Published by IOP Publishing for SISSA
Received: *September 18, 2008*
Revised: *January 16, 2009*
Accepted: *February 20, 2009*
Published: *March 13, 2009*

Dirac Born Infeld (DBI) cosmic strings

Eugeny Babichev,[a] **Philippe Brax,**[b,c] **Chiara Caprini,**[b] **Jérôme Martin**[c] **and Danièle A. Steer**[a,d]

[a] *APC (Astroparticules et Cosmologie), UMR 7164 (CNRS, Université Paris 7, CEA, Observatoire de Paris), 10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France*
[b] *Institut de Physique Théorique, CEA, IPhT, CNRS, URA 2306, F-91191Gif/Yvette Cedex, France*
[c] *Institut d'Astrophysique de Paris, UMR 7095-CNRS, Université Pierre et Marie Curie, 98bis boulevard Arago, 75014 Paris, France*
[d] *CERN Physics Department, Theory Division, CH-1211 Geneva 23, Switzerland*
E-mail: babichev@apc.univ-paris7.fr, philippe.brax@cea.fr, chiara.caprini@cea.fr, jmartin@iap.fr, steer@apc.univ-paris7.fr

Abstract: Motivated by brane physics, we consider the non-linear Dirac-Born-Infeld (DBI) extension of the Abelian-Higgs model and study the corresponding cosmic string configurations. The model is defined by a potential term, assumed to be of the mexican hat form, and a DBI action for the kinetic terms. We show that it is a continuous deformation of the Abelian-Higgs model, with a single deformation DBI parameter depending on a dimensionless combination of the scalar coupling constant, the vacuum expectation value of the scalar field at infinity, and the brane tension. By means of numerical calculations, we investigate the profiles of the corresponding DBI-cosmic strings and prove that they have a core which is narrower than that of Abelian-Higgs strings. We also show that the corresponding action is smaller than in the standard case suggesting that their formation could be favoured in brane models. Moreover we show that the DBI-cosmic string solutions are non-pathological everywhere in parameter space. Finally, in the limit in which the DBI model reduces to the Bogomolnyi-Prasad-Sommerfield (BPS) Abelian-Higgs model, we find that DBI cosmic strings are no longer BPS: rather they have positive binding energy. We thus argue that, when they meet, two DBI strings will not bind with the corresponding formation of a junction, and hence that a network of DBI strings is likely to behave as a network of standard cosmic strings. On the other hand, we also find that, if the BPS condition is no longer satisfied and the coupling constant is less than twice the charge squared of the scalar field, DBI strings can change their behaviour from type I to type II depending on the DBI parameter.

Keywords: Strings and branes phenomenology

ArXiv ePrint: 0809.2013

doi:10.1088/1126-6708/2009/03/091

Contents

1 Introduction

The (Wilkinson Microwave Anisotropy Probe) WMAP5 results [1–6] give strong indication in favour of cosmic inflation over other mechanisms for the production of primordial fluctuations [7]. Since inflation generally takes place at high energy, recently there has been a flurry of activity in constructing models inspired by or derived from string theory (for recent reviews see e.g. refs. [8–11]). In a large category of these models, particularly brane-antibrane inflation [12–20] and D3/D7 inflation [21, 22], the end result of the inflationary phase is the creation of D-strings (as well as potentially F-strings [23]), interpreted from the four-dimensional point of view as cosmic strings. Since cosmic strings are strongly ruled out as the main originator of primordial fluctuations, the D-(and indeed F-) string tension is severely constrained and (under certain assumptions) such that $G_{\rm N}\mu \lesssim 10^{-6}$ in order to preserve the features of the WMAP5 results [24]. Nevertheless, the existence and possible detection of the effects of D-strings in the aftermath of an era of brane inflation could be a testable prediction of string theory.

D-strings themselves have been conjectured to be in correspondence with the D-term strings of supergravity [25]. One of their remarkable properties is that they satisfy a Bogomolnyi-Prasad-Sommerfield (BPS) condition, i.e. they have no binding energy and preserve 1/2 of the original supersymmetries. They also carry fermionic zero modes and are therefore vorton candidates, leading to possible interesting phenomenological consequences [26].

The identification between D-term strings and D-strings has been made in the low energy limit, when field gradients are small. Inspired by the case of open string modes which can be effectively described by a non-linear action of the Dirac-Born-Infeld (DBI) type, in this paper we construct models of cosmic strings which depart from the low energy approximation and generalise the Abelian-Higgs model to a non-linear one. We will call the resulting topological objects 'DBI-cosmic strings', and they are exact solutions of the generalised non-linear DBI action. The action we consider is very different from others which have been discussed in the literature, refs. [27–32], and in particular does not lead to pathological configurations. In the limit of small field gradients our DBI strings reduce to Abelian-Higgs strings. We construct the DBI string solutions numerically in a broad range of parameter space, using two numerical methods: a relaxation method and a shooting algorithm. In this way, we show, in particular, that DBI strings with a potential term corresponding to the BPS limit of the Abelian-Higgs model are no-longer BPS. More specifically, $\mu_{2n} \geq 2\mu_n$, where μ_n is the action per unit time and length for a string with a winding number n: the equality only holds in the low-energy limit. Borrowing language from the standard cosmic string literature [33–35], the strings are therefore in the type II regime (though the deviations from BPS are small, in a sense we will quantify). The network of strings produced will therefore not contain junctions, and all the strings will have the same tension $\mu_{n=1}$. In the cosmological context we therefore expect the DBI-string network to evolve in the standard way e.g. [36], containing infinite strings and loops, radiating energy through gravitational waves, and eventually reaching a scaling solution.

The paper is organised as follows. In section 2 we recall the properties of Abelian-Higgs cosmic strings. In section 3, we first briefly review the different non-linear actions which have been put forward so far in the literature, and then present our proposed non-linear action for cosmic strings. In section 4, we study the DBI-cosmic strings solutions analytically and numerically. In subsection 4.1, we present simple analytical estimates which allow us to roughly guess the form of the DBI string profiles and, in subsection 4.2, we compute them numerically by means of two different methods (shooting and over relaxation). In section 5 we briefly summarise our main findings and discuss our conclusions. Finally, in A, we outline how Abelian-Higgs cosmic strings can be realized in string theory in the D3/D7 system and in B, we give the full non-linear structure of the DBI cosmic string action and discuss the motivation for this action, which we expect to be applicable when field gradients are large.

2 Abelian-Higgs cosmic strings

We begin by recalling briefly the properties of standard Abelian-Higgs cosmic strings, and at the same time introduce our notation following ref. [34] though we use the signature $(-+++)$.

The Abelian-Higgs model is governed by the action

$$S = -\int \mathrm{d}^4x\sqrt{-g}\left[(D_\mu\Sigma)(D^\mu\Sigma)^\dagger + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + V(|\Sigma|)\right] , \tag{2.1}$$

JHEP03(2009)091

where the potential is given by

$$V(|\Sigma|) = \frac{\lambda}{4}\left(|\Sigma|^2 - \eta^2\right)^2 . \tag{2.2}$$

In eq. (2.1), D_μ denotes the covariant derivative defined by $D_\mu \equiv \partial_\mu - iqA_\mu$ with A_μ the vector potential, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and q is the gauge coupling. The potential is characterised by two free parameters: the coupling $\lambda > 0$ and an energy scale η. It is useful to introduce the dimensionless coupling

$$\beta \equiv \frac{\lambda}{2q^2} = \frac{m_{\rm s}^2}{m_{\rm g}^2}, \tag{2.3}$$

where the Higgs mass is $m_{\rm s} = \sqrt{\lambda}\eta/\sqrt{2}$ and the vector mass $m_{\rm g} = q\eta$.

Due to the non-trivial topology of the vacuum manifold, after gauge symmetry breaking the model possesses vortex (or cosmic string) solutions for which the scalar field can be expressed as

$$\Sigma(r,\theta) = \eta X(\rho)\,{\rm e}^{in\theta}, \tag{2.4}$$

where we have used the cylindrical coordinates, and the cosmic string is aligned along the z-axis. Here n is the winding number proportional to the quantised magnetic flux on the string, and we have defined a rescaled radial coordinate

$$\rho \equiv \lambda^{1/2}\eta r, \tag{2.5}$$

with $X(\rho) \to 1$ at infinity, while $X(0) = 0$. In the radial gauge, the only non-vanishing component of the vector potential A_μ is $A_\theta(\rho)$ with $A_\theta(0) = 0$. We define

$$Q \equiv n - qA_\theta, \tag{2.6}$$

so that the tension, defined to be the action per unit time and length $\mu = -S/{\rm d}t{\rm d}z$, can be expressed as

$$\begin{aligned}\mu_n(\beta) &= 2\pi\eta^2 \int_0^{+\infty} {\rm d}\rho\rho \left[\left(\frac{{\rm d}X}{{\rm d}\rho}\right)^2 + \frac{Q^2X^2}{\rho^2} + \frac{\beta}{\rho^2}\left(\frac{{\rm d}Q}{{\rm d}\rho}\right)^2 + \frac{1}{4}\left(X^2-1\right)^2\right] \\ &\equiv 2\pi\eta^2 g_n\left(\beta^{-1}\right). \end{aligned} \tag{2.7}$$

The function $g_n\left(\beta^{-1}\right)$ is plotted in figure 1 (left panel).

In the BPS case, $\beta = 1$, the tension given in eq. (2.7) can be re-expressed as

$$\mu_n = 2\pi\eta^2 \int_0^{+\infty} {\rm d}\rho\rho \left\{\left(\frac{{\rm d}X}{{\rm d}\rho} - \frac{QX}{\rho}\right)^2 + \left[\frac{1}{\rho}\frac{{\rm d}Q}{{\rm d}\rho} - \frac{1}{2}(X^2-1)\right]^2 + \frac{1}{\rho}\frac{\rm d}{{\rm d}\rho}\left[Q(X^2-1)\right]\right\}, \tag{2.8}$$

and is minimised for cosmic strings that are solutions of the BPS equations

$$\frac{{\rm d}X}{{\rm d}\rho} = \frac{XQ}{\rho}, \quad \frac{{\rm d}Q}{{\rm d}\rho} = \frac{\rho}{2}(X^2-1). \tag{2.9}$$

JHEP03(2009)091

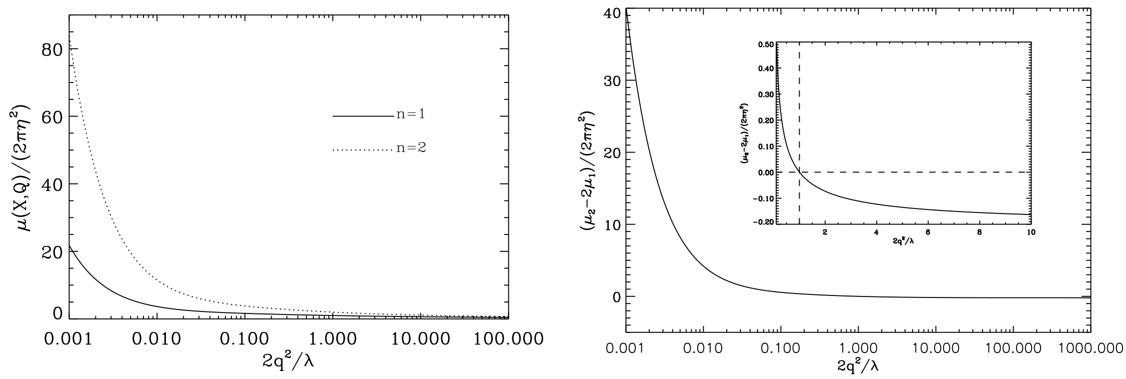

Figure 1. Left panel: the tension of $n = 1$ and $n = 2$ Abelian-Higgs strings as a function of $\beta^{-1} = 2q^2/\lambda$. Right panel: the binding energy $\mu_{2n} - 2\mu_n$ for $n = 1$ Abelian-Higgs strings as a function of β^{-1}. When the BPS condition $\beta = 1$ is satisfied, $\mu_n = 2\pi n\eta^2$ so that $\mu_{2n} - 2\mu_n = 0$ as can be verified in the figure.

