Particle Accelerators, 1990, Vol. 31, pp. 21–26
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PLASMA PHYSICS AT THE FINAL FOCUS

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Abstract Particle-in-cell simulations are used to determine the final radius that results when electron and positron bunches counterstream through each other. We find that the beams both initially pinch strongly and then relax to an equilibrium radius in less than a relativistic plasma period. The final radius squared is found to decrease by a factor of 25. This would indicate that the luminosity enhancement factor H_d should asymptotically approach this value for large D as found by Chen and Yokoya[1]. We also find considerable emittance growth as predicted by Fawley and Lee[2]. Simple analytic models are presented.

INTRODUCTION

The ever increasing need for higher luminosity in e^+e^- colliders has led to beam densities in the regime where the collective effects of one beam on the other are significant. It is therefore critical that the plasma physics which arises during a collision be understood. When e^+-e^- bunches pass through each other, the fields of one bunch cause the other bunch to pinch since the space charge and $\vec{v}\times\vec{B}$ self-forces of each bunch are almost in balance. As the bunches pinch, their density increases resulting in a desirable increase in the collision's luminosity. The self-pinching process is given the name disruption. The luminosity enhancement which results from disruption depends on the dimensionless parameter D[3]. The disruption parameter D has a simple plasma physics interpretation[3]. The square root of disruption parameter, $\sqrt{D}$, measures the number of plasma oscillations which occur during a collision. By convention, $D \equiv \frac{\omega_p^2}{c^2}\sigma_{\|}^2\sqrt{\frac{\pi}{2}}$ where $\sigma_{\|}$ is half the longitudinal length of the bunch where ω_p is the relativistic plasma frequency. If the beams are gaussian in shape with azimuthal symmetry, then it can be shown that $D = \frac{N r_e \sigma_{\|}}{\gamma \sigma_o^2}$, where r_e is the classical electron radius.

Most of the effort on disruption has been directed towards understanding how the luminosity enhancement, $H_d \equiv \frac{L}{L_o}$, scales with D[1,2,3,]. This work has relied on numerical modeling since disruption is highly nonlinear. From their numerical work, Chen and Yokoya[1] have identified three regimes for D. The regimes are identified by the qualitative shapes of the $\frac{dH_d}{dt}$ vs t curves. They are called the weak-focusing, transition and confinement regimes. In weak-focusing, the disruption parameter is less than unity so the bunches do not even completely pinch during a collision. In the transition regime, the collision time is on the order of a plasma period. As a result, the beams fully pinch once causing a spike in the $\frac{dH_d}{dt}$ vs t curve. In the confinement regime, the collision time covers many plasma oscillations. The bunches therefore have time to reach an equilibrium radius. If the time to reach this radius is short compared to the collision time, then the luminosity enhancement will approximately be the ratio of the square of the initial radius to the square of the equilibrium radius. Future TeV linear colliders as well as $B\bar{B}$ factories may[4] operate in the large D (confinement) regime.

In this paper, we attempt to study the confinement regime using PIC simulations. It is not feasible to study high D cases directly using electromagnetic PIC codes. The reason is that in order to resolve both electromagnetic and plasma time scales it is necessary to use relatively small values of γ. When γ is small, the paraxial ray approximation is violated because as the beams pinch their parallel velocity falls substantially below the speed of light, c. Consequently, we study the high D regime indirectly as follows. Since the luminosity enhancement is determined primarily by the final radius, then we use PIC simulations to determine the equilibrium (asymptotic radius). Similar reasoning was used by Fawley and Lee[2]. We assume translational invariance in the longitudinal direction. Computational time is gained while convolutional information is lost. The assumption is that the equilibrium radius obtained by letting the e^+ and e^- bunches evolve identically will be the same as that of the central part of the beams when they pass through each other.

In the simulations, the bunches have gaussian profiles in the radial direction with $\sigma_o = 6.25\, c / \omega_{po}$. ($\omega_{po}$ is the non-relativistic plasma frequency). The cell size is .02 σ_o and there are 10 particles in each cell. The γ of the beams is varied and in one run some initial emittance was included while in another run the positrons were held fixed for a plasma period as might occur at the head of the electron bunch.

