

Determination of the CKM-Angle γ with Tree-Level Processes at LHCb

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Abstract

Recent results on studies sensitive to the CKM-angle γ are combined to give an overall estimate of the precision expected at LHCb, assuming the detector performance of the DC04 data challenge. A result of $1.9\text{--}2.7^\circ$ is obtained for 10 fb^{-1} of data, with the variation arising from the dependence on the physics parameters involved. Brief discussion is given to other, as yet unexplored, methods for improving this precision still further.

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1 Introduction

Studies performed with the so-called ‘Data Challenge 04’ (DC04) simulation production have yielded estimates of the sensitivity of LHCb to the CKM-angle γ in a variety of channels. In this note we combine these results to estimate the overall precision on γ expected at LHCb, taking care to apply a uniform set of assumptions and the best available knowledge that exists of relevant external parameters. It is assumed that experimental systematic uncertainties will be significantly smaller than the statistical errors – discussion on the use of control channels to achieve this goal can be found in the referenced studies.

We consider only measurements where the dependence on γ enters through processes which are dominated by tree-level graphs, with no pollution from loop diagrams. The value of γ extracted in this manner is expected to have negligible influence from new physics contributions and to provide a benchmark against which other measurements in the unitarity triangle can be compared. This ‘Standard Model’ γ may also be compared against the result of other determinations available from LHCb, using modes where new physics may well enter, for instance $B \rightarrow hh$ decays [1].

Section 2 discusses the precision on γ available from methods involving $B \rightarrow DK$ decays. Section 3 summarises the expectations from time-dependent measurements. In both cases brief consideration is also given to other, as yet unexplored, strategies which may add to the knowledge of γ . The overall sensitivity is presented in Section 4.

2 Measurements of γ with $B \rightarrow DK$ Strategies

The most powerful strategy to measure γ in tree-level processes is through the family of $B \rightarrow DK$ decays. Figure 1 shows the contributing diagrams for the charged B case. These diagrams will interfere if the D meson is reconstructed in a mode accessible to both D^0 and \bar{D}^0 ¹. In this case the decay rate, or the kinematical distribution of the D decay products in an n -body decay ($n \geq 3$), will have a dependence on the following parameters:

- The CKM-angle γ ;
- A strong phase difference δ_B between the two diagrams;
- The relative magnitudes of the two-diagrams, r_B ;
- Parameters related to the particular D decay under study.

Comparison of the rates or kinematical distributions between the B^+ and B^- decays allows these parameters to be determined (provided that a sufficient number of observables are included in the analysis). This strategy has two clear advantages. Firstly, all analyses, irrespective of the D decay, will have at least three parameters in common,

¹In this note D signifies either a D^0 or a \bar{D}^0 . Furthermore, unless the context suggests otherwise, the charge conjugated process is also implied in addition to the stated decay.

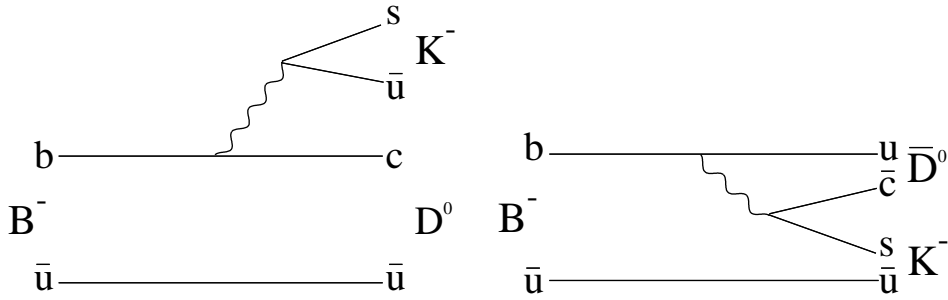


Figure 1: The diagrams for $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$. There is a relative phase of $\delta_B - \gamma$ between the two amplitudes, and a relative magnitude of r_B .

namely γ , δ_B and r_B . This property can be exploited through making a global fit to all D decay modes under consideration, as will be demonstrated in the present study. Secondly, as no flavour-tagging is required, all decays can be used in the analysis, thus enabling LHCb to exploit fully its statistical power.

The $B \rightarrow DK$ strategy may also be pursued with B^0 decays, with the strange meson here being the K^{*0} , which is reconstructed in the self-tagging final state $K^+\pi^-$. The principal decay parameters are now γ , δ_{B^0} and r_{B^0} . In this case both B decay diagrams are colour suppressed, as opposed to the charged B decay where this is true only for the $b \rightarrow u$ graph. This feature suppresses the overall decay rate by an order of magnitude, but enhances the interference effects ($r_{B^0} > r_B$).

All results discussed here neglect the effect of $D^0 - \bar{D}^0$ mixing, which induces a negligible bias in the value of γ [2], and can, if necessary, be accommodated in the analysis. Furthermore, CP violation is assumed to be negligible in D meson decays.

2.1 Inputs

Several studies have been conducted with DC04 simulated data which have resulted in predicted signal and background yields together with γ sensitivity estimates. These studies are as follows:

1. Selection of $B^- \rightarrow D^0(hh)K^-$ events ($D^0 \rightarrow K^\pm\pi^\mp, K^+K^-, \pi^+\pi^-$) [3] and γ extraction with this sample alone [4] through a combined ADS/GLW [5, 6] technique;
2. Selection of $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ events [7] and γ extraction in conjunction with the $B^- \rightarrow D^0(hh)K^-$ sample through a combined ADS/GLW analysis [3];
3. Selection of $B^0 \rightarrow D^0(hh)K^{*0}$ events ($D^0 \rightarrow K^\pm\pi^\mp, K^+K^-, \pi^+\pi^-$ and $K^{*0} \rightarrow K^+\pi^-$) and γ extraction [8] through the B^0 analogue [9] of the ADS/GLW method;
4. Selection of $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ events [10] and γ extraction using both the GGSZ approach [11, 12] of an unbinned log-likelihood fit to the $K_S^0\pi^+\pi^-$ Dalitz space [10], and a model independent binned fit [13] as proposed in [12] and developed further in [14];

5. Selection of $B^- \rightarrow D^0(K^+K^-\pi^+\pi^-)K^-$ events [7] and γ extraction [15] using a four-body amplitude analysis as proposed in [16].

