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TRANSIENT RESISTIVE-WALL EFFECTS OF A BUNCHED ELECTRON BEAM IN A WIGGLER

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Abstract The effects of transverse resistive-wall instability on the operation of a free-electron laser (FEL) are analyzed. The equation of motion for a bunched beam was previously solved analytically and a steady-state solution shown to exist. The possibility that the transient state could dominate this steady state is now considered. The maximum transient amplitude is obtained for the case where the focusing force dominates the resistive-wall force. An analytic expression for this amplitude when the bunches come in with random initial displacements is also derived. Results are compared with numerical simulations and are applied to various FEL's to determine if the transverse resistive-wall instability could pose a problem in their operation.

INTRODUCTION

When an electron bunch travels through a pipe of small radius in the wiggler of a free-electron laser (FEL), it generates a wakefield behind it if the pipe is not perfectly conducting. This wakefield affects the transverse motion of the bunches that follow and can lead to beam loss. This possibility was considered in our earlier papers[1,2]. Transverse displacement of the bunches was shown to attain a steady-state value. Criteria were established to determine when this steady-state displacement could grow exponentially with the length of the wiggler leading to instability. However, only a heuristic treatment of the transient state (that precedes the steady state) was given. Since the transient state could have a large displacement leading to beam loss even if the steady state is well behaved, it is important to analyze the transient state in a more careful fashion. This is the goal of this paper.

The analysis is restricted to the case where the dominant effect is from the first mode in an expansion of the wakefield. This single-mode solution is a good approximation to the complete solution when the beam-pipe thickness is small. First, an integral representation of the solution is obtained. Second, this integral representation is used to derive analytic formulae characterizing the transient state for two different cases - the case where only the first bunch has an initial displacement (called the "single pulse" case henceforth) and the case where bunches come in with random initial displacements. Results are compared with numerical simulations and are also applied to various proposed FEL's to determine whether the transient state could lead to beam breakup. Because of space limitations, proofs of equations are not given. These details can be found in Ref. 3.

INTEGRAL REPRESENTATION OF SINGLE MODE SOLUTION

The beam is considered to be a series of bunches traveling with speed v (in the z direction) in a pipe of inner radius b, outer radius d, thickness $\tau(= d - b)$, and conductivity σ. Each bunch carries a charge q and there is a fixed time interval Δ between any two bunches. The transverse displacement from the axis of the Kth bunch is denoted by $\xi(z = ct, K)$. Thus $t = 0$ denotes the time when the bunch enters pipe ($z = 0$).

The equation of motion for $\xi(t, K)$ has been derived earlier[2]. If the first mode dominates, it is given as

$$\frac{d^2\xi(t,K)}{dt^2} + \omega_0^2\xi(t,K) = G\sum_{l=0}^{K-1}\exp(-(K-l)\Delta/T)\xi(t,l) \tag{1}$$

where

$$G = \frac{eqv}{m\gamma\pi\sigma\tau b^3}, \tag{2}$$

$$T = \frac{2\pi\sigma b\tau}{c^2}. \tag{3}$$

Here, the factor G incorporates resistive-wall effects and T is the characteristic time of diffusion of the magnetic field through the wall of the beam pipe. The quantity ω_0 in the above expressions is the frequency of the slow betatron motion in the wiggler and γ is the usual relativistic factor. The sum over l in Eq. (1) represents a sum of the interactions between the Kth bunch and the wakefields of all bunches ahead of it.