In this paper, we will first give a few simple analytical models in order to determine relevant time scales and then present and discuss the simulation results.

SHEET MODEL

In order for the luminosity enhancement to be related to the final radius, then the beams must approach equilibrium in a time small compared to their length. Before the bunches can reach a final radius, they must pinch once. In fact, the simulations show that the beams pinch robustly once and then relax to the equilibrium radius in less than one relativistic plasma oscillation. We derive expressions for the time to this first compression for both flat top and gaussian radial profiles. We show that in the first case the sheets never cross while in the second case they eventually cross.

In the sheet model, the bunch is modeled as a bunch of concentric rings, which we refer to as sheets. If the sheets do not cross then the force on a sheet depends only on the sheets instantaneous position and the amount of charge enclosed by the given sheet initially. If the radial distribution is initially flat, then the radial displacement, ζ, of a given sheet from its initial position r_o therefore satisfies the differential equation.

$$\ddot{\zeta} + \frac{\omega_{po}^2}{\gamma} \frac{r_o^2}{\zeta + r_o} = 0 \qquad (1)$$

Note we have assumed that there is zero initial angular momentum (or no emittance). Eq. (1) can be integrated once to give

$$\frac{\dot{\zeta}^2}{2} + \frac{\omega_{po}^2}{\gamma} r_o^2 \ln(1+\frac{\zeta}{r_o}) = 0 \qquad (2)$$

If no sheets cross, then the time for a sheet to move a fraction $x = \frac{\zeta}{r_o}$ of its initial position is given by

$$\int_o^x \frac{dx'}{\sqrt{\ln\frac{1}{1+x'}}} = -\frac{\omega_{po}}{\sqrt{\gamma}} t_x \qquad (3)$$

The time to reach the origin, t_{ro}, is therefore

$$\omega_{po} t_{ro} = \sqrt{\gamma}\int_{-1}^{0} \frac{dx}{\sqrt{\ln\frac{1}{1+x}}} = \sqrt{\gamma}\Gamma\left[\frac{3}{2}\right] = \sqrt{\gamma}\sqrt{\frac{\pi}{2}} \qquad (4)$$

and it is independent of r_o This demonstrates that for flat top profiles the sheets all arrive at the origin at the same time $\omega_{po}t = \frac{\sqrt{\gamma\pi}}{2}$ so that they never cross. This time is also considerably less than a relativistic plasma period $\omega_{po}\tau_r = 2\pi\sqrt{\gamma}$. If the radial profile is gaussian then the starting differential equation is

$$\ddot{\zeta} + \frac{\omega_{po}^2}{\gamma}\frac{\sigma(1-e^{-r_o^2/\sigma^2})}{\zeta+r_o} = 0 \qquad (5)$$

Integrating once as before gives

$$\frac{\dot{\zeta}^2}{2} + \frac{\omega_{po}^2}{\gamma}\sigma^2(1-e^{-r_o^2/\sigma^2})\ln(1+\frac{\zeta}{r_o}) = 0 \qquad (6)$$

As long as the sheets do not cross, then as before, we can calculate the time it takes for $\frac{\zeta}{r_o}$ to reach a value x and it is

$$\int_o^x \frac{dx'}{\sqrt{\ln 1/1+x'}} = \frac{\omega_{po}}{\sqrt{\gamma}}\frac{\sigma}{r_o}\sqrt{(1-e^{-r_o^2/\sigma^2})}\, t_x \qquad (7)$$

The critical time t_{ro} is therefore given by

$$\omega_{po} t_{ro} = \frac{\sqrt{\pi}}{2}\sqrt{\gamma}\frac{r_o}{\sigma}\frac{1}{\sqrt{1-e^{-r_o^2/\sigma^2}}} \qquad (8)$$

and it is now a function of r_o. It can easily be shown that $\frac{\partial t_{ro}}{\partial r_o} < 1$ for all r_o. This indicates that every sheet begins to cross those sheets inside of it and this will result in a distortion of the radial profile. Furthermore, since the sheets cross, their motion cannot be periodic. It can also be shown that if x is << 1, then the sheets do not cross indicating that sheet crossing occurs when they have moved a substantial fraction of their initial position. A rough estimate of the time when the bunches first pinch is obtained for $r_o \leq \sigma$ which gives $\omega_{po} t_{ro} \sim \sqrt{\frac{\pi}{2}} \sqrt{\gamma}$; the same result obtained for flat beams.