In the present combination we do not include 5. This is because it is not convenient to integrate the amplitude fit into the combination procedure explained in Section 2.2, and the systematic uncertainties associated with the knowledge of the $D \rightarrow K^+K^-\pi^+\pi^-$ decay have not yet been assessed. The expected γ precision from $B^- \rightarrow D^0(K^+K^-\pi^+\pi^-)K^-$ alone is expected to be 18° for 2 fb^{-1} of data which makes it less sensitive than the other modes.

In the following we summarise the anticipated sensitivity of each channel, paying particular attention to those measurements external to LHCb which will be of use in the γ determination.

2.1.1 $B^- \rightarrow D^0(hh)K^-$

Reference [4] describes how γ can be extracted from $B^- \rightarrow D^0(hh)K^-$ events through a least-squared fit to the event rates of the modes under consideration. In addition to γ the following parameters are determined: r_B , δ_B , $\delta_D^{K\pi}$ and an overall normalisation factor. Here $\delta_D^{K\pi}$ is the CP-conserving strong phase difference between doubly Cabibbo suppressed (DCS) and Cabibbo favoured (CF) decays of neutral D mesons to the $K\pi$ final state. It is assumed that $r_D^{K\pi}$, the ratio of the magnitudes of the DCS to the CF D decay amplitudes, and the relative values of the different D branching ratios are known sufficiently well that they introduce negligible uncertainty into the fit results. Additional power comes from using the information on the value of $\delta_D^{K\pi}$ available from quantum-correlated D production at CLEO-c. The study reported in [4] assumes that $\cos \delta_D^{K\pi}$ is known to ± 0.20 . With 2 fb^{-1} of data it is found that a precision of $8 - 10^\circ$ is obtainable on γ , assuming $r_B = 0.077$, $r_D^{K\pi} = 0.06$ and considering different values of $\delta_D^{K\pi}$ in the range -25° to 25° .

We have developed a standalone simulation and fit program similar to that described in [3, 4]. Under the same inputs it produces consistent results to those reported in [4]. For the studies shown here, however, we have modified some of the assumptions:

- We have set $r_B = 0.10$ to be identical to the value used in the other $B \rightarrow DK$ studies, and to be consistent with the present world average of this quantity according to [17], which is $r_B = 0.10 \pm 0.02$;
- We have set $r_D = 0.0616$ following the values of the relevant branching ratios reported in [18];
- We have considered a range of values for $\delta_D^{K\pi}$ centred around -180° , rather than 0° as in [4]. CLEO-c reports the result $\delta_D^{K\pi} = (22_{-16}^{+14})^\circ$ [19]. However, as explained in Appendix A, the CP convention used in the charm measurements is different from that of the $B \rightarrow DK$ studies. When using the CLEO-c result in the ADS/GLW analysis a phase shift of -180° is required. In the fit we constrain $\delta_D^{K\pi}$ to lie within $(_{-16}^{+14})^\circ$ of the input value of -158° .

$\delta_D^{K\pi}$ ($^\circ$)	-190	-174	-158	-144	-130
σ_γ ($^\circ$)	12.7	10.8	13.8	12.6	10.8

Table 1: The expected sensitivity to γ from $B^- \rightarrow D(hh)K^-$ decays with data corresponding to an integrated luminosity 2 fb^{-1} . The sensitivities for different values of $\delta_D^{K\pi}$ are given.

The results for the precision on γ are given in Table 1, for an integrated luminosity of 2 fb^{-1} . The mean value of the uncertainty is somewhat higher than for the results presented in [4]. This is mainly due to the different range of $\delta_D^{K\pi}$ considered which leads to a reduced asymmetry between the B^+ and B^- modes.

2.1.2 $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$

The results reported in [3] come from an analysis which takes a set of inputs that include the relative rates of the four $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ channels. This analysis, however, is idealised, as it assumes that the DCS and CF D decays each proceed through a single amplitude. In reality, intermediate resonances contribute which mean that the formalism used in [3] is not valid.

The additional resonances in the decay lead to a richer interference structure which, in principle, could be exploited in an amplitude fit. If however, the analysis integrates over phase space, then the consequence is to dilute the interference effects, as in general the contributing amplitudes will not be in phase with each other. The consequences of this scenario have been explored in [20]. The expressions for the four decay rates take the following form:

$$\Gamma(B^- \rightarrow (K^- \pi^+ \pi^+ \pi^-)_D K^-) \propto 1 + r_B r_D^{K3\pi} + 2r_B r_D^{K3\pi} R_{K3\pi} \cos(\delta_B - \delta_D^{K3\pi} - \gamma), \quad (1)$$

$$\Gamma(B^- \rightarrow (K^+ \pi^- \pi^+ \pi^-)_D K^-) \propto r_B^2 + (r_D^{K3\pi})^2 + 2r_B r_D^{K3\pi} R_{K3\pi} \cos(\delta_B + \delta_D^{K3\pi} - \gamma), \quad (2)$$

$$\Gamma(B^+ \rightarrow (K^+ \pi^- \pi^+ \pi^-)_D K^+) \propto 1 + r_B r_D^{K3\pi} + 2r_B r_D^{K3\pi} R_{K3\pi} \cos(\delta_B - \delta_D^{K3\pi} + \gamma), \quad (3)$$

$$\Gamma(B^+ \rightarrow (K^- \pi^+ \pi^+ \pi^-)_D K^+) \propto r_B^2 + (r_D^{K3\pi})^2 + 2r_B r_D^{K3\pi} R_{K3\pi} \cos(\delta_B + \delta_D^{K3\pi} + \gamma). \quad (4)$$

These differ from the expressions assumed in [3] through the inclusion of the *coherence factor*, $R_{K3\pi}$, which appears in front of the interference term and has a value between 0 and 1. The strong phase difference $\delta_D^{K3\pi}$ now is an effective phase averaged over all amplitudes.

