

N -body nuclear forces at short distances in holographic QCDKoji Hashimoto,^{1,*} Norihiro Iizuka,^{2,†} and Takashi Nakatsukasa^{1,‡}¹*RIKEN Nishina Center, Wako, Saitama 351-0198, Japan*²*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

(Received 17 November 2009; published 7 May 2010)

We provide a calculation of N -body ($N \geq 3$) nucleon interactions at short distances in holographic QCD. In the Sakai-Sugimoto model of large- N_c massless QCD, N baryons are described by N Yang-Mills instantons in 5 spacetime dimensions. We compute a *classical* short-distance interaction Hamiltonian for N 't Hooft instantons. This corresponds to N baryons sharing identical *classical* spins and isospins. We find that genuine N -body nuclear forces turn out to vanish for $N \geq 3$, at leading order. This suggests that classical N -body forces are always suppressed compared with 2-body forces.

DOI: 10.1103/PhysRevD.81.106003

PACS numbers: 21.30.-x, 11.25.-w, 11.25.Tq, 21.45.Ff

I. INTRODUCTION

Recent developments in computational nuclear physics reveal that the microscopic description of nucleus in terms of nucleon degrees of freedom requires three-nucleon interactions. In fact, although the nuclear three-body interaction is significantly weaker than the two-body interaction, the binding energies of light nuclei [1] and the saturation density of nuclear matter [2] cannot be understood without taking into account the three-body terms. This is due to a large cancellation of the kinetic energy and two-body attraction. The main component of the three-body interaction is associated with two-pion exchange, such as the Fujita-Miyazawa force [3]. However, in addition to this, a repulsive three-body interaction of a short range is required for a quantitative description of nuclear systems [1,4]. In particular, the short-range three-nucleon interaction, which is assumed to be spin/isospin independent in many cases, is important for determination of the nuclear equation of state at high density [2,5].

The two-body interactions adopted in those many-body calculations are determined by the phase-shift analysis of nucleon-nucleon scattering data. However, much less information is available for the N -body forces ($N \geq 3$). Of course, we know that, in principle, the nuclear properties should be derived from QCD [6]. However, QCD is strongly coupled at the nuclear energy scale, which leads to a huge gap between QCD and nuclear many-body problems.

Recent progress in string theory can bridge this gap, analytically. It is called holographic QCD, an application of gauge/string duality [7] to strongly coupled QCD. We apply the holographic QCD to N -body nuclear force ($N \geq 3$).

In holographic QCD, one of the most successful D-brane models is the Sakai-Sugimoto model (SS model) [8,9]. The theory, which is a $U(N_f)$ Yang-Mills-Chern-Simons (YM-CS) theory in a warped 5-dimensional spacetime, was conjectured to be dual to low energy massless QCD with N_f flavors, in the large- N_c and large- λ limits ($\lambda \equiv N_c g_{\text{QCD}}^2$ is a 't Hooft coupling of QCD). Modes of the gauge fields correspond to meson degrees of freedom and this model reproduces surprisingly well various expected features of hadrons, incorporating very nicely the nature of chiral Lagrangians.

Baryons are identified with soliton solutions localized in the spatial 4-dimensions [8]. This is quite analogous to that in pion effective theory; baryons are identified with Skyrmions [10]. Quantization of a single soliton in the SS model [11,12] gives baryon spectra, and also chiral properties such as charge radii and magnetic moments [13] (for other approaches to baryons, see [14]). Meson-baryon-baryon couplings [13] give a basis of a 2-body nuclear force at long distances, *à la* the one-meson-exchange picture. Short-distance nucleon-nucleon forces were computed [15], which generates a repulsive core with analytic formula for potentials in the large- N_c limit. A key is that the warping can be absorbed into the rescaling of the YM-CS theory and brings the string scale to QCD scale. Furthermore, when two solitons are close to each other, the warping factor is almost constant; therefore the effects of the curved geometry can be ignored so that an exact two-soliton solution is available.

In this paper, we compute N -body nuclear forces for arbitrary N , with *exact* N -instanton solutions, generalizing the method in Ref. [15]. The exact treatment is in contrast to the Skyrmion and other chiral soliton models, in which multisoliton solutions are quite difficult to obtain.