This simple calculation shows that the bunches self-pinch within a fraction of a relativistic plasma oscillation. As a result it is conceivable that a state of equilibrium could be reached in less than a plasma oscillation or equivalently in large D collisions. We point out than even if some angular momentum was included it would change the equilibrium radius but it would have little effect on the collapse time since in the integral on the left hand side of Eq. (6) has little contribution for large |x|.

Simulation Results

The simulation results are summarized in Fig. 1 where plots of $\int dr\, r\, n^2(r)$ are plotted vs. t for various simulations. The time scale is normalized to ω_{po}^{-1} and we define the average radius squared as $<r^2> \equiv (\int dr\, r\, n^2(r))^{-1}$.

In Fig. 1a the result is from a simulation in which $\gamma = 4096$. The curve exhibits the shape which is characteristic in almost all the simulations. A robust initial collapse is followed by a rapid decay to equilibrium. The initial collapse occurs at $\omega_{po} t \cong 80$ or $\omega_p t = 1.25$ and this is close to $\omega_{po} t_\sigma = 71$ or $\omega_p t_\sigma = 1.115$ which are obtained form Eq. (7) with $r_o = \sigma$. In other simulations the first peak occurred for $\omega_p t = 1.3$ for γ's ranging from 32 to 512. This demonstrates dramatically that the inverse of the relativistic plasma frequency is the natural time scale.

After the bunches initially collapse they begin to relax outwards to an equilibrium radius. The equilibrium is reached by $\omega_p t \cong \sqrt{\pi}$. This is still a fraction of a complete plasma period, indicating that a confined radius in indeed possible for large values of D. The asymptotic radius is smallest for the larger γ cases. Similar behavior is seen for the dependence of the radius at the initial collapse with γ. In both cases the variation of $<r^2>$ with γ is due to the violation of the paraxial ray approximation for small γ's. The ratio of the square of the initial radius to the square of the asymptotic radius, $\frac{<r^2>_i}{<r^2>_f}$, varies from 16 for $\gamma = 32$ to 26 for $\gamma = 4096$. The change in this ratio is slight when γ is changed from 512 to 4096 so we believe that 26 is near a maximum value. When $\frac{<r^2>_i}{<r^2>_f} = 26$ the bunches are now only a skin depth in radius which is 6 cells across so there is no resolution problem. Furthermore, at peak compression $<r^2>$ is reduced by a factor of 130 and this was resolved.

If D is large enough that the contribution of the initial pinching is negligible, then the luminosity enhancement, H_d, should simply be the ratio of the square of the initial radius to the square of the asymptotic radius. This would imply that $H_d = 26$ for large values of D. This is in remarkable agreement with the work of Chen and Yokoya[1] who found $H_d = 26$ for their largest value of D(100) and their smallest amount of emittance.

Fawley and Lee[2] have argued both analytically and numerically that the square of the ratio the asymptotic to initial radius cannot increase by more than 2.7 which is a factor of ten less than 26. We considered one possible source of disagreement was the definition of $<r^2>$. Fawley and Lee used $<r^2> = \int dr\, r\, r^2 n(r)$ while we have used the luminosity relevant definition $<r^2> = (\int dr\, r\, n^2(r))^{-1}$. However, we have checked that the ratio of these two definitions is insensitive to the function n(r). For example if $n(r) = e^{-(r/\sigma)^m}$ then the ratio of these definitions is $\frac{m}{2\pi} 2^{2/m} \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{2}{m}+\frac{1}{2})}{\Gamma(\frac{2}{m})}$ which reduces to unity for $m = 2$ and $m \to \infty$. At present we have no satisfactory explanation of the discrepancy between our numerical work and that of Fawley and Lee.