Knowledge concerning $R_{K3\pi}$ and $\delta_D^{K3\pi}$ comes from the analysis of coherent D production at CLEO-c. A preliminary study [21] reports the following three results:

$$R_{K3\pi} \cos \delta_D^{K3\pi} = -0.60 \pm 0.19 \pm 0.24, \quad (5)$$

$$(R_{K3\pi})^2 = -0.20 \pm 0.23 \pm 0.09, \quad (6)$$

$$R_{K3\pi} \cos(\delta_D^{K\pi} - \delta_D^{K3\pi}) = 0.00 \pm 0.16 \pm 0.07, \quad (7)$$

which leads to the allowed region in $R_{K3\pi} - \delta_D^{K3\pi}$ space shown in Figure 2 (which in exploiting result of Ref. 7 also uses the CLEO-c determination of $\delta_D^{K\pi}$). The most probable point in this space is $R_{K3\pi} = 0.2$ and $\delta_D^{K3\pi} = 144^\circ$. The indication for this decay, therefore, is that the interference effects integrated over all phase space are small. Such a property would mean that the four rates 1–4 have little sensitivity to γ , but instead assume importance in a global analysis because of the information they bring on the parameter r_B .

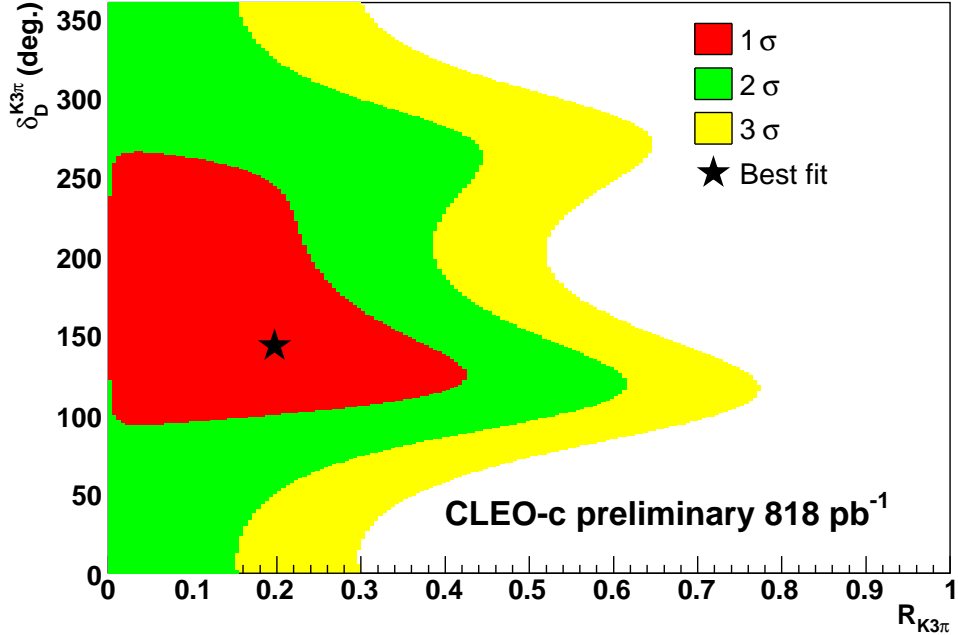


Figure 2: The 1σ , 2σ and 3σ confidence contours for $R_{K3\pi}$ and $\delta_D^{K3\pi}$ obtained from CLEO-c data.

In the study presented here $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ events are generated in a standalone simulation according to the expected yields and background levels reported in [7]. The parameter $r_D^{K3\pi}$ is set to 0.0568 [18]. In the global fit discussed in Section 2.2 the two free parameters specific to this decay are $R_{K3\pi}$ and $\delta_D^{K3\pi}$. The measurements given in expressions 5–7 are applied as external Gaussian constraints.

The sensitivities to γ from $B^- \rightarrow D(hh)K^-$ with and without $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ data and with and without CLEO-c measurements are shown in Figure 3; the yields assumed are those expected from 2 fb^{-1} . The impact of CLEO-c measurements on the sensitivity of LHCb to γ is significant; the CLEO-c constraints are equivalent to a doubling of the LHCb data set.

