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THE BEAM LOSSES AFTER INJECTING INTO SYNCHROTRON UNDER THE DC MAGNETIC FIELD

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Abstract in order to study some beam loss mechanisms after the beam injecting into the electron synchrotron, a series of experiments have been performed in several synchrotrons.

INTRODUCTION

A series of experiments of beam losses have been performed in the INS–ES 1.3 GeV synchrotron, IMS 600 MeV synchrotron, Japan and the NBS storage ring, U.S.A. with the DC magnetic field mode. At first, it is discovered that there is a rather large discrepency between the traditional formula of the beam lifetime vs vacuum pressure and the experimental results. By considering the beam distribution and the different loss prerequisites between the radial and vertical directions, and quoting the concepts of emittance and acceptance, a modified beam lifetime formula due to residual gas scattering is deduced. The estimated lifetime is in good agreement with the experimental results. Then, the influences of RF accelerating voltage on the survival beam intensity and the beam lifetime have also been measured. Due to phase motion, the energy spread of the accumulated beam is increased, causing a decrement of the effective radial aperture of the vacuum doughnut. The deduced formulas and the estimated results with above formulas can explain both of these experiments to a certain extent.

BEAM LIFETIME DUE TO RESIDUAL GAS SCATTERING

After fast loss the beam lifetime has been measured in the INS–ES synchrotron and the NBS storage ring with 10, 15 MeV injecting energy and DC mode. The experiments in the INS–ES show that the beam lifetime is not dependent on the accumulated beam intensity in the experimental range as shown in Fig.1. and Fig.2. And the fact that there is only one gradient in the curve family of the lifetime vs time shown in Fig.1 illustrates that there is only one dominant loss mechanism. Obviously, in this energy range the beam lifetime is determined by an average pressure $\overline{P}$, on the basis of the residual gas scattering. However, with the traditional formula of beam lifetime the estimated beam lifetime is always several times larger than the measured value.

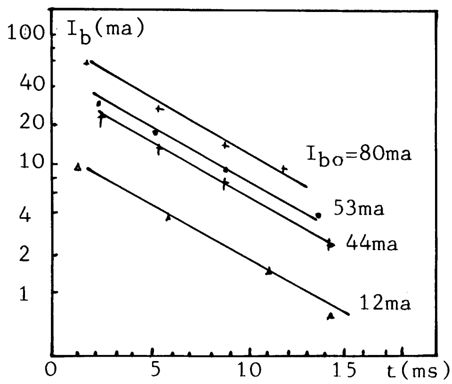

FIGURE 1 Beam intensity vs time with several accumulated beams

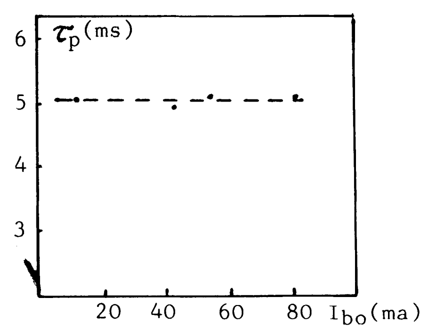

FIGURE 2 Beam lifetime τ_p vs beam intensity

The above discrepency is due to the traditional formula considering the beam being concentrated on centre orbit. In order to solve the problem, in this paper by considering a Gauss–distribution of the beam in the doughnut and the different loss prerequisites between radial and vertical directions, and quoting the concepts of emittance and acceptance, a modified beam lifetime formula is deduced.

Deduction of Beam Lifetime Formula[1]

The different cross section due to elastic Coulomb scattering is given by

$$\sigma(\theta)d\omega = Z^2 e^4 dw / 4E_o^2 sin^4(\theta/2) \tag{1}$$

As a result of the residual gas scattering the electron horizontal velocity angle x' becomes $x' + tg\theta cos\Phi$ and the vertical velocity angle y' becomes $y' + tg\theta sin\Phi$. Then the loss critical conditions of the electron are given by

$$\gamma_x(s)x^2 + 2\alpha_x(s)x(x' + tg\theta cos\Phi) + \beta_x(s)(x' + tg\theta cos\Phi)^2 \geqslant A_x^2 \tag{2}$$

$$\gamma_y(s)y^2 + 2\alpha_y(s)y(y' + tg\theta sin\Phi) + \beta_y(s)(y' + tg\theta sin\Phi)^2 \geqslant A_y^2 \tag{3}$$

where α,β,γ are Twiss parameters of the synchrotron, A_x^2, A_y^2 are radial and vertical acceptance respectively

$$A_x^2 = \left\{\frac{(R_x - COD_x)^2 - \eta_m^2 \bar{\varepsilon}^2}{\alpha_x(s)}\right\}_{min} \qquad A_y^2 = \left\{\frac{(R_y - COD_y)^2}{\alpha_y(s)}\right\}_{min} \tag{4}$$