II. NUCLEAR FORCE AT SHORT RANGE

Baryons, including nucleons, are identified with solitonic solutions in the SS model [8], and we provide a brief

*koji@riken.jp

†norihiro.iizuka@cern.ch

‡nakatsukasa@riken.jp

review of the construction of the soliton and the 2-body nuclear force at short range for two flavors ($N_f = 2$) computed in [15].

The following rescaling of the coordinates [11] can allow one to understand the system as a $1/\lambda$ perturbation around a flat space, which is suitable for studying the instanton solution:

$$\tilde{x}^M = \lambda^{1/2} x^M \quad (M = 1, 2, 3, 4), \quad \tilde{x}^0 = x^0, \quad (1)$$

and accordingly $\tilde{A}_0(t, \tilde{x}) = A_0(t, \tilde{x})$ and $\tilde{A}_M(t, \tilde{x}) = \lambda^{-1/2} A_M(t, \tilde{x})$. In the following we omit the tilde for simplicity. In these new variables, there are essentially two deviations from the YM theory in the flat space, at the leading order in $1/\lambda$ expansion: (i) the effect of the CS term, and (ii) the effect of the space weakly curved along the x^4 direction. The additional Hamiltonians are

$$H_{\text{pot}}^{U(1)} \equiv \frac{-aN_c}{2} \int \prod_M dx^M \hat{A}_0 \hat{A}_0, \quad (2)$$

$$H_{\text{pot}}^{SU(2)} \equiv \frac{aN_c}{6} \int \prod_M dx^M (x^4)^2 \text{tr}(F_{MN})^2, \quad (3)$$

respectively, with $a \equiv 1/(216\pi^3)$. Note that we work in the unit $M_{\text{KK}} = 1$, where M_{KK} is the unique scale parameter appearing in the model, and it can be fixed by fitting the ρ meson mass, giving $M_{\text{KK}} = 949$ [MeV].

In particular, for a single baryon, the leading order solution is a single instanton in the flat 4-dimensional space, that is BPST instanton [16]. The instanton has moduli parameters: the instanton location X^M , the size ρ , and the orientation in $SU(2)$. These Hamiltonians induce potentials in the moduli space of the instanton, and ρ and X^4 prefer particular values classically,

$$(\rho_{\text{cl}})^2 = \frac{1}{8\pi^2 a} \sqrt{\frac{6}{5}}, \quad X_{\text{cl}}^4 = 0.$$

For multi-instantons, there appears a potential for the moduli representing the distance between the instantons, which is in fact the nuclear force.

In Ref. [15], this 2-body nuclear force was evaluated explicitly. Multi-instanton solutions are available in flat space, while in this particular curved space it is difficult to find them. However, when instantons are close enough to each other, the effect of the curved space can be neglected, and as a leading order solution we can use the multi-instanton solutions in the flat space. Therefore, the distance r_{ij} between the i th and the j th nucleons allowed in this approximation is $|r_{ij}| < M_{\text{KK}}^{-1}$ ($|r_{ij}| < \lambda^{1/2} M_{\text{KK}}^{-1}$) in the original (rescaled) coordinates. Thus we probe only the short range for the nuclear force.

The construction of the two-instanton solution owes to the renowned ADHM (Atiyah-Drinfeld-Hitchin-Manin) method [17,18]. The moduli parameters of generic N instanton solutions are completely encoded in the real $N \times N$

matrix function $L(x; X, \dots)$. Osborn's formula [19] tells us the instanton density

$$\text{tr}(F_{MN})^2 = \square^2 \log \det L, \quad (4)$$

where $\square \equiv \partial_M \partial_M$. Using this expression, the equation of motion for the $U(1)$ part of the gauge field which is sourced by the instanton density is solved as [11]

$$\hat{A}_0 = \frac{1}{32\pi^2 a} \square \log \det L. \quad (5)$$

With this explicit dependence on the instanton moduli parameters in L , one can compute the Hamiltonians (2) and (3) as functions of them. Then, the expectation value of the Hamiltonians for given baryon states (the wave functions are written by the moduli parameters) gives the nuclear force at short range [15].

There is the third contribution to the additional Hamiltonians, H_{kin} , which is present only in the multi-instanton case. This comes from the metric of the instanton moduli space. In Ref. [15], it was shown that it is higher order in $1/N_c$ compared to the other two Hamiltonians (2) and (3), so we need not compute it in this paper.

