PHYSICAL REVIEW D **79,** 013001 (2009)

Building the full Pontecorvo-Maki-Nakagawa-Sakata matrix from six independent Majorana-type phases

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(Received 6 October 2008; published 7 January 2009)

In the framework of three light Majorana neutrinos, we show how to reconstruct, through the use of 3×3 unitarity, the full PMNS matrix from six independent Majorana-type phases. In particular, we express the strength of Dirac-type *CP* violation in terms of these Majorana-type phases by writing the area of the unitarity triangles in terms of these phases. We also study how these six Majorana phases appear in *CP*-odd weak-basis invariants as well as in leptonic asymmetries relevant for flavored leptogenesis.

DOI: 10.1103/PhysRevD.79.013001 PACS numbers: 14.60.Pq, 11.30.Er

I. INTRODUCTION

The discovery of neutrino oscillations [1] providing evidence for nonvanishing neutrino masses and leptonic mixing, is one of the most exciting recent developments in particle physics. At present, it is not known whether neutrinos are Dirac or Majorana fermions. The latter possibility has the special appeal of providing, through the seesaw mechanism [2–6] an elegant explanation of why neutrinos are much lighter than the other known fermions. It is well known that the presence of Majorana neutrinos introduces some novel features in leptonic *CP* violation, like the possibility of having *CP* violation in the case of two Majorana neutrinos as well as having *CP* breaking even in the limit of three exactly degenerate neutrinos [7]. These features reflect the fact that in the presence of Majorana neutrinos, the simplest nontrivial rephasing invariant functions of the leptonic mixing matrix elements, are bilinears and not quartets, as is the case for Dirac particles. We designate ‘‘Majorana-type phases’’ the arguments of these rephasing invariant bilinears. Physically, these phases correspond to the orientation in the complex plane of the sides of the Majorana unitarity triangles. Recall that in the case of the quark sector and in general for Dirac particles, the orientations of the unitarity triangles have no physical meaning, reflecting the fact that Dirac unitarity triangles rotate under rephasing of the quark fields. The leptonic mixing matrix, often called Pontecorvo, Maki, Nakagawa, and Sakata (PMNS) matrix, is usually parametrized by a 3×3 unitary matrix containing three mixing angles, one Dirac phase, and two Majorana phases, for a total of six independent parameters. It should be emphasized that these Majorana phases are related to but do not coincide with the above defined Majorana-type phases. The crucial point is that Majorana-type phases are rephasing invariants which are measurable quantities and do not depend on any particular parametrization of the PMNS matrix.

In this paper we adopt the arguments of these rephasing invariant bilinears as fundamental parameters and we show that, in the framework of three light Majorana neutrinos and in presence of Dirac-type *CP* violation, the full PMNS matrix can be reconstructed from six independent Majorana-type phases. We also study how these Majorana-type phases appear in neutrinoless double beta decay, as well as in *CP*-odd weak-basis invariants and in leptonic asymmetries relevant for flavored leptogenesis. We conclude that, in this framework, all low energy leptonic physics is encoded into six leptonic masses and six Majorana-type phases. In the case of one massless neutrino one of these Majorana-type phases may be fixed (e.g., chosen to be equal to zero), without changing the lengths of the sides and internal angles of the unitary triangles.

This paper is organized as follows. In Sec. II we set the notation and present our framework. In Sec. III, we choose six independent Majorana-type phases and show how the full unitary PMNS matrix can be constructed from these six input phases. We also show how to express the strength of Dirac-type *CP* violation in terms of the six Majorana-type phases and analyze the unitarity triangles, taking into account the present experimental data. In Secs. IV, V, and VI we show how Majorana-type phases appear in the elements of the effective mass matrix, in *CP*-odd weak-basis invariants, and in leptogenesis, respectively. Finally our conclusions are contained in Sec. VII.

II. FRAMEWORK AND NOTATION

We consider an extension of the stardard model (SM) consisting of the addition of an arbitrary number of right-

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handed neutrinos leading to three light Majorana neutrinos, through the seesaw mechanism. The leptonic mixing matrix V is a $3 \times (3 + n_R)$ matrix connecting the charged leptons to the three light neutrinos and the n_R heavy neutrinos. This mixing matrix V is, of course, a submatrix of a $(3 + n_R) \times (3 + n_R)$ unitary matrix. In this work, we are especially interested in the low energy limit of the theory, where the leptonic mixing matrix reduces to the 3×3 PMNS matrix connecting charged leptons to the light neutrinos. Let us choose, for the low energy limit, the physical basis where both the charged lepton mass matrix, m_l and the neutrino mass matrix m_ν are diagonal and real:

$$m_l = \mathrm{diag}(m_e, m_\mu, m_\tau), \qquad m_\nu = \mathrm{diag}(m_1, m_2, m_3). \tag{1}$$

In this basis, there is still the freedom to rephase the charged lepton fields:

$$l_j \to l'_j = \exp(i\phi_j) l_j, \tag{2}$$

with arbitrary ϕ_js, which leaves the charged lepton mass terms $m_j \bar{l}_j l_j$ invariant. Because of the Majorana nature of the neutrinos the rephasing

$$\nu_k \to \nu'_k = \exp(-i\psi_k)\nu_k \tag{3}$$

with arbitrary ψ_ks is not allowed, since it would not keep the Majorana mass terms $\nu_{Lk}^T C^{-1} m_k \nu_{Lk}$ invariant. Note however that one can still make the rephasing of Eq. (2) for $\psi_k = (n_k \pi)$ with n_k an integer.

In the mass eigenstate basis, the low energy weak charged current can be written as

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{l}_{jL} \gamma_\mu U_{jk} \nu_{kL} + \mathrm{H.c.}, \tag{4}$$

where

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}. \tag{5}$$

So far, we have not introduced the constraints of unitarity. As a result, U is characterized by nine moduli and six phases, since three of the nine phases of U can be eliminated through the rephasing of Eq. (2). If we assume 3×3 unitarity, it is well known that U is characterized by six parameters which, as mentioned above, are usually taken as three mixing angles and three CP violating phases.