Equation (1) has been solved exactly[3]. For the sake of simplicity, we give below only the solution for the case when all the bunches are assumed to have zero initial velocity:

$$\xi(t,K) = \sum_{k=0}^{K}\exp(-k\Delta/T)\sum_{n=0}^{\infty}\left(\frac{\pi\,\omega_0\,t}{2}\right)^{1/2}\left(\frac{G\,t}{2\,\omega_0}\right)^n\frac{1}{n!}\binom{k-1}{k-n} \times J_{n-\frac{1}{2}}(\omega_0 t)\,\xi(0,K-k). \tag{4}$$

To obtain a transient state solution, it is convenient to transform the above result into an integral representation (see Ref. 3 for the derivation):

$$\xi(t,K) = \sum_{k=0}^{K} (-1)^k \exp(-k\Delta/T)\, R_1(t,k)\, \xi(0,K-k) \tag{5}$$

where

$$R_1(t,k) = \frac{1}{2\pi i} \oint du \, \frac{1}{u^{2k+1}} \cos\left[\left(\omega_0^2 + \frac{G u^2}{1+u^2}\right)^{1/2} t\right]. \tag{6}$$

The contour in Eq. (6) encloses the origin and $| u |< 1$ everywhere on the contour. Equation (5) is now in a form that can be used to derive the transient state solution.

TRANSIENT STATE FOR SINGLE PULSE

In this Section, the transient state is derived for a beam where the first bunch is displaced off-axis by an amount d and the subsequent bunches follow on-axis. This case will be referred to as the "single pulse" case. It is assumed that $\omega_0^2 \gg G$, an assumption valid in most practical situations. The results obtained are compared with numerical simulation.

Setting $\xi(0,0) = d$, $\xi(0,k) = 0$ for all $k > 1$ in Eq. (5), the integral representation of $\xi(t,K)$ for a single pulse is given as follows

$$\xi(t,K) = d(-1)^K \exp(-K\Delta/T)\, \frac{1}{2\pi i} \oint du \, \frac{1}{u^{2k+1}} \cos\left[\left(\omega_0^2 + \frac{G u^2}{1+u^2}\right)^{1/2} t\right]. \tag{7}$$

A saddle point evaluation of the integral gives[3]:

$$\xi(t,K) \cong \frac{d}{K\sqrt{8\pi}} \left(\frac{2KGt}{\omega_0}\right)^{1/4} \exp\left[-\frac{K\Delta}{T} + \left(\frac{KGt}{\omega_0}\right)^{1/2}\right]$$
$$\times \operatorname{Re}\left\{\exp\left[i\omega_0 t - i\frac{\pi}{8} + i\frac{Gt}{4\omega_0} - i\left(\frac{KGt}{\omega_0}\right)^{1/2}\right]\right\}. \tag{8}$$

At a given location $z = vt$ along the wiggler, $\xi(t,K)$ reaches a maximum when

$$K_m^{1/2} \cong \left(\frac{Gt}{4\omega_0}\right)^{1/2} \frac{T}{2\Delta} + \left[\left(\frac{Gt}{4\omega_0}\right) - \frac{3\Delta}{T}\right]^{1/2} \frac{T}{2\Delta} \tag{9}$$

The normalized (setting $d = 1$) maximum transient displacement for a single pulse is found to be

$$\xi_m(t,K_m) \cong \frac{1}{K_m\sqrt{8\pi}} \left(\frac{2K_m Gt}{\omega_0}\right)^{1/4} \exp\left[-\frac{K_m\Delta}{T} + \left(\frac{K_m Gt}{\omega_0}\right)^{1/2}\right]. \tag{10}$$

