

On the consistency between the observed amount of CP violation in the K and B_d systems within minimal flavor violation

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We reappraise the question of whether the standard model, and minimal flavor violating (MFV) models at large, can simultaneously describe the observed CP violation in the K and B_d systems. We find that CP violation in the B_d system, measured most precisely through $(\sin 2\beta)_{J/\psi K_S}$, implies $|\epsilon_K^{\text{SM}}| = 1.78(25) \times 10^{-3}$ for the parameter ϵ_K , measuring indirect CP violation in the K system, to be compared with the experimental value $|\epsilon_K^{\text{exp}}| = 2.23(1) \times 10^{-3}$. To bring this prediction to 1σ agreement with experiment, we then explore the simplest new-physics possibility not involving new phases, namely, that of MFV scenarios with no new effective operators besides the standard model ones. We emphasize the crucial input and/or measurements to be improved in order to probe this case. In particular, we point out that this tension could be removed in this framework, with interesting signatures, e.g. correlated suppression patterns for rare K decay branching ratios. On the other hand, MFV contributions from new operators appear, in the calculable case of the MSSM, to worsen the situation. We finally explore some well-motivated new-physics scenarios beyond MFV models, like those involving generic new contributions in Z penguins.

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I. INTRODUCTION

Forty-four years after its discovery in the decay $K_L \rightarrow \pi\pi$ [1], CP violation leaves plenty of open questions. In the standard model (SM) CP violation is generated by the physical phase appearing in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which in turn governs all flavor-violating interactions. While this picture of flavor and CP violation cannot be viewed as a fundamental theory of flavor, it turns out to be a very successful parametrization of intergenerational quark interactions, in which the hierarchies in CP -violating phenomena predicted in K , B_d , B_s and D decays are strongly correlated with the hierarchies of CP -conserving, but flavor-violating decays [2]. At the root of these correlations is the uniqueness of the CKM phase. In extensions of the SM, because of the natural presence of new flavor-violating interactions as well as CP -violating phases, such a delicate pattern is, in general, badly destroyed. Therefore, probing it to the best possible accuracy provides one of the most crucial SM tests.

Our knowledge of CP -violating phenomena is based on the following measurements: (i) the parameter ϵ_K (indirect CP violation) in $K_L \rightarrow \pi\pi$ and $K_L \rightarrow \pi\ell\nu$ decays; (ii) the parameter ϵ' (direct CP violation) in $K_L \rightarrow \pi\pi$ decays; (iii) the parameter $\sin 2\beta$ (CP violation in the interference between mixing and decay), very precisely determined from $B \rightarrow J/\psi K_S$ decays and with still significant theoretical and experimental uncertainties in several additional modes; (iv) direct CP violation in various hadronic B decays, again with still substantial uncertainties [3].

On the other hand, no evidence exists to date for CP violation in the D and B_s systems, which in the SM is predicted to be tiny, so that precisely these two systems would offer the most crucial probes of non-SM CP -violating effects.

Because of the theoretical and/or experimental uncertainties involved, it may still take some time until measurements in (ii) and (iv) above become important as tests of the CKM picture at the quantitative level. Instead, a major insight into the CKM correlation between ϵ_K and $\sin 2\beta$ could become possible in the coming years through

- (1) an improved determination of $\sin 2\beta$ and, in particular, of the CKM angle γ through tree-level decays;
- (2) improved calculations of the nonperturbative parameter \hat{B}_K , which crucially enters the formula for ϵ_K .

Based on existing analyses of the unitarity triangle (UT), the measured value of $\sin 2\beta$, dominated by the measurement of the time-dependent asymmetry in $B \rightarrow J/\psi K_S$, and the value of ϵ_K are regarded as consistent with each other within the CKM picture of flavor and CP violation. It should, however, be stressed that this $\sin 2\beta - \epsilon_K$ correlation is still far from being accurate at the theoretical level. Indeed, as seen in any plot of the UT, while the $\sin 2\beta$ constraint in the $(\bar{\rho} - \bar{\eta})$ plane is very strong, the corresponding one from ϵ_K is fairly weak. Confidence that the size of CP violation in the B_d system ($\sin 2\beta$) and in the K system (ϵ_K) are consistent with each other is only at the 15% level. This fundamental test of consistency of CP violation across different generations is, by the way, the only one available at present.

In a recent paper [4] we have raised the possibility that the SM prediction of $|\epsilon_K|$ implied by the measured value of $\sin 2\beta$ may be too small to agree with experiment. Two main ingredients, absent in the existing UT analyses to date, led to the above hypothesis:

- (a) a decrease of \hat{B}_K to the value [5] (see also [6])

$$\hat{B}_K = 0.720 \pm 0.013 \pm 0.037, \quad (1)$$

lower by 5%–10% with respect to the values used in existing UT fits [7,8];

- (b) the observation [4] that effects neglected in the usually adopted formula for ϵ_K amount to an additional suppression, which can be parametrized as a multiplicative factor, estimated within the SM as

$$\kappa_\epsilon = 0.92 \pm 0.02. \quad (2)$$

Because $\epsilon_K \propto \hat{B}_K \kappa_\epsilon$, the total suppression of ϵ_K with respect to the commonly adopted formulas is potentially of the order of 20%. These facts motivated us, in [4], to look in more detail into the $\epsilon_K - \sin 2\beta$ correlation, in particular at the \hat{B}_K range implied by the assumption that the correlation be fully described by the SM. It should be mentioned that our study has been inspired by a complementary analysis of Lunghi and Soni [9], who, assuming no new physics in ϵ_K and using the value of \hat{B}_K from [5], found, even in the limit $\kappa_\epsilon = 1$, values for $\sin 2\beta$ visibly larger than $(\sin 2\beta)_{J/\psi K_S}$.