III. RECONSTRUCTION OF THE FULL UNITARY PMNS MATRIX FROM SIX MAJORANA-TYPE PHASES

The study of rephasing invariant quantities is of special importance for the analysis of mixing and CP violation both in the quark and lepton sectors. In the quark sector, the simplest rephasing invariant quantities are the nine moduli of the elements of the Cabibbo-Kobayashi-Maskawa matrix, V_{CKM}, and the arguments of rephasing invariant quartets, like, for example, $\arg(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$. The assumption of 3×3 unitarity of V_{CKM} leads to a series of exact relations among various rephasing invariant quantities [8] which provide an important test of the SM. Unitarity also allows for various parametrizations of V_{CKM} which can be taken as three moduli and one invariant phase, as in the so-called standard parametrization [9], four independent moduli [10], or four independent invariant phases [11]. The novel feature of the low energy limit of the leptonic sector with Majorana neutrinos is the existence of rephasing invariant bilinears of the type $U_{l\alpha} U_{l\beta}^*$ where $\alpha \neq \beta$ and no summation on repeated indices is implied. We designate $\arg(U_{l\alpha} U_{l\beta}^*)$ "Majorana-type phases." These are the minimal CP-violating quantities in the case of Majorana neutrinos [12–17]. Note that in order for these phases to be precisely defined we work with real, nonzero neutrino masses corresponding to Majorana fields which satisfy Majorana conditions that do not contain phase factors. It can be readily seen, from their definition, that there are only six independent Majorana-type phases even in the general case where unitarity is not imposed on U. All the other Majorana-type phases in U can be obtained from these six phases. This reflects the freedom one has to rephase the three charged lepton fields. This would still be true for the matrix U in a general framework including an arbitrary number of right-handed neutrinos ($n_R \geq 3$) together with, for instance, an arbitrary number of vectorlike charged leptons.

We choose the six independent Majorana-type phases to be

$$\begin{aligned} &\beta_1 \equiv \arg(U_{e1} U_{e2}^*), && \beta_2 \equiv \arg(U_{\mu 1} U_{\mu 2}^*), \\ &\beta_3 \equiv \arg(U_{\tau 1} U_{\tau 2}^*), && \gamma_1 \equiv \arg(U_{e1} U_{e3}^*), \\ &\gamma_2 \equiv \arg(U_{\mu 1} U_{\mu 3}^*), && \gamma_3 \equiv \arg(U_{\tau 1} U_{\tau 3}^*). \end{aligned} \tag{6}$$

Let us now consider Dirac-type phases, which correspond to the arguments of rephasing invariant quartets. It can be readily seen that the 3×3 U matrix contains four independent Dirac-type phases. Again, this result is completely general; in particular, it does not depend on the number of right-handed neutrinos ($n_R \geq 3$) or the eventual presence of vectorlike charged leptons. We choose the following four independent Dirac-type invariant phases:

$$\sigma_{e\mu}^{12} \equiv \arg(U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*), \tag{7}$$

$$\sigma_{e\tau}^{12} \equiv \arg(U_{e1} U_{\tau 2} U_{e2}^* U_{\tau 1}^*), \tag{8}$$

$$\sigma_{e\mu}^{13} \equiv \arg(U_{e1} U_{\mu 3} U_{e3}^* U_{\mu 1}^*), \tag{9}$$

$$\sigma_{e\tau}^{13} \equiv \arg(U_{e1} U_{\tau 3} U_{e3}^* U_{\tau 1}^*). \tag{10}$$

It is clear that these four Dirac-type phases can be obtained from the six Majorana-type phases:

$$\sigma_{e\mu}^{12} = \beta_1 - \beta_2, \tag{11}$$

$$\sigma_{e\tau}^{12} = \beta_1 - \beta_3, \tag{12}$$

$$\sigma_{e\mu}^{13} = \gamma_1 - \gamma_2, \tag{13}$$

$$\sigma_{e\tau}^{13} = \gamma_1 - \gamma_3. \tag{14}$$

It follows from these expressions that, in the framework of Majorana neutrinos, Dirac-type *CP* violation in the leptonic sector, necessarily implies Majorana-type *CP* violation.

Now, we assume unitarity of the 3×3 PMNS matrix and show that in this limit, it is possible to fully reconstruct the unitarity mixing matrix from the six Majorana-type phases, β_j, γ_j provided there is Dirac-type *CP* violation. This can be shown making use of the standard parametrization [9]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P, \tag{15}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, with all θ_{ij} in the first quadrant, δ is a Dirac-type phase, and $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$ with α and β denoting the phases associated with the Majorana character of neutrinos.

The extraction of the angles θ_{ij} and δ is done through the unitarity triangles. There are two types of unitarity triangles, those obtained by multiplication of two different rows and those obtained by multiplication of two different columns. It has been pointed out [14] that these triangles are fundamentally different. Those of the first type were designated as "Dirac triangles" and have similar properties to those built in the quark sector. Their orientation has no physical meaning since, under rephasing transformations of the charged lepton fields these triangles rotate in the complex plane. Those of the second type were designated as "Majorana triangles." Under the allowed rephasing, these triangles do not rotate in the complex plane since the orientations of all their sides correspond to the arguments of rephasing invariants. As a result, the orientation of Majorana triangles is physically meaningful [14]. Of course, all the six triangles share a common area $A = 1/2|\mathrm{Im} U_{ij}U_{kj}^* U_{kl} U_{il}^*|$ (no sum in repeated indices, $k \neq i$, $l \neq j$). The Majorana triangles provide the necessary and sufficient conditions for *CP* conservation, to wit, vanishing of their common area A and orientation of all collapsed Majorana triangles along the direction of the real or imaginary axes. The vanishing of A implies that the Dirac phase δ of the parametrization of Eq. (15) equals zero or π. The three different Majorana triangles are