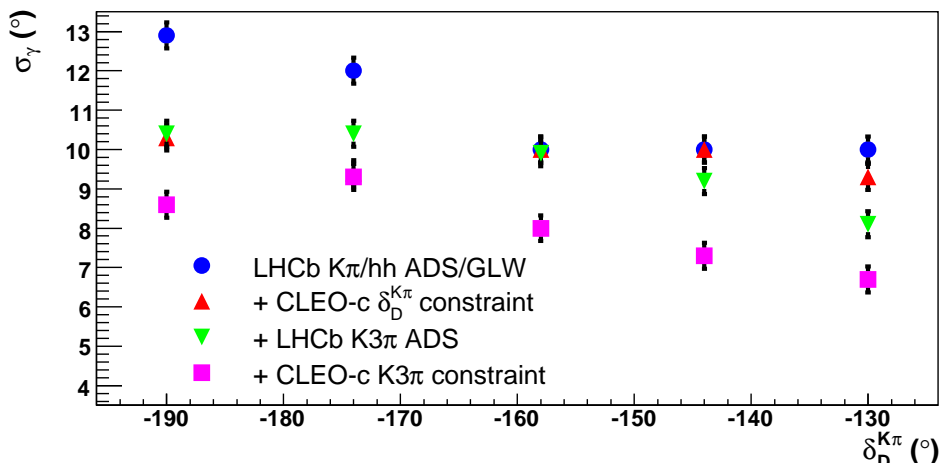


Figure 3: The sensitivity to γ from $B^- \rightarrow D(hh)K^-$ and $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ for four different scenarios as a function of $\delta_D^{K\pi}$ and assuming 2 fb^{-1} of data. The four scenarios are: (blue circles) $B^- \rightarrow D(hh)K^-$ alone without CLEO-c constraint on $\delta_D^{K\pi}$; (red triangles) $B^- \rightarrow D(hh)K^-$ alone with CLEO-c constraint on $\delta_D^{K\pi}$; (green inverted triangles) $B^- \rightarrow D(hh)K^-$ and $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ with CLEO-c constraint on $\delta_D^{K\pi}$ but without CLEO-c constraints on $R_{K3\pi}$ and $\delta_D^{K3\pi}$, and; (magenta squares) $B^- \rightarrow D(hh)K^-$ and $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ with CLEO-c constraints on $\delta_D^{K\pi}$, $R_{K3\pi}$ and $\delta_D^{K3\pi}$.

2.1.3 $B^0 \rightarrow D^0(hh)K^{*0}$

A standalone program has been written to generate $B^0 \rightarrow D^0(hh)K^{*0}$ events ($D^0 \rightarrow K^\pm\pi^\mp, K^+K^-, \pi^+\pi^-$ and $K^{*0} \rightarrow K^\pm\pi^\mp$) and fit the parameters of interest, which in this case are γ , r_{B^0} and δ_{B^0} . Here r_{B^0} and δ_{B^0} are the B^0 analogues of r_B and δ_B . It is expected that $r_{B^0} > r_B$, since in the B^0 system both interfering decay diagrams experience the same colour suppression. In this analysis, following [8], we set $r_{B^0} = 0.4$, and reproduce the results of the earlier study.

There exists a non-experimental systematic uncertainty associated with this mode which arises from the possibility that other amplitudes may pollute the $D^0K\pi$ final state in addition to the $D^0K^{*0}(892)$ signal mode. Additional amplitude pollution would introduce a coherence factor in an identical manner to that discussed above for $D \rightarrow K^\pm\pi^\mp\pi^+\pi^-$. This scenario has been considered in [22] where a value for this factor of 0.95 ± 0.03 is estimated. For all studies presented here we therefore reduce the effective value of r_{B^0} by a factor 0.95. The systematic uncertainty on γ from this source is found to be very small (0.7°) and is neglected at present.

As is the case for $B^- \rightarrow D^0(hh)K^-$ (see Section 2.1.1), the analysis presented in [8] considers an inappropriate range of $\delta_D^{K\pi}$. In Table 2, therefore, we show updated results for a range matched to the formalism that is assumed (see Appendix A), and using a

δ_{B^0} ($^\circ$)	0	45	90	135	180
σ_γ ($^\circ$)	6.2	10.8*	12.7*	9.5	5.2

Table 2: The expected sensitivity to γ from $B^0 \rightarrow D(hh)K^{*0}$ decays with data corresponding to an integrated luminosity 2 fb^{-1} . The sensitivities for different values of δ_{B^0} are given. The * indicates values of δ_{B^0} where the distribution of γ fit results was non-Gaussian; the R.M.S. of the distributions are quoted for these cases.

diluted value of r_{B^0} .

2.1.4 $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$

Within LHCb the sensitivity to γ of $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ events has been evaluated both with an unbinned model-dependent amplitude fit [10] and with a binned model-independent method [13]. In the present study we adopt the latter approach as it has no significant theoretical uncertainty and is straightforward to include in the global analysis. In [13] various scenarios are considered for the background composition. We take the most conservative of these possibilities, which is where the background is dominated by correctly reconstructed D mesons together with combinatorial kaons. The statistical error from the fit in this case is 12.8° for 2 fb^{-1} of data.

The model-independent approach requires external input coming from measurements made with quantum-correlated charm decays at the $\psi(3770)$. Analyses made with data from CLEO-c and BES-III will determine the cosine and sine of the strong phase difference in the Dalitz space bins of interest. The statistical uncertainty on these quantities propagates through to the γ measurement. It has been estimated [14] that using the CLEO-c dataset alone the associated error on γ in the $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ fit will be 5° , an assumption which is adopted in the present study and propagated to all results which are presented.

2.2 Global Fit: Procedure and Results

A standalone program has been used to simulate and fit samples of $B^- \rightarrow D^0(hh)K^-$, $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$, $B^0 \rightarrow D^0(hh)K^{*0}$ and $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ events. The fit parameters are summarised in Table 3. The external inputs assumed in the fit come from CLEO-c and are as follows: the measurement of $\delta_D^{K\pi}$; the measurements sensitive to $R_{K3\pi}$ and $\delta_D^{K3\pi}$; and the measurements of the cosine and sine of the strong phase differences in $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays. The distributions of the fitted results are shown in Figure 4 for an example position in parameter space. In general the results exhibit a near Gaussian behaviour, although the nature of the external CLEO-c constraints means this is not true for $R_{K3\pi}$ and $\delta_D^{K3\pi}$. The resulting sensitivities are summarised in Table 4. These sensitivities are shown as a function of δ_{B^0} , as this parameter is completely unconstrained by present measurements. A value of $\delta_{B^0} \sim 45^\circ$ leads to worse γ precision than when $\delta_{B^0} \sim 180^\circ$ on account of the reduced level of

Parameter	Value / Range	Constraint
γ	60°	/
δ_B	130°	/
r_B	0.10	/
δ_{B^0}	$0^\circ \rightarrow 180^\circ$	/
r_{B^0}	0.40	/
$\delta_D^{K\pi}$	-158°	$({}^{+22}_{-16})^\circ$
$R_{K3\pi}$	0.20	Expressions
$\delta_D^{K3\pi}$	144°	5,6 and 7

Table 3: Summary of fitted parameters, giving value (or range considered) and external constraint applied. In addition to those parameters listed there are also two normalisation factors: one for the B^- ADS/GLW analysis, and one for the B^0 ADS/GLW analysis.

