

## INITIAL CONDITIONS, TIME EVOLUTION AND BE CORRELATIONS IN $e^+e^-$ ANNIHILATION\*

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*(Received February 4, 2009)*

Bose–Einstein correlations of identical charged-pion pairs produced in hadronic Z decays are analyzed in terms of various parametrizations. The  $\tau$ -model with a one-sided Lévy proper-time distribution provides a good description, enabling the source function to be reconstructed.

PACS numbers: 13.38.Dg, 13.66.Bc

### 1. Introduction

In particle and nuclear physics intensity interferometry provides a direct experimental method for the determination of sizes, shapes and lifetimes of particle-emitting sources (for recent reviews see [1–3]). In particular, boson interferometry provides a powerful tool for the investigation of the space-time structure of particle production processes, since Bose–Einstein correlations (BEC) of two identical bosons reflect both geometrical and dynamical properties of the particle radiating source.

For our analysis we use a sample of about 500 thousand two-jet events, selected by the Durham algorithm [6] with  $y_{\text{cut}} = 0.006$ , from  $e^+e^-$  annihilation data collected by L3 at a center-of-mass energy of 91.2 GeV.

### 2. Parametrizations of BEC

The two-particle correlation function is defined as:

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)}, \quad (1)$$

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\* Presented by T. Novák at the IV Workshop on Particle Correlations and Femtoscopy, Kraków, Poland, September 11–14, 2008.

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<sup>‡</sup> Visitor from Budapest, Hungary, sponsored by the Scientific Exchange between Hungary (OTKA) and The Netherlands (NWO), project B64-27/N25186.

where  $\rho_2(p_1, p_2)$  is the two-particle invariant momentum distribution,  $\rho_1(p_i)$  the single-particle invariant momentum distributions and  $p_i$  the four-momentum of particle  $i$ . Since we are only interested in BEC, the product of single particle densities is replaced by the so-called reference sample,  $\rho_0(p_1, p_2)$ , the two-particle density that would occur in the absence of Bose–Einstein interference. Here we use mixed events as a reference sample [7].

After some assumptions [1, 2], this two-particle correlation function is related to the Fourier transformed source distribution. In this case

$$R_2(p_1, p_2) = 1 + |\tilde{f}(Q)|^2, \quad (2)$$

where  $Q$  is the invariant four-momentum difference,  $Q = \sqrt{-(p_1 - p_2)^2}$  and  $\tilde{f}(Q)$  is the Fourier transform of the density of the source,  $f(x)$ .

### 2.1. Gaussian distributed source

The simplest assumption is that the source has a symmetric Gaussian distribution, in which case  $\tilde{f}(Q) = \exp\left(i\mu Q - \frac{(RQ)^2}{2}\right)$  and

$$R_2(Q) = \gamma [1 + \lambda \exp(-(RQ)^2)] (1 + \delta Q), \quad (3)$$

where the parameter  $\gamma$  is a constant of normalization,  $\lambda$  is an intercept or incoherence factor, which measures the strength of the correlation, and  $(1 + \delta Q)$  is introduced to parametrize possible long-range correlations not adequately accounted for in the reference sample.

A fit of Eq. (3) to the data results in an unacceptably low confidence level [7] from which we conclude that the shape of the source deviates from a Gaussian. The fit is particularly bad at low  $Q$  values.

### 2.2. Lévy distributed source

Adopting Nolan's  $S(\alpha, \beta = 0, \gamma, \delta; 1)$  convention [8] for the symmetric Lévy stable distribution with rescaling of the scale parameter  $\gamma$  to  $R$  and the location parameter  $\delta$  to  $x_0$ , the Fourier transform (characteristic function)  $\tilde{f}(Q)$  has the following general form:

$$\tilde{f}(Q) = \exp(iQx_0 - |RQ|^\alpha). \quad (4)$$

The index of stability,  $\alpha$ , satisfies the inequality  $0 < \alpha \leq 2$ . The case  $\alpha = 2$  corresponds to a Gaussian source distribution. For more details see [8].

Then  $R_2$  has the following, relatively simple form [9]:

$$R_2(Q) = \gamma [1 + \lambda \exp(-(RQ)^\alpha)] (1 + \delta Q). \quad (5)$$

After fitting Eq. (5) to the data it is clear that the correlation function is far from Gaussian:  $\alpha \approx 1.3$ . The confidence level, although improved compared to the fit of Eq. (3), is still unacceptably low [7].