$$U_{e1}U_{e2}^* + U_{\mu1}U_{\mu2}^* + U_{\tau1}U_{\tau2}^* = 0, \tag{16}$$

$$U_{e1}U_{e3}^* + U_{\mu1}U_{\mu3}^* + U_{\tau1}U_{\tau3}^* = 0, \tag{17}$$

$$U_{e2}U_{e3}^* + U_{\mu2}U_{\mu3}^* + U_{\tau2}U_{\tau3}^* = 0. \tag{18}$$

Some of the general features of the three Majorana triangles are worth pointing out. The three internal angles of the first Majorana triangle corresponding to Eq. (16) are given by $\pi - (\beta_i - \beta_j)$ with $i \neq j$ both indices ranging from 1 to 3. Similarly, for the internal angles of the second triangle corresponding to Eq. (17) with β's replaced by γ's. From the internal angles of two Majorana triangles one can readily obtain the internal angles of the third triangle. Obviously, there are only four independent combinations of $(\beta_i - \beta_j)$ and $(\gamma_i - \gamma_j)$ which can be taken as those given by Eqs. (7) to (10). The internal angles of the three different Dirac triangles are also given in terms of these four independent combinations. It is sufficient to know the internal angles of two of the triangles in order to know all internal angles of all unitarity triangles.

Next we show how to obtain the full PMNS matrix from the knowledge of β_i, γ_i. Through the law of sines we obtain

$$\tan^2\theta_{12} = \frac{|U_{e2}|^2}{|U_{e1}|^2} = \frac{|\sin(\gamma_1 - \gamma_2)||\sin(-\beta_2 + \gamma_2 + \beta_3 - \gamma_3)||\sin(\gamma_1 - \gamma_3)|}{|\sin(-\beta_1 + \gamma_1 + \beta_2 - \gamma_2)||\sin(\gamma_2 - \gamma_3)||\sin(-\beta_1 + \gamma_1 + \beta_3 - \gamma_3)|}, \tag{19}$$

$$\tan^2\theta_{23} = \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2} = \frac{|\sin(\gamma_1 - \gamma_3)||\sin(-\beta_1 + \gamma_1 + \beta_3 - \gamma_3)||\sin(\beta_1 - \beta_2)|}{|\sin(-\beta_1 + \gamma_1 + \beta_2 - \gamma_2)||\sin(\gamma_1 - \gamma_2)||\sin(\beta_1 - \beta_3)|}, \tag{20}$$

$$\tan^2\theta_{13}\frac{1}{\sin^2\theta_{12}} = \frac{|U_{e3}|^2}{|U_{e2}|^2} = \frac{|\sin(\gamma_2 - \gamma_3)||\sin(\beta_1 - \beta_3)||\sin(\beta_1 - \beta_2)|}{|\sin(\gamma_1 - \gamma_3)||\sin(\gamma_1 - \gamma_2)||\sin(\beta_2 - \beta_3)|}. \tag{21}$$

From Eqs. (19)–(21) we can easily extract the angles θ_{ij} from the knowledge of the Majorana phases. Finally the phase δ can be obtained by computing the common area of the triangles. For instance, from the second triangle we obtain

$$A = \tfrac{1}{2}|U_{e1}U_{e3}^*||U_{\mu 1}U_{\mu 3}^*||\sin(\gamma_1 - \gamma_2)|. \tag{22}$$

From the law of sines we replace $|U_{\mu 1}U_{\mu 3}^*|$ by

$$|U_{\mu 1}U_{\mu 3}^*| = |U_{e1}U_{e3}^*|\frac{|\sin(\gamma_1 - \gamma_3)|}{|\sin(\gamma_2 - \gamma_3)|}, \tag{23}$$

which leads to

$$A = \frac{1}{2}|c_{12}c_{13}s_{13}|^2|\sin(\gamma_1 - \gamma_2)|\frac{|\sin(\gamma_1 - \gamma_3)|}{|\sin(\gamma_2 - \gamma_3)|}. \tag{24}$$

Since the θ_{ij} are obtained from β_i, γ_i, using Eqs. (19)–(21), it follows that Eq. (24) gives us the common area of the triangles, in terms of Majorana phases. The phase δ, entering in the standard parametrization, is readily obtained by recalling that $A = 1/2\,\mathrm{Im}Q$ where Q denotes any rephasing invariant quartet. One obtains

$$A = \tfrac{1}{16}|\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\cos(\theta_{13})\sin\delta|. \tag{25}$$

From Eqs. (24) and (25) one obtains δ in terms of Majorana phases. The quadrant of δ and the angles α and β of Eq. (15) are obtained by inspection.

A. The strength of Dirac-type *CP* violation

As we have seen, in the limit of 3×3 unitarity, the six Majorana-type phases completely fix the PMNS mixing matrix and therefore the strength of Dirac-type *CP* violation, which is given by $|\mathrm{Im}Q|$ where Q denotes any rephasing invariant quartet of the PMNS matrix, like for example $Q = (U_{e2}U_{\mu 3}U_{e3}^*U_{\mu 2}^*)$. Note that in the framework of 3×3 unitarity, one can infer the size of $|\mathrm{Im}Q|$ even without the direct measurement of any *CP* violating observable. Indeed, as it was shown for the quark sector [10], $|\mathrm{Im}Q|$ can be expressed in terms of four independent moduli of the PMNS matrix. From the present experimental data, one cannot infer the size of Dirac-type leptonic *CP* violation, which can range from zero, for instance in the case of vanishing U_{e3}, to a significant value, of order 10^{-2}, therefore much larger than the corresponding value in the quark sector where $|\mathrm{Im}Q|(\text{quark}) \sim 10^{-5}$.

