-radial, and the fields in the positive and negative field magnets made different functions of radius (the negative field magnet can even be designed to have zero field). The magnet edges, radial or non-radial, can be tipped in the same direction, approaching the spiral sector design. Machines made with these modifications do not seem to show any strong advantages with perhaps the following exception. It is conceivable, using backwindings, to transform from a spiral sector at the outside edge of the machine, with a small circumference factor where it is needed, to a radial sector at the inside edge, with a large vertical aperture for injection. Such a design would have the advantages of both types with, however, a considerable increase in magnet complexity.

Another modification is the spiral sector constant frequency cyclotron. In this machine, the frequency of revolution of the particles can be made independent of energy even at relativistic energies, but the orbits in this case do not scale, and the number of betatron oscillations, ν_x and ν_z cannot be kept constant.

LIST OF REFERENCES

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