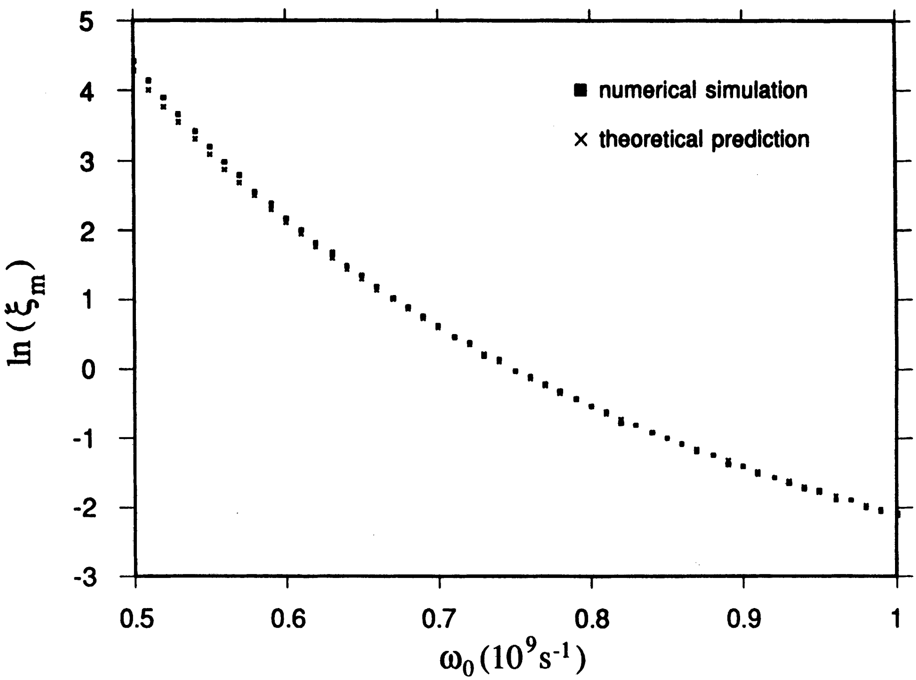

FIGURE 1 Variation of the logarithm of normalized maximum transverse displacement ξ_m in the hypothetical FEL as a function of the betatron frequency ω_0 for the case of a single pulse. The results obtained using the analytical formula (Eq. (10)) are compared with those obtained using numerical simulations.

Five proposed FEL's were studied in Ref. 3. In all these cases, the steady state was found to dominate the transient state. And since the steady state is well behaved in all these cases, resistive wall instability should not be a problem.

To verify the validity of Eq. (10) when the transient state is dominant, a hypothetical FEL with extreme values of parameters is considered. This FEL has $I = 4.6$A, pipelength = 600m, $b = 0.18$cm, $d = 0.198$cm, $T = 0.46\mu$s, and $G = 1.4 \times 10^{13}\text{s}^{-2}$. Figure 1 gives the comparison between numerical simulations and Eq. (10) for different values of the focusing strength (i.e. ω_0). The two results are seen to be in good agreement for large ω_0's. However they start to diverge slightly for small ω_0's. This is to be expected since Eq. (10) is valid only for focusing forces large compared to the resistive-wall forces. As an additional check, the above results were compared with those obtained earlier. When the nonexponential terms in Eq. (8) are ignored, our results[3] agree with the less general results derived by Neil and Whittum[4]. We include the nonexponential terms in our analysis since they can contribute significant corrections to the maximum amplitude.

TRANSIENT STATE FOR A RANDOM INITIAL DISPLACEMENT

In this Section, we consider the case where the bunches come in with random initial displacements. An expression for the root-mean-squared displacement is derived and is compared with numerical simulations.

First, we rewrite the solution for a single pulse in the following form:

$$\xi(t,K) \cong \xi_0 \left[f(t,K) + f^*(t,K)\right] \tag{11}$$

where

$$f(t,K) \cong \frac{1}{K\sqrt{32\pi}} \left(\frac{2KGt}{\omega_0}\right)^{1/4} \exp\left[-\frac{K\Delta}{T} + \left(\frac{KGt}{\omega_0}\right)^{1/2}\right] \times \exp\left[i\omega_0 t - i\frac{\pi}{8} + i\frac{Gt}{4\omega_0} - i\left(\frac{KGt}{\omega_0}\right)^{1/2}\right]. \tag{12}$$

and $f^*(t,K)$ is the complex conjugate of $f(t,K)$. When bunches come in with random initial displacements, $\xi(t,K)$ can be written as a linear superposition of solutions of the above form[5]. Thus,

$$\xi(t,K) \cong \sum_{k=0}^{K} [f(t,k) + f^*(t,k)]\, \xi(0, K-k). \tag{13}$$

After considerable manipulations including use of the facts that initial displacements of two successive bunches is uncorrelated and that $f(t,k)$ oscillates only very slowly with k, the final result is obtained as[3]:

$$\frac{<\xi^2(t,\infty)>}{<\xi_0^2>} \cong 2\pi^{1/2} \left(\frac{\omega_0}{Gt}\right)^{1/4} K_m^{3/4} \mid \xi_m(t,K_m) \mid^2 . \tag{14}$$

where $<\xi^2(t,\infty)>$ is the rms value of $\xi(t,K)$ as $K \to \infty$, $<\xi_0^2>$ is the rms initial displacement, and K_m and $\xi_m(t,K_m)$ are given by Eqs. (9) and (10).