For a electron with $x,x'$and y,y', its integral cross section due to gas scattering can be found by integrating the formula (1), the limits of the integration being determined by Eqs.(2) and (3). By taking some appropriate approximations, the integrating result can be expressed as

$$\sigma \approx \frac{4\pi Z^2 r_e^2}{\gamma^2}\left\{\frac{\beta_x(A_x^2+a_x^2-\frac{2}{\beta_x}x^2)}{2(A_x^2-a_x^2)}+\frac{\beta_y(A_y^2+a_y^2-\frac{2}{\beta_y}y^2)}{2(A_y^2-a_y^2)}\right\} \tag{5}$$

Assume that the beam distribution function of the betatron–oscillation amplitude invariant satisfies a modified Gauss–distribution[2]

$$f(a_x^2)=\frac{1}{<a_x^2>}exp\left(-\frac{a_x^2}{<a_x^2>}\right)F\left(\frac{a_x^2}{<a_x^2>},\frac{A_x^2}{<a_x^2>}\right) \tag{6}$$

According to the above distribution, integrating the formula (5) for a statistical mean value, one can obtain the following expression of the cross section due to elastic scattering

$$\sigma_c=\frac{4\pi Z^2 r_e^2}{\gamma^2}\frac{<\beta_x>}{A_x^2}\frac{1}{2}\left[C(k_x^2)+\frac{A_x^2<\beta_y>}{A_y^2<\beta_x>}C(k_y^2)\right] \tag{7}$$

where $k_x^2=A_x^2/<a_x^2>$, $k_y^2=A_y^2/<a_y^2>$ and $C(k^2)$ is a complicate integration, its relationship with the k^2 value being shown in Fig.3.

At last, the beam lifetime due to residual gas scattering is given by

$$\tau_p=\frac{1.037\times10^{-5}\gamma^2}{\bar{P}}\frac{A_x^2}{<\beta_x>}\left[\frac{1}{2}C(k_x^2)+\frac{1}{2}\frac{A_x^2<\beta_y>}{A_y^2<\beta_x>}C(k_y^2)\right] \tag{8}$$

Comparing the deduced formula with the traditional formula, $\tau_p=1.037\times10^{-5}\gamma^2\theta_c^2/\bar{P}$, one can find that the traditional formula multiplies an additional factor of $\left[0.5C(k_x^2)+0.5C(k_y^2)A_x^2<\beta_y>/A_y^2<\beta_x>\right]$.

Comparsion Between Calculated Value and Experimental Result

Using the formula (8) and Fig. 3, we calculate the beam lifetime of the INS–ES synchrotron and NBS storage ring and compare these values with the measured results. The parameters of the INS–ES are $k_x^2=2.8\sim4, k_y^2=3.3\sim5.0, <\beta_x> = <\beta_y> = 2.43$, $Z^2=100$. The comparing result is shown in table 1. Form Table I, it is clear that the calculated results by deduced formula are in good agreement with the experimental results.

TABLE I Comparsion between Calculated value and Experimental Result

Machine	Energy of beam E_e (MeV)	Pressure $\bar{P}$ (Torr)	Traditional formula	Authors' formula	Experimental result
INS–ES	15	6×10^{-10}	68ms.	3.8~7.0ms.	5ms.
NBS	10	9.6×10^{-9}	69s.	10~17s.	10s.
NBS	15	7.3×10^{-9}	204ms.	30~47s.	47s.

BEAM SURVIVING RATE VS RF ACC. VOLTAGE

The influence of RF acc. voltage on the surviving rate of the accumulated beam has also been measured in the INS–ES with the DC mode. It is observed that the surviving beam intensity declines gradually as the increment of the RF acc. voltage. Fig. 4 shows the experimental curve of the beam surviving rate vs RF voltage.