CP asymmetry in the $B^0 \rightarrow D^0(hh)K^{*0}$ analysis at this point in parameter space. If the value of r_B turns out to be larger (smaller) than that assumed in the study, the resulting precision on γ will be correspondingly better (worse), with a $\sigma_\gamma \propto 1/r_B$ dependence expected.

The improvement in sensitivity to γ obtained through dealing with the common parameters by the global fit approach is illustrated by considering the fit with and without the $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ data. For 2 fb^{-1} of data and $\delta_{B^0} = 90^\circ$ the statistical sensitivity to γ improves from 6.6° to 5.5° when $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ is included (in this example we do not include the residual error associated with the finite CLEO-c statistics). This is equivalent to adding an uncorrelated measurement with statistical uncertainty of 10.1° , which can be compared to the uncertainty of 12.8° for $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$ alone [13].

2.3 Other Channels and Future Improvements

The ultimate sensitivity of LHCb to γ (and the other parameters listed in Table 3) through $B \rightarrow DK$ may be better than reported above. Other modes can be included in the analysis, and external knowledge of D^0 decay properties may improve beyond the assumptions used in this study. Some possible additional modes and possible improvements are listed below.

- It has been noted [23] that $B^- \rightarrow D^*K^-$ with $D^* \rightarrow D^0\gamma$ or $D^0\pi^0$ has particular power for the γ determination, providing that the two D^* decay modes are distinguished in the analysis. A preliminary LHCb study [24] shows promising results, but is limited in its conclusions on account of inadequate Monte Carlo background statistics.
- The channel $D^0 \rightarrow K^\pm\pi^\mp\pi^0$ is a promising ADS mode because of its high branching ratio and an expected high level of coherence. The coherence factor and mean

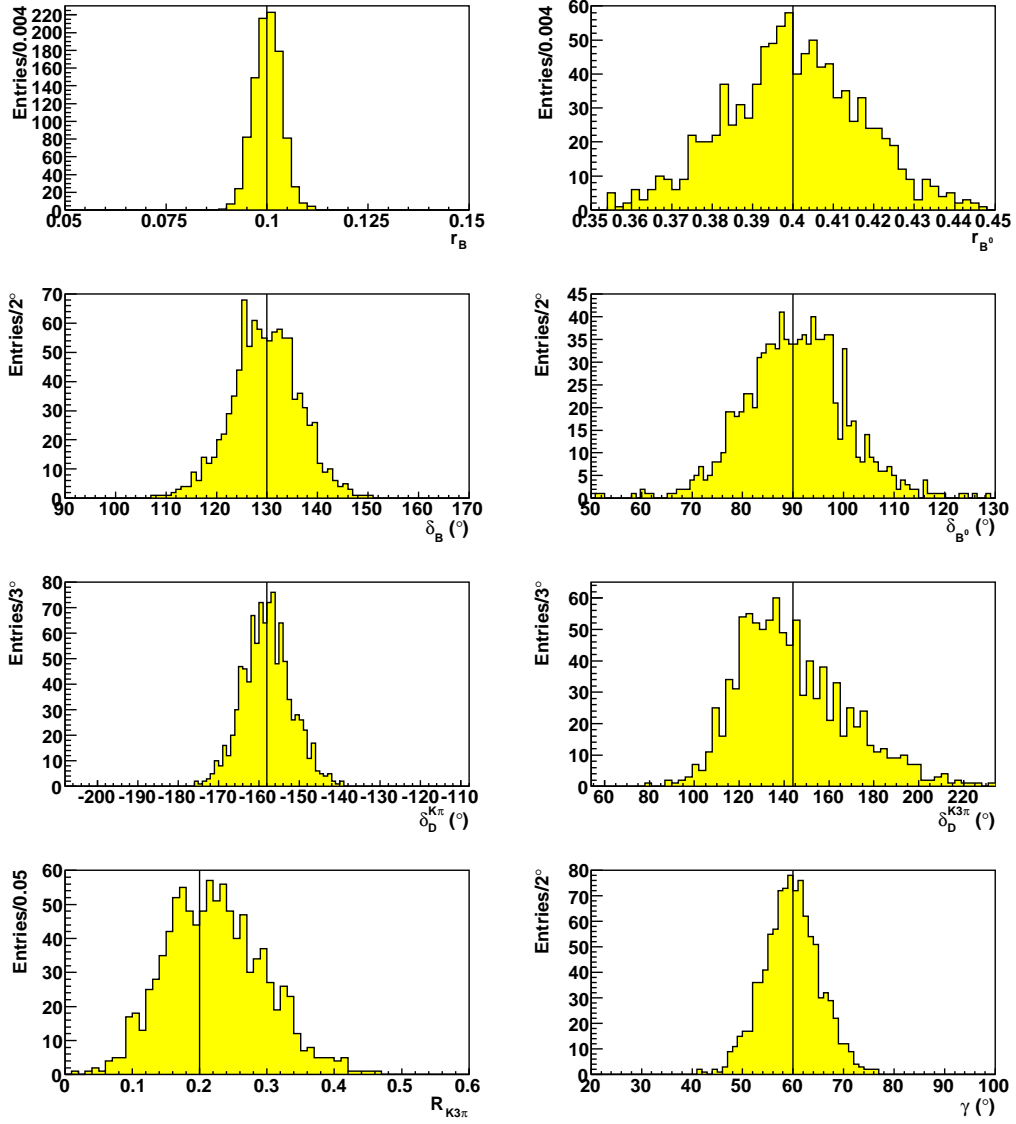


Figure 4: An example of the global fit results for all fit parameters. All measurements and CLEO-c constraints are included and $\delta_{B^0} = 90^\circ$. The yields assumed are for an integrated luminosity of 2 fb^{-1} . The vertical line indicates the input value of the parameter.