With present data, no statement above the 2σ level can be made [4,9,10]. However, an improvement in the relevant input, e.g. an independent lattice determination of \hat{B}_K confirming point (a), has, in our opinion, a concrete potential to uncover an inconsistency between ϵ_K and $\sin 2\beta$ within the SM.

The purpose of the present paper is to provide additional arguments for the above possibility and to comment on how the $\epsilon_K - \sin 2\beta$ correlation—along with additional observables in the flavor sector—is modified within the simplest extensions of the SM, within and beyond minimal flavor violation (MFV).

II. ϵ_K IN THE STANDARD MODEL

Let us first recall that within the SM

- (i) $S_{J/\psi K_S} = \sin 2\beta$ measures directly the phase β (see [11–13] for corrections to this relation);
- (ii) with the implied precise value of β , $|\epsilon_K|$ can be predicted in terms of the remaining three parameters of the CKM matrix, which we choose to be $|V_{us}|$, $|V_{cb}|$ and the UT side R_t , the rest of the parametric dependence being in the loop functions, in \hat{B}_K and in κ_ϵ .

From point (ii) and Eq. (13) of [4] one easily gets

$$|\epsilon_K|^{\text{SM}} = \kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 |V_{us}|^2 \left(\frac{1}{2} |V_{cb}|^2 R_t^2 \sin 2\beta \eta_{tt} S_0(x_t) + R_t \sin \beta (\eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c) \right),$$

$$\text{with } C_\epsilon = \frac{G_F^2 F_K^2 M_{K^0} M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}, \quad (3)$$

where the SM loop functions S_0 (see e.g. [14]) depend on $x_i = \bar{m}_i^2(m_i)/M_W^2$. The residual approximations involved in Eq. (3) are well below 1%. Using the parametric input reported in Table I (cf. [4])—implying $R_t = 0.914 \pm 0.031$ through $\Delta M_d/\Delta M_s$ —and the result of [5] for \hat{B}_K ,¹ we find

$$|\epsilon_K|^{\text{SM}} = (1.78 \pm 0.25) \times 10^{-3}, \quad (4)$$

to be compared with

$$|\epsilon_K|^{\text{exp}} = (2.229 \pm 0.012) \times 10^{-3}. \quad (5)$$

The 15% error in Eq. (4) can be understood most simply in terms of the three main sources of uncertainty in Eq. (3), namely \hat{B}_K , $|V_{cb}|^4$ and R_t^2 , the latter two components entering the top-top contribution to ϵ_K^{SM} , which in turn constitutes about 75% of the full result. A natural question is whether the discrepancy between Eqs. (4) and (5) may be due to short-distance physics, which is encoded in the loop functions and in the κ_ϵ factor. Correspondingly, in the next sections we will explore the kind of new physics required in S_0 and in κ_ϵ to bring Eq. (4) to 1σ agreement with experiment, and the impact on other observables. Needless to say, a simple solution to the tension between (4) and (5) is an increased value of $\sin 2\beta$, which would imply new phases in $B_d^0 - \bar{B}_d^0$ mixing. This solution has been analyzed in detail already in [4,9] and we will not consider it here.

Barring all these possibilities, one is led to the conclusion that better agreement between (4) and (5) requires higher values for \hat{B}_K , R_t or $|V_{cb}|$. The fate of the test of the $\epsilon_K - \sin 2\beta$ correlation within the SM depends crucially on these three inputs.

III. NEW PHYSICS IN THE $\Delta F = 2$ LOOP FUNCTIONS

Let us first address the possibility of a modification in the loop functions S_0 , assuming that the mechanism of flavor violation (encoded in the CKM matrix) along with the set of relevant operators stay the same as in the SM. This set of assumptions embodies what is called constrained minimal flavor violation (CMFV) [32–34]. Since the pure top contribution in ϵ_K^{SM} [first term in the parentheses of Eq. (3)] amounts to roughly 75% of the total, it is reasonable to assume that new-physics contributions mostly affect this part. Now, for Eq. (4) to recover 1σ

¹ \hat{B}_K has been estimated by various other lattice collaborations [26–31]. We choose the value of [5] since the involved systematics should be minimal (cf. [5], caption of Fig. 4).

TABLE I. Input parameters. Quantities lacking a reference are taken from [15].

$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$	$\eta_{cc} = 1.43(23)$ [16]
$M_W = 80.398(25) \text{ GeV}$	$\eta_{ct} = 0.47(4)$ [16]
$M_t = 172.6(1.4) \text{ GeV}^a$ [17]	$\eta_{tt} = 0.5765(65)$ [18]
$m_c(m_c) = 1.270(17) \text{ GeV}$	$F_K = 0.1561(8) \text{ GeV}$ [19]
$M_{B_d} = 5.2795(3) \text{ GeV}$	$M_{K^0} = 0.497614 \text{ GeV}$
$M_{B_s} = 5.3663(6) \text{ GeV}$	$\Delta M_K = 0.5292(9) \times 10^{-2}/\text{ps}$
$\Delta M_d = 0.507(5)/\text{ps}$	$ \epsilon_K = 2.229(12) \times 10^{-3}$
$\Delta M_s = 17.77(12)/\text{ps}$ [20]	$\kappa_\epsilon = 0.92(2)$ [4]
$\xi_s = 1.21(4)$ [21]	$\phi_\epsilon = 43.5(7)^\circ$
$\lambda = 0.2255(7)$ [19]	$\epsilon'/\epsilon = 1.65(26) \times 10^{-3}$ [15,22,23]
$ V_{cb} = 41.2(1.1) \times 10^{-3}$	$\sin 2\beta = 0.675(26)$ [24]