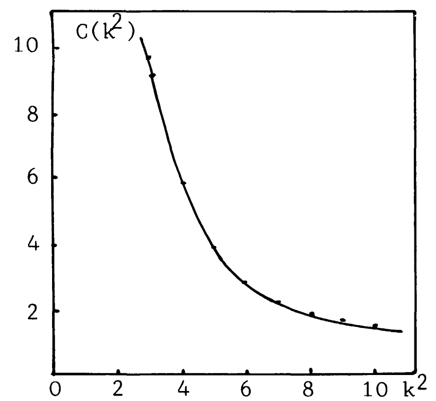

FIGURE 3 Curve of $C(k^2)$ vs k^2

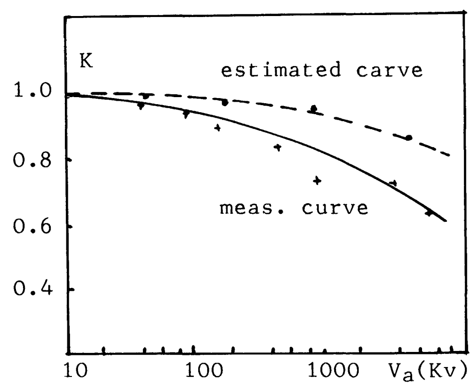

FIGURE 4 Beam surviving rate vs RF acc. voltage

Deduction of Beam Surviving Rate Formula

After injecting, as a result of phase motion, the energy spread of the accumulated beam is increased. The higher the RF acc. voltage is, the larger the achievable maximum energy spread becomes. Due to the actions of η – function and the limitation of radial acceptance, some parts of the accumulated beam will be lost. It belongs to a fast loss. And it must be pointed out that when th RF voltage is relatively lower, most of the particles of the beam are not in the phase capture region.

When a synchrotron operates with the DC mode, equilibrium phase being equal to 90°, during the phase motion process the maximum energy spread of the electron with an injecting energy spread ε_i and a intial phase Φ_i can be determined by the following formula:

$$\varepsilon_m = D\left\{1 - sin\Phi_i + \varepsilon_i^2 / D^2\right\}^{\frac{1}{2}} \tag{9}$$

where $D = (eV_a sin\Phi_s)^{\frac{1}{2}} / (\pi\alpha h E_o)^{\frac{1}{2}}$, V_a is amplitude of the RF acc. voltage, h is harmonical number, α is orbit compaction factor, E_o is the total energy of the electron. Set the distribution function of the energy spread of the injecting beam satisfy Gauss–distribution

$$f(\varepsilon_i) = \frac{1}{\sqrt{2\pi}\,\sigma_\varepsilon} exp(-\varepsilon_i^2 / 2\sigma_\varepsilon^2) \tag{10}$$

And the beam satisfying the following inequality will be lost

$$\beta_x(s)a_x^2 + [\varepsilon_m \eta(s)]^2 \geqslant \beta_x A_x^2 \tag{11}$$

According to Eq. (11), a critical energy spread with which the beam will be lost is given by

$$\varepsilon_{mo} = \left\{ \sqrt{(A_x^2 - a_x^2)\beta_x(s)} \,/\, \eta(s) \right\}_{min} \tag{12}$$

Then after injecting, the beam loss propablity G can be determined

$$G = \int_0^{A_x^2} B(a_x^2)da_x^2 \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{d\Phi_i}{\pi} \int_{\varepsilon_{oi}}^{\varepsilon_{2i}} f(\varepsilon_i)d\varepsilon_i \,/\, A \tag{13}$$

where ε_{2i} is the allowable maximum energy spread of the synchrotron, $\varepsilon_{2i} = (R_x - COD)$, $\varepsilon_{oi} = \left[\varepsilon_{mo}^2 - (1 - sin\Phi_i)/D\right]^{\frac{1}{2}}$, $B(a_x^2)$ is the distribution function of betatron–oscillation amplitude invariant. Set $B(a_x^2) = a_o / (a_x^2)^{\frac{1}{2}}$.[3] So the beam surviving rate is equal to

$$K = 1 - G \tag{14}$$

Comparsion Between Estimated Value And Experimental Result

For the INS–ES, $\sigma_\varepsilon = 2\%$, $R_x = 27$mm, COD = 1mm, $\eta_m = 1.4$m, $\beta_{xm} = 4.1$m, according to Eqs. (13) and (14), the curve of K—V_a can be determined as shown in Fig.4. It can qualitatively explain the experimental curve in Fig.4.

BEAM LIFETIME VS RF ACC. VOLTAGE

The experimental curve of the beam lifetime vs RF acc. voltage in the INS–ES is shown in Fig. 5. At the present situation of the relatively lower beam energy, the beam life–time is still determined by the beam resi–dual gas scattering, and calculated by Eq. (8). But the influence of the RF voltage must be considered. The influence can be ascribed to increase the mean energy spread $\overline{\varepsilon^2}$ of the beam due to the RF voltage.