On inserting back into eq. (2.8) one finds

$$\mu_n = 2\pi\eta^2 n \tag{2.10}$$

so that $g_n(1) = n$.

The functions $g_{n=1}(\beta^{-1})$ and $g_{n=2}(\beta^{-1})$ are plotted in the left hand panel of figure 1 whereas the binding energy $\mu_2 - 2\mu_1$ is shown in the right hand panel. In the BPS limit, $g_n(1) = n$, and the force between vortices vanishes [37]. For $\beta < 1$, the strings are type I with a negative binding energy, while for $\beta > 1$ they are type II with positive binding energy. Type I string therefore attract and can form bound states or 'zippers' [38] linked by junctions. Zippers may form (in a certain regime of parameter space) when two strings in a network collide, refs. [39, 40]. A network of type II strings, on the other hand, contains no junctions and the strings all have the same tension $\mu_{n=1}$. In A, we outline how BPS Abelian-Higgs cosmic strings form in certain string theory models of inflation.

We now discuss possible non-linear extensions of the Abelian-Higgs system and focus on a specific, well defined form. We then analyse the departure from the BPS case induced by the higher order terms. Indeed, we will see that DBI strings with $\beta = 1$ have a positive binding energy and hence repel each other.

3 DBI cosmic strings

3.1 Non-standard actions for cosmic strings

We are interested in situations in which gauged cosmic strings form when higher order corrections to the kinetic terms in the action cannot be neglected. In the absence of an explicit derivation from, say, string theory, we take a phenomenological approach (which, however, is strongly inspired by string theory, see B). We will construct an action [given in eqs. (3.10) and (B.7)] which satisfies the following two criteria: Firstly, the Abelian-Higgs limit should be recovered when gradients are small. In particular, the action should be a continuous deformation of the Abelian-Higgs model. Secondly, the resulting cosmic string solutions should have no pathological and/or singular behaviour as the model becomes more and more non-linear (in field gradients).

In the remainder of this subsection, before turning to the action advocated in the present article, we discuss the other non-linear actions which have been considered in the literature and which do not satisfy the above criteria.

In refs. [27, 28, 30] an attempt to construct a non-linear model for (electrically) charged vortices in (2+1) dimensions uses an hybrid approach with a (truncated) Born-Infeld action for the gauge field, a standard linear action for the Higgs field, and a Chern-Simons term:[1]

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\sigma^2\left(\sqrt{1+\frac{1}{2\sigma^2}F_{\mu\nu}F^{\mu\nu}}-1\right)+\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho}+\frac{1}{2}|D\phi|^2+V(\phi)\right], \quad (3.1)$$

where σ is a parameter of dimension two and $|D\phi|^2=(D_\mu\phi)(D^\mu\phi)^\dagger$. At a threshold $\sigma=\sigma_{\rm c}$ corresponding to the very non-linear regime, the gauge field becomes singular at the origin of the vortex whilst, below the threshold, no solution exists. Incorporating the Higgs kinetic terms into the square root, while dropping the Chern-Simons term, leads to the following expression

$$\begin{aligned}S=\int \mathrm{d}^4x\sqrt{-g}\Bigg\{\sigma^2\Bigg[\Bigg(1+\frac{1}{2\sigma^2}F_{\mu\nu}F^{\mu\nu}+\frac{1}{\sigma^2}\left|D\phi\right|^2+\frac{1}{\sigma^4}\tilde{F}_\mu\tilde{F}_\nu D^\mu\phi^\dagger D^\nu\phi\\ -\frac{1}{2\sigma^4}\left|D\phi\right|^2F_{\mu\nu}F^{\mu\nu}\Bigg)^{1/2}-1\Bigg]-V\left(\phi\right)\Bigg\},\end{aligned} \quad (3.2)$$

where $\tilde{F}_\mu\equiv\epsilon_{\mu\nu\rho}F^{\nu\rho}/2$. With such a very particular form (which differs from the one we will propose shortly), one does not find finite energy solutions.

A model for (global) cosmic strings was proposed in ref. [29] with an action given by

$$S=-\int \mathrm{d}^4x\sqrt{-g}\left[\sqrt{1+|\partial\phi|^2}-1+V(\phi)\right] \quad (3.3)$$

[this is identical to eq. (3.2) in the global limit]. For this model the solutions become multi-valued and undefined at the origin as soon as the magnitude of $V(\phi)$ becomes sufficiently large [29].

In view of these negative approaches where singularities and pathologies abound, one could be tempted to think that non-linear cosmic string actions all lead to these problems. Fortunately, a well-behaved action has been suggested by Sen [41] and studied in ref. [42] in the case of D-strings obtained at the end of D-$\bar{\rm D}$ inflation. In such a system, hybrid inflation occurs and the rôle of the waterfall field is played by the open string tachyon T with a charge ±1 under the U(1) gauge groups of the D3- (respectively $\bar{\rm D}$3-) brane. When the two branes coincide the effective action reads

$$S=-T_3\int \mathrm{d}^4x\,V\left(T,T^\dagger\right)\left[\sqrt{\det\left(-g^+\right)}+\sqrt{\det\left(-g^-\right)}\right], \quad (3.4)$$

[1]No charged vortex solutions exist when the Chern-Simons term is absent.

where $g^{\pm}_{\mu\nu} = g_{\mu\nu} \pm \ell_s^2 F_{\mu\nu} + \left(D_\mu T D_\nu T^\dagger + D_\mu T^\dagger D_\nu T\right)/2$ and V is the tachyon potential. Since $\det(-g^+) = \det(-g^-)$ (see the appendix), the action reduces to

$$S = -2T_3 \int \mathrm{d}^4x V\left(T, T^\dagger\right)\sqrt{\det\left(-g^+\right)}, \tag{3.5}$$

and it admits BPS vortex solutions.

Topological defects in models with general non-linear kinetic terms were studied in refs. [31, 32]. The action proposed in [31] for global topological defects differs from the tachyon action above (in the global limit) as the potential is added to the generalised kinetic terms:

$$S = \int \mathrm{d}^4x \left[M^4 K\left(\frac{X}{M^4}\right) - V(\phi)\right]. \tag{3.6}$$

Here

$$X \equiv \frac{1}{2}(\partial_\mu \phi_a \partial^\mu \phi_a) \tag{3.7}$$

is the standard kinetic term, $K(X)$ is some non-linear function, ϕ_a is a set of scalar fields, M has dimensions of mass, and the potential term provides the symmetry breaking term. One of the main restrictions imposed in [31] on the form of the non-linear function $K(X)$ is that $K(X)$ should have a canonical asymptotic form, $K(X) \sim X$ as $X \to 0$. However, for large gradients $K(X)$ could deviate considerably from the canonical kinetic terms. The former requirement implies a non-pathological behaviour of solutions far from the defect core, while the different possibilities for $K(X)$ at infinity leads to deviations of the defect from the standard case inside the core. The action (3.6) leads to non-pathological solutions for so-called k-defects — domain walls, cosmic strings and monopoles — whose properties can differ considerably from those of standard defects.

A gauge version of action (3.6) was considered in ref. [32]: for a complex scalar field

$$S = \int \mathrm{d}^4x \left[M^4 K\left(\frac{X}{M^4}\right) - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right] \tag{3.8}$$

with

$$X \equiv \frac{1}{2}(D_\mu\phi)(D^\mu\phi)^\dagger. \tag{3.9}$$

It was shown that non-pathological cosmic string solutions exist at least for some choices of the non-linear function $K(X)$ [32].

In the following we study a non-linear extension of Abelian-Higgs model which retains some of the properties of the tachyon and k-defect models. The potential will be additive as in the k-defect case while the kinetic terms have a DBI form as in the tachyon case. However, the kinetic terms are *not* a function solely of X anymore: they differentiate between the radial and angular gradients of the defects.

3.2 A DBI action for cosmic strings

We now turn to the action that we propose in this article, namely

$$S = -T\int \mathrm{d}^4x \left\{\sqrt{-\det\left[g_{\mu\nu} + (\mathcal{D}_{(\mu}\Phi)(\mathcal{D}_{\nu)}\Phi)^\dagger + \ell_s^2\mathcal{F}_{\mu\nu}\right]} - \sqrt{-g} + \sqrt{-g}\frac{V(\sqrt{T}|\Phi|)}{T}\right\}. \tag{3.10}$$

Here T has dimension 4 and ℓ_s has dimension of -1. In the extra dimensional context motivated in B, T corresponds to a brane tension and ℓ_s to the string scale. Notice that, in the above equation, the potential [i.e. $V(x)$ as a function of x] is still given by the expression (2.2) and, therefore, contains the parameter λ. When the complex scalar field Φ vanishes, eq. (3.10) describes Born-Infeld electrodynamics [43].

We now discuss action (3.10) in detail. In particular we compare its properties to those of the Abelian-Higgs action (2.1) discussed in section 2, and then we construct the static cosmic string solutions of the action.

A first important property of eq. (3.10) is that, to leading order in derivatives, it reduces to the standard action (2.1) on identifying

$$\Sigma = \sqrt{T}\Phi \,, \tag{3.11}$$

and redefining the charge according to the following expression

$$q = \frac{\hat{q}}{\sqrt{T}\ell_s^2} \tag{3.12}$$

together with the gauge field

$$\mathcal{A}_\mu = \frac{A_\mu}{\sqrt{T}\ell_s^2} \,. \tag{3.13}$$

Hence, if the spatial derivatives characterising DBI-strings are small (we shall discuss whether or not this is the case below), their properties should to be very similar to Abelian Higgs strings. More generally, however, and as discussed in detail in the appendix where we calculate the determinant explicitly, eq. (3.10) contains terms of higher order in covariant derivatives as well as numerous different mixing terms between $\mathcal{F}$ and $\mathcal{D}$ (suitably contracted). These extra terms could significantly change the string solution and the resulting strings' properties relative to the Abelian Higgs case. It follows from this that our action is very different from that considered by Sarangi in ref. [29], even in the global case. As a consequence we will find non-pathological cosmic strings solutions with a continuous limit to Abelian-Higgs strings.

