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IMPEDANCES AND INSTABILITIES OF THE AGS BOOSTER*

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Abstract In this paper we review calculations done recently to demonstrate the stability of beam bunches in the AGS Booster for both cases of protons and heavy ions.

INTRODUCTION

The AGS Booster project,[1] presently under construction at Brookhaven National Laboratory, and expected to be ready for commissioning during 1991, has two major objectives:

(i) Increase the proton intensity in the AGS by a factor of four (to 6×10^{13} ppp). The Booster will operate at the repetition rate of 10 Hz and produce 3 bunches per cycle each of 5×10^{12} protons.

(ii) Accelerate heavy ions up to gold in the Booster for AGS and eventually for RHIC. In this mode of operation the injection energy is very low, ($\beta \sim 0.05$).

Because of the large proton intensity and the low velocity at injection of the heavy ions and their large charge state, it is important to demonstrate that beam bunches are stable against collective effects that can cause coherent oscillations both longitudinal as well as transverse.

In the following we shall make first an estimate of the longitudinal and transverse coupling impedance and then evaluate the beam stability.

LONGITUDINAL COUPLING IMPEDANCE

The energy range of the AGS Booster is limited. The energy does not exceed 1.5 GeV for protons and it is only a fraction of a GeV/nucleon for heavy ions. The injection energy though is quite low; thus we expect that the space charge provides the largest contribution to the total longitudinal coupling impedance which is large and essentially capacitive.

We wrote a small program (ZOVERN) which calculates and adds the contributions from several wall components, like: wall resistivity (stainless steel), bellows, beam position monitors and variations of the vacuum chamber cross–section. An example of the total result is shown in Fig. 1 for $\gamma = 2.6$. As one

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can see, the imaginary part is very large. The real part is constant and below 10 ohm. To observe also that the cut–off harmonic number is relatively low, of only few thousand.

Independent estimates of the impedance due to injection/extraction kicker magnets and to the vacuum ports have also been made. They seem to lead to a small value which add insignificantly to the results shown in Fig. 1.

The contribution of the rf cavities is not taken into account here.

Table I gives the space charge contribution alone, because it is so dominant, for different ion species. As one can see, it can be as large as 4 kohm in some cases.

TRANSVERSE COUPLING IMPEDANCE

There are four major contributions to the transverse coupling impedance in the AGS Booster.

The space charge. The expression for the contribution is

$$Z_\perp = \frac{iRZ_0}{\beta^2\gamma^2} \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

where $Z_0 = 377\Omega$, a is the average beam radius and b is the vacuum chamber radius. For protons

$$Z_\perp \approx i\ 53 \ \ M\Omega/m \ \text{ at injection}$$
$$\approx i\ 14 \ \ M\Omega/m \ \text{ at extraction}$$

The resistivity of the vacuum chamber

$$Z_\perp = \frac{(1-i)R}{b^3} \left(\frac{2RZ_0\rho}{\beta(n-\nu)}\right)^{1/2}$$

where R is the average radius of the closed orbit, ρ the wall resistivity, and ν the betatron tune number. The contribution is small compared to the space–charge effect. For protons again, for $(n-\nu) = 5$ - 4.8,

$$Z_\perp = (1-i)0.058 \ M\Omega/m \ \text{ at injection}$$
$$0.045 \ M\Omega/m \ \text{ at extraction}$$

In the circular approximation, by virtue of the deflection theorem, the longitudinal coupling impedance estimate can be translated into an equivalent transverse coupling impedance

$$Z_\perp = \frac{2R}{\beta b^2} \ \frac{Z_{||}}{n-\nu}$$

If we take $|Z_{||}/n| \approx 10\Omega$ as previously estimated, then

$$|Z_\perp| \approx 0.2\ M\Omega/m$$

Finally there are transverse modes proper due to several resonanting structures, like rf cavities, but they are difficult to estimate and can usually be observed with the beam itself.

LONGITUDINAL SINGLE BUNCH INSTABILITY

To estimate the longitudinal stability of individual bunches in the AGS Booster one calculates the following complex quantity

$$U' - iV' = -i\frac{2eI_p\beta^2(Z/n)}{\pi|\eta|E(\Delta E/E)_{FWHM}}\frac{Q}{A}$$

where e = charge of the electron; Q = particle charge state; A = particle atomic mass number; β = ratio of particle velocity to speed of light; Z = the complex beam– environment coupling impedance; n = the harmonic number of the instability; E = the total energy *per nucleon* of the particle; $(\Delta E/E)_{FWHM}$ = the full–width half–maximum relative bunch energy spread.

Also

$$\eta = \gamma_T^{-2} - \gamma^{-2}$$

where $\gamma = E/E_0$; E_0 = the particle rest energy *per nucleon*; and γ_T is evaluated at the Booster transition energy. ($\gamma_T = 4.88$).