This result has been compared with numerical simulations for our hypothetical FEL. Figure 2 shows the results of numerical simulation for $\omega_0 = 9 \times 10^8 \mathrm{s}^{-1}$ with $<\xi_0^2>^{1/2} = 1\mathrm{mm}$. It gives a value of $\sim 1.2\mathrm{cm}$ for $<\xi^2(t,\infty)>^{1/2}$. From Eq. (14) and Figure 1 we obtain a theoretical value of $\sim 1.3\mathrm{cm}$ for $<\xi^2(t,\infty)>^{1/2}$. Thus we get a good agreement between theory and simulation.

SUMMARY

The transient state of a bunched electron beam in a wiggler was analysed in detail using the integral representation of the complete solution. Analytic expressions for

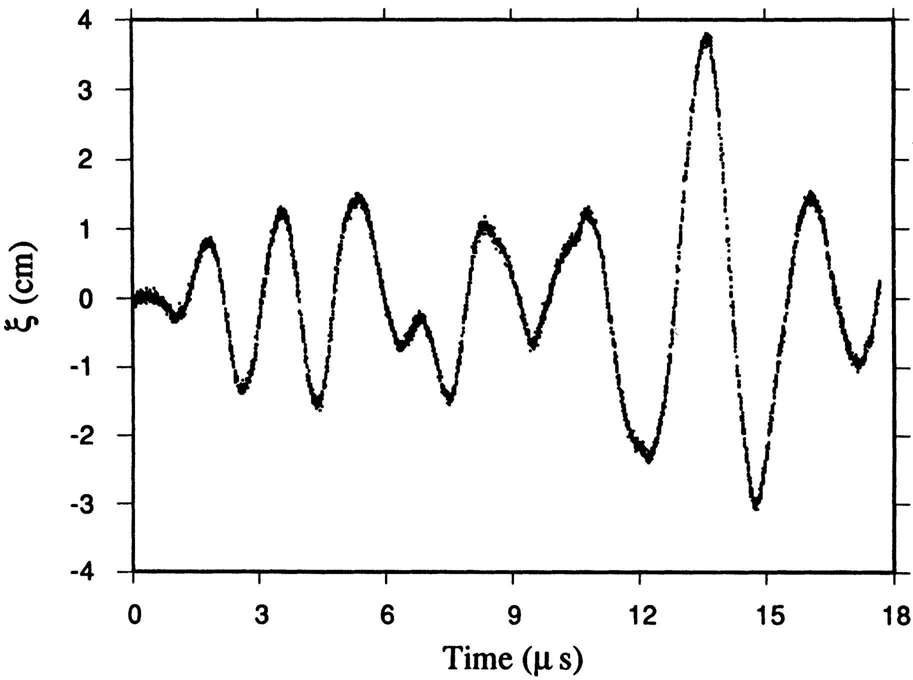

FIGURE 2 Numerical simulation of the transverse displacement of the beam in the hypothetical FEL when the bunches come in with random initial displacements. The rms value of the initial displacements was taken to be 1mm. The betatron frequency has the value $9 \times 10^8 s^{-1}$.

the maximum transient amplitude were derived for the case where only the first bunch is displaced off-axis and for the case where bunches come in with random initial displacements. This led to the conclusion that transient state could lead to beam loss if the normalized maximum amplitudes given in Eqs. (10) and (14) exceed unity. However, no such instability was found in the five proposed FEL's that were studied.

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