We now focus on the cosmic string solutions of eq. (3.10). For this purpose, first it is useful to pass to dimensionless variables, denoted with a hat. Explicitly we set

$$\hat{\Phi} \equiv \frac{\Phi}{\ell_s} \,, \quad \hat{\mathcal{F}}_{\mu\nu} \equiv \ell_s^2 \mathcal{F}_{\mu\nu} \,, \quad \hat{\mathcal{D}}_\mu \equiv \ell_s \mathcal{D}_\mu \,, \quad \hat{\eta} \equiv \frac{\eta}{\sqrt{T}\ell_s} \,, \quad \hat{r} \equiv \frac{r}{\ell_s} \,, \tag{3.14}$$

as well as

$$\hat{V}(|\hat{\Phi}|) \equiv \frac{\hat{\lambda}}{4}\left(\hat{\Phi}^2 - \hat{\eta}^2\right)^2 \,, \tag{3.15}$$

where $\hat{\lambda} \equiv \lambda T \ell_s^4$, so that the action becomes

$$S = -T\ell_s^4 \int d^4x \left\{ \sqrt{-\det\left[g_{\mu\nu} + (\hat{\mathcal{D}}_{(\mu}\hat{\Phi})(\hat{\mathcal{D}}_{\nu)}\hat{\Phi})^\dagger + \hat{\mathcal{F}}_{\mu\nu}\right]} - \sqrt{-g} + \sqrt{-g}\hat{V}\left(|\hat{\Phi}|\right) \right\}. \tag{3.16}$$

We now follow the procedure outlined in section 2 for Abelian-Higgs cosmic strings, however for action (3.16). In dimensionless cylindrical coordinates, $\mathrm{d}s^2 = -\mathrm{d}\hat{t}^2 + \mathrm{d}\hat{r}^2 + \hat{r}^2\mathrm{d}\theta^2 + \mathrm{d}\hat{z}^2$ and in the radial gauge ($\hat{A}_{\hat{r}} = 0$), the cosmic string profile is

$$\hat{\Phi} = \hat{\eta}X(\rho)\mathrm{e}^{in\theta}\,, \quad Q(\rho) = n - \hat{q}\hat{\mathcal{A}}_\theta\left(\hat{r}\right)\,, \tag{3.17}$$

where we have defined a new radial coordinate ρ by the following expression

$$\rho \equiv \hat{\lambda}^{1/2}\hat{\eta}\hat{r} \tag{3.18}$$

which should be compared to eq. (2.5). The boundary conditions on the fields are

$$\lim_{\rho\to 0} X = 0\,, \quad \lim_{\rho\to 0} Q = n\,, \quad \lim_{\rho\to\infty} X = 1\,, \quad \lim_{\rho\to\infty} Q = 0\,. \tag{3.19}$$

Substituting eqs. (3.17) and (3.18) into (3.16) as well as using (3.14), we find that[2]

$$\left(-\det g_{\mu\nu}\right)\gamma^{-2} = -\det\left[g_{\mu\nu} + (\hat{\mathcal{D}}_{(\mu}\hat{\Phi})(\hat{\mathcal{D}}_{\nu)}\hat{\Phi})^\dagger + \hat{\mathcal{F}}_{\mu\nu}\right]\,, \tag{3.20}$$

where we have defined the γ factor by

$$\gamma^{-2} \equiv \left[1 + \alpha\left(\frac{\mathrm{d}X}{\mathrm{d}\rho}\right)^2\right]\left(1 + \frac{\alpha Q^2X^2}{\rho^2}\right) + \frac{\alpha\beta}{\rho^2}\left(\frac{\mathrm{d}Q}{\mathrm{d}\rho}\right)^2\,. \tag{3.21}$$

Hence the string tension defined as $-S/\ell_\mathrm{s}^2\mathrm{d}\hat{z}\mathrm{d}\hat{t}$ is given by

$$\mu_n = \frac{4\pi\eta^2}{\alpha}\int_0^{+\infty}\mathrm{d}\rho\rho\left\{\sqrt{\left[1+\alpha\left(\frac{\mathrm{d}X}{\mathrm{d}\rho}\right)^2\right]\left(1+\alpha\frac{Q^2X^2}{\rho^2}\right)+\frac{\alpha\beta}{\rho^2}\left(\frac{\mathrm{d}Q}{\mathrm{d}\rho}\right)^2}-1+\frac{\alpha}{8}(X^2-1)^2\right\}\,, \tag{3.22}$$

where

$$\alpha \equiv 2\hat{\lambda}\hat{\eta}^4\,, \tag{3.23}$$

and, as in the Abelian-Higgs case,

$$\beta = \frac{\hat{\lambda}}{2\hat{q}^2} = \frac{\lambda}{2q^2}\,. \tag{3.24}$$

Eq. (3.22) is the main result of this section and represents the non-linear DBI generalisation of the linear Abelian-Higgs model: it should be compared to eq. (2.7). Notice that it involves the single additional parameter α which measures the deformation from the Abelian-Higgs model, since eq. (3.22) reduces to the tension of Abelian Higgs strings in the linear limit $\alpha \to 0$. As discussed in section 2, Abelian-Higgs strings are BPS when $\beta = 1$, and hence for $\beta = 1$, DBI-cosmic strings with tension given by eq. (3.22) are a continuous deformation of the BPS Abelian-Higgs strings. This property is, of course, very important and constitutes an additional motivation for the action given in eq. (3.16).

Finally, we note that the argument of the cosmic string profile ρ is also identical to its counterpart in the Abelian-Higgs case, whatever the value of α. Hence we will be able to find continuous deformations of the cosmic string profiles parameterised by α and depending on the universal variable ρ.

[2] Note that $\gamma^{-2} = \mathcal{D}$ where $\mathcal{D}$ is discussed in the appendix.

JHEP03(2009)091

4 DBI string solutions

4.1 Analytical estimates

Having established the model and its action, we now turn to the solutions of the equations of motion. The DBI cosmic string equations follow from eq. (3.22) and read

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\left[\rho\gamma\left(1+\frac{\alpha Q^2X^2}{\rho^2}\right)\frac{\mathrm{d}X}{\mathrm{d}\rho}\right] = \frac{\rho}{2}(X^2-1)X+\gamma\frac{Q^2X}{\rho}\left[1+\alpha\left(\frac{\mathrm{d}X}{\mathrm{d}\rho}\right)^2\right], \tag{4.1}$$

$$\mathit{frac}\mathrm{d}\mathrm{d}\rho\left(\frac{\gamma}{\rho}\frac{\mathrm{d}Q}{\mathrm{d}\rho}\right) = \frac{\gamma Q}{\beta\rho}\left[1+\alpha\left(\frac{\mathrm{d}X}{\mathrm{d}\rho}\right)^2\right]X^2\,, \tag{4.2}$$

for the scalar field and gauge fields, respectively, where γ is defined in eq. (3.21). In the Abelian Higgs limit, $\alpha=0$, eqs. (4.1) and (4.2) reduce to the standard cosmic string field equations for which $\gamma=1$. Deviations from Abelian Higgs strings will occur if $\gamma<1$. Notice that here the fields are purely space-dependent. For time-dependent fields, and particularly in DBI inflationary cosmology with inflaton $\phi(t)$ whose dynamics is described by action (3.16) in the global limit, then γ is a generalisation of the cosmological Lorentz factor. Indeed, as can be seen from eq. (3.21) in the case when $g_{\mu\nu}$ describes an homogeneous and isotropic manifold, $\gamma^{-2}=1-\dot{\phi}^2/T(\phi)$ where $T(\phi)$ is related to the metric of the extra-dimensions. The difference in sign between spatial and temporal derivatives is responsible for the fact that deviations from standard cosmology ($\gamma=1$) occur here when $\gamma\to+\infty$ rather than $\gamma\ll1$.

Unfortunately, as is clear from eqs. (4.1) and (4.2), the DBI cosmic string equations cannot be solved exactly. We have therefore carried out a full numerical integration of the equations of motion. For convenience, we will focus mainly on the deformed BPS case where $\beta=1$ and $\alpha\neq0$ (though $\beta\neq1$ and $\alpha\neq0$ can also been done with the numerical methods used here, see figure 7).

Before discussing the numerical results, we analyse the asymptotic behaviour of the fields in order to obtain a rough understanding of the solution. We will consider the two limits $\rho\to0$ and $\rho\to\infty$ and will address two issues. The first one is the non-existence of singularities in the core of the cosmic string. The second one will be the determination of the shape of the cosmic string profile both at the origin and at infinity. In particular we will find that the functional form of the string profile is similar to the Abelian-Higgs case inside the core, the only difference springing from α-dependent factors.

Consider first the $\rho\to0$ limit and let us examine the possibility of singular DBI strings deep in the string core. As already discussed, the DBI features of the solutions depend on γ. In particular, extreme deviations from the Abelian-Higgs case would appear if $\gamma\to0$ at the origin. This can only happen if the derivative of X and/or Q become extremely large, i.e. the string becomes singular. Let us first assume that the gradient of Q becomes large and dominates the γ factor, i.e. $\gamma\sim\rho\mathrm{d}\rho/\left(\sqrt{\alpha}\mathrm{d}Q\right)$. The gauge equation becomes nonsensical as the left-hand of (4.2) vanishes and the right-hand side does not. Hence there is no regime where the gradient Q is arbitrarily large leading to $\gamma\to0$ at the origin. We now examine the possibility that X becomes singular at the origin with a large gradient.

In this limit, we find

$$\gamma \sim \frac{\rho}{\alpha X\left(\mathrm{d}X/\mathrm{d}\rho\right)}\left[1-\frac{1}{2}\frac{1}{\alpha\left(\mathrm{d}X/\mathrm{d}\rho\right)^2}-\frac{1}{2n^2}\frac{\rho^2}{\alpha X^2}\right], \tag{4.3}$$

where $Q \sim n$ close to the origin and we have expanded γ in $1/\left[\alpha\left(\mathrm{d}X/\mathrm{d}\rho\right)^2\right] \ll 1$ and $\rho^2/\left(\alpha X^2\right) \ll 1$, this last condition being the only one compatible with the condition on the derivative of X. Working to first order in these two parameters, the profile is determined by

$$\frac{\mathrm{d}}{\mathrm{d}\rho}\left\{X\left[\frac{1}{n^2}\frac{\rho^2}{\alpha X^2}-\frac{1}{\alpha\left(\mathrm{d}X/\mathrm{d}\rho\right)^2}\right]\right\}=\frac{\mathrm{d}X}{\mathrm{d}\rho}\left[\frac{1}{\alpha\left(\mathrm{d}X/\mathrm{d}\rho\right)^2}-\frac{1}{n^2}\frac{\rho^2}{\alpha X^2}\right]. \tag{4.4}$$

Notice that to zeroth order in the two small parameters, the equation is tautological. In the limit $\rho \to 0$ with an ansatz $X \sim \rho^\delta$ the equation of motion is satisfied for $\delta^2 = n^2$. The only solution satisfying $X(0) = 0$ is obtained for $\delta = n$ which has finite derivative at the origin. This contradicts our premises and, as a result, we conclude that singular DBI strings do not exist.