The explicit expression for $|\mathrm{Im}Q|$ in terms of the six Majorana-type phases is given by

$$|\mathrm{Im}Q| = I_1I_2I_3I_4I_5I_6I_7I_8I_9/D^2, \tag{26}$$

with

$$\begin{aligned} D = {}& |\sin(\beta_1 - \beta_2)||\sin(\beta_1 - \beta_3)||\sin(\gamma_2 - \gamma_3)||\sin(-\beta_2 + \gamma_2 + \beta_3 - \gamma_3)| + |\sin(\gamma_1 - \gamma_2)||\sin(\gamma_1 - \gamma_3)| \\ & \times |\sin(\beta_2 - \beta_3)||\sin(-\beta_2 + \gamma_2 + \beta_3 - \gamma_3)| + |\sin(-\beta_1 + \gamma_1 + \beta_2 - \gamma_2)| \\ & \times |\sin(-\beta_1 + \gamma_1 + \beta_3 - \gamma_3)||\sin(\gamma_2 - \gamma_3)||\sin(\beta_2 - \beta_3)| \end{aligned} \tag{27}$$

and $I_1I_2I_3I_4I_5I_6I_7I_8I_9$ denoting the product of the sines of the nine internal angles of the three Majorana triangles, or else of the three Dirac triangles:

$$\begin{aligned} I_1I_2I_3I_4I_5I_6I_7I_8I_9 = {}& |\sin(\beta_1 - \beta_2)||\sin(\beta_1 - \beta_3)||\sin(\beta_2 - \beta_3)| \times |\sin(\gamma_1 - \gamma_2)||\sin(\gamma_1 - \gamma_3)||\sin(\gamma_2 - \gamma_3)| \\ & \times |\sin(-\beta_1 + \gamma_1 + \beta_2 - \gamma_2)||\sin(-\beta_1 + \gamma_1 + \beta_3 - \gamma_3)||\sin(-\beta_2 + \gamma_2 + \beta_3 - \gamma_3)|. \end{aligned} \tag{28}$$

The case of no Dirac-type *CP* violation is a singular case, where all unitarity triangles collapse to a line and the matrix U can be written as a real unitary matrix with two factored out phases which are usually called Majorana phases in the standard parametrization. In this case the phases of Majorana bilinears decouple from the size of mixing angles and Eqs. (19)–(21) become indetermination relations of the form 0/0 due to the equality modulo π among all β_j Majorana-type phases as well as equality modulo π of all γ_j among themselves.

We address now the question of finding the values of the six fundamental Majorana phases which lead to a maximal value of $|\mathrm{Im}Q|$. It can be readily seen that the following choice of β_j, γ_j leads to a maximal value of Dirac-type *CP* violation:

$$\beta_k = \frac{2\pi}{3}k, \qquad \gamma_k = \frac{4\pi}{3}k, \qquad k = 1, 2, 3. \tag{29}$$

This choice of the six different Majorana-type phases, together with the adoption of a specially convenient phase convention leads to the following PMNS matrix:

$$U_M = \begin{pmatrix} \frac{1}{\sqrt{3}}w & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}w^* \\ \frac{1}{\sqrt{3}}w^* & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}w \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{30}$$

where $w = \exp i\frac{2\pi}{3}$. All unitarity triangles corresponding to U_M are equilateral and the maximal value of *CP* violation corresponds to

$$|\mathrm{Im}Q| = \frac{1}{9}\frac{\sqrt{3}}{2}. \tag{31}$$

B. Unitarity triangles and present experimental data

The current experimental bounds on neutrino masses and leptonic mixing are [9]

$$\Delta m^2_{21} = (8.0 \pm 0.3) \times 10^{-5}\ \text{eV}^2, \tag{32}$$

$$\sin^2(2\theta_{12}) = 0.86^{+0.03}_{-0.04}, \tag{33}$$

$$|\Delta m^2_{32}| = (1.9 \quad \text{to } 3.0) \times 10^{-3}\ \text{eV}^2, \tag{34}$$

$$\sin^2(2\theta_{23}) > 0.92, \tag{35}$$

$$\sin^2(2\theta_{13}) < 0.19, \tag{36}$$

with $\Delta m^2_{ij} \equiv m^2_j - m^2_i$, where m_j's denote the neutrino masses. The angle θ_{23} may be maximal, meaning 45°, while θ_{12} is already known to deviate from this value. At the moment there is an experimental upper bound on the angle θ_{13}. Recently, there are hints of $\theta_{13} > 0$ from global neutrino data analysis, which provides the global estimate [18]

$$\sin^2\theta_{13} = 0.016 \pm 0.010 \quad (1\sigma). \tag{37}$$

Present experimental data suggest that in leading order the leptonic mixing matrix may be approximated by the Harrison, Perkins, and Scott (HPS) mixing matrix [19]

$$\begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \tag{38}$$

which is often designated as tribimaximal mixing and corresponds to $\tan\theta_{12} = 1/\sqrt{2}$, $\theta_{23} = \pi/4$, and $\theta_{13} = 0$

From the point of view of leptonic low energy phenomenology, a value of θ_{13} not far from its present experimental bound would have interesting experimental implications and would allow for the possibility of Dirac-type *CP* violation to be detected experimentally in the near future provided the value of the phase δ is not suppressed.

We address now the question of what unitarity triangles correspond to a perturbation of the HPS matrix which consists of keeping the values for θ_{12} and θ_{23} fixed and choosing δ and θ_{13} that maximize the area of the unitarity triangle, with θ_{13} within the experimentally allowed values (i.e., $\sin\theta_{13} = 0.22$ and $\delta = \pi/2$). It follows from Eq. (15) that this perturbation spoils the exact trimaximal mixing of the second column of the HPS matrix. In this case we have for the first Majorana triangle

$$U_{e1}U^*_{e2} = 0.448, \qquad U_{\mu1}U^*_{\mu2} = -0.224 - 0.11i, \qquad U_{\tau1}U^*_{\tau2} = -0.224 + 0.11i, \tag{39}$$

where two sides are equal in length and the internal angles of the triangle are 26.1° (for two of the angles) and 127.8°. For the second Majorana triangle we have

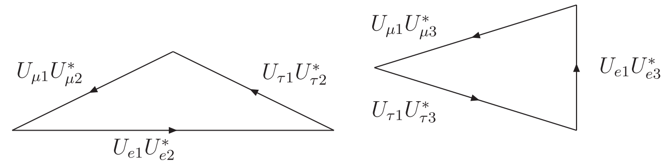

FIG. 1. First and second Majorana unitarity triangles, corresponding to Eqs. (39) and (40).