$\delta_{B^0} (\text{°})$	0	45	90	135	180
0.5 fb ⁻¹					
σ_γ without CLEO-c constraints (°)	11.5	12.9	13.1	12.5	9.7
σ_γ with CLEO-c constraints (°)	9.0	12.0	10.7	11.1	8.6
2 fb ⁻¹					
σ_γ without CLEO-c constraints (°)	5.8	8.3	7.8	8.4	5.0
σ_γ with CLEO-c constraints (°)	4.6	6.1	5.7	6.0	4.3
10 fb ⁻¹					
σ_γ without CLEO-c constraints (°)	2.6	5.4	3.5	4.8	2.4
σ_γ with CLEO-c constraints (°)	2.3	3.5	2.9	3.2	2.2

Table 4: The expected sensitivity to γ from $B \rightarrow DK$ strategies for data sets corresponding to integrated luminosities of 0.5, 2 and 10 fb⁻¹. The sensitivity “without CLEO-c constraints” corresponds to ADS/GLW measurements of γ from $B^- \rightarrow D(hh)K^-$, $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$ and $B^0 \rightarrow D(hh)K^{*0}$ without constraints on $\delta_D^{K\pi}$, $R_{K3\pi}$ and $\delta_D^{K3\pi}$. The sensitivity “with CLEO-c constraints” corresponds to ADS/GLW measurements of γ from $B^- \rightarrow D(hh)K^-$, $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$, $B^0 \rightarrow D(hh)K^{*0}$ and $B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$, with constraints on $\delta_D^{K\pi}$, $R_{K3\pi}$ and $\delta_D^{K3\pi}$. In the latter the assumed CLEO-c measurements of the strong phase differences in $D^0 \rightarrow K_S^0\pi^+\pi^-$ are used as input, and a 5° uncertainty propagated to the resulting value of γ .

strong phase difference can be determined in $\psi(3770)$ decays.

- The channel $D^0 \rightarrow K_S^0 K^+ K^-$ may contribute to the γ measurement through a Dalitz analysis. Although the branching ratio is $\sim 20\%$ that of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ the background level may be lower, and the systematics associated with knowledge of the D^0 decay will be uncorrelated.
- Interference effects in the channel $B^- \rightarrow D^0(K^+ K^- \pi^+ \pi^-)_D K^-$ can be probed in a four-body amplitude analysis, as explored in [15].
- The ADS/GLW analysis of $B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)_D K^-$ events can be extended to consider separate bins in D decay phase space, each of which will have higher levels of coherence than for the inclusive decay. Ultimately, the maximum sensitivity for this channel may come through a full amplitude analysis.
- External constraints and inputs on D decay properties will improve with the final analyses available from CLEO-c, and with the results from the larger datasets expected at BES-III.
- The family of decays $B^- \rightarrow D^0 K^{*-}$, $K^{*-} \rightarrow K_S^0 \pi^-$ can be included in the analysis.
- Other D^0 decays can be added to the $B^0 \rightarrow D^0(hh)K^{*0}$ analysis ².

²It may naively be expected that using $D^0 \rightarrow K^\pm\pi^\mp\pi^+\pi^-$ decays in an ADS/GLW analysis would

- Decays exploiting B_s^0 mesons, particularly the untagged strategy involving $B_s^0 \rightarrow D^0 \phi$ [25], warrant attention.

3 Time-dependent Measurements of γ

Two methods have been investigated with the DC04 data which give sensitivity to γ through time-dependent CP asymmetries. These use the channels $B_s \rightarrow D_s^\mp K^\pm$ and $B^0 \rightarrow D^\mp \pi^\pm$. As all relevant information on these analyses may be found in the referenced LHCb studies, and because the conventional analysis strategies involve no other parameters or systematics in common apart from γ , the discussion is more brief than was possible in Section 2.

3.1 $B_s \rightarrow D_s^\mp K^\pm$

Measuring the time-dependent CP asymmetries in $B_s \rightarrow D_s^\mp K^\pm$, and extracting the additional information available in the untagged decay rates accessible through the non-zero value of $\Delta\Gamma_s$ ($|\Delta\Gamma_s/\Gamma_s| = 0.121_{-0.090}^{+0.083}$ [18]), allows $\gamma - 2\beta_s$ to be determined [26]. From this measurement γ can be extracted with essentially identical precision, since β_s will be very well constrained through studies made in $B_s \rightarrow J/\psi \phi$ decays [27]. This measurement is impossible to perform at the B-factories, and may well turn out to be unique to LHCb.

The physics reach of LHCb in $B_s \rightarrow D_s^\mp K^\pm$ is detailed in [28, 29]. We assume a statistical sensitivity to γ of 10.3° in 2 fb^{-1} , with no significant source of systematic uncertainty. This result is based on the analysis of both flavour-tagged and untagged events and assumes $\Delta\Gamma_s/\Gamma_s = 0.10$, $\Delta m_s = 17.5 \text{ ps}^{-1}$ and a relative magnitude of 0.37 and strong phase difference of zero between the two interfering tree diagrams.