^aThe $\overline{\text{MS}}$ mass value $m_t(m_t) = 162.7(1.3)$ is derived using [25].

agreement with Eq. (5), one needs, under our assumptions, a $+10\%$ shift in $S_0(x_t)$. Would this shift be visible elsewhere? The function S_0 also enters the SM formulas for the mass differences in the $B_{d,s} - \bar{B}_{d,s}$ systems, respectively $\Delta M_{d,s}^{\text{SM}}$. However, the latter still suffer from substantial uncertainties, exceeding 20%, in the relevant lattice input $F_{B_q}^2 \hat{B}_q$, $q = d, s$. As an example, taking $F_{B_s} \approx 0.245 \text{ GeV}$, $\hat{B}_s \approx 1.30$ and $\xi_s \equiv (F_{B_s} \sqrt{\hat{B}_s}) / (F_{B_d} \sqrt{\hat{B}_d}) \approx 1.21$, and further including the assumed $\delta S_0 = +10\%$ shift in the S_0 function, parametrized as

$$S_0(x_t) \rightarrow S_0(x_t)(1 + \delta S_0), \quad (6)$$

one would get the CMFV predictions

$$\begin{aligned} \Delta M_d^{\text{CMFV}} &\approx (0.638 \pm 20\%)/\text{ps}, \\ \Delta M_s^{\text{CMFV}} &\approx (21.6 \pm 20\%)/\text{ps}. \end{aligned} \quad (7)$$

Comparing with the experimental results reported in Table I, one notices that both central values in Eq. (7) exceed experiment by about 20%, but errors are also of this size. It is clear that the sensitivity of $\Delta M_{d,s}^{\text{SM}}$ to an S_0 shift will only be possible when the mentioned lattice input is controlled to a matching accuracy. In general, with lower errors on Eqs. (7), increased values of S_0 would have to be compensated by decreased values of $F_{B_q}^2 \hat{B}_q$ in order for the CMFV predictions to be in agreement with the experimental $\Delta M_{d,s}$ reported in Table I. This point can be appreciated quantitatively in Fig. 1. This figure displays the values of $F_{B_s} \sqrt{\hat{B}_s}$ vs \hat{B}_K required by the agreement of the ΔM_s^{CMFV} and ϵ_K^{CMFV} predictions with the experimental data. The scattered points are obtained by assuming that the theoretical input (other than $F_{B_s} \sqrt{\hat{B}_s}$ and \hat{B}_K) obeys normal distributions according to the values listed in Table I. The case of central values on all the input is reported as a red (solid) line. Superimposed on the latter are also the values of δS_0 [see definition (6)]. The \hat{B}_K range (1) is reported as well, as a horizontal band. For reference, unquenched

determinations of $F_{B_s} \sqrt{\hat{B}_s}$ are in the ballpark of 0.245–0.281 GeV with about a 10% quoted uncertainty [35–37] (see also Refs. [21,38] for a collection of results). As the simultaneous agreement with ϵ_K and ΔM_s corresponds to the overlap of the blue and red bands in Fig. 1, the downward shift of $F_{B_s} \sqrt{\hat{B}_s}$ mentioned before is clearly seen, although, in view of the large lattice errors, it cannot be appreciated at present.

The ratio of ΔM_d^{SM} to ΔM_s^{SM} also affects the SM UT side R_t , with substantially smaller, O(5%) lattice uncertainties, since those on ΔM_d^{SM} and ΔM_s^{SM} are largely correlated. However, within CMFV, a shift in S_0 affects ΔM_d and ΔM_s universally, thereby exactly canceling in R_t [32]. Thus, in the case of CMFV models, the only route for prediction (4) to get closer to (5) via a shift in the loop

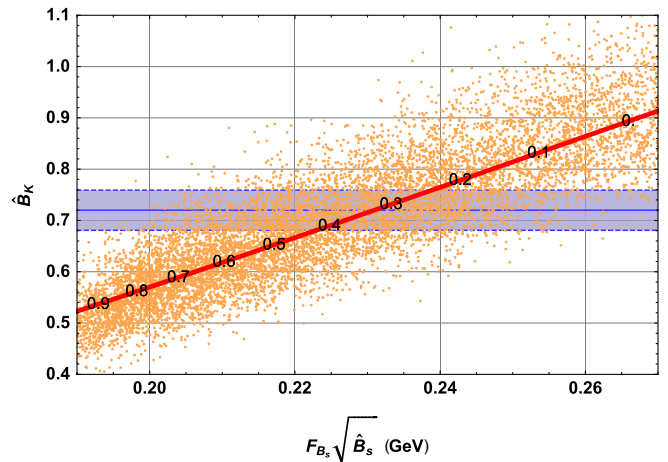


FIG. 1 (color online). Values of $F_{B_s} \sqrt{\hat{B}_s}$ vs \hat{B}_K required by ΔM_s and ϵ_K (see text for details). The scattered points are obtained by assuming the theoretical input to ΔM_s and ϵ_K to be normally distributed around the values listed in Table I. The red (solid) line corresponds to the case of central values on all input. Superimposed on the red line is the δS_0 shift. The horizontal band reports the \hat{B}_K range (1).

functions can come from $\delta S_0 > 0$. Within CMFV models $\delta S_0 > 0$ is actually the most likely possibility [39,40].