$$U_{e1}U^*_{e3} = 0.175i, \qquad U_{\mu1}U^*_{\mu3} = -0.2815 - 0.0875i, \qquad U_{\tau1}U^*_{\tau3} = 0.2815 - 0.0875i, \tag{40}$$

and once again two of the sides are equal in length. In this case two of the internal angles are equal to 72.7° and the other one to 34.6°. Finally for the third Majorana triangle we have

$$U_{e2}U^*_{e3} = 0.1238i, \qquad U_{\mu2}U^*_{\mu3} = 0.3980 - 0.0619i, \qquad U_{\tau2}U^*_{\tau3} = -0.3980 + 0.0619i. \tag{41}$$

Two sides have equal length, leading to two internal angles of 81.2° and another angle of 17.6°. Of the three triangles thus obtained, this is the one with a smallest internal angle. Note that all three triangles are isosceles, which results from the fact that there is equality of moduli between rows two and three of the mixing matrix. This is due to the particular values of θ_{23} and δ. Perturbations around the HPS values for θ_{12} and θ_{23} in the range still allowed by experiment would not alter significantly the shape of these triangles. Figure 1 depicts the first two Majorana triangles.

An alternative generalization of the tribimaximal form was considered in Ref. [20] where the exact trimaximal mixing of the second column is maintained and unitarity is imposed by construction, with U_{e3} now different from zero and possibly complex. In this construction small deviations from the HPS values of θ_{12} and θ_{23} occur and μ-τ reflection symmetry [21,22] is broken for $\text{Re}(U_{e3})$ different from zero. The Majorana-type triangles thus obtained, involving orthogonality relations with the second column become specially simple. The third Majorana triangle has the interesting feature of one of the sides being simply proportional to U_{e3}. In this approach θ_{13} and θ_{12} are related by the constraint of trimaximal mixing in the second column. For the maximal θ_{13} still allowed experimentally ($\sin\theta_{13} = 0.22$) together with $|U_{e2}|$ fixed as $1/\sqrt{3}$ which is a consequence of imposing trimaximal mixing in the second column, and with unitarity we are lead to

$$\sin^2(2\theta_{12}) = 0.91, \tag{42}$$

a value for θ_{12} which is already disfavored, as can be seen from Eq. (33).

So far, in this section we assumed unitarity of the PMNS matrix, together with the presence of Dirac-type *CP* violation, which in turn allowed for its reconstruction from six

Majorana phases. Yet, it should be noted that deviations from unitarity naturally arise in a variety of extensions of the SM, involving the lepton and or quark sectors. Actually, in the context of standard seesaw the 3×3 PMNS matrix is not exactly unitary. However, in this framework deviations from unitarity cannot be detected experimentally due to the extreme degree of their suppression. On the other hand, there are extensions of the SM where experimentally detectable deviations from unitarity may arise. Examples include models with vectorlike quarks [23–32] as well as models with heavy Majorana neutrinos with masses of order 1 TeV or lower [33,34]. Majorana neutrino singlets with no new gauge interactions might be produced within the reach of the LHC, up to masses of order 200 Gev [34]. The possibility of having extensions of the SM with natural violations of 3×3 unitarity raises the question of how to test experimentally the validity of the unitarity hypothesis. A set of exact relations connecting measurable quantities were derived for the quark sector [8] providing tests of unitarity of the V_{CKM} matrix. Similar relations can be derived in the leptonic sector. Examples of such relations, derived from the Majorana-type triangles, are

$$\frac{|U_{e1}U_{e2}^*|}{\sin(\beta_2-\beta_3)} = \frac{|U_{\mu 1}U_{\mu 2}^*|}{\sin(\beta_1-\beta_3)} = \frac{|U_{\tau 1}U_{\tau 2}^*|}{\sin(\beta_1-\beta_2)}, \tag{43}$$

$$\frac{|U_{e1}U_{e3}^*|}{\sin(\gamma_2-\gamma_3)} = \frac{|U_{\mu 1}U_{\mu 3}^*|}{\sin(\gamma_1-\gamma_3)} = \frac{|U_{\tau 1}U_{\tau 3}^*|}{\sin(\gamma_1-\gamma_2)}, \tag{44}$$

$$\frac{|U_{e2}U_{e3}^*|}{\sin(-\beta_2+\beta_3+\gamma_2-\gamma_3)} = \frac{|U_{\mu 2}U_{\mu 3}^*|}{\sin(-\beta_1+\beta_3+\gamma_1-\gamma_3)} = \frac{|U_{\tau 2}U_{\tau 3}^*|}{\sin(-\beta_1+\beta_2+\gamma_1-\gamma_2)}, \tag{45}$$

with analogous relations for the Dirac-type triangles. Although relations (43)–(45) are exact predictions of the PMNS framework, relating physically measurable quantities, their experimental test is a great challenge which would require the experimental discovery of leptonic *CP* violation of Dirac type [35,36].

Whenever the length of the largest side of the triangle is smaller than the sum of the lengths of the other two, several possibilities arise. Either there is Dirac-type *CP* violation or violation of unitarity of the PMNS matrix or both. In Ref. [37] a set of measurements is suggested which will, in principle, allow one to measure all sides of the e-μ Dirac unitarity triangle.

We have previously emphasized that the orientation of Majorana triangles has physical meaning since they are related to the size of certain Majorana-type phases. This raises the question of which observables would in principle be sensitive to these orientations. It is well known that neutrino oscillations are only sensitive to Dirac-type *CP* violation and thus its experimental discovery only provides information about differences of Majorana phases, like $\beta_1 - \beta_2$ or $\gamma_1 - \gamma_2$, but not on the individual values of β_i, γ_i. As a result, no knowledge about the orientation of Majorana triangles can be obtained from the detection of Dirac-type *CP* violation.

In the next sections we discuss the question of how neutrinoless double beta decay as well as leptogenesis [38] when flavor effects matter [39–44] are sensitive to the Majorana-type phases.