3.2 $B^0 \rightarrow D^\mp \pi^\pm$

Measurement of the time dependent CP asymmetries in $B^0 \rightarrow D^\mp \pi^\pm$ allows $\gamma + \phi_d$ to be determined [30]. As ϕ_d will be very well measured in $B^0 \rightarrow J/\psi K_S^0$ decays [31], γ can be extracted in isolation.

Although the formalism of this study is identical to that of the $B_s \rightarrow D_s^\mp K^\pm$ analysis there are two important practical differences:

- The magnitude of the ratio between the interfering tree diagrams, $r_{D\pi}$ is very small ($\sim 2\%$) and cannot be fitted from the data. Instead $r_{D\pi}$ must be estimated from external measurements involving other channels and SU(3) symmetry arguments [32]. The uncertainty on this estimate is presently at the $\sim 20\%$ level [33], but is expected to improve as the precision on the input measurements improve.

be beneficial in constraining the value of r_{B^0} , in an analogous manner to that found to be so for r_B in B^- decays. Initial studies, however, suggest this mode is much less useful here, presumably due to the much larger value of r_{B^0} , which therefore can be well determined from the $D^0 \rightarrow hh$ decays alone.

- There is an 8-fold ambiguity in the extracted value of γ . This problem can in principle be ameliorated by either comparing the results of related channels, such as $B^0 \rightarrow D^{*\pm}\pi^\mp$, or by performing a U-spin analysis in conjunction with $B_s \rightarrow D_s^\mp K^\pm$ [34]. The U-spin approach does not require any external knowledge of $r_{D\pi}$. The uncertainty associated with the U-spin symmetry assumption needs to be better estimated, but will also depend on which region of parameter space the values of the phases lie.

The expected event yield in $B^0 \rightarrow D^\mp\pi^\pm$ at LHCb has been reported in [35], and the sensitivity to γ in [36], using both the ‘conventional’ and U-spin based approaches. The γ sensitivity in both strategies has a significant dependence on the strong phases assumed. In the present study we assume precision of 10° for 10 fb^{-1} , scaling by ‘ $1/\sqrt{N}$ ’ for smaller datasets. This sensitivity is representative of what can be achieved using the conventional study. We choose here not to use the U-spin results because of the large correlation with the γ measurement coming from the standalone $B_s \rightarrow D_s^\mp K^\pm$ study, and the current absence of a reliable estimate for U-spin breaking effects. However, it must be emphasised that the U-spin approach shows promise, in particular for its ability to eliminate ambiguous solutions.

3.3 Other Channels and Future Improvements

Other measurements may be made involving time-dependent asymmetries which will contribute to the overall LHCb γ sensitivities. Below are listed some promising possibilities.

- The statistics in the $B_s \rightarrow D_s^\mp K^\pm$ analysis can be significantly improved by exploiting other D_s decay modes apart from $K^+K^-\pi^+$, such as $\pi^+\pi^-\pi^+$, $K^+\pi^+\pi^-$ and $K^+K^-\pi^+\pi^0$.
- The mode $B^0 \rightarrow D^{*\pm}\pi^\mp$ is known to be feasible at LHCb [37] but has not been investigated recently. As the strong phase is likely to be different from that in $B^0 \rightarrow D^\pm\pi^\mp$, the combination of the two channels is likely to be useful in reducing the number of ambiguous solutions, as well as increasing the statistical precision. A U-spin combination of $B^0 \rightarrow D^{*\pm}\pi^\mp$ and $B_s \rightarrow D_s^{*\mp}K^\pm$ is also an attractive analysis option.
- The U-spin pair of channels $B^0 \rightarrow D^\pm\rho^\mp$ and $B_s \rightarrow D_s^\mp K^{*\pm}$ is another interesting, although more challenging, possibility. Decays involving other excited meson states can be investigated, such as $B_s \rightarrow D_s^\mp K_1(1270)^\pm$ [38].
- A time-dependent Dalitz plot analysis of $B^0 \rightarrow D^\mp K^0\pi^\pm$ decays as proposed in [39], and recently explored experimentally in [40], offers the possibility of accessing larger interference effects than those available with the $B^0 \rightarrow D^{(*)\mp}\pi(\rho)^\mp$ modes.

Channel	Signal	Background
$B^\pm \rightarrow D(K^\pm \pi^\mp)K^\pm$	56k	35k
$B^+ \rightarrow D(K^- \pi^+)K^+$	680	780
$B^- \rightarrow D(K^+ \pi^-)K^-$	400	780
$B^+ \rightarrow D(K^+ K^- + \pi^+ \pi^-)K^+$	3.3k	7.2k
$B^- \rightarrow D(K^+ K^- + \pi^+ \pi^-)K^-$	4.4k	7.2k
$B^\pm \rightarrow D(K^\pm \pi^\mp \pi^+ \pi^-)K^\pm$	61k	40k
$B^+ \rightarrow D(K^- \pi^+ \pi^+ \pi^-)K^+$	470	1.2k
$B^- \rightarrow D(K^+ \pi^- \pi^+ \pi^-)K^-$	350	1.2k
$B^0 \rightarrow D(K^+ \pi^-)K^{*0}, \bar{B}^0 \rightarrow D(K^- \pi^+)\bar{K}^{*0}$	3.4k	1.7k
$B^0 \rightarrow D(K^- \pi^+)K^{*0}$	350	850
$\bar{B}^0 \rightarrow D(K^+ \pi^-)\bar{K}^{*0}$	230	850
$B^0 \rightarrow D(K^+ K^- + \pi^+ \pi^-)K^{*0}$	150	500
$\bar{B}^0 \rightarrow D(K^+ K^- + \pi^+ \pi^-)\bar{K}^{*0}$	550	500
$B^\pm \rightarrow D(K_S^0 \pi^\pm \pi^\mp)K^\pm$	5k	4.7k
$B_s, \bar{B}_s \rightarrow D_s^\mp K^\pm$	6.2k	4.3k
$B^0, \bar{B}^0 \rightarrow D^\mp \pi^\pm$	1,300k	290k

Table 5: Summary of signal and background yields for 2 fb^{-1} . In those rows where more than one channel is specified (eg. $B^\pm \rightarrow D(K^\pm \pi^\mp)K^\pm$ or $B^+ \rightarrow D(K^+ K^- + \pi^+ \pi^-)K^+$), the yields correspond to the *sum* over all indicated modes. All physics parameters are as reported in Table 3; δ_{B^0} is set to 90° . The numbers derive from the studies reported in [3, 7, 8, 10, 28, 29, 35].