Having shown that the strings are not singular, we will now show that the functional form of the solutions is similar to the ones in the Abelian-Higgs case. Let us assume that, in the limit $\rho \to 0$, the DBI solutions are of the form

$$X(\rho) = A_{\rm DBI}\rho^p\,, \quad Q(\rho) = n - B_{\rm DBI}\rho^q\,, \tag{4.5}$$

where p and q are two constants which we will determine below, while $A_{\rm DBI}$ and $B_{\rm DBI}$ are two constants to be obtained by numerical integration; and we have taken into account the boundary conditions at $\rho = 0$: $X(0) = 0$ and $Q(0) = n$. By direct substitution of the asymptotic form (4.5) into the equations of motion eqs. (4.1), (4.2) and taking the limit $\rho \to 0$, one can check that the correct asymptotic form for X and Q reads,

$$X(\rho) = A_{\rm DBI}\rho^n\,, \quad Q(\rho) = n - B_{\rm DBI}\rho^2\,. \tag{4.6}$$

Thus $p = n$ and $q = 2$ and, as guessed above, the only difference between DBI cosmic strings and Abelian-Higgs cosmic strings close to the origin is in the numerical values of the prefactors $A_{\rm DBI}$ and $B_{\rm DBI}$ which are α-dependent. In particular, these coefficients become large for large α implying that away from the origin but for reasonable and finite values the gamma factor becomes noticeably different from one, i.e. the cosmic strings are in a mild DBI regime.

From eqs. (3.21) and (4.6) it immediately follows that γ is always finite at $\rho = 0$. This a salient point as it confirms that the cosmic strings constructed with a DBI action are non-singular at the origin. This is of course yet another argument supporting the fact that eq. (3.22) represents the natural DBI generalisation for cosmic strings.

More precisely we find that as $\rho \to 0$ there are two possible regimes: the standard regime where $|1-\gamma| \ll 0$ and a mild DBI regime where $\gamma < 1$ but finite. Let us first analyse the global string, for which $Q = 0$. It is clear from eqs. (3.21) and (4.6) that for $n \geq 2$ the mild DBI regime cannot be realised around $\rho \to 0$. Note, however, that for

$\alpha \gg 1$ we find numerically that $A_{\rm DBI} \gg 1$, which implies that away from the origin the gradient $\mathrm{d}X/\mathrm{d}\rho$ becomes large, so that the solution is in the mild DBI regime. For $n = 1$ the mild DBI regime is valid starting from $\rho = 0$, if $\alpha \gg 1$. For $\alpha \ll 1$ the regime is always of non-DBI type, independently of n.

In the case of gauge strings, the situation is similar in the limit $\rho \to 0$. Again for $\alpha \ll 1$ the non-DBI regime is realised. For $\alpha \gg 1$ we find numerically that $A_{\rm DBI}$ and $B_{\rm DBI}$ in eq. (4.6) are large. Thus the gradient $\mathrm{d}Q/\mathrm{d}\rho$ is large too, while the terms proportional to $\mathrm{d}X/\mathrm{d}\rho$ and to Q^2X^2 are large only for $n = 1$, and they are small in a small region around $\rho = 0$ for $n \geq 2$. However, these terms become large away from the origin, since a large constant $A_{\rm DBI}$ implies that $\mathrm{d}X/\mathrm{d}\rho$ becomes large at some point. In conclusion, we find that for α large enough, the cosmic strings are in a mild DBI regime for finite values of ρ. This is confirmed numerically.

Finally let us notice that at infinity, independently of α, both the gradients $\mathrm{d}X/\mathrm{d}\rho$ and $\mathrm{d}Q/\mathrm{d}\rho$ are small, and the cosmic string matches the standard behaviour. This is in agreement with the general findings for topological defects with a non-canonical kinetic term.

In summary, the difference between the Abelian-Higgs and DBI strings will be small very far from the core of the string, while the DBI string can differ from the Abelian-Higgs one inside the core of the string: the larger α the larger the difference inside the core.

4.2 Numerical solutions

As mentioned above, the equations of motion (4.1) and (4.2) cannot be solved analytically. For this reason, we now turn to a full numerical integration.

As is well-known, the numerical integration is non-trivial because the boundary conditions are not fixed at the same point. The solutions discussed is this article have been obtained by means of two independent methods: a relaxation method [44–46] and a shooting method. More precisely, the former is in fact the over relaxation method. The over relaxation method differs from the relaxation method (also known as the Newton iteration method) by the fact that the Newtonian iteration step is multiplied by a factor of ω. In the standard case, convergence for the over relaxation method is guaranteed provided the over relaxation parameter $\omega < 2$ and, therefore, a good choice is for instance $\omega \sim 1.99$. Here, the highly non-linear nature of the equation of motions may render the over relaxation method unstable. To deal with this problem, we have considered a "step-dependent" over relaxation parameter ω interpolating from $\omega \sim 1$, close to the origin, and to $\omega \to 1.3$ at "infinity". As already mentioned the choice $\omega = 1.3 \ll 1.99$ is due to the highly non linear behaviour of the equations. We have observed very severe instabilities for higher values of ω. On the other hand, the shooting method can be directly implemented in its standard formulation in the case of global strings, since there is only one integration constant to be obtained, $A_{\rm DBI}$. While in the gauge case the presence of two "shooting" constants, $A_{\rm DBI}$ and $B_{\rm DBI}$, makes the direct implementation of the standard scheme impossible, we have thus modified the shooting method appropriately. All in all, the two different numerical procedures, a relaxation and a shooting method, give the same numerical solutions, up to small numerical errors.

Our numerical results depend on the DBI parameter α which, as already mentioned, is given by $\alpha = 2\lambda\eta^4/T$ where η is the vev of the charged scalar field at infinity and T is

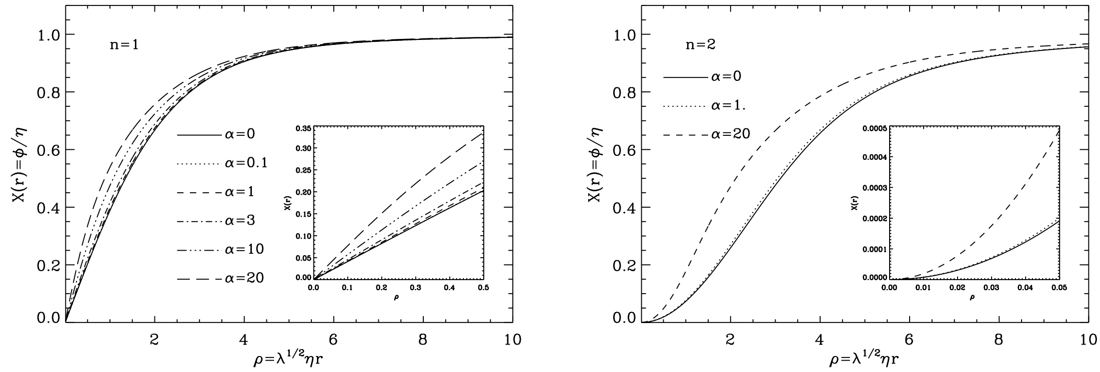

Figure 2. Left panel: the profiles of global DBI strings for $n = 1$ and for various values of the parameter α defined in eq. (3.23). Right panel: same as left panel but with $n = 2$

analogous to a brane tension. In our particular setting, these two parameters are free and unrelated. Therefore α is not constrained and can take values which are not theoretically determined. On the other hand, DBI strings deviate from Abelian-Higgs strings when α increases from zero. Hence it is natural to compare DBI strings and Abelian-Higgs strings in the small α limit and then extend to larger values of α. When α is significantly different from zero, we have observed noticeable deviations from Abelian-Higgs strings which will be spelt out in the rest of this section. Numerically we have gone up to $\alpha \sim 1.5$ which is a non-perturbative regime where the square root in the DBI cannot be approximated by the first few terms in a gradient expansion. Hence we have probed the full DBI regime. Larger values of α are more difficult to handle numerically: it is either very time consuming or (and) instabilities can appear due to the highly non linear nature of the equations. So reaching values of order $\alpha \gtrsim \mathcal{O}(5-10)$ is already highly non trivial, numerically speaking. A thorough study of the large α limit in conjunction with varying β will be the subject of a future publication.

Numerical integration of the equations of motion (4.1) and (4.2) are presented and discussed below.

Firstly, in figures 2, we consider global DBI strings (that is to say without the gauge field) for, respectively, winding numbers $n = 1$ (left panel) and $n = 2$ (right panel) and different α's. This figure confirms the qualitative statements made in the previous subsection. We notice that, even for "non-perturbative" values of α, i.e. $\alpha > 1$, the difference between the standard and the DBI profiles remains quite small. Moreover, as announced, the maximum difference lies at a (dimensionless) radius ρ of order one, namely half way from the origin and the region where $X \to 1$. Another remark is that the DBI profiles are always above the standard profiles. This is of course expected since the DBI regime means larger derivatives which, in the present context, implies the above mentioned property. Finally, one can check that the asymptotic behaviours discussed in the previous subsection are clearly observed in figures 2. Indeed, for $n = 1$, we notice that $X(\rho) \sim A_{\rm DBI}\rho$ where $A_{\rm DBI}$ is clearly a function of α (see in particular the zoom in the left panel). The same remark applies for $n = 2$, where $X(\rho) \sim \rho^2$.

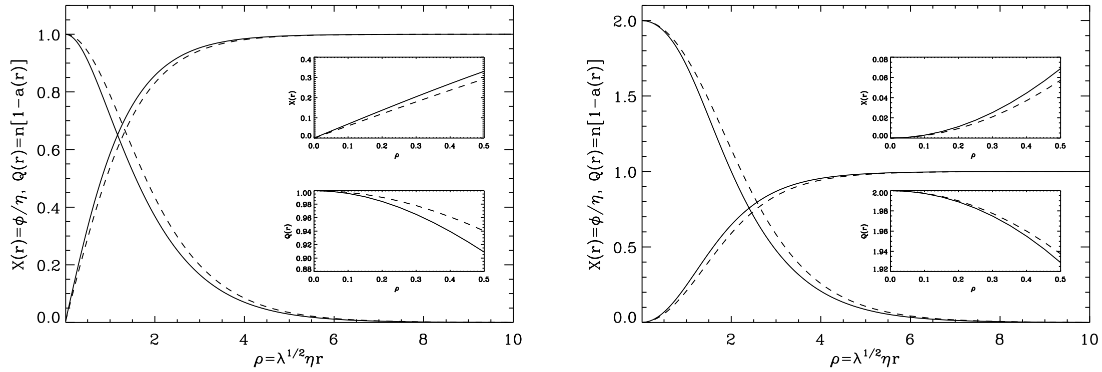

Figure 3. Left panel: the solid lines represent the profiles of the scalar and gauge fields of a DBI strings with $n = 1$ and $\alpha = 1$ while the dashed lines are the profiles of the scalar and gauge fields of a standard Abelian-Higgs string (i.e. $\alpha = 0$). Right panel: same as left panel but with $n = 2$. Notice that, on the y-axis, we have used the notation $a \equiv \hat{q}\hat{A}_\theta/n$.

	$n=1$		$n=2$		$n=3$	
α	$A_{\rm DBI}$	$B_{\rm DBI}$	$A_{\rm DBI}$	$B_{\rm DBI}$	$A_{\rm DBI}$	$B_{\rm DBI}$
0.1	0.611	0.523	0.481	0.252	0.449	0.168
1	0.694	0.752	0.570	0.275	0.557	0.183
3	1.020	2.836	0.949	0.349	1.055	0.234
5	1.984	19.66	2.253	0.491	3.052	0.333

Table 1. Shooting parameters $A_{\rm DBI}$ and $B_{\rm DBI}$ for different n and α, and $\beta = 1$.