IV. MAJORANA PHASES AND THE ELEMENTS OF THE NEUTRINO EFFECTIVE MASS MATRIX

In the leptonic low energy limit and in the weak basis where the mass matrix of the charged leptons is real and diagonal, the effective neutrino mass matrix m_{eff} is complex and symmetric, with nine independent parameters. Although it may in principle be fully reconstructed from experiment, it has been pointed out that it is not possible, in practice, to fully reconstruct m_{eff} without ambiguities from a set of feasible experiments. This has motivated several authors to introduce some input from theory in order to allow for this reconstruction [45,46].

In the seasaw framework the effective Majorana mass matrix is given by

$$m_{\mathrm{eff}} = -m_D \frac{1}{M_R} m_D^T, \tag{46}$$

where m_D is the Dirac-type mass matrix and M_R is the Majorana mass matrix for the right-handed neutrino singlets. With this notation the connection among light neutrino masses and the elements of the PMNS matrix, starting from the weak basis specified above, is established through the relation

$$U^\dagger m_{\mathrm{eff}} U^* = d = \mathrm{diag}(m_1, m_2, m_3). \tag{47}$$

Note that at this stage we are considering the limit where U is a unitary matrix. From this equation it is clear that each entry of m_{eff}, to be denoted in what follows by m_{ij}, can be fully expressed in terms of observable quantities—neutrino masses, mixing angles, and phases. The absolute value of the element (11) of m_{eff} is especially interesting experimentally since, in the absence of additional lepton number violating interactions other than those generated by the charged currents involving Majorana neutrinos, it can be measured in neutrinoless double beta decay experiments [47–49].

From Eq. (47) we obtain

$$\begin{aligned} |m_{11}|^2 = {} & m_1^2|U_{e1}|^4 + m_2^2|U_{e2}|^4 + m_3^2|U_{e3}^2|^4 \\ & + 2m_1m_2|U_{e1}|^2|U_{e2}|^2\cos(2\beta_1) \\ & + 2m_1m_3|U_{e1}|^2|U_{e3}|^2\cos(2\gamma_1) \\ & + 2m_2m_3|U_{e2}|^2|U_{e3}|^2\cos[2(\beta_1-\gamma_1)]. \end{aligned} \tag{48}$$

The angle $(\gamma_1 - \beta_1)$ is the argument of $U_{e1}^* U_{e2} U_{e1} U_{e3}^*$, which is not a rephasing invariant Dirac-type quartet. The corresponding product in the quark sector, in terms of elements of $V_{\rm CKM}$, would not be a rephasing invariant. It is the Majorana character of the neutrinos that gives physical meaning to the phase of this fourfold product. If we were to rewrite Eq. (48) using the parametrization of the PMNS matrix given by Eq. (15), the Dirac phase δ would appear explicitly. On the other hand, it is always possible to eliminate the explicit dependence on δ from $|m_{11}|$ by redefining the factorizable phase β in such a way that the phase δ only appears on the second and third rows of the PMNS matrix. This may seem paradoxical, but it has a simple explanation. There is Dirac-type *CP* violation only when the PMNS matrix contains nonfactorizable Majorana-type phases. The measurement of $|m_{11}|$ is only sensitive to one row of the PMNS matrix. When one single row of the PMNS matrix is considered it is always possible to factor out all physical phases on the right-hand side of the matrix. It is necessary to combine information from other rows in order to extract information on the possible presence of nonfactorizable phases. Provided we know the masses of each of the light neutrinos, once we measure the modulus of m_{11} we can infer whether or not there are relative phases among each term and therefore whether or not there is Majorana-type *CP* violation. On the other hand it is also clear from Eq. (48) that neutrinoless double beta decay, although sensitive to the possible existence of Majorana-type *CP* violation, can also occur in the limit of no *CP* violation.

Unfortunately, there are no known feasible experiments that would allow us to measure directly the modulus of other entries of $m_{\rm eff}$. For the off-diagonal entries we have

$$\begin{aligned}|m_{ij}|^2 = {}& m_1^2|U_{i1}|^2|U_{j1}|^2 + m_2^2|U_{i2}|^2|U_{j2}|^2 \\ &+ m_3^2|U_{i3}|^2|U_{j3}|^2 \\ &+ 2m_1m_2|U_{i1}||U_{i2}||U_{j1}||U_{j2}|\cos(\beta_i + \beta_j) \\ &+ 2m_1m_3|U_{i1}||U_{i3}||U_{j1}||U_{j3}|\cos(\gamma_i + \gamma_j) \\ &+ 2m_2m_3|U_{i2}||U_{i3}||U_{j2}||U_{j3}|\cos(\gamma_i + \gamma_j \\ &- \beta_i - \beta_j). \end{aligned} \tag{49}$$

This expression combines information involving two rows of the PMNS matrix where again the Majorana-type phases appear in combinations that are not Dirac-type and that would not be rephasing invariant for Dirac neutrinos. The measurement of one of the $|m_{ij}|$, together with the knowledge of the three neutrino masses, would only give information on the sum of two β_j and the sum of two γ_j without allowing one to determine whether or not the Majorana phases are factorizable.

V. MAJORANA PHASES AND *CP*-ODD WEAK-BASIS INVARIANTS

We have seen that leptonic *CP* violation at low energies requires the presence of complex Majorana-type bilinears, which are defined in terms of entries of the PMNS matrix. The information on whether or not a Lagrangian violates *CP* is also encoded in the fermionic mass matrices written in a weak basis. Unlike the physical basis, weak bases are not unique and, as a result, there is an infinite number of sets of fermion mass matrices corresponding to the same physics. It is often practical to analyze the *CP* properties of the Lagrangian in terms of *CP*-odd weak-basis invariants. Different weak bases (WB) invariants are sensitive to different *CP*-violating phases in different physical scenarios. The general strategy to build such WB invariants was outlined for the first time in Ref. [50], and in Ref. [51] several relevant examples are given together with additional references.