More interesting can, in principle, be the case of a completely general MFV [41], where one just requires that any new flavor structure inherits from the SM Yukawa couplings (see also [42,43]). In this framework, the occurrence of new contributions proportional to operators other than those relevant within the SM is not forbidden, and indeed they arise in e.g. the two-Higgs-doublet extension of the SM [41], relevant also to the minimal supersymmetric standard model (MSSM). With regards to meson system mass differences, the largest contributions from operators other than the SM $(V - A) \otimes (V - A)$ structure are due to scalar operators. The latter, being proportional to the quark masses of the external states, are negligible in ϵ_K and affect ΔM_d and ΔM_s differently, hence they are potentially visible in R_t . However, in the calculable case of the MSSM, MFV effects not accounted for by CMFV will shift R_t *beneath* the SM value [44,45], since their dominant impact is to add destructively to the SM contributions in ΔM_s [46,47] (for a very recent reappraisal of this issue, see [48]). On the other hand, improved agreement with the ϵ_K constraint would require R_t values *above* the SM one.

In short, the $\epsilon_K - \sin 2\beta$ correlation can be improved with respect to the SM already by invoking MFV new-physics contributions universal to all meson mixings, as in CMFV. This possibility is, however, tested at a level presently not better than 20% and cries out progress in the $F_{B_q}^2 \hat{B}_q$ estimations. If instead one is after MFV effects not accounted for within CMFV, i.e. from non-SM operators, then they would most likely come from SM extensions other than the MSSM, as the latter appears to increase the tension between (4) and (5).

IV. NEW PHYSICS IN κ_ϵ

Let us now address the possibility that κ_ϵ is different from the value in (2), in particular higher, as required to recover 1σ agreement between Eqs. (4) and (5). For the reader's convenience, we briefly summarize here the origin of this correction factor in ϵ_K . The ϵ_K parameter can be calculated through the general formula [4]

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right), \quad (8)$$

where

$$\xi = \frac{\text{Im}A_0}{\text{Re}A_0}, \quad (9)$$

with A_0 the 0-isospin amplitude in $K \rightarrow \pi\pi$ decays, $M_{12}^K = \langle K | \mathcal{H}_{\Delta F=2}^{\text{full}} | \bar{K} \rangle$, ΔM_K the $K - \bar{K}$ system mass difference, and the phase $\phi_\epsilon = (43.5 \pm 0.7)^\circ$ (see Table I). The approximate ϵ_K formula typically used in phenomenological analyses can be recovered from (8) by setting

$\phi_\epsilon = \pi/4$ and $\xi = 0$. Since deviations from $\phi_\epsilon = \pi/4$ and $\xi = 0$ can be regarded as perturbations, one can parametrize their combined effect as an overall factor κ_ϵ in ϵ_K , namely

$$\kappa_\epsilon = \frac{\sin\phi_\epsilon}{1/\sqrt{2}} \times \bar{\kappa}_\epsilon, \quad (10)$$

with $\bar{\kappa}_\epsilon$ parametrizing the effect of $\xi \neq 0$ through

$$\bar{\kappa}_\epsilon = 1 + \frac{\xi}{\sqrt{2}|\epsilon_K|} \equiv 1 + \Delta_\epsilon, \quad (11)$$

where Δ_ϵ has been introduced for later convenience. As discussed in detail in [4], a *direct* calculation of ξ is subject, at present, to very large hadronic uncertainties, as no consensus exists on the value of the nonperturbative parameter B_6 , describing QCD-penguin operators that dominate ξ . Much more reliable is the indirect strategy where one evaluates the EW-penguin contribution to ϵ'/ϵ and uses the experimental ϵ'/ϵ value to determine ξ [49–51]. Allowing for a 25% error in this estimate, one arrives within the SM at $\kappa_\epsilon = 0.92 \pm 0.02$ [4], as given in Eq. (2). Hence the like sign of the two corrections in Eq. (10) turns out to build up a -8% total correction with respect to the approximate ϵ_K formula.

However, the EW-penguin contribution to ϵ'/ϵ can be affected by non-SM physics. Within the SM and for MFV models at large, the EW-penguin contributions are generally dominated by Z-penguin diagrams [52], so that the simplest expectation for new-physics contributions is a shift in the Z-penguin amplitude (see [53] for an updated discussion). We would like to address the question of how this shift may alter ξ . This can be done with a strategy, to be described in the next paragraph, entirely analogous to the indirect route to ξ mentioned above. In Sec. V we will comment on how this strategy deals with a more general modification in ϵ'/ϵ from new physics.

We start from the following convenient formula for evaluating ϵ'/ϵ within the SM [54,55],

$$\frac{\epsilon'}{\epsilon} = \text{Im}\lambda_t \cdot F_{\epsilon'}(x_t), \quad (12)$$

where $\lambda_t = V_{ts}^* V_{td}$, x_t has already been introduced, and $F_{\epsilon'}$ is given by

$$F_{\epsilon'}(x_t) = P_0 + P_X X_0(x_t) + P_Y Y_0(x_t) + P_Z Z_0(x_t) + P_E E_0(x_t), \quad (13)$$

with X_0 , Y_0 , Z_0 , and E_0 combinations of Inami-Lim functions [56]. The coefficients P_i in Eq. (13) are defined as [54,55]

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8. \quad (14)$$

Here $r_i^{(0)}$, $r_i^{(6)}$, and $r_i^{(8)}$ encode the information on the Wilson-coefficient functions of the $\Delta S = 1$ effective Hamiltonian at the next-to-leading order [57–60], and their

TABLE II. The coefficients $r_i^{(0)}$, $r_i^{(6)}$, and $r_i^{(8)}$ of formula (14) for various $\Lambda_{\overline{\text{MS}}}^{(4)}$ in the NDR scheme, taken from Ref. [54].