### 3. The $\tau$ -model

A model of strongly correlated phase-space was developed [12] to explain the experimentally found invariant relative momentum  $Q$  dependence of Bose–Einstein correlations in  $e^+e^-$  reactions. This model also predicts a specific transverse mass dependence of  $R_2$ , that we subject to an experimental test here. In this model, it is assumed that the average production point  $\bar{x}^\mu$  of particles with a given momentum  $k^\mu$  is given by

$$\bar{x}^\mu(k^\mu) = dk^\mu. \tag{6}$$

In the case of two-jet events,  $d = \tau/m_t$ , where  $\tau = \sqrt{t^2 - k_z^2}$  is the longitudinal proper-time and  $m_t = \sqrt{m^2 + p_t^2}$  is the transverse mass. The second assumption is that the distribution of  $x^\mu(k^\mu)$  about its average,  $\delta_\Delta(x^\mu(k^\mu) - \bar{x}^\mu(k^\mu))$ , is narrower than the proper-time distribution. Then the emission function of the  $\tau$ -model is

$$S(x, k) = \int_0^\infty d\tau H(\tau) \delta_\Delta(x - dk) N_1(k), \tag{7}$$

where  $H(\tau)$  is the longitudinal proper-time distribution, the factor  $\delta_\Delta(x - dk)$  describes the strength of the correlations between coordinate space and momentum space variables and  $N_1(k)$  is the experimentally measurable single-particle spectrum. In the plane-wave approximation the Yano–Koonin formula [13] gives the following two-pion multiplicity distribution:

$$\rho_2(k_1, k_2) = \int d^4x_1 d^4x_2 S(x_1, k_1) S(x_2, k_2) (1 + \cos [(k_1 - k_2)(x_1 - x_2)]). \tag{8}$$

Approximating the  $\delta_\Delta$  function by a Dirac delta function, the argument of the cosine becomes

$$(k_1 - k_2)(\bar{x}_1 - \bar{x}_2) = -0.5(d_1 + d_2)Q^2. \tag{9}$$

Then the two-particle Bose–Einstein correlation function is obtained as

$$R_2(k_1, k_2) = 1 + \lambda \text{Re} \tilde{H}^2 \left( \frac{Q^2}{2m_t} \right), \tag{10}$$

where  $\tilde{H}(\omega) = \int d\tau H(\tau) \exp(i\omega\tau)$  is the Fourier transform of  $H(\tau)$ . Thus an invariant relative momentum dependent BEC appears.

Guided by the result of the previous section, we use a one-sided Lévy distribution for the longitudinal proper-time density. The corresponding

BEC function has an analytic form [9–11]:

$$R_2(Q^2, \bar{m}_t) = \gamma \left[ 1 + \lambda \cos \left( \frac{\tau_0 Q^2}{\bar{m}_t} + A \left( \frac{\Delta\tau Q^2}{\bar{m}_t} \right)^\alpha \right) \exp \left( - \left( \frac{\Delta\tau Q^2}{\bar{m}_t} \right)^\alpha \right) \right] B, \quad (11)$$

where the parameter  $\tau_0$  is the proper-time of the onset of particle production,  $\Delta\tau$  is a measure of the width of the proper-time distribution,  $A = \tan(\frac{\alpha\pi}{4})$  and  $B = (1 + \delta Q)$ .

Assuming that particle production starts immediately and defining an effective radius,  $R$ , [11]  $R_2$  simplifies to

$$R_2(Q) = \gamma [1 + \lambda \cos [(R_a Q)^{2\alpha}] \exp(-(RQ)^{2\alpha})] (1 + \delta Q), \quad (12)$$

where  $R_a$  is related to  $R$  by  $R_a^{2\alpha} = \tan(\alpha\pi/2)R^{2\alpha}$ .

The fit of Eq. (12) to the data is statistically acceptable [7]. The data are well described by the fit. For  $Q$  between 0.5 GeV and 1.5 GeV the data points go below the level of the long-range correlations extrapolated to lower  $Q$  values. These data points indicate an anti-correlation in the  $Q \approx 1$  GeV region. This property of the data is well reproduced by the fitted curve, which also goes below unity as a result of the cosine term in Eq. (12), which comes from the asymmetric Lévy assumption.

After fitting Eq. (11) for various  $\bar{m}_t$  intervals we find that the quality of the fits is statistically acceptable and the fitted values of the model parameters are stable and within errors the same in all investigated  $m_t$  intervals, confirming the  $m_t$ -dependence predicted by the  $\tau$ -model. The  $\tau$ -model with a one-sided Levy proper-time distribution describes the data with parameters  $\tau_0 = 0$  fm,  $\alpha \approx 0.43 \pm 0.03$  and  $\Delta\tau \approx 1.8 \pm 0.4$  fm (the difference in  $m_t$  of the two pions is required to be less than 0.2 GeV).