III. 3-BODY NUCLEAR FORCE

The 2-body nuclear force computed in Ref. [15] is for generic spin/isospin components. But since an explicit generic N instanton solution is not available, we consider a special solution called 't Hooft instanton which has $5N - 3$ moduli parameters (while a generic instanton solution has $8N - 3$ moduli parameters). It is important to notice that once we restrict our moduli space by hand like this, we cannot get the generic expression for the nuclear force for given baryon states. Instead, what we will obtain is a classical analog of the nuclear force.

The moduli parameters of the 't Hooft instantons are only the size ρ_i and the location X_i^M of each instanton ($i = 1, 2, \dots, N$). The missing parameters, the orientations of the instantons in $SU(2)$, are responsible for the spin/isospin wave functions of the baryons. Thus our analysis with the 't Hooft instantons is restricted to "classical" baryons, where all the spin/isospins of the baryons are classically identical.

First, let us show that $H_{\text{pot}}^{SU(2)}$ given in Eq. (3) is irrelevant to the three-body nuclear forces. We can use the generic formula obtained in Appendix C of Ref. [15],

$$\int d^4x (x^4)^2 \text{tr}(F_{MN})^2 = 8\pi^2 \sum_{i=1}^N (2(X_i^4)^2 + \rho_i^2), \quad (6)$$

for the N 't Hooft instantons. The expression consists of just a sum of each instanton sector, which means that there is no term involving the internucleon distance, that is, no contribution to the nuclear force. Therefore, we compute the other Hamiltonian (2) in this paper. (The contribution

from H_{kin} is suppressed as in the case of the 2 instantons [20].)

In this section, we concentrate on the case for $N = 3$, i.e. the 3-body force.

For three 't Hooft instantons, which correspond to nucleons sharing classically identical spins/isospins, we have

$$L = \begin{pmatrix} (x - X_1)^2 + \rho_1^2 & \rho_1 \rho_2 & \rho_1 \rho_3 \\ \rho_1 \rho_2 & (x - X_2)^2 + \rho_2^2 & \rho_2 \rho_3 \\ \rho_1 \rho_3 & \rho_2 \rho_3 & (x - X_3)^2 + \rho_3^2 \end{pmatrix}, \quad (7)$$

where we omit the index M and denote $(x^M)^2$ by x^2 . Then Osborn's formula becomes particularly simple,

$$\text{logdet}L = \sum_i \log(x - X_i)^2 + \text{log}f, \quad (8)$$

with

$$f \equiv 1 + \sum_i \frac{\rho_i^2}{(x - X_i)^2}.$$

This gives the $U(1)$ gauge field

$$\hat{A}_0 = \frac{1}{32\pi^2 a} \left[\square \sum_i \log(x - X_i)^2 + \frac{\square f}{f} - \frac{(\partial_M f)^2}{f^2} \right]. \quad (9)$$

The first term in Eq. (8) is a self-energy which was already computed, and f is a harmonic function, i.e., $\square f = 0$. Thus, all we need to evaluate is only the last term in Eq. (9), $(\partial_M f)^2/f^2$.

For three instantons, we can expand the expression for $(x - X_1)^2 \ll (x - X_2)^2, (x - X_3)^2$. In particular, we can approximate $(x - X_2)^2 \sim (X_1 - X_2)^2 \equiv X_{12}^2$, and a similar expression for X_{13} . Furthermore, for simplicity we put $X_1^M = 0$. Then, the expansion is

$$\begin{aligned} \hat{A}_0 = & \frac{1}{8\pi^2 a} \left[\frac{1}{x^2} \left(1 - \frac{\rho_1^4}{(x^2 + \rho_1^2)^2} \right) + \frac{1}{X_{12}^2} + \frac{1}{X_{13}^2} \right. \\ & + \frac{2\rho_1^4}{(x^2 + \rho_1^2)^3} \left(\frac{\rho_2^2}{X_{12}^2} + \frac{\rho_3^2}{X_{13}^2} \right) \\ & + \frac{-3\rho_1^4 x^2}{(x^2 + \rho_1^2)^4} \left(\frac{\rho_2^2}{X_{12}^2} + \frac{\rho_3^2}{X_{13}^2} \right)^2 \\ & \left. + \frac{-2\rho_1^2}{(x^2 + \rho_1^2)^2} \left(\frac{\rho_2^2 x \cdot X_{12}}{X_{12}^4} + \frac{\rho_3^2 x \cdot X_{13}}{X_{13}^4} \right) + \text{higher} \right]. \end{aligned} \quad (10)$$