i	$\Lambda_{\overline{\text{MS}}}^{(4)} = 310 \text{ MeV}$ ($\alpha_s(M_Z) = 0.117$)			$\Lambda_{\overline{\text{MS}}}^{(4)} = 340 \text{ MeV}$ ($\alpha_s(M_Z) = 0.119$)			$\Lambda_{\overline{\text{MS}}}^{(4)} = 370 \text{ MeV}$ ($\alpha_s(M_Z) = 0.121$)		
	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$
0	-3.574	16.552	1.805	-3.602	17.887	1.677	-3.629	19.346	1.538
X_0	0.574	0.030	0	0.564	0.033	0	0.554	0.036	0
Y_0	0.403	0.119	0	0.392	0.127	0	0.382	0.134	0
Z_0	0.714	-0.023	-12.510	0.766	-0.024	-13.158	0.822	-0.026	-13.855
E_0	0.213	-1.909	0.550	0.202	-2.017	0.589	0.190	-2.131	0.631

numerical values for different choices of $\Lambda_{\overline{\text{MS}}}^{(4)}$ at $\mu = m_c$ in the NDR renormalization scheme are displayed in Table II. On the other hand, $R_{6,8}$, defined as

$$R_6 \equiv B_6^{(1/2)} \left[\frac{121 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2, \quad (15)$$

$$R_8 \equiv B_8^{(3/2)} \left[\frac{121 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right]^2,$$

encode, through the ‘‘ B parameters’’ $B_6^{(1/2)}$ ($B_8^{(3/2)}$), the information on the operator matrix elements $\langle Q_6 \rangle_0$ ($\langle Q_8 \rangle_2$) between a K meson and a $\pi\pi$ state with isospin $I = 0$ ($I = 2$). Equations (12)–(14) assume the $\Delta S = 1$ operator basis Q_{1-10} (see [61]), wherein Q_6 (Q_8) represents the most important QCD-penguin (EW-penguin) operator. On the impact of the additional magnetic penguins $Q_{11,12}$ we will add comments in the next section. Concerning R_8 , we assume the reasonable range

$$R_8 = 1.0 \pm 0.2, \quad (16)$$

which encompasses various estimates reviewed in [54]. On the other hand, in view of the huge theoretical uncertainties mentioned, we make no assumption on R_6 . Its range, necessary for the estimation of ξ , hence κ_ϵ , will instead be extracted indirectly by demanding equality of the theoretical ϵ'/ϵ formula with $(\epsilon'/\epsilon)_{\text{exp}} = (1.65 \pm 0.26) \times 10^{-3}$ (see Table I), within its 1σ range.

More explicitly, once the R_6 range has been estimated, the entailed range for the correction Δ_ϵ , hence ξ [see Eq. (11)], can be obtained from the following approximate, but quite accurate formula,

$$\Delta_\epsilon \approx -\frac{1}{\omega} \text{Im}\lambda_t \cdot F_{\epsilon'}(x_t)|_{R_8 \rightarrow 0} \quad (17)$$

where $\omega = \text{Re}A_2/\text{Re}A_0 = 0.045$. In order to derive this approximate expression for Δ_ϵ , let us recall the basic formula for ϵ'/ϵ (see e.g. [14]),

$$\frac{\epsilon'}{\epsilon} = -\omega \Delta_\epsilon (1 - \Omega), \quad (18)$$

where $(-\omega \Delta_\epsilon)$ represents, by definition, the sum of the $\Delta I = 1/2$ contributions to ϵ'/ϵ , whereas Ω is the absolute value of the ratio between the $\Delta I = 3/2$ and the $\Delta I = 1/2$ contributions. We note that the right-hand side (r.h.s.) of Eq. (17) includes in the Δ_ϵ estimate the contributions from the coefficients $r_i^{(0)}$ [see Eq. (14)], which consist of a $\Delta I = 3/2$ component along with the $\Delta I = 1/2$ one. The former component is not separated away in Eq. (17). Using the results of Ref. [61], one can, however, convince oneself that this approximation amounts to overestimating $|\Delta_\epsilon|$ by less than 10%, even for O(50%) new physics in Z penguins (i.e. $\delta C = 0.5$; see below). Therefore, effectively, the limit $R_8 \rightarrow 0$ in the P_i coefficients (14) corresponds to $\Omega \rightarrow 0$ in (18), hence the possibility to estimate Δ_ϵ from the simple relation (17).

With this strategy at hand, we can now study how ξ may be affected by new physics in Z -penguin contributions. The latter arise from the $\bar{s}Zd$ effective Lagrangian interaction that reads (‘t Hooft-Feynman gauge)

$$\mathcal{L}^Z = \frac{G_F}{\sqrt{2}} \frac{g_2}{2\pi^2} \frac{M_W^2}{\cos\theta_w} Z_{ds} \bar{s}(\gamma_\mu)_L d Z^\mu + \text{H.c.}, \quad (19)$$

with the complex ‘‘coupling’’ Z_{ds} given in the SM by

$$Z_{ds}^{\text{SM}} = \lambda_t C_0(x_t). \quad (20)$$

One can now parametrize the presence of non-SM contributions in Z_{ds} through the replacement [62,63]

$$Z_{ds}^{\text{SM}} \rightarrow Z_{ds} = \lambda_t C_0(x_t) (1 + \delta C e^{i\phi_{\text{NP}}}), \quad (21)$$

with arbitrary δC and ϕ_{NP} . It should be remarked that, since the interaction in Eq. (19) is gauge dependent, so is the coupling Z_{ds} in Eq. (21). In the SM, this gauge dependence is rather weak, as it enters only in terms that are subleading in m_t and is canceled in the functions X_0 , Y_0 , and Z_0 [Eq. (13)], which are linear combinations of the gauge-dependent C_0 and other photon-penguin and box diagrams [64]. Since in any known extension of the SM the latter diagrams receive subdominant contributions with respect to those affecting Z penguins, we expect that the