#### 4. Reconstruction of the emission function

In order to reconstruct the space-time picture of the emitting process we assume that the emission function can be factorized in the following way:

$$S(r, z, t) = I(r)G(\eta)H(\tau), \quad (13)$$

where  $I(r)$  is the single-particle transverse distribution,  $G(\eta)$  is the space-time rapidity distribution of particle production, which approximately coincides with the single-particle rapidity distribution, and  $H(\tau)$  is the observed proper-time distribution.

With these assumptions one can reconstruct the longitudinal part of the emission function integrated over the transverse distribution. It is plotted as a function of  $t$  and  $z$  in Fig. 1. It exhibits the typical boomerang shape

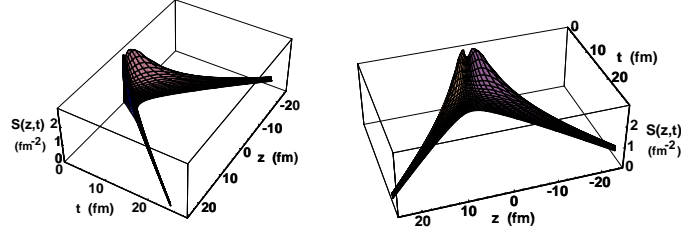


Fig. 1. Two views of the longitudinal part of the source function normalized to the average number of pions per event.

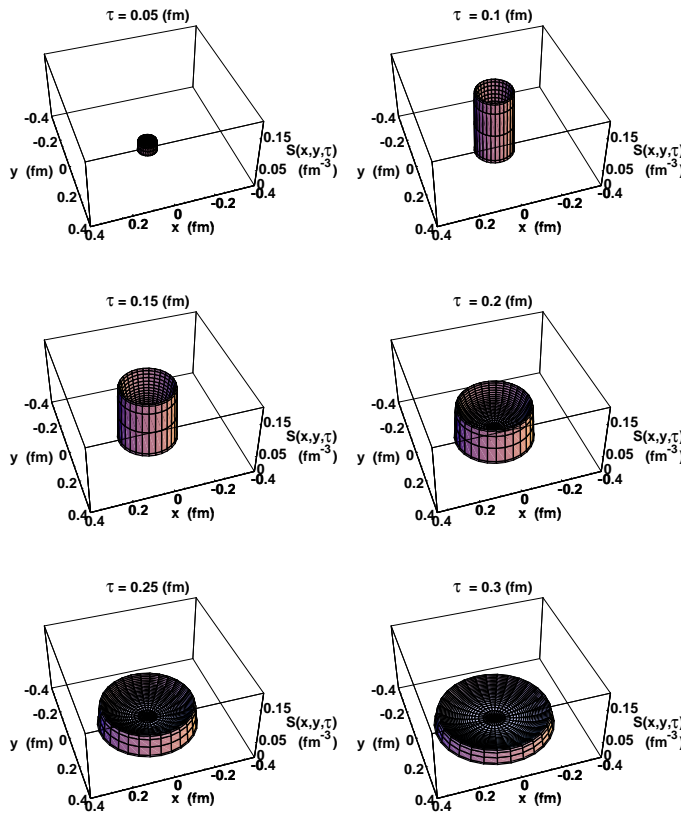


Fig. 2. The source function normalized to the average number of pions per event for various proper-times.

with a maximum at low  $t$  and  $z$  but with tails reaching out to very large  $t$  and  $z$  values.

The transverse profile, which follows from Eq. (7), is given by

$$\frac{d^4n}{d\tau d^3r} = \frac{m_t^3}{\tau^3} H(\tau) N_1 \left( k = \frac{m_t r}{\tau} \right). \quad (14)$$

This equation describes the particle production in coordinate space as a function of the proper-time  $\tau$ . It describes the expansion of the source as the proper-time increases. The particle production probability is proportional to the proper-time distribution  $H(\tau)$ . Fig. 2 shows the transverse part of the emission function for various proper-times. Particle production starts immediately, increases rapidly and decreases slowly. A ring-like structure, similar to the expanding, ring-like wave created by a pebble in a pond, is reconstructed from L3 data. An animated gif file that shows this effect is available from [16].

This research was supported by the OTKA grants NK73143 and TO49466, as well as by the exchange program of the Hungarian Academy of Sciences and the Polish Academy of Arts and Sciences.

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