We like to compute the potential (2). As seen from the expression for \hat{A}_0 , the leading term of the nontrivial three-body force has the form $\frac{1}{X_{12}^2 X_{13}^2}$. This means that, we should expect the nuclear force appearing *proper to the 3-body*, at the leading order,

$$H = \mathcal{O}\left(\frac{N_c}{\lambda^2}\right) \frac{1}{X_{12}^2 X_{13}^2}, \quad (11)$$

in the original coordinates. Let us consider only terms of this leading form (11). We obtain

$$\begin{aligned} H_{\text{pot}}^{U(1)}|_{3\text{-body}} = & \frac{-aN_c}{2(8\pi^2 a)^2} \int d^4x \left[\frac{1}{x^2} \left(1 - \frac{\rho_1^4}{(x^2 + \rho_1^2)^2} \right) \square \frac{-6\rho_1^4 x^2}{(x^2 + \rho_1^2)^4} \frac{\rho_2^2 \rho_3^2}{X_{12}^2 X_{13}^2} + \frac{-6\rho_1^4 x^2}{(x^2 + \rho_1^2)^4} \frac{\rho_2^2 \rho_3^2}{X_{12}^2 X_{13}^2} \square \frac{1}{x^2} \left(1 - \frac{\rho_1^4}{(x^2 + \rho_1^2)^2} \right) \right. \\ & + \frac{1}{X_{12}^2 X_{13}^2} \left(1 + \frac{2\rho_1^4 \rho_2^2}{(x^2 + \rho_1^2)^3} \right) \square \left(1 + \frac{2\rho_1^4 \rho_3^2}{(x^2 + \rho_1^2)^3} \right) + \frac{1}{X_{12}^2 X_{13}^2} \left(1 + \frac{2\rho_1^4 \rho_3^2}{(x^2 + \rho_1^2)^3} \right) \square \left(1 + \frac{2\rho_1^4 \rho_2^2}{(x^2 + \rho_1^2)^3} \right) \\ & \left. + (1 \rightarrow 2 \rightarrow 3) + (1 \rightarrow 3 \rightarrow 2) \right]. \end{aligned} \quad (12)$$

Performing the derivatives, and using the following integration formulas:

$$\int d^4x \frac{x^{2(N-j)}}{(x^2 + \rho_1^2)^{N+5}} = \frac{\pi^2(j+2)!(N-j+1)!}{\rho_1^{2j+6}(N+4)!}, \quad (13)$$

we find that the right-hand side of Eq. (12) vanishes. Therefore, the leading term of the order $1/(X_{12}^2 X_{13}^2)$ vanishes. This means that the expansion starts from the next-to-leading order,

$$H_{\text{pot}}^{U(1)}|_{3\text{-body}} = \frac{-N_c}{128\pi^4 a} \mathcal{O}\left(\frac{(\rho)^4}{X_{12}^2 X_{13}^4}, \frac{(\rho)^4}{X_{12}^3 X_{13}^3}, \dots\right), \quad (14)$$

where ... represents terms obtained by permutation for the

indices 1, 2, 3. Here the dependence on ρ_i ($i = 1, 2, 3$) is fixed to be $(\rho)^4$ by a dimensional analysis. The expectation value of this $(\rho)^4$ at the leading order in large- N_c is given by the classical value given before. Then, rescaling the coordinates back as $X_{12} \rightarrow \lambda^{1/2} X_{12}$ and writing it as the 3-dimensional internucleon distance r_{12} since we substitute the classical value $X_i^4 = 0$, we obtain, at the leading order in $1/N_c$,

$$H_{\text{pot}}^{U(1)}|_{3\text{-body}} = \frac{N_c}{\lambda^3} \mathcal{O}\left(\frac{1}{r_{12}^2 r_{13}^4}, \frac{1}{r_{12}^3 r_{13}^3}, \dots\right), \quad (15)$$

where again ... represents the term obtained by permutation for the indices 1, 2, 3.