Secondly, in figures 3, we display the profiles for DBI local strings, i.e. for the scalar field and the gauge field, in the case where $\alpha = 1$, $n = 1$ (left panel) and $n = 2$ (right panel). The same remarks as before apply. In particular, the scalar field profile always lies above its Abelian-Higgs counter part and, on the contrary, the DBI gauge field profile always lies inside the standard profile. As already discussed, this is because, in the DBI regime, the gradients are, by definition, larger than in the standard case. This means that a DBI string has a core smaller than an Abelian-Higgs string. As before, one can also check that the asymptotic behaviours are those discussed in the previous subsection. This is true in particular for the gauge field for which we always see that $Q \sim n - \rho^2$ at the origin.

Thirdly, additional information on the profiles can be gained from figures 4. In the left panel, we have compared the local and global profiles. One can notice that the global profile is less concentrated than the local one. Another remark is that the difference between the Abelian-Higgs and DBI profiles is more important in the local case than in the global one. In the right panel, we have compared the slopes at the origin. In the standard case, one has $X(\rho) \sim A_{\rm standard}\rho^n$ and it has been argued before that in the DBI case, one also has $X(\rho) \sim A_{\rm DBI}\rho^n$. We have represented the ratio $A_{\rm DBI}/A_{\rm standard}$ for various values of α and n. In table 1 we give different values for the shooting parameters $A_{\rm DBI}$ and $B_{\rm DBI}$. One

JHEP03(2009)091

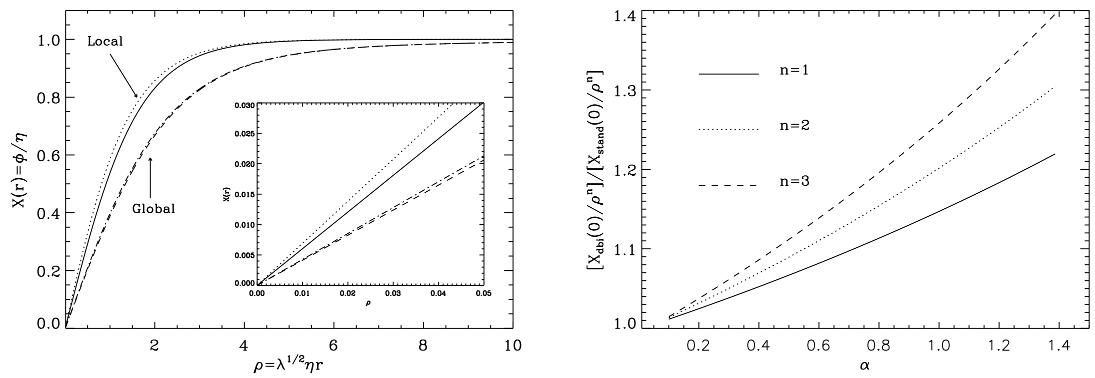

Figure 4. Left panel: comparison of the cosmic string profiles for global and local DBI strings with $n = 1$ and $\alpha = 1$. The solid line represents the scalar field profile for a local Abelian-Higgs string while the dotted line is the corresponding DBI profile. On the other hand, the dashed line represents the scalar field profile for an Abelian-Higgs global string whereas the dotted-dashed line is the corresponding DBI profile still in the global case. Right panel: ratio $A_{\rm DBI}/A_{\rm standard}$ (see the text) as a function of the parameter α for different values of the winding number n.

notices that the larger α, the steeper the DBI slope, which seems natural since the value of the parameter α controls how important the DBI effects are. We also remark that the same trend is observed when one increases n rather than α. In conclusion, from these two figures, one confirms that the deeper one penetrates into the DBI regime, the narrower the core of a cosmic string is. The effect is larger in the local case than in the global one and for large winding numbers than for small ones.

Fourthly, given the numerical solutions presented above it is straightforward to calculate their tension which, from the action given by eq. (3.22), takes the form

$$\mu_n\left(X,Q\right) = 2\pi\eta^2 f_n(\alpha)\,, \tag{4.7}$$

where $f_n(0) = n$ in the Abelian-Higgs case. In figure 5 (left panel), we plot the universal functions $f_n(\alpha)$ for the DBI local strings. We notice that the DBI action is, for any n and/or α, smaller than the corresponding standard action. Moreover, at a fixed value of α, the (absolute) difference between the DBI and Abelian-Higgs actions increases with the winding number. The fact that the DBI action is smaller than the standard one is likely to have important physical consequences, in particular with regards to the formation of DBI cosmic strings. Indeed, if their energy is smaller than in the standard case, one can legitimately expect their formation to be favoured as compared to the Abelian-Higgs case.

In the right panel in figure 5, we have represented the DBI string binding energy $\mu_{2n} - 2\mu_n$ as a function of the parameter α for different values of the winding number n. We observe that this quantity is always positive but small in comparison to one. Moreover, as expected since one has $\left(\mu_{2n} - 2\mu_n\right)(\alpha = 0) = 0$, it increases with α. We conclude that when $\alpha \neq 0$, the DBI cosmic string is no longer a BPS object. The fact that $\mu_{2n} > 2\mu_n$ means that, when they meet, two DBI strings will not constitute a new single string with

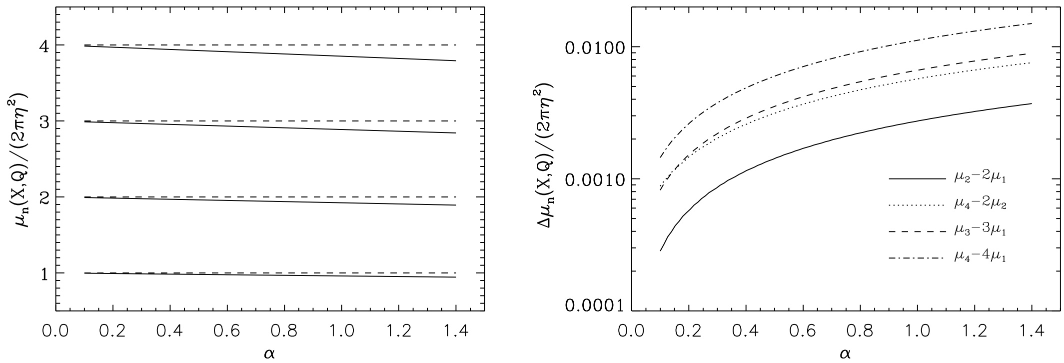

Figure 5. Left panel: the solid lines represent the DBI string tension as a function of α for different values of the winding number n (from $n = 1$ to $n = 4$ going from the bottom to the top of the plot). The dashed lines corresponds to the Abelian-Higgs tension, namely $\mu_n = 2\pi\eta^2 n$, and are thus horizontal lines located at the y-coordinate n. Right panel: the DBI string binding energy $\mu_{2n} - 2\mu_n$ for various n as a function of the parameter α.

winding number $2n$ since this appears to be disfavoured from the energy point of view. This has important consequences for cosmology since the above discussion implies that the behaviour of a network of DBI cosmic strings will be similar to the behaviour of a network of Abelian-Higgs strings. This means that the cosmological constraints derived, for instance in refs. [24], also apply to the present case.

In figure 6, we represent the energy density as a function of the dimensionless radial coordinate ρ for $n = 1$ (left panel) and $n = 2$ (right panel) for different values of the parameter α. We notice that the DBI energy densities are usually more peaked than the Abelian-Higgs ones. Moreover, the larger α, the more peaked the distributions. The case $n = 2$ is particularly interesting. One observes that, as α increases, the peaks of the distribution are displaced towards the left, i.e. towards smaller values of ρ. This is probably due to the fact that, as discussed at the beginning of this subsection, the difference between the DBI and Abelian-Higgs profiles is maximum for intermediate values of ρ.

So far, we have discussed the case $\beta = 1$. It is also interesting to investigate what happens if $\beta \neq 1$. In figure 7, we have represented the quantity $(\mu_2 - 2\mu_1) / \left(2\pi\eta^2\right)$ versus the parameter α for different values of β. For $\beta < 1$, we can see that the binding energy vanishes for a non zero value of α. It is explicitly seen for the case $\beta = 0.98$ (the critical value of α being $\alpha_{\rm cri} \sim 0.85$ in this case) but it seems clear that this is true for any value of $\beta < 1$. In figure 7, it cannot be seen for smaller values of $\beta < 1$ because the corresponding values of $\alpha_{\rm cri}$ is larger which makes these cases difficult to handle numerically. Hence, for $\beta < 1$ there are both type I and type II strings depending on the value of α. For $\beta > 1$, on the contrary, there are only type II strings. This can be noticed in figure 7 where the corresponding curves never vanish, see for instance the case $\beta = 1.1$. We conclude that the DBI cosmic strings possess an additional remarkable feature, namely their type can change according to the value of the parameter α, provided $\beta < 1$. This behaviour differs from

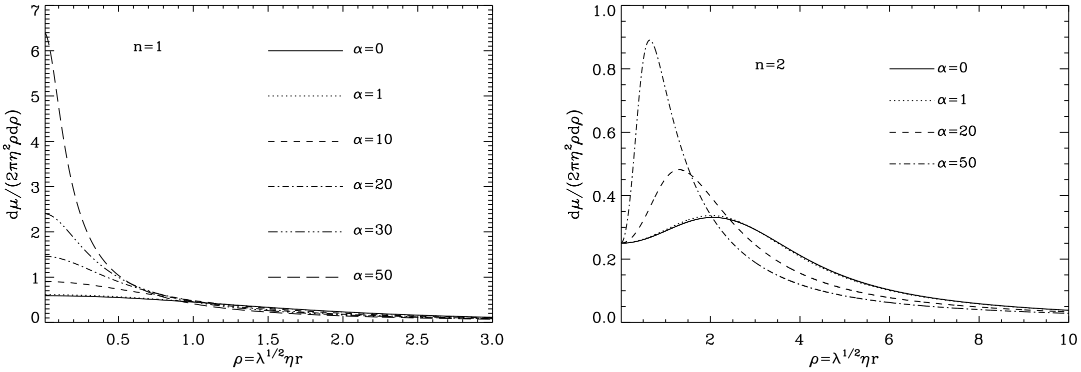

Figure 6. Left panel: the energy density for DBI strings as a function of ρ for $n = 1$ for different values of the parameter α. Right panel: same as right panel but for $n = 2$.