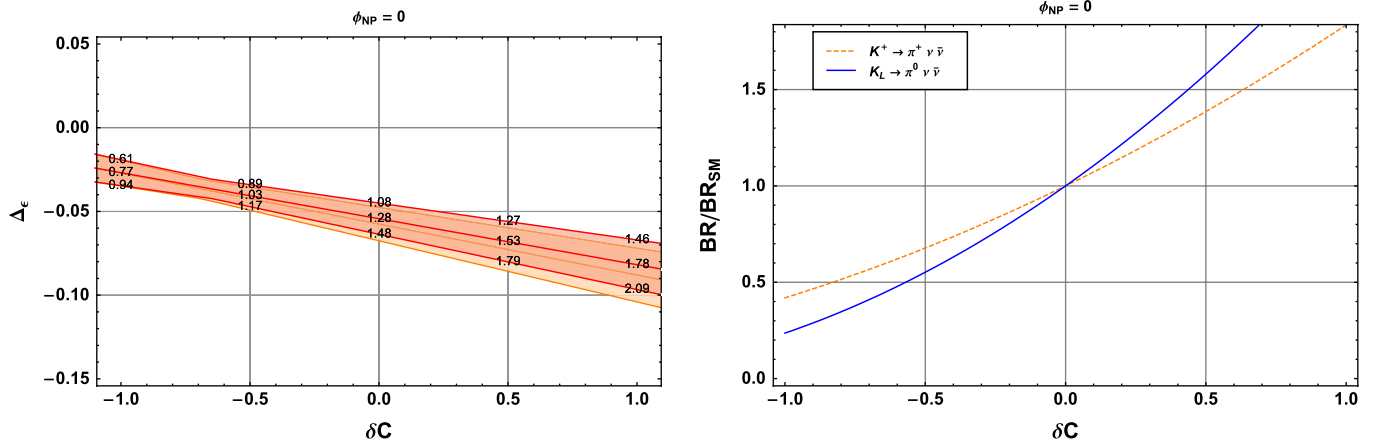


FIG. 2 (color online). Left panel: Correction Δ_ϵ as a function of the new-physics shift δC in the $\bar{s}Zd$ vertex and $\phi_{\text{NP}} = 0$. The darker (red) area refers to the choice $\Lambda_{\overline{\text{MS}}}^{(4)} = 310$ MeV for the coefficients in Table II. The lighter (orange) area corresponds instead to the choice $\Lambda_{\overline{\text{MS}}}^{(4)} = 370$ MeV. Displayed on the $\{0, \pm 1\}\sigma$ solid lines are the values of R_6 corresponding to the given Δ_ϵ . Right panel: Rate of enhancement in the branching ratios for the $K \rightarrow \pi \nu \bar{\nu}$ decays as a function of δC .

gauge dependence of new-physics contributions to Z_{ds} is also very weak and that it is a very good approximation to parametrize the new-physics contributions by the modification of Z_{ds} only [62]. Arguments for new physics modifying dominantly Z penguins are given in [52].

To study the impact of the new-physics modification (21) on ϵ'/ϵ , and in turn on ξ , let us first focus on the case of CMFV, where one additionally demands $\phi_{\text{NP}} = 0$. The left panel of Fig. 2 shows the modification of a shift $\delta C \in [-1, 1]$ on Δ_ϵ , as defined in Eq. (11). For $\delta C = 0$, one can read $\Delta_\epsilon \approx -0.06$ [4]. Note that the chosen range for δC is quite generous, taking into account the constraints implied within MFV by other flavor observables [65] as well as by $Z \rightarrow b\bar{b}$ pseudo-observables [53]. In particular, positive shifts in δC , suppressing κ_ϵ even further below unity, are of no interest in this discussion.

We observe that, as expected, in order to increase κ_ϵ , or equivalently ξ , while keeping the experimental value of ϵ'/ϵ fixed, the magnitude of EW contributions to ϵ'/ϵ has to be decreased with respect to the SM case. This is apparent by noting, from Table II, that the main contributions to QCD penguins (dominating ξ) and EW penguins, respectively $r_0^{(6)}$ and $r_Z^{(8)}$, come with opposite signs. From the left panel of the figure, one can note that, for κ_ϵ to be outside the range in Eq. (2), new physics in EW penguins must be non-negligible with respect to the SM contribution. For example, even a δC shift as large as -0.5 would imply $\bar{\kappa}_\epsilon \approx 0.96$, whence, using Eq. (10), one would arrive at $\kappa_\epsilon \approx 0.93$.

We observe in addition that the new physics required to increase κ_ϵ would generally suppress the branching ratios for rare K decays. With our parametrization (21), this can be explored numerically by using the formulas of Ref. [66] (with parametric input taken from [67–70]). The rate of suppression is displayed in the right panel of Fig. 2 for the

decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. A suppression on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ seems disfavored in the light of present knowledge [71], but data are definitely premature to draw any conclusion on this point.

As a further remark, even in the case where EW-penguin contributions are suppressed to zero, one would have $\kappa_\epsilon = 0.94 \pm 0.01$. The decrease in the error in this case is related to the fact that, in the absence of EW contributions to ϵ'/ϵ , the relative error on ξ is the same as that in $(\epsilon'/\epsilon)_{\text{exp}}$.²

V. BEYOND MINIMAL FLAVOR VIOLATION

We would now like to shortly address the case of new physics beyond MFV. Concerning non-MFV contributions to the $\Delta F = 2$ loop functions, very little can be said with present errors on the relevant lattice matrix elements. Indeed, as we have seen in Sec. III, even a universal CMFV shift in the top contribution, producing the predictions (7), is consistent with experiment as long as lattice matrix elements allow for a 20% uncertainty.