Note that we are working in a regime $\lambda^{-1/2} \ll X_{12,13} \ll 1$ in the unit $M_{\text{KK}} = 1$. The natural scale for the 2-body force [15] is $\mathcal{O}(N_c/\lambda X_{12}^2)$. So, if we consider a natural separation of the nucleons as $X_{ij} \sim 1/M_{\text{KK}}$, the 3-body force is suppressed compared to the 2-body force. We conclude that the 3-body force at short range is small, for baryons carrying classical and equal spin/isospins.

IV. N -BODY NUCLEAR FORCE

We can easily extend the analysis in the previous section to N 't Hooft instantons. The result for the leading term vanishes again, as we explain briefly below.

The quantity necessary for computing \hat{A}_0 is

$$\begin{aligned} \frac{(\partial_M f)^2}{f^2} &= \left[\frac{4\rho_1^4}{x^6} + \sum_{i=2}^N \frac{4\rho_i^2 \rho_i^2 x \cdot X_{1i}}{x^4 X_{1i}^4} + \sum_{i=2}^N \frac{4\rho_i^4}{X_{1i}^6} \right. \\ &\quad \left. + \sum_{i \neq j} \frac{4\rho_i^2 \rho_j^2 X_{1i} \cdot X_{1j}}{X_{1i}^4 X_{1j}^4} \right] \left[1 + \frac{\rho_1^2}{x^2} + \sum_{i=2}^N \frac{\rho_i^2}{X_{1i}^2} \right]^{-2}. \end{aligned} \quad (16)$$

The expansion analogous to the 3-body case is

$$\begin{aligned} \left[1 + \frac{y_1^2}{x^2} + \sum_{i=2}^N \frac{y_i^2}{X_{1i}^2} \right]^{-2} &= \frac{x^4}{(x^2 + y_1^2)^2} \left(1 - 2 \frac{x^2}{x^2 + y_1^2} \sum_i \frac{y_i^2}{X_{1i}^2} \right. \\ &\quad \left. + 3 \left(\frac{x^2}{x^2 + y_1^2} \sum_i \frac{y_i^2}{X_{1i}^2} \right)^2 \right. \\ &\quad \left. - 4 \left(\frac{x^2}{x^2 + y_1^2} \sum_i \frac{y_i^2}{X_{1i}^2} \right)^3 + \dots \right), \end{aligned} \quad (17)$$

which implies that, apparently, the leading order of the short-range nuclear force in proper to the N -body would be

$$H_{\text{pot}}^{U(1)}|_{N\text{-body}} = \frac{N_c}{\lambda^{N-1}} \mathcal{O} \left(\prod_{i=2}^N \frac{1}{X_{1i}^2}, \dots \right). \quad (18)$$

Again, we have rescaled back the coordinates to the original coordinates and \dots represents permutation terms. In the previous section, we showed that for $N = 3$ this leading contribution vanishes, for the 't Hooft instantons. In this section, we prove that for any N this leading contribution vanishes.

First, we consider

$$\int d^4x \frac{(\partial_M f)^2}{f^2} \square \frac{(\partial_P f)^2}{f^2} \quad (19)$$

in the integral (2). The only possibility to get the leading expression (18) from this integral is to pick up a term (we name it c_1) having $\prod_{i=2}^l (1/X_{1i}^2)$ from the first $(\partial_M f)^2/f^2$ and also a term (we name it c_2) having $\prod_{i=l+1}^N (1/X_{1i}^2)$ from the second $(\partial_P f)^2/f^2$. In view of the original expression (16), we find

$$\begin{aligned} c_1 &= (l-1)! \frac{4\rho_1^4}{x^6} \frac{x^4}{(x^2 + \rho_1^2)^2} (-1)^l l \left(\frac{x^2}{x^2 + \rho_1^2} \right)^{l-1} \prod_{i=2}^l \frac{1}{X_{1i}^2}, \\ c_2 &= (N-l+1)! \frac{4\rho_1^4}{x^6} \frac{x^4}{(x^2 + \rho_1^2)^2} (-1)^{N-l} (N-l+1) \\ &\quad \times \left(\frac{x^2}{x^2 + \rho_1^2} \right)^{N-l} \prod_{i=l+1}^N \frac{1}{X_{1i}^2}. \end{aligned} \quad (20)$$