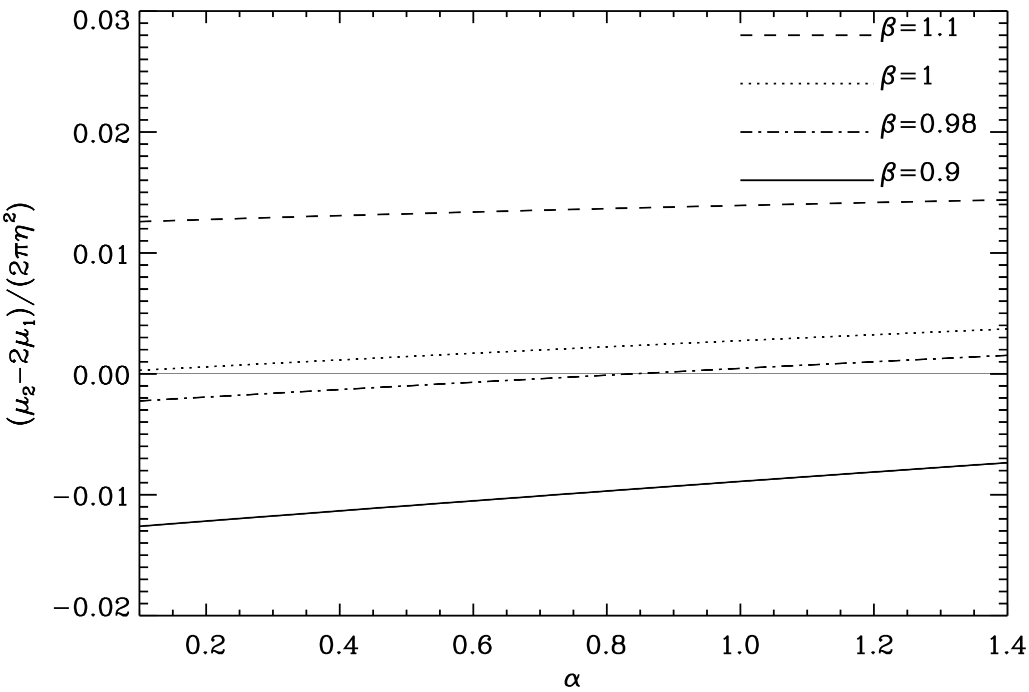

Figure 7. DBI string binding energy density for various β as a function of the parameter α

the standard case where no corresponding phenomenon is observed.

5 Conclusions

We have considered a natural DBI generalisation of the Abelian-Higgs model whereby the kinetic terms of the Higgs fields do not lead to a linear differential operator in the equations of motion. The particular form of the action is motivated by a specific extra-dimensional model where the Higgs field becomes a complex direction normal to a D3-brane. Although this model leads to nice cosmic string properties, it is not directly related to a string theory model. As such, the closest model of string theory which could have led to such a DBI action is the D3/D7 system where BPS cosmic strings are formed at the end of an hybrid-like inflation phase. Unfortunately, the charged fields associated to the open string joining the D3- and D7-branes have no obvious geometric meaning and therefore do not lead to our DBI action. It would certainly be very interesting to see if our construction can be embedded within string theory.

As a four-dimensional model of non-canonical type, the DBI model of cosmic strings does not suffer from any pathology such as divergences or non-single-valuedness of the field profiles (typical of other non-linear actions which have been proposed in the literature). Indeed we find that DBI strings can be continuously deformed to their Abelian-Higgs analogue. In fact, the main difference from the Abelian-Higgs case appears in the BPS case where the DBI strings show a small departure from the BPS property. In particular, we find that the string tension is reduced, a property which may have some phenomenological significance in order to relax the bound on the string tension coming from Cosmic Microwave Background (CMB) data. Moreover we find that the BPS DBI strings have a positive binding energy implying that the string coalescence is energetically disfavoured, leading to the likely formation of networks with singly-wound strings and statistical properties akin to the usual Abelian-Higgs ones. In the non BPS case, and in the subcritical case $\beta < 1$, we obtain that the sign of the binding energy can change depending on the DBI parameter α, implying that the DBI strings can change from type I to type II.

In the present article, we have not tackled some important aspects of DBI string dynamics such as string scattering (for which we expect that the higher order terms discussed in the appendix may play an important rôle), gravitational back reaction, and the coupling to fermions and their zero modes. This is currently under investigation.

In summary, we have introduced DBI cosmic strings as non-singular solutions derived from a non-linear Lagrangian. We have studied the solutions numerically and found that they differ significantly from their Abelian-Higgs analogues. The network properties of these strings depend on the DBI parameter α and the ratio of the Higgs mass over the gauge boson mass. When the latter is smaller than one, the string type can either be type I or II depending on the DBI parameter α, certainly resulting in different network features. It would be very interesting to analyse networks of DBI strings. This is left for future work.

Acknowledgments

We would like to thank Patrick Peter and Christophe Ringeval for useful discussions, particularly on the numerical aspect of this work. We also thank David Langlois and Sébastien Renaux-Petel, as well as Renata Kallosh for enlightening discussions on the stringy aspect of this work. The work of E. B. was supported by the EU FP6 Marie Curie Research and Training Network "UniverseNet" (MRTN-CT-2006-035863).

A BPS Abelian-Higgs strings in the D3-D7 system

In string theory models, cosmic strings can form after the end of inflation [8–11]. In the D3/D7 system [21, 22] in particular, the two branes attract during the inflationary period and then eventually coalesce forming D-strings. The whole picture (inflation and string formation) can be described in terms of the field theoretical D-term hybrid inflation [25]. In this language the D-strings have been conjectured to be analogous to D-term strings, and furthermore — as we now outline — the strings are BPS Abelian-Higgs strings.

In the D3/D7 system there are three complex fields [21, 22]: the inflaton ϕ and the waterfall fields $\phi^{\pm}$. In string theory, ϕ is the interbrane distance and $\phi^{\pm}$ are in correspondence with the open strings between the branes. In the supersymmetric language, the Kähler potential is

$$K = -\frac{1}{2}\left(\phi - \phi^{\dagger}\right)^2 + |\phi^+|^2 + |\phi^-|^2 \,, \tag{A.1}$$

leading to canonically normalised fields. Notice that the inflation direction is invariant under the real shift symmetry $\phi \to \phi + c$, thus guaranteeing the flatness of the inflaton direction at the classical level [47]. The superpotential is

$$W = \lambda \phi \phi^+ \phi^- \,. \tag{A.2}$$

During inflation, the U(1) symmetry under which the waterfall fields have charges ± 1 is not broken, i.e $\phi^{\pm} = 0$. The scalar potential is flat and picks up a slope at the one loop level. This is enough to drive inflation. As ϕ decreases, it goes through a threshold after which the waterfall field ϕ^+ condenses and the inflaton vanishes. This corresponds to the coalescence of the D3- and D7-branes. The effective potential describing the condensation is given by the D-term potential (the F-terms all vanish when $\phi = 0,\ \phi^- = 0$)

$$V_D = \frac{g^2}{2}\left(|\phi^+|^2 - \xi\right)^2 . \tag{A.3}$$

The term ξ is called a Fayet-Iliopoulos (FI) term [48]. As ϕ^+ condenses and $\langle \phi^+ \rangle = \sqrt{\xi}$, cosmic strings form interpolating between a vanishing field in the core and $\sqrt{\xi}$ at infinity. These cosmic strings are BPS objects preserving one half of the original supersymmetries. Their tension is known to be $\mu_n = 2\pi n \xi$ [25]. As a consequence, there is no binding energy as $\mu_{2n} = 2\mu_n$.

In fact [25], the D-term string model of D-string formation is nothing but an Abelian-Higgs model with particular couplings

$$\mathcal{L} = D_\mu \phi^+ \left(D^\mu \phi^+\right)^{\dagger} + \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + V_D \,, \tag{A.4}$$

where $D_\mu \phi^+ = (\partial_\mu - i A_\mu)\phi^+$. Upon rescaling $A_\mu \to A_\mu / g$ and comparing with (2.1), this leads to the identification

$$q = g \,, \quad \lambda = 2g^2 \,, \quad \xi = \eta^2 \,, \tag{A.5}$$

corresponding to $\beta = 1$ and, hence, a BPS system. This explains why one recovers $\mu_{2n} = 2\mu_n$.

Moreover, the energy scale $\sqrt{\xi}$ can be given a stringy interpretation. Indeed, one can show that the Fayet-Iliopoulos term is related to internal fluxes on D7-branes [49]. For this purpose, let us consider a ten-dimensional metric in the form

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu + g_{pq}\mathrm{d}X^p \mathrm{d}X^q + g_{ij}\mathrm{d}X^i \mathrm{d}X^j \,, \tag{A.6}$$

with $\mu = 0,\ldots,3$, $p = 4,\ldots,7$ and $i = 8,9$. The quantity $g_{\mu\nu}$ is the four-dimensional metric and g_{pq} and g_{ij} the compactification six-dimensional metric. This corresponds to the metric on $K^3 \times T^2$ compactifications for instance. The internal dimensions of the D7-brane are the coordinates $a \equiv (\mu, p)$ while the D3-brane lies along the μ coordinates. We denote by T_7 the brane tension and g_s the string coupling. The four-dimensional gauge coupling is given by

$$\frac{1}{g^2} = \frac{T_7 V_4 \ell_\mathrm{s}^4}{g_\mathrm{s}} \,, \tag{A.7}$$

where the string length is $\ell_\mathrm{s} = 1/\sqrt{2\pi\alpha'}$ and $V_4 = \int \mathrm{d}^4x \sqrt{\det g_{pq}}$ is the volume of the four compact internal dimensions of the D7-brane. Consider now a dimensionless magnetic flux F_{pq} along the internal dimensions $(p,q) = 4,\ldots,7$, of a D7-brane. Then

$$\xi^2 = \frac{T_7}{2 g_\mathrm{s} g^2} \int \mathrm{d}^4x \sqrt{-g_{pq}} F_{pq} F^{pq} \,. \tag{A.8}$$

Notice that the absolute value of the FI term is not fixed, it can be decomposed as $\xi^2 = \zeta/\ell_s^2$ where the prefactor depends on F_{pq}.

In this paper, we have considered cosmic string models for which the canonical kinetic terms have been replaced by a non-linear term of the DBI type. In effective actions describing string theory phenomena, and particularly brane dynamics, such a replacement is mandatory as soon as the gradient terms in the effective action become large. Indeed, the DBI action usually describes the dynamics of the open strings in correspondence with the brane motion (such as the 3-3 and 7-7 open strings in the D3/D7 system). As we have recalled, the formation of cosmic strings in the D3/D7 system is governed by the 3-7 strings of no obvious geometric significance. In such a situation, and assuming that there could be higher order terms correcting the lowest order Lagrangian, the effect of the higher order corrections to the kinetic terms (terms in $|D\phi^+|^{2p}$, $p > 1$) would be to induce modifications of the cosmic string profile and of the tension.

B A Non-linear action for cosmic strings

Consider now a brane model in which cosmic strings appear as deformations of a brane. (In a sense the brane becomes curved with a puncture at the location of the string, as we discuss.) To do so, consider a ten-dimensional setting as is natural for brane models derived from or inspired by string theory. We choose a non-warped compactification and write the ten-dimensional metric in cylindrical form

$$\mathrm{d}s_{10}^2 \equiv g_{AB}^{10}\mathrm{d}X^A \mathrm{d}X^B = \mathrm{d}s_4^2 + 2 g_{\alpha\bar\beta}\mathrm{d}Z^\alpha \mathrm{d}\bar Z^{\bar\beta} \,, \tag{B.1}$$

JHEP03(2009)091

where

$$ds_4^2 \equiv g_{\mu\nu} dX^\mu dX^\nu = -(dX^0)^2 + dR^2 + R^2 d\Theta^2 + dZ^2 \,. \tag{B.2}$$

The metric along the internal dimensions $g_{\alpha\bar{\beta}}$ ($\alpha = 5, 6, 7$) is kept arbitrary, i.e. Hermitian and positive definite, and we have assumed that the six-dimensional manifold is complex (it could be a Calabi-Yau manifold) therefore having complex coordinates. The complex coordinates are crucial to analyse cosmic strings.