Last we mention that a simulation was done with some initial emittance. Results from this simulation are shown in Fig. 1c. The major differences are that both the radius at collapse and the equilibrium radius are both increased by emittance. In addition the radius of the bunch evolves more smoothly in time with initial emittance. These differences are easily seen by comparing Fig. 1b to Fig. 1c. Interestingly, the final emittance for the cold beam is larger than the final emittance of the warm beam by a factor of 2. Crudely speaking we find that emittance is important if a particle transits the bunch transversely before the initial compression.

CONCLUSION

We have investigated the behavior of the self-pinching of e^+ and e^- beams. We find that the beams pinch rapidly and then relax to an equilibrium radius in a fraction of a relativistic plasma period. The minimum equilibrium radius which was observed corresponds to an $H_d = 26$ for large values of D. This is in agreement with Chen and Yokoya[1] but in disagreement with Lee.[2] Resolving this discrepancy as well as quantifying how the equilibrium radius depends on the initial emittance are areas for future work.

This work is supported by DOE contract no. DE-AS03-83-ER40120 and NSF.

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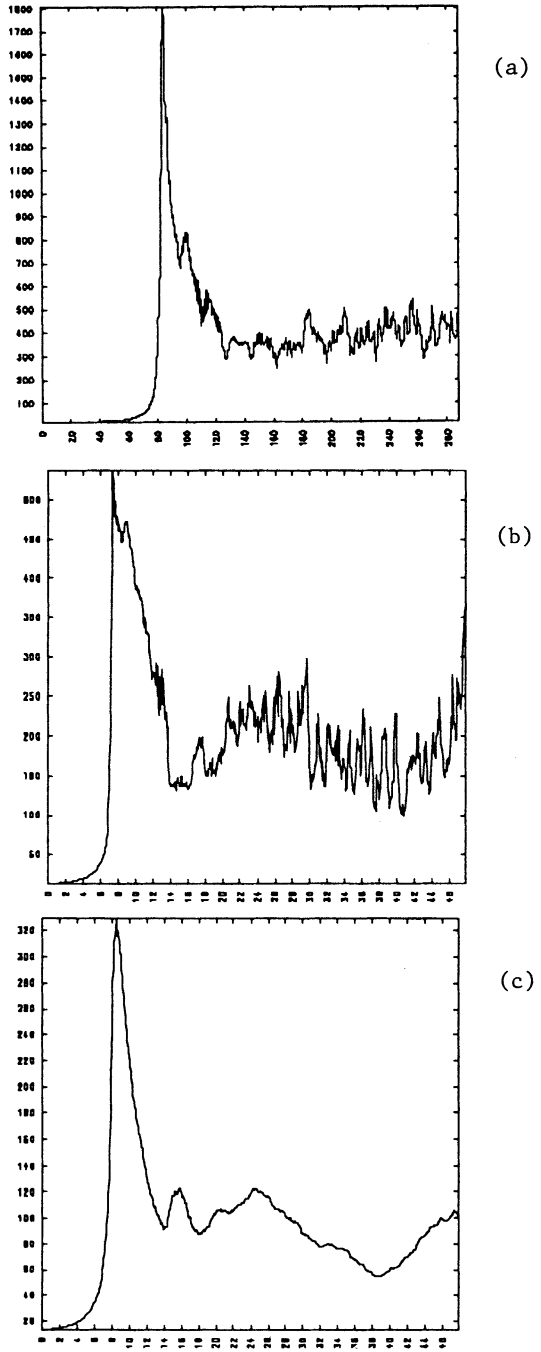

FIGURE 1.

We plot $\int dr\, r\, n^2(r)$ vs $\omega_{po}t$ for a) $\gamma = 4096$, no emittance, b) $\gamma = 32$, no emittance and c) $\gamma = 32$, $\varepsilon = .45$. In simulation units, $\int dr\, r\, n^2(r) = 13.4$ at $\omega_{po}t = 0$.