4 Global Precision on γ

The assumptions for the signal and background yields that underlie the results used in this note are summarised in Table 5. The results presented in Sections 2 and 3 are uncorrelated and may be combined to give the values presented in Table 6. Table 7 lists the relative weight of each contributing analysis in a dataset of 2 fb^{-1} for those values of δ_{B^0} which give the smallest ($\delta_{B^0} = 0^\circ$) and largest ($\delta_{B^0} = 45^\circ$) uncertainty on γ .

5 Conclusions

A combination has been performed of existing studies of the sensitivity of LHCb to the CKM angle γ . It is found that a precision of $1.9\text{--}2.7^\circ$ is achievable with 10 fb^{-1} of data, where the variation is associated with the possible range in the value of δ_{B^0} . This result has been calculated from knowledge of the expected LHCb statistical precision and the existing constraints on various external parameters. No attempt has been made to include contributions from any experimental systematic uncertainties, although these are not expected to be dominant. Other strategies wait to be explored which may improve this sensitivity still further.

δ_{B^0} ($^\circ$)	0	45	90	135	180
σ_γ for 0.5 fb^{-1} ($^\circ$)	8.1	10.1	9.3	9.5	7.8
σ_γ for 2 fb^{-1} ($^\circ$)	4.1	5.1	4.8	5.1	3.9
σ_γ for 10 fb^{-1} ($^\circ$)	2.0	2.7	2.4	2.6	1.9

Table 6: The expected combined sensitivity to γ from $B \rightarrow DK$ and time-dependent measurements for data sets corresponding to integrated luminosities of 0.5, 2 and 10 fb^{-1} .

Analysis	$\delta_{B^0} = 0^\circ$	$\delta_{B^0} = 45^\circ$
$B^- \rightarrow D^0(hh)K^-, B^- \rightarrow D^0(K^\pm\pi^\mp\pi^+\pi^-)K^-$	25	38
$B^- \rightarrow D^0(K_S^0\pi^+\pi^-)K^-$	12	25
$B^0 \rightarrow D^0(hh)K^{*0}$	44	8
$B_s \rightarrow D_s^\mp K^\pm$	16	24
$B^0 \rightarrow D^\mp\pi^\pm$	3	5

Table 7: The relative weight (in percent) of each contributing analysis in the overall γ determination for two values of δ_{B^0} and a dataset of 2 fb^{-1} .

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A CP Formalism and the Value of $\delta_D^{K\pi}$

The convention assumed to describe the CP operation on the D^0 has non-trivial consequences in the definition of phase differences. The CP operator acting on the D^0 can result in either:

$$CP|D^0\rangle = |\bar{D}^0\rangle \quad (8)$$

or

$$CP|D^0\rangle = -|\bar{D}^0\rangle. \quad (9)$$

The ratio of DCS to CF amplitudes is defined as:

$$\frac{\langle K^+\pi^-|\mathcal{H}|D^0\rangle}{\langle K^-\pi^+|\mathcal{H}|D^0\rangle} = r_D^{K\pi} e^{-i\delta_D^{K\pi}}, \quad (10)$$

where $r_D^{K\pi}$ is the absolute ratio of the DCS and CF amplitudes, $\delta_D^{K\pi}$ is the difference in the strong phases between the CF and DCS amplitudes and \mathcal{H} is the Hamiltonian that describes the transition. No CP violation is assumed in the D decay.

To obtain the relation between amplitudes for D^0 and \bar{D}^0 decaying to the same $K\pi$ final state the CP relations in Equations 8 and 9 are used. For the formalism described

by Equation 8 the ratio becomes:

$$\frac{\langle K^+\pi^-|\mathcal{H}|D^0\rangle}{\langle K^-\pi^+|\mathcal{H}|\bar{D}^0\rangle} = \frac{\langle K^+\pi^-|\mathcal{H}|D^0\rangle}{\langle K^-\pi^+|\mathcal{H}CP|\bar{D}^0\rangle} = \frac{\langle K^+\pi^-|\mathcal{H}|D^0\rangle}{\langle K^-\pi^+|CP\mathcal{H}|\bar{D}^0\rangle} = \frac{\langle K^+\pi^-|\mathcal{H}|D^0\rangle}{\langle K^+\pi^-|\mathcal{H}|\bar{D}^0\rangle} = r_D^{K\pi} e^{-i\delta_D^{K\pi}}, \quad (11)$$

but for the formalism given by Equation 9 the ratio is:

$$\frac{\langle K^+\pi^-|\mathcal{H}|D^0\rangle}{\langle K^+\pi^-|\mathcal{H}|\bar{D}^0\rangle} = -r_D^{K\pi} e^{-i\delta_D^{K\pi}} = r_D^{K\pi} e^{-i(\pi+\delta_D^{K\pi})}. \quad (12)$$

The CLEO-c measurement of $\delta_D^{K\pi}$ [19] uses the CP formalism given by Equation 9 which results in Equation 12 whereas the ADS formalism uses the CP formalism given by Equation 8 which results in Equation 11. The consequence of this is that the measured value of $\delta_D^{K\pi} = (22_{-16}^{+14})^\circ$ must be offset by 180° when input to the ADS analysis.

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