Consider the DBI action for a three-brane embedded along the first four coordinates

$$S = -T \int d^4x \sqrt{-\det\left(\tilde{g}_{\mu\nu} + \ell_s^2 \mathcal{F}_{\mu\nu}\right)} - \int d^4x \sqrt{-g}\, V\left(\sqrt{T} Z^\alpha\right) , \tag{B.3}$$

where T is the brane tension, $\mathcal{F}_{\mu\nu}$ is the field strength on the brane (and has dimension two), distances have dimension minus one and $\mathcal{A}_\mu$ has dimension one. We have included a potential for the deformations Z^α of the normal directions to the three-branes. As suitable when the normal directions are charged under the world-volume gauge group [in this case the local U(1) on the brane], we include a covariant derivative in the definition of the induced metric

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + g_{\alpha\bar{\beta}} \left(\mathcal{D}_\mu Z^\alpha \mathcal{D}_\nu \bar{Z}^{\bar{\beta}} + \mathcal{D}_\mu \bar{Z}^{\bar{\beta}} \mathcal{D}_\nu Z^\alpha\right) \tag{B.4}$$

with

$$\mathcal{D}_\mu = \partial_\mu - i\hat{q}\mathcal{A}_\mu \,. \tag{B.5}$$

Clearly, when the gauge fields vanish, $\tilde{g}_{\mu\nu}$ is simply the induced metric on the brane. A similar extension of the induced metric to charged fields has already been introduced in the context of N-coinciding D-branes [50] with the corresponding non-Abelian SU(N) gauge theory. There the brane coordinates are in the adjoint representation and have kinetic terms involving the SU(N) covariant derivative [50]. We extend this procedure to the DBI cosmic string situation with a U(1) gauge group[3]

When the six-dimensional metric is nearly flat $g_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}}$ locally, the action becomes

$$S = -T \int d^4x \left\{ \sqrt{-\det\left[g_{\mu\nu} + \left(\mathcal{D}_\mu Z^\alpha \mathcal{D}_\nu \bar{Z}_{\bar{\alpha}} + \mathcal{D}_\mu \bar{Z}^{\bar{\alpha}} \mathcal{D}_\nu Z_\alpha\right) + \ell_s^2 \mathcal{F}_{\mu\nu}\right]} - \sqrt{-g} \right\} - \int d^4x \sqrt{-g}\, V\left(\sqrt{T} Z^\alpha\right) , \tag{B.6}$$

where, as usual, we have subtracted the action of the "flat" brane so that the Abelian-Higgs model is recovered when gradients are small. In the following we suppose that only one normal direction is excited and define $\Phi \equiv Z^1$. The resulting action is given by

$$S = -T \int d^4x \left\{ \sqrt{-\det\left[g_{\mu\nu} + (\mathcal{D}_{(\mu}\Phi)(\mathcal{D}_{\nu)}\Phi)^\dagger + \ell_s^2 \mathcal{F}_{\mu\nu}\right]} - \sqrt{-g} + \sqrt{-g}\frac{V(\sqrt{T}|\Phi|)}{T} \right\} . \tag{B.7}$$

[3] In the D-brane context, the brane fields do not carry any U(1) charge as they belong to the adjoint representation.

We now study in more detail the general form of the action (B.7) and/or (3.10) considered in this article. Eq. (B.7) can be rewritten as

$$S = -T \int \mathrm{d}^4x\sqrt{-g}\left[\left(\sqrt{\mathcal{D}}-1\right)+\frac{V(\sqrt{T}|\Phi|)}{T}\right], \tag{B.8}$$

where $\mathcal{D}$ is defined by

$$\mathcal{D} \equiv \det\left[\delta^\mu{}_\nu + (\mathcal{D}^\mu\Phi)(\mathcal{D}_\nu\Phi)^\dagger + (\mathcal{D}^\mu\Phi)^\dagger(\mathcal{D}_\nu\Phi) + \ell_\mathrm{s}^2\mathcal{F}^\mu{}_\nu\right], \tag{B.9}$$

T has dimensions of (energy)^4 and ℓ_s is a length scale. As before, $\mathcal{D}_\mu = \partial_\mu - i\hat{q}\mathcal{A}_\mu$.

Our goal is to compute and simplify eq. (B.9) for $\mathcal{D}$. As it is clear from its definition, this will allow us to derive a more compact formula for our action in the general case. In eq. (3.22) we have evaluated the action (B.8) for a cylindrically symmetric static string profile. In this case it takes a simple form. However, when there is time dependence and less symmetry — as occurs for example in string scattering — it is important to know the general form of the action.

First define the following quantities

$$N^\nu \equiv \mathcal{D}^\mu\Phi\,, \quad S^\mu{}_\nu \equiv N^\mu\bar{N}_\nu + \bar{N}^\mu N_\nu\,, \quad R^\mu{}_\nu \equiv S^\mu{}_\nu + \mathcal{F}^\mu{}_\nu\,, \tag{B.10}$$

where are bar denotes complex conjugation and we set $\ell_s = 1$ in this appendix. Note that by definition $S^{\mu\nu}$ is a symmetric matrix and $\mathcal{F}^{\mu\nu}$ is antisymmetric, while $S^\mu{}_\nu$ and $\mathcal{F}^\mu{}_\nu$ are in general neither symmetric nor antisymmetric. Denote by S the matrix with components $S^\mu{}_\nu$, while $\mathcal{F}$ is the matrix with components $\mathcal{F}^\mu{}_\nu$. For integer n and p

$$\mathrm{tr}\left(S^p\mathcal{F}^{2n+1}\right) = 0\,. \tag{B.11}$$

On the other hand, we also have

$$\begin{aligned}\mathcal{D} &= \det\left(\delta^\mu{}_\nu + R^\mu{}_\nu\right) &\text{(B.12)}\\ &= -\frac{1}{4!}\varepsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\varepsilon^{\beta_1\beta_2\beta_3\beta_4}\left(\delta^{\alpha_1}{}_{\beta_1}+R^{\alpha_1}{}_{\beta_1}\right)\left(\delta^{\alpha_2}{}_{\beta_2}+R^{\alpha_2}{}_{\beta_2}\right)\\ &\quad\times\left(\delta^{\alpha_3}{}_{\beta_3}+R^{\alpha_3}{}_{\beta_3}\right)\left(\delta^{\alpha_4}{}_{\beta_4}+R^{\alpha_4}{}_{\beta_4}\right) &\text{(B.13)}\end{aligned}$$

which, on using the identity

$$\varepsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}\varepsilon^{\alpha_1\cdots\alpha_j\beta_{j+1}\cdots\beta_4} = -\,(4-j)!j!\delta^{[\beta_{j+1}}_{\alpha_{j+1}}\cdots\delta^{\beta_4]}_{\alpha_4}\,, \tag{B.14}$$

gives

$$\mathcal{D} = 1 + R^\alpha{}_\alpha + R^{[\alpha}{}_\alpha R^{\beta]}{}_\beta + R^{[\alpha}{}_\alpha R^\beta{}_\beta R^{\gamma]}{}_\gamma + R^{[\alpha}{}_\alpha R^\beta{}_\beta R^\gamma{}_\gamma R^{\delta]}{}_\delta\,. \tag{B.15}$$

We now evaluate each term in the above equation. For the first (linear in R), it follows from eqs. (B.10) and (B.11) that

$$R^\alpha{}_\alpha = S^\alpha{}_\alpha = 2\bar{N}_\alpha N^\alpha = 2\left(\mathcal{D}^\mu\Phi\right)\left(\mathcal{D}_\mu\Phi\right)^\dagger\,. \tag{B.16}$$

The quadratic term is given by

$$R^{[\alpha}{}_{\alpha}R^{\beta]}{}_{\beta} = S^{[\alpha}{}_{\alpha}S^{\beta]}{}_{\beta} + 2S^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta]}{}_{\beta} + \mathcal{F}^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta]}{}_{\beta}\,, \tag{B.17}$$

$$= \frac{1}{2}\left[\mathrm{tr}^2(S) - \mathrm{tr}\left(S^2\right)\right] - \frac{1}{2}\mathrm{tr}\left(\mathcal{F}^2\right)\,, \tag{B.18}$$

$$= \left(\bar{N}_\alpha N^\alpha\right)^2 - \left(N_\alpha N^\alpha\right)\left(\bar{N}_\beta \bar{N}^\beta\right) - \frac{1}{2}\mathrm{tr}\left(\mathcal{F}^2\right)\,, \tag{B.19}$$

where to get from eq. (B.18) to eq. (B.19) we have used eq. (B.10). Notice that these terms are compatible with the U(1) invariance of the action. The next step is to calculate the cubic term. It is given by

$$R^{[\alpha}{}_{\alpha}R^{\beta}{}_{\beta}R^{\gamma]}{}_{\gamma} = S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}S^{\gamma]}{}_{\gamma} + 3S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}\mathcal{F}^{\gamma]}{}_{\gamma} + 3S^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma]}{}_{\gamma} + \mathcal{F}^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma]}{}_{\gamma}\,. \tag{B.20}$$

The term in S^3 vanishes for the single complex scalar field studied here since, on using eq. (B.10), it contains the contraction of an antisymmetric tensor with a symmetric one. Similarly $S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}\mathcal{F}^{\gamma]}{}_{\gamma} = 0 = \mathcal{F}^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma]}{}_{\gamma}$ on using eq. (B.11). Therefore, the cubic term takes the form

$$R^{[\alpha}{}_{\alpha}R^{\beta}{}_{\beta}R^{\gamma]}{}_{\gamma} = 3S^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma]}{}_{\gamma} = \frac{1}{2}\left[-\mathrm{tr}(S)\,\mathrm{tr}\left(\mathcal{F}^2\right) + 2\mathrm{tr}\left(S\mathcal{F}^2\right)\right]\,. \tag{B.21}$$

Finally, the quartic term can be expressed as

$$R^{[\alpha}{}_{\alpha}R^{\beta}{}_{\beta}R^{\gamma}{}_{\gamma}R^{\delta]}{}_{\delta} = S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}S^{\gamma}{}_{\gamma}S^{\delta]}{}_{\delta} + 4S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}S^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} + 4S^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} + 6S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} + \mathcal{F}^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} \tag{B.22}$$

$$= 6S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} + \mathcal{F}^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta}\,, \tag{B.23}$$

since the terms on the first line in the above equations vanish, on using the same arguments as above. Also

$$S^{[\alpha}{}_{\alpha}S^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} = \frac{1}{4!}\left\{4\mathrm{tr}(S)\,\mathrm{tr}\left(S\mathcal{F}^2\right) - 4\mathrm{tr}\left(\mathcal{F}^2S^2\right) - 2\mathrm{tr}\left(\mathcal{F}S\mathcal{F}S\right) + \left[\mathrm{tr}\left(S^2\right) - \mathrm{tr}^2(S)\right]\mathrm{tr}\left(\mathcal{F}^2\right)\right\}. \tag{B.24}$$

$$\mathcal{F}^{[\alpha}{}_{\alpha}\mathcal{F}^{\beta}{}_{\beta}\mathcal{F}^{\gamma}{}_{\gamma}\mathcal{F}^{\delta]}{}_{\delta} = \frac{1}{4!}\left[-6\mathrm{tr}\left(\mathcal{F}^4\right) + 3\mathrm{tr}^2\left(\mathcal{F}^2\right)\right] \tag{B.25}$$

Therefore, in the end, one obtains the following expression for $\mathcal{D}$

$$\mathcal{D} = 1 + \mathrm{tr}(S) - \frac{1}{2}\mathrm{tr}\left(\mathcal{F}^2\right) + \frac{1}{8}\left[\mathrm{tr}^2\left(\mathcal{F}^2\right) - 2\mathrm{tr}\left(\mathcal{F}^4\right)\right] + \frac{1}{2}\left[\mathrm{tr}^2(S) - \mathrm{tr}\left(S^2\right)\right] + \frac{1}{2}\left[2\mathrm{tr}\left(S\mathcal{F}^2\right) - \mathrm{tr}(S)\,\mathrm{tr}\left(\mathcal{F}^2\right)\right] + \frac{1}{4}\left[\mathrm{tr}\left(S^2\right) - \mathrm{tr}^2(S)\right]\mathrm{tr}\left(\mathcal{F}^2\right) + \mathrm{tr}(S)\,\mathrm{tr}\left(S\mathcal{F}^2\right) - \mathrm{tr}\left(\mathcal{F}^2S^2\right) \tag{B.26}$$

The three terms of the first line in eq. (B.26), when substituted in eq. (B.8) and on expanding the square-root, give the standard Abelian-Higgs model. The last two terms of eq. (B.26) are the standard terms of Born-Infeld electro-dynamics. Finally, as discussed in the main text, the factor $\mathcal{D}$ and, hence, our action defined by eq. (B.7), contains terms higher order in covariant derivatives as well as mixing terms between $\mathcal{F}^2$ and the covariant derivatives.