On the other hand, much more can be said on new-physics contributions beyond MFV to κ_ϵ . A first comment concerns the possible impact of the magnetic operators $Q_{11,12}$ [72] and new physics therein. These operators affect, in principle, our strategy in two ways. First, they add the unknown parameters $B_{11}^{(1/2)}$, $B_{12}^{(1/2)}$, and $B_{12}^{(3/2)}$. However, since Q_{11} contributes only to the $\Delta I = 1/2$ amplitude, within our strategy its effect is accounted for as a mere shift in the central value of R_6 . Concerning the Q_{12} matrix

²Figure 2 shows that the point with minimum Δ_ϵ error is at $\delta C \approx -0.7$: this is where the EW-penguin contributions are exactly zero, thereby eliminating the R_8 contribution to the Δ_ϵ error. The difference with respect to the naive expectation $\delta C = -1$ is due to $r_{0,E_0}^{(8)} \neq 0$ (see Table II).

elements, they are very suppressed, if not vanishing. Hence they can safely be set to zero [72]. Second, Q_{11} and Q_{12} mix—at the two-loop level—with Q_{1-10} . In Ref. [72], the mixing with the QCD-penguin operators has been estimated as a roughly 10% increase in the $\Delta I = 1/2$ part of ϵ'/ϵ . In our case this effect can be lumped into the R_6 estimate. In other words, similarly to the SM and new-physics effects of the operators Q_{1-6} , those of the operators $Q_{11,12}$ are taken into account by leaving R_6 as a free parameter. Concerning the two-loop QCD mixing between

$Q_{11,12}$ and Q_{7-10} [73,74], as well as the QED one [75,76], to our knowledge no analysis exists exploring their possible impact on ϵ_K . However, we expect this impact to be well within the theoretical error associated with our procedure.

A second issue is the possible presence of new phases. With regards to Z penguins, this would amount to $\phi_{\text{NP}} \neq 0$ in our parametrization (21). The ensued effect on Δ_ϵ is displayed in Fig. 3, which is analogous to Fig. 2, but for the Z-penguin new-physics phase chosen as

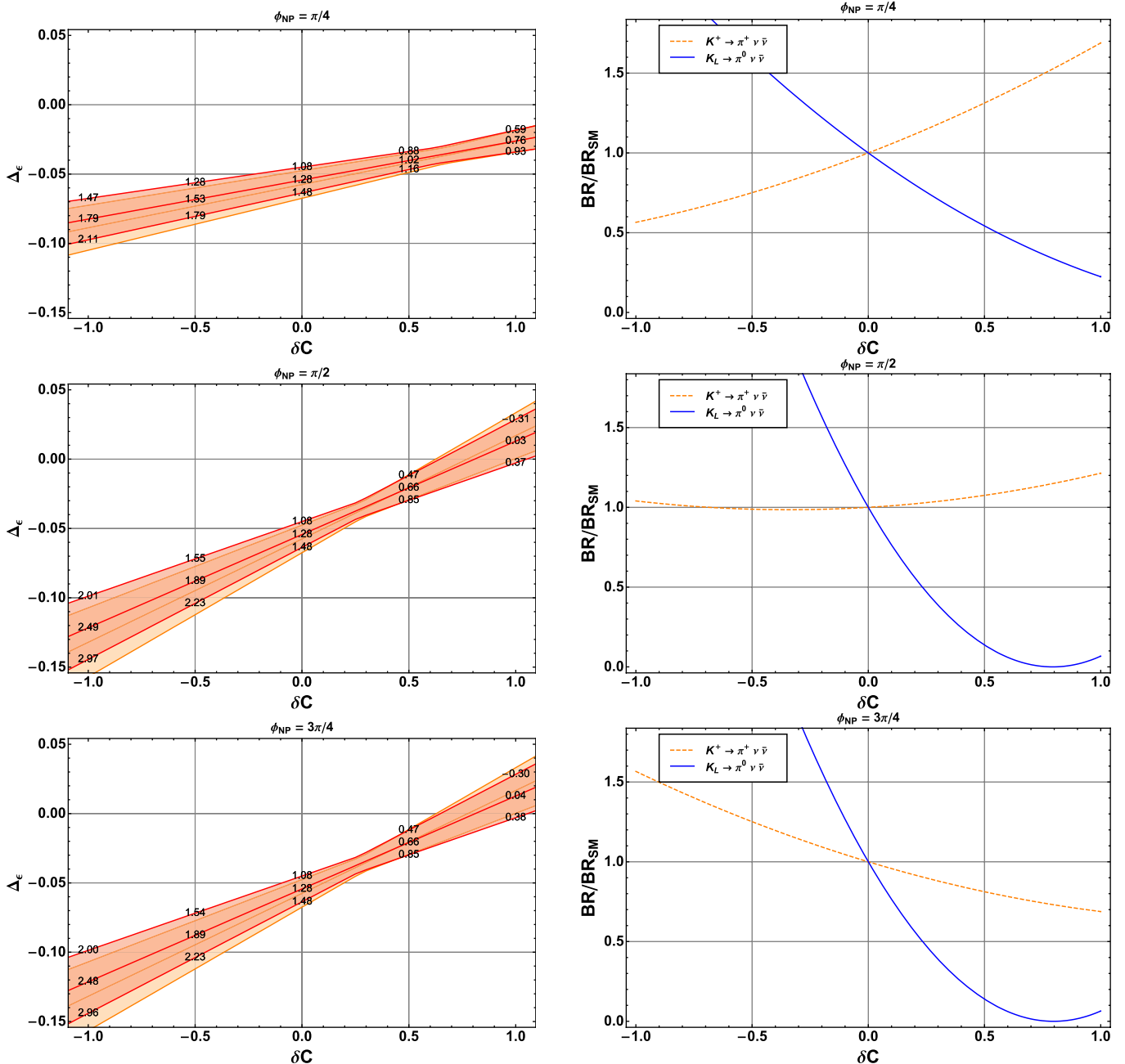


FIG. 3 (color online). Same as Fig. 2, but for the Z-penguin new-physics phase chosen as $\phi_{\text{NP}} \in \{\pi/4, \pi/2, 3\pi/4\}$ in the upper, central, and lower panels, respectively.