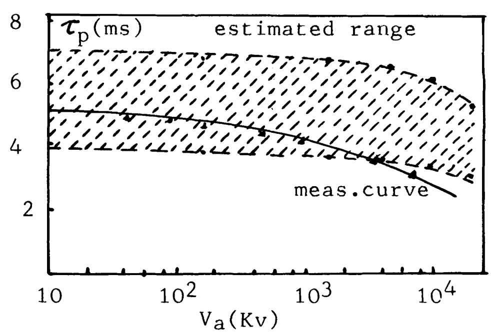

FIGURE 5 Beam lifetime vs RF acc. voltage

Deduction of Expression Of Beam Lifetime vs RF Voltage

The key for solving the problem is to find a expression of $\overline{\varepsilon^2}$ vs V_a. After the fast loss, in the $\varepsilon \sim \Phi$ phase space the probability of an achievable max. energy spread E_m which is larger than a certain value E_{mo} is equal to

$$F(E_m \geqslant E_{mo}) = \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{d\Phi_i}{\pi} \int_{\varepsilon_{oi}}^{\varepsilon_{2i}} f(\varepsilon_i)d\varepsilon_i \tag{15}$$

where $f(\varepsilon_i)$is still the distribution function of energy spread of the injecting beam, being determined by the Eq. (10). $g_2 = D^{-1}(R_x / \eta_m)^2 - (1 - sin\Phi_i)$,when $g_2 > 0$, $\varepsilon_{2i} = Dg_2^{\frac{1}{2}}$ and when $g_2 < 0$, $\varepsilon_{2i} = 0$. $g_1 = \varepsilon_{mo}^2 D^{-2} - (1 - sin\Phi_i)$,when $g_1 > 0$, $\varepsilon_{oi} = Dg_1^{\frac{1}{2}}$ and when $g_1 < 0$, $\varepsilon_{oi} = 0$.

So the distribution function of energy spread of the beam can be determined by the following expression

$$\Phi(\varepsilon_{mo}) = \frac{dF(\varepsilon_m \geqslant \varepsilon_{mo})}{d\varepsilon_{mo}} \tag{16}$$

Therefore one can obtain a statistical mean square value of the beam energy spread $\overline{\varepsilon^2}$

$$\overline{\varepsilon^2} = \int_o^{R_x/\eta_m} \varepsilon_{mo}^2 \Phi(\varepsilon_{mo}) d\varepsilon_{mo} \Big/ \int_o^{R_x/\eta_m} \Phi(\varepsilon_{mo}) d\varepsilon_{mo} \tag{17}$$

Comparsion Between Estimated Value And Experimental Result

Substituting the parameters of the INS–ES into Eq. (17), we calculated the relation of $\overline{\varepsilon^2}$ vs V_a as shown in Table II

TABLE II Calculated Relation of $\overline{\varepsilon^2}$ vs V_a

V_a (Kv)	0	0.189	1.69	4.70	9.24	18.9
$\overline{\varepsilon^2} \times 10^3$	0.110	0.111	0.114	0.128	0.141	0.173

And $A_x^2 = A_{xo}^2 - \overline{\varepsilon^2}\eta_m^2 / \beta_{xm}$. However by reason of the change of A_x^2, the value of the factor of $[0.5C(k_x^2) + 0.5C(k_y^2)A_x^2 < \beta_y > / A_y^2 < \beta_x >]^{-1}$is also influenced. By taking all these factors into account and applying $\overline{\varepsilon^2}$ value, Eqs. (8) and (17), one can estimate the influence of RF voltage V_a on the beam lifetime $\tau_p$as shown in Table III and Fig. 5. The tendency of the estimated result is identical with the experimental result.

TABLE III Estimated Result of Beam Lifetime vs V_a

V_a (Kv)	0	0.189	1.69	4.70	9.24	18.9
τ_p (ms)	3.84–6.98	3.82–6.96	3.78–6.92	3.63–6.55	3.49–6.20	3.20–5.23

REFERENCES

1. Guozi Liu, Master Thesis,Tsinghua University, China, (1983)
2. A.W. Chao, IEEE Trans. Nuclear Sci. NS24, 1885(1977)
3. Y.Miyahara and T. Kawakubo, Beam Loss Due to Momentum Blow Up In KEK Booster, KEK–78–25 (1978)