A straightforward calculation shows

$$\begin{aligned} \square c_2 &= 16\rho_1^4 (-1)^{N-l} (N-l-1) x^{N-l-2} \\ &\quad \times (x-2 + \rho_1^2)^{-N+l-4} ((N-l)(N-l-1)\rho_1^4 \\ &\quad - 6(N-l)x^2\rho_1^2 + 6x^4) \prod_{i=l+1}^N \frac{1}{X_{1i}^2}. \end{aligned} \quad (21)$$

We compute the term $c_1 \square c_2$ included in the integrand of (19) as

$$\begin{aligned} N-1 C_{l-1} \sum_{i=2}^N c_1 \square c_2 &= (N-1)! 64\rho_1^8 (-1)^{N-1} x^{2(N-4)} (x^2 + \rho_1^2)^{-N-5} \prod_{i=2}^N \frac{1}{X_{1i}^2} \\ &\quad \times \sum_{i=1}^N l(N-l+1) [(N-l)(N-l-1)\rho_1^4 \\ &\quad - 6(N-l)x^2\rho_1^2 + 6x^4]. \end{aligned} \quad (22)$$

Here, the factor $N-1 C_{l-1}$ is for choosing a set of $l-1$ elements among $\{2, \dots, N\}$, for c_1 having $l-1$ multiples of $1/X_{1i}^2$. Using the formula (13), this can be easily integrated with $\int d^4x$ to give 0 as

$$\int d^4x_{N-1} C_{l-1} \sum_{i=2}^N c_1 \square c_2 = 0. \quad (23)$$

In the Hamiltonian (18), there are additional terms

$$- \int d^4x \left(\frac{4}{x^2} + \sum_{i=2}^N \frac{4}{X_{1i}^2} \right) \square \frac{(\partial_M f)^2}{f^2} \quad (24)$$

coming from the first term in Eq. (9). For the leading contribution of the form (18), it is enough to pick up c_2 with $l=1$ (for the integral with $1/x^2$) and $l=2$ (for the integral with $\sum_{i=2}^N 1/X_{1i}^2$). Using (21), the integrals can be evaluated, and they are found to vanish.

Therefore, we conclude that the leading order N -body nuclear force (18) vanishes, for arbitrary N . Note that the 2-body nuclear force does not vanish at the leading order, as the $N=2$ computation is exceptional.

V. SUMMARY AND DISCUSSIONS

Using the SS model of holographic QCD, we have found that the N -body nuclear force at short range ($N \geq 3$) is of the order of $N_c (\lambda r^2)^{-N}$, for nucleons sharing identical

classical spin/isospins. This is small compared to the 2-body force which is $\mathcal{O}(N_c(\lambda r^2)^{-1})$ in contrast, and it leads to a hierarchy of the $(N+1)$ -body/ N -body ratio $V^{(N+1)}/V^{(N)} \sim 1/(\lambda r^2) \ll 1$ for $N \geq 3$, in the unit $M_{\text{KK}} = 1$. This suppression is consistent with our empirical knowledge.

Effects of the short-range many-body interaction become more prominent for higher-density nuclear matter. Therefore, for physics of neutron stars and supernovae, for instance, properties of N -body interactions such as what are revealed in this paper are important, even if qualitative.

Our computation is not fully satisfactory since the quantum spin/isospin states of each baryon have not been incorporated. The wave function of the classical spin/isospin is a delta function of the $SU(2)$ orientational moduli of the instantons; thus it is difficult to relate it with the quantum spin/isospins. Nevertheless, it is quite remarkable that the generic N -body nuclear force can be obtained by analytic computations. The successful performance of this computation owes, in particular, to the simplicity of the SS model, in contrast to other chiral soliton models.

Furthermore, the theory on which our computations of the nuclear force is based is not a phenomenological model but obtained by a D-brane construction in string theory with the gauge/string duality. Therefore, in principle, we can try to address theoretically what is different from QCD and what is inherited from it. The present computations go beyond the limitations of arguments using universality of chiral symmetry breaking.