The strength of Dirac-type *CP* violation can be obtained from the following low energy WB invariant:

$$\mathrm{Tr}\,[h_{\rm eff}, h_l]^3 = -6i\Delta_{21}\Delta_{32}\Delta_{31}\,\mathrm{Im}\{(h_{\rm eff})_{12}(h_{\rm eff})_{23}(h_{\rm eff})_{31}\}, \tag{50}$$

where $h_{\rm eff} = m_{\rm eff}m_{\rm eff}^\dagger$, $h_l = m_l m_l^\dagger$, and $\Delta_{21} = (m_\mu^2 - m_e^2)$ with analogous expressions for Δ_{31}, Δ_{32}. The right-hand side of this equation is the computation of this invariant in the special WB where the charged lepton masses are real and diagonal. An analogous invariant is relevant for the quark sector [50]. This WB invariant can be fully expressed in terms of physical observables since

$$\mathrm{Im}\{(h_{\rm eff})_{12}(h_{\rm eff})_{23}(h_{\rm eff})_{31}\} = -\Delta m_{21}^2\Delta m_{31}^2\Delta m_{32}^2\,\mathrm{Im}Q, \tag{51}$$

where ImQ is the imaginary part of a rephasing invariant quartet of the leptonic mixing matrix U and signals the presence of Dirac-type *CP* violation.

It is also possible to construct WB invariants that are sensitive to Majorana-type phases.

It has been shown [12] that the condition

$$\mathrm{Im}\ \mathrm{tr}F = 0, \tag{52}$$

with $F = h_l m_{\rm eff} m_{\rm eff}^* m_{\rm eff} h_l^* m_{\rm eff}^*$ is a necessary condition for *CP* invariance in the leptonic sector, for an arbitrary number of light Majorana neutrinos. This *CP*-odd invariant is sensitive to Majorana-type phases and it may not vanish even in the case where there is no Dirac-type *CP* violation. In order to see that this is the case, it is useful to compute it in terms of lepton masses, mixing angle, and *CP*-violating phase in the simple case of two generations, where there is no Dirac-type *CP* violation but Majorana-type *CP* violation occurs. One obtains

$$\mathrm{Im}\ \mathrm{tr}F = \tfrac{1}{4}m_1m_2(m_2^2 - m_1^2)(m_\mu^2 - m_e^2)^2\sin^2 2\theta\sin^2 2\gamma, \tag{53}$$

where the 2×2 leptonic mixing matrix is parametrized as

$$K = \begin{bmatrix} \cos\theta & -\sin\theta e^{i\gamma} \\ \sin\theta e^{-i\gamma} & \cos\theta \end{bmatrix}. \tag{54}$$

It is the Majorana character of the neutrinos that prevents the phase γ in Eq. (54) to be rotated away. The phase $(-\gamma)$ is in fact the argument of the Majorana bilinears $(K_{11}K_{12}^*)$ and $(K_{21}K_{22}^*)$, modulo π.

Another peculiar aspect of Majorana neutrinos is the fact that for three Majorana neutrinos there is CP violation even in the limit of exact degeneracy of neutrino masses. In this limit, a necessary and sufficient condition [7] for CP invariance is

$$G \equiv \mathrm{Tr}[m_{\mathrm{eff}}^* \cdot h_l \cdot m_{\mathrm{eff}}, h_l^*]^3 = 0. \tag{55}$$

Therefore, this WB invariant condition must be sensitive to Majorana-type CP violation even in the absence of Dirac-type CP violation, both in the case of degenerate and nondegenerate neutrino masses. By analogy to Eq. (50) we may write

$$G = -6i\Delta_{21}\Delta_{32}\Delta_{31} \times \mathrm{Im}\{(m_{\mathrm{eff}}^* \cdot h_l \cdot m_{\mathrm{eff}})_{12} \times (m_{\mathrm{eff}}^* \cdot h_l \cdot m_{\mathrm{eff}})_{23}(m_{\mathrm{eff}}^* \cdot h_l \cdot m_{\mathrm{eff}})_{31}\}. \tag{56}$$

It can be checked that G is indeed sensitive to Majorana bilinears, by writing each factor of the form $(m_{\mathrm{eff}}^* \cdot h_l \cdot m_{\mathrm{eff}})_{ij}$, $i \neq j$, explicitly in terms of masses and mixing, with the help of Eq. (47). It is the presence of the matrix h_l between m_{eff}^* and m_{eff} that makes this CP-odd invariant fundamentally different from the one in Eq. (50). The terms in $m_i m_j$ with $i \neq j$ which are generated once we expand the above factors, always appear multiplied by Majorana bilinears and also by the square of a charged lepton mass. These three different factors prevent the possibility of simplification among these terms which otherwise would add to zero due to unitarity. If the charged leptons were degenerate in mass only the terms in m_j^2 of the expansion would survive. These terms do not depend on Majorana bilinears.

VI. MAJORANA PHASES AND LEPTOGENESIS

CP violation in the leptonic sector may play a fundamental role in the generation, via leptogenesis, of the observed baryon number asymmetry of the universe [52]:

$$\frac{n_B}{n_\gamma} = (6.1^{+0.3}_{-0.2}) \times 10^{-10}. \tag{57}$$

In this framework a CP asymmetry is generated through out-of-equilibrium L-violating decays of heavy Majorana neutrinos [38] leading to a lepton asymmetry which, in the presence of $(B + L)$-violating but $(B - L)$-conserving sphaleron processes [53], produces a baryon asymmetry.

In the single flavor approach, with three singlet heavy neutrinos N_i, thermal leptogenesis is insensitive to the CP-violating phases appearing in the PMNS matrix. In this case there is complete decoupling among the phases responsible for CP violation at low energies and those responsible for leptogenesis [54,55].