References

[1] WMAP collaboration, B. Gold et al., *Five-year Wilkinson Microwave Anisotropy Probe (WMAP[1]) observations: galactic foreground emission*, Astrophys. J. Suppl. **180** (2009) 265 [arXiv:0803.0715] [SPIRES].

[2] WMAP collaboration, R.S. Hill et al., *Five-year Wilkinson Microwave Anisotropy Probe (WMAP[1]) observations: beam maps and window functions*, Astrophys. J. Suppl. **180** (2009) 246 [arXiv:0803.0570] [SPIRES].

[3] WMAP collaboration, G. Hinshaw et al., *Five-year Wilkinson Microwave Anisotropy Probe (WMAP[1]) observations:data processing, sky maps, & basic results*, Astrophys. J. Suppl. **180** (2009) 225 [arXiv:0803.0732] [SPIRES].

[4] WMAP collaboration, M.R. Nolta et al., *Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: angular power spectra*, Astrophys. J. Suppl. **180** (2009) 296 [arXiv:0803.0593] [SPIRES].

[5] WMAP collaboration, J. Dunkley et al., *Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: likelihoods and parameters from the WMAP data*, Astrophys. J. Suppl. **180** (2009) 306 [arXiv:0803.0586] [SPIRES].

[6] WMAP collaboration, E. Komatsu et al., *Five-year Wilkinson Microwave Anisotropy Probe (WMAP[1]) observations:cosmological interpretation*, Astrophys. J. Suppl. **180** (2009) 330 [arXiv:0803.0547] [SPIRES].

[7] M. Lemoine, J. Martin and P. Peter eds., *Inflationary cosmology*, *Lect. Notes Phys.* **738**, Springer, Berlin Heidelberg Germany (2008).

[8] L. McAllister and E. Silverstein, *String cosmology: a review*, Gen. Rel. Grav. **40** (2008) 565 [arXiv:0710.2951] [SPIRES].

[9] C.P. Burgess, *Lectures on cosmic inflation and its potential stringy realizations*, Class. Quantum Grav. **24** (2007) S795 [*PoS* **P 2GC** (2006) 008] [arXiv:0708.2865] [SPIRES].

[10] R. Kallosh, *On inflation in string theory*, Lect. Notes Phys. **738** (2008) 119 [hep-th/0702059] [SPIRES].

[11] S.H. Henry Tye, *Brane inflation: string theory viewed from the cosmos*, *Lect. Notes Phys.* **737** (2008) 949 [hep-th/0610221] [SPIRES].

[12] E. Silverstein and D. Tong, *Scalar speed limits and cosmology: acceleration from D-cceleration*, Phys. Rev. **D 70** (2004) 103505 [hep-th/0310221] [SPIRES].

[13] M. Alishahiha, E. Silverstein and D. Tong, *DBI in the sky*, Phys. Rev. **D 70** (2004) 123505 [hep-th/0404084] [SPIRES].

[14] X. Chen, *Multi-throat brane inflation*, *Phys. Rev.* **D 71** (2005) 063506 [hep-th/0408084] [SPIRES].

[15] X. Chen, *Inflation from warped space*, *JHEP* **08** (2005) 045 [hep-th/0501184] [SPIRES].

[16] L. Lorenz, J. Martin and C. Ringeval, *Brane inflation and the WMAP data: a Bayesian analysis*, *JCAP* **04** (2008) 001 [arXiv:0709.3758] [SPIRES].

[17] L. Lorenz, J. Martin and C. Ringeval, *Constraints on kinetically modified inflation from WMAP5*, *Phys. Rev.* **D 78** (2008) 063543 [arXiv:0807.2414] [SPIRES].

[18] L. Lorenz, J. Martin and C. Ringeval, *K-inflationary power spectra in the uniform approximation*, *Phys. Rev.* **D 78** (2008) 083513 [arXiv:0807.3037] [SPIRES].

[19] D. Langlois, S. Renaux-Petel, D.A. Steer and T. Tanaka, *Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation*, *Phys. Rev.* **D 78** (2008) 063523 [arXiv:0806.0336] [SPIRES].

[20] D. Langlois, S. Renaux-Petel, D.A. Steer and T. Tanaka, *Primordial fluctuations and non-Gaussianities in multi-field DBI inflation*, *Phys. Rev. Lett.* **101** (2008) 061301 [arXiv:0804.3139] [SPIRES].

[21] K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, *D3/D7 inflationary model and M-theory*, *Phys. Rev.* **D 65** (2002) 126002 [hep-th/0203019] [SPIRES].

[22] K. Dasgupta, J.P. Hsu, R. Kallosh, A. Linde and M. Zagermann, *D3/D7 brane inflation and semilocal strings*, *JHEP* **08** (2004) 030 [hep-th/0405247] [SPIRES].

[23] E.J. Copeland, R.C. Myers and J. Polchinski, *Cosmic F- and D-strings*, *JHEP* **06** (2004) 013 [hep-th/0312067] [SPIRES].

[24] L. Pogosian, S.H.H. Tye, I. Wasserman and M. Wyman, *Observational constraints on cosmic string production during brane inflation*, *Phys. Rev.* **D 68** (2003) 023506 [hep-th/0304188] [SPIRES].

[25] G. Dvali, R. Kallosh and A. Van Proeyen, *D-term strings*, *JHEP* **01** (2004) 035 [hep-th/0312005] [SPIRES].

[26] P. Brax, C. van de Bruck, A.C. Davis and S.C. Davis, *Cosmic D-strings and vortons in supergravity*, *Phys. Lett.* **B 640** (2006) 7 [hep-th/0606036] [SPIRES].

[27] E. Moreno, C. Núñez and F.A. Schaposnik, *Electrically charged vortex solution in Born-Infeld theory*, *Phys. Rev.* **D 58** (1998) 025015 [hep-th/9802175] [SPIRES].

[28] Y.S. Yang, *Classical solutions in the Born-Infeld theory*, *Proc. Roy. Soc. Lond.* **A 456** (2000) 615 [SPIRES].

[29] S. Sarangi, *DBI global strings*, *JHEP* **07** (2008) 018 [arXiv:0710.0421] [SPIRES].

[30] Y. Brihaye and B. Mercier, *Classical solutions in the Einstein-Born-Infeld-Abelian-Higgs model*, *Phys. Rev.* **D 64** (2001) 044001 [hep-th/0102002] [SPIRES].

[31] E. Babichev, *Global topological k-defects*, *Phys. Rev.* **D 74** (2006) 085004 [hep-th/0608071] [SPIRES].

[32] E. Babichev, *Gauge k-vortices*, *Phys. Rev.* **D 77** (2008) 065021 [arXiv:0711.0376] [SPIRES].

[33] M.B. Hindmarsh and T.W.B. Kibble, *Cosmic strings*, *Rept. Prog. Phys.* **58** (1995) 477 [hep-ph/9411342] [SPIRES].

JHEP03(2009)091

[34] A. Vilenkin and E.P.S. Shellard, *Cosmic stringas and other topological defects*, Cambridge University Press, Cambridge U.K. (1994).

[35] V. Rubakov, *Classical theory of gauge fields*, Cambridge University Press, Cambridge U.K. (2002).

[36] N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, *Fitting CMB data with cosmic strings and inflation*, *Phys. Rev. Lett.* **100** (2008) 021301 [astro-ph/0702223] [SPIRES].

[37] L. Jacobs and C. Rebbi, *Interaction energy of superconducting vortices*, *Phys. Rev.* **B 19** (1979) 4486 [SPIRES].

[38] L.M.A. Bettencourt and T.W.B. Kibble, *Nonintercommuting configurations in the collisions of type-I* U(1) *cosmic strings*, *Phys. Lett.* **B 332** (1994) 297 [hep-ph/9405221] [SPIRES].

[39] E.J. Copeland, T.W.B. Kibble and D.A. Steer, *Collisions of strings with Y junctions*, *Phys. Rev. Lett.* **97** (2006) 021602 [hep-th/0601153] [SPIRES].

[40] P. Salmi et al., *Kinematic constraints on formation of bound states of cosmic strings - field theoretical approach*, *Phys. Rev.* **D 77** (2008) 041701 [arXiv:0712.1204] [SPIRES].

[41] A. Sen, *Dirac-Born-Infeld action on the tachyon kink and vortex*, *Phys. Rev.* **D 68** (2003) 066008 [hep-th/0303057] [SPIRES].

[42] Y. Kim, B. Kyae and J. Lee, *Global and local D-vortices*, *JHEP* **10** (2005) 002 [hep-th/0508027] [SPIRES].

[43] M. Born and L. Infeld, *Foundations of the new field theory*, *Proc. Roy. Soc. Lond.* **A 144** (1934) 425 [SPIRES].

[44] S.L. Adler and T. Piran, *Relaxation methods for gauge field equilibrium equations*, *Rev. Mod. Phys.* **56** (1984) 1 [SPIRES].

[45] P. Peter, *Superconducting cosmic string: equation of state for space - like and time - like current in the neutral limit*, *Phys. Rev.* **D 45** (1992) 1091 [SPIRES].

[46] C. Ringeval, *Fermionic currents flowing along extended objects*, Ph.D. Thesis, University Paris 6, France (2002).

[47] J.P. Hsu, R. Kallosh and S. Prokushkin, *On brane inflation with volume stabilization*, *JCAP* **12** (2003) 009 [hep-th/0311077] [SPIRES].

[48] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology*, *Class. Quant. Grav.* **21** (2004) 3137 [hep-th/0402046] [SPIRES].

[49] C.P. Burgess, R. Kallosh and F. Quevedo, *de Sitter string vacua from supersymmetric D-terms*, *JHEP* **10** (2003) 056 [hep-th/0309187] [SPIRES].

[50] M.R. Garousi and R.C. Myers, *World-volume interactions on D-branes*, *Nucl. Phys.* **B 542** (1999) 73 [hep-th/9809100] [SPIRES].