Although the model at hand is for large- N_c QCD, these two properties of baryons in holographic QCD would be sufficiently strong motivations for studying holographic QCD and its relation to nuclear physics further.

ACKNOWLEDGMENTS

K. H. is grateful to M. Oka for helpful discussions. K. H. and N. I. would like to thank KITP, Santa Barbara, and YITP at Kyoto University. N. I. also thanks the Aspen Center for Physics and RIKEN. K. H. and T. N. are partly supported by the Grant-in-Aid for Scientific Research (No. 19740125, No. 21340073), of the Japan Ministry of Education, Culture, Sports, Science and Technology.

-
- [1] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, *Phys. Rev. C* **64**, 014001 (2001).
 - [2] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, *Phys. Rev. C* **58**, 1804 (1998).
 - [3] J. Fujita and H. Miyazawa, *Prog. Theor. Phys.* **17**, 360 (1957); **17**, 366 (1957).
 - [4] S. A. Coon, M. T. Pena, and D. O. Riska, *Phys. Rev. C* **52**, 2925 (1995).
 - [5] S. Nishizaki, Y. Yamamoto, and T. Takatsuka, *Prog. Theor. Phys.* **105**, 607 (2001).
 - [6] For recent lattice QCD analyses, see N. Ishii, S. Aoki, and T. Hatsuda, *Phys. Rev. Lett.* **99**, 022001 (2007).
 - [7] J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998); *Int. J. Theor. Phys.* **38**, 1113 (1999).
 - [8] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **113**, 843 (2005).
 - [9] T. Sakai and S. Sugimoto, *Prog. Theor. Phys.* **114**, 1083 (2005).
 - [10] T. H. R. Skyrme, *Proc. R. Soc. A* **260**, 127 (1961); **262**, 237 (1961); *Nucl. Phys.* **31**, 556 (1962).
 - [11] H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, *Prog. Theor. Phys.* **117**, 1157 (2007).
 - [12] H. Hata and M. Murata, *Prog. Theor. Phys.* **119**, 461 (2008).
 - [13] K. Hashimoto, T. Sakai, and S. Sugimoto, *Prog. Theor. Phys.* **120**, 1093 (2008).
 - [14] D. K. Hong, M. Rho, H. U. Yee, and P. Yi, *Phys. Rev. D* **76**, 061901 (2007).
 - [15] K. Hashimoto, T. Sakai, and S. Sugimoto, *Prog. Theor. Phys.* **122**, 427 (2009).
 - [16] A. A. Belavin, A. M. Polyakov, A. S. Shvarts, and Yu. S. Tyupkin, *Phys. Lett.* **59B**, 85 (1975).
 - [17] M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld, and Yu. I. Manin, *Phys. Lett.* **65A**, 185 (1978).
 - [18] E. Corrigan and P. Goddard, *Ann. Phys. (N.Y.)* **154**, 253 (1984).
 - [19] H. Osborn, *Nucl. Phys.* **B159**, 497 (1979).
 - [20] Let us explain briefly why $H_{\text{kin}} \equiv -\nabla^2/16\pi^2 a N_c$ gives smaller contributions compared to the other two Hamiltonians. Here ∇^2 is the metric on the multi-instanton moduli space. We evaluate the expectation value of ∇^2 with the baryon states. The dimension of ∇^2 is $[\text{length}]^{-2}$. The leading 2-body forces are $\propto 1/X_{12}^2$, so the remaining factor should be dimensionless. Thus ∇^2 originating in the metric is written only by using a dimensionless operator $y \cdot \partial/\partial y$ twice, where y is the moduli ρ and the $SU(2)$ orientations. Then it was found in Ref. [15] that $\langle \nabla^2 \rangle = \mathcal{O}(N_c)$, so, in total, after rescaling back $|X|^2 \rightarrow \lambda|X|^2$, one obtains $\langle H_{\text{kin}} \rangle = \mathcal{O}(\lambda^{-1})$, which is smaller than $\langle H^{SU(2)} \rangle \sim \langle H^{U(1)} \rangle \sim \mathcal{O}(N_c/\lambda)$ by the factor of $1/N_c$. This suppression by $1/N_c$ is expected to any instanton number, that is, general N -body force, and so in this paper we do not consider H_{kin} .