From Eq. (47) and the definition of m_{eff} one can write m_D in the Casas and Ibarra parametrization [56] as

$$m_D = iU\sqrt{d}R\sqrt{D}. \tag{58}$$

The matrix R is a general complex orthogonal matrix, and d and D are diagonal matrices for the light and the heavy neutrino masses, respectively. Clearly low energy physics cannot provide any information on R since this matrix cancels out in m_{eff}. The lepton number asymmetry resulting from the decay of heavy Majorana neutrinos, ε_{N_j}, was computed, in the single flavor approach, by several authors [57–59]. The result is proportional to $\sum_{k\neq j}\mathrm{Im}(m_D^\dagger m_D)_{jk} \times (m_D^\dagger m_D)_{jk}$ with an additional factor depending on the ratio of the masses of the two heavy neutrinos k and j, $x_k = \frac{M_k^2}{M_j^2}$. The matrix U cancels out in the combination $m_D^\dagger m_D$ and in this case leptogenesis only depends on CP violation present in R. This is a consequence of having summed up into all charged lepton indices $l_i^\pm$ $(i = e, \mu, \tau)$ resulting from the decay of the heavy neutrino. Therefore, in the case of unflavored leptogenesis the leptonic asymmetry is not sensitive to low energy CP violating phases. As a result, the Majorana-type phases do not play any role.

Flavor effects matter when washout processes are sensitive to the different leptonic flavors produced in the decay of heavy Majorana neutrinos [60]. In this particular case the single flavor approach ceases to be valid and the separate asymmetry produced in each decay has to be considered.

The separate lepton i family asymmetry $\varepsilon_{N_j}^i$ generated from the decay of the jth heavy Majorana neutrino is given by [40]

$$\varepsilon_{N_j}^i = \frac{g^2}{M_W^2}\frac{1}{16\pi}\sum_{k\neq j}\left[I(x_k)\frac{\mathrm{Im}((m_D^\dagger m_D)_{jk}(m_D^*)_{ij}(m_D)_{ik})}{|(m_D)_{ij}|^2} + \frac{1}{1-x_k}\frac{\mathrm{Im}((m_D^\dagger m_D)_{kj}(m_D^*)_{ij}(m_D)_{ik})}{|(m_D)_{ij}|^2}\right], \tag{59}$$

with

$$I(x_k) = \sqrt{x_k}\left[1 + \frac{1}{1-x_k} + (1 + x_k)\ln\frac{x_k}{1+x_k}\right]. \tag{60}$$

Clearly, when one works with separate flavors the matrix U does not cancel out and one is lead to the interesting possibility of having viable leptogenesis even in the case of R being a real matrix [61–64]. If we were to sum over all charged leptons, the first term in Eq. (59) would lead to the expression obtained for the total lepton number asymmetry in the case of unflavored leptogenesis, while the second term would become real.

Assuming R to be real, from Eq. (58) we obtain

$$\mathrm{Im}((m_D^\dagger m_D)_{jk}(m_D^*)_{ij}(m_D)_{ik}) = (m_D^\dagger m_D)_{jk}\sqrt{d_l}R_{lj}\sqrt{D_j}\sqrt{d_s}R_{sk} \times \sqrt{D_k}|U_{il}||U_{is}|\sin(\arg(U_{il}^* U_{is})). \quad (61)$$

The only indices that are summed up are l and s and each term in this sum is proportional to the sine of a β_i, a γ_i, or a $(\beta_i - \gamma_i)$, which are pure Majorana-type phases. The second term, $\mathrm{Im}((m_D^\dagger m_D)_{kj}(m_D^*)_{ij}(m_D)_{ik})$, only differs from this one by the structure of indices of $(m_D^\dagger m_D)$. Furthermore we conclude that in the limit $R = 1$ flavored leptogenesis may be viable even without Dirac-type CP violation, i.e., even if CP violation is not observed in neutrino oscillations where, in principle, low energy CP violation could be detected. Flavoured leptogenesis is sensitive to each one of the different Majorana-type phases alone and, in the general case of complex R, it will depend on the additional phases present in this matrix.

VII. CONCLUSIONS

We have emphasized that in the case of Majorana neutrinos, the arguments of rephasing invariant bilinears, designated Majorana-type phases, are the fundamental quantities in the study of CP violation in the leptonic sector. If one further assumes 3×3 unitarity of the PMNS matrix, we have shown that in general the full PMNS matrix can be derived using as input six independent Majorana-type phases. The presence of nonfactorizable Majorana-type phases in the PMNS matrix signals the presence of Dirac-type CP violation which might be observable in future neutrino oscillation experiments. As a result, Dirac-type CP violation requires the existence of Majorana-type CP violation. Obviously the converse is not true. We have shown how to relate the strength of Dirac-type CP violation to these Majorana-type phases by writing the area of the unitarity triangles in terms of these phases. We have also studied how these Majorana-type phases appear in the elements of the neutrino mass matrix, as well as in flavored leptogenesis.

Observables that should be sensitive to the Majorana-type phases, even in the absence of Dirac-type CP violation, include neutrinoless double beta decay and possibly leptogenesis. Neutrino-antineutrino oscillation processes can also in principle be used to measure CP-violating Majorana phases [65]. Other manifestly CP-violating physical processes are leptonic electric dipole moments [66]. An extensive review of issues related to flavor phenomena and CP violation in the leptonic sector and the potential for their discovery in the LHC and possible future experiments is provided in Ref. [67].

It is clear that the application of our results to perform practical tests of the PMNS paradigm is severely restricted by the scarcity of data on leptonic mixing and CP violation, leading to the dreadful situation that the neutrino mass matrix cannot be fully reconstructed from a set of presently conceived feasible experiments. One possible hope is having a significant development in our understanding of flavor, in particular, of leptonic flavor. If a theory of flavor implies, for example, direct constraints on the Majorana-type phases, then the relations we have derived, connecting these phases to other leptonic observables, would be of paramount importance.

ACKNOWLEDGMENTS

This work was partially supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal) through the projects PDCT/FP/63914/2005, PDCT/FP/63912/2005, POCI/81919/2007, and CFTP-FCT UNIT 777 which are partially funded through POCTI (FEDER), and by the Marie Curie RTNs MRTN-CT-2006-035505 and MRTN-CT-2004-503369. G. C. B. would like to thank Francisco Botella for the kind hospitality at Universitat de València-CSIC during his sabbatical leave. M. N. R. would like to thank Francisco Botella for the warm welcome during the short visit to Universitat de València supported by Acções de Mobilidade Portugal/Espanha 2008. The authors are grateful for the warm hospitality of the CERN Physics Department (P. H.) Theory Division (T. H.) where part of this work was done.

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