$\phi_{\text{NP}} \in \{\pi/4, \pi/2, 3\pi/4\}$ in the three rows of figures. Plots with ϕ_{NP} values in the third and fourth quadrants can obviously be obtained by just flipping the δC axis.

The right panels of each row in Fig. 3 demonstrate the strong sensitivity of the rate of enhancement for the decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ to the possible presence of a new phase in Z penguins (cf. [63,77]). The flip side of the coin is, however, the loss of correlation with the Δ_ϵ modification, as compared to the $\phi_{\text{NP}} = 0$ case of Fig. 2. However, a feature that can be read from both Figs. 2 and 3 is that, if one advocates Z -penguin contributions to decrease $|\Delta_\epsilon|$, this implies a ratio between $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ larger than in the SM, where this ratio is about 3.

Concerning Δ_ϵ , one can in addition notice the change in ‘‘slope’’ as a function of the δC shift with respect to the left panel of Fig. 2. This is easy to understand from the following approximate numerical relation for ϵ'/ϵ :

$$\frac{1}{\omega} \frac{\epsilon'}{\epsilon} \approx \dots + 0.047R_6 - 0.018R_8 + (-0.027 \cos \phi_{\text{NP}} + 0.067 \sin \phi_{\text{NP}})R_8 \delta C, \quad (22)$$

where dots denote other terms, e.g. constant ones, unimportant in this discussion. One can see that, for $\phi_{\text{NP}} = 0$, an increase in δC implies an increase in R_6 [recall that the r.h.s. of Eq. (22) is required to be numerically within the experimental ϵ'/ϵ range], i.e. in $|\Delta_\epsilon|$. However, already for $\phi_{\text{NP}} = \pi/4$, the terms in the parentheses on the r.h.s. of Eq. (22) have roughly flipped sign, and now an increase in δC means a decrease in R_6 , hence in $|\Delta_\epsilon|$.

VI. CONCLUSIONS

We have reconsidered the test of compatibility between CP violation in the K and the B_d systems within the SM, by analyzing the ϵ_K prediction implied by $\sin 2\beta$. As already hinted at by the analysis in [4], ϵ_K^{SM} can explain only about 80% of the experimental result, potentially signaling an inconsistency, presently masked by a 15% input uncertainty.

Assuming that the problem is not in the parametric input relevant to ϵ_K , we have addressed the question of whether the mentioned tension could be removed without going beyond the MFV framework. The most efficient solution to the tension in question is realized in CMFV, i.e. without advocating operator structures besides those relevant in the SM. This solution proceeds through a positive shift in the $\Delta F = 2$ top-top loop function, and implies $\Delta M_{d,s}$ predictions roughly 20% above experiment. Therefore, with improved determinations of the relevant lattice input, this shift would have to be compensated by decreased values

of $F_{B_q}^2 \hat{B}_q$ in order for the CMFV predictions to be in agreement with the experimental $\Delta M_{d,s}$. This is illustrated in Fig. 1.

Another avenue would be an increase of the factor κ_ϵ in ϵ_K , which in the SM we estimated to be $\kappa_\epsilon = 0.92(2)$. We showed that, within the framework of CMFV, the needed increase in κ_ϵ is correlated, through ϵ'/ϵ , with a suppression in the branching ratios of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which is not supported by present—however limited—data on the latter decay mode [71]. Even admitting this case, we find $\kappa_\epsilon \lesssim 0.95$, once other relevant CMFV constraints [53] are taken into account, the upper bound holding for a new-physics contribution of $O(1)$ with respect to the SM one. Therefore, we conclude that our SM estimate of κ_ϵ is robust also within CMFV at large. Solution to the tension, within the CMFV frameworks, would be a positive shift in the loop function S_0 .

In general MFV frameworks, where new operators matter, addressing the tension between $(\sin 2\beta)_{J/\psi K_S}$ and ϵ_K is a model-dependent issue. However, this tension appears to increase in the case of the MFV MSSM, where contributions from new operators arise for large $\tan \beta$.

Beyond MFV, agreement between ϵ_K and $(\sin 2\beta)_{J/\psi K_S}$ can of course be achieved through appropriate new-physics contributions to the $\Delta F = 2$ Hamiltonians, which are, in general, different in the K and B_d systems, and/or through an increase in κ_ϵ . Figure 3 shows the implications on rare K decays for a scenario where the κ_ϵ increase is due to new-physics dominantly in Z penguins.

The possibility to really probe all the above options rests, however, on improved values of \hat{B}_K , V_{cb} —on which ϵ_K carries strong sensitivity—as well as of $F_{B_q}^2 B_q$, crucial instead for ΔM_q . The accuracy on these input quantities parametrically rules the accuracy of the consistency test of CP violation between the K and the B_d systems within MFV frameworks. A complementary route would be an alternative, direct measurement of the phase in the CKM matrix. That of the UT angle γ from tree-level decays will be a crucial step forward in this direction.

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