Finally I_p is the bunch peak current

$$I_p = \frac{NQe\beta c}{\sqrt{2\pi}\sigma}$$

N = the number of particles in a bunch; σ = rms bunch length. The bunches are assumed to have a bi–gaussian distribution.

In our notation Z = X + iY with X > 0 a resistance and Y is positive for a capacitive reactance and negative for an inductive reactance.

If there is no resistance, the reactance being positive (capacitive) and the accelerating cycle always below the Booster transition energy, the individual bunches are always stable.

Only the presence of a resistance in the coupling impedance can cause the bunches to be unstable. We can calculate the tolerances on X/n. The results are given in Table II. We show the values of U' with space charge at injection and extraction for each case. Based on the stability diagram shown in Fig. 2 we can then infer the maximum allowed values for V'. Since we are below the transition energy, sign $(K_0) > 0$.

The choice of V' depends critically on the shape of the energy distribution.

The range of U' for the proton beam during the acceleration cycle is shown in Fig. 2. With the exception of a truncated cosine distribution (8) and a first–order parabolic distribution (9), the beam bunch is always stable provided $V' < 0.4$, the limit being set by a second–order parabolic distribution (7) at top energy. This corresponds to the resistive impedance limit $X/n < 60\ \Omega$.

TRANSVERSE SINGLE BUNCH

In the worst case of no Landau damping, the growth rate of an instability within an individual bunch in the transverse plane is given by

$$\tau^{-1} = \frac{I_p r_0}{e\nu\gamma Z_0}\ Re\left(Z_\perp\right)$$

This formula is valid in the case that the growth rate is larger than a synchrotron oscillation period (fast head–tail instability). For a slower rate, one would recover the conventional head–tail instability that can be controlled by letting the chromaticity take a slightly negative value. (We are below the transition energy at all times!) Only the real part of the transverse coupling impedance $Z_\perp$ gives a contribution to the instability growth rate.

We could identify only the resistivity of the wall as the major contributor to the real part of $Z_\perp$, especially in the low frequency range when $n \approx \nu$. Inserting the values, we obtain for $\gamma \sim 1$, at injection, $\tau \approx 1$ msec.

The imaginary part enters in the stability criterion that can be written as

$$|Z_\perp| < \left(\frac{E_0}{e}\right)\frac{\pi\nu\beta\gamma}{I_p R}\ |(n-\nu)\eta + \xi|\ \frac{\Delta p}{p}$$

where $\Delta p/p$ is the bunch momentum spread, ξ the chromaticity. Because of the large contribution of space charge to the coupling impedance, the stability criterion cannot be easily satisfied. Nevertheless, in analogy to the longitudinal microwave instability, we expect that only perturbations with wavelengths shorter than the bunch length can cause instability. This corresponds to a harmonic number large enough when the coupling impedance is greatly reduced and, the growth rate is substantially smaller.

COUPLED BUNCHED INSTABILITIES

Using codes like ZAP and with analytical calculations, we found that the beam in the booster is *stable* against longitudinal bunch–to–bunch coherent motion. This is true for both the smooth wall components and the parasitic resonating modes.

The beam in the booster is *unstable* against transverse bunch–to–bunch motion. The instability is induced by the wall resistivity, with $n - \nu = 0.2$ being the predominant mode. There is not enough Landau damping to make the beam stable, especially at low energy, because of the very large space–charge forces. The estimated growth time is 3 ms. There is a need of a transverse damper acting on any bunch–to–bunch mode providing a damping rate of at least 300 s^{-1}.

Higher order parasitic modes do not seem again to cause any harm to the beam transverse stability.

REFERENCES

1. W.T. Weng, invited talk to this conference.

TABLE I Space Charge Z/n for Heavy Ion Beams in kΩ

	Carbon	Sulfur	Copper	Iodine	Gold
A	12	32	63	127	197
Q	6	14	21	29	33
$N \times 10^9$	22	6.7	4.7	3.2	2.2
$\beta_{injection}$	0.1262	0.1002	0.0782	0.0595	0.0478
$\beta_{extraction}$	0.8714	0.8716	0.8534	0.7900	0.6868
Z/n at injection	1.47	1.86	2.40	3.16	3.93
at extraction	0.23	0.23	0.27	0.40	0.65

TABLE II Bunch Stability Requirement in the Booster

	U′			
	Injection	Extraction	V′	X/n
Proton	0.67	0.90	0.4	60 Ω
Carbon	0.11	0.066	0.5	5.3 Ω
Sulfur	0.066	0.041	0.5	8.5
Copper	0.052	0.033	0.5	10.6
Iodine	0.034	0.021	0.5	16.7
Gold	0.019	0.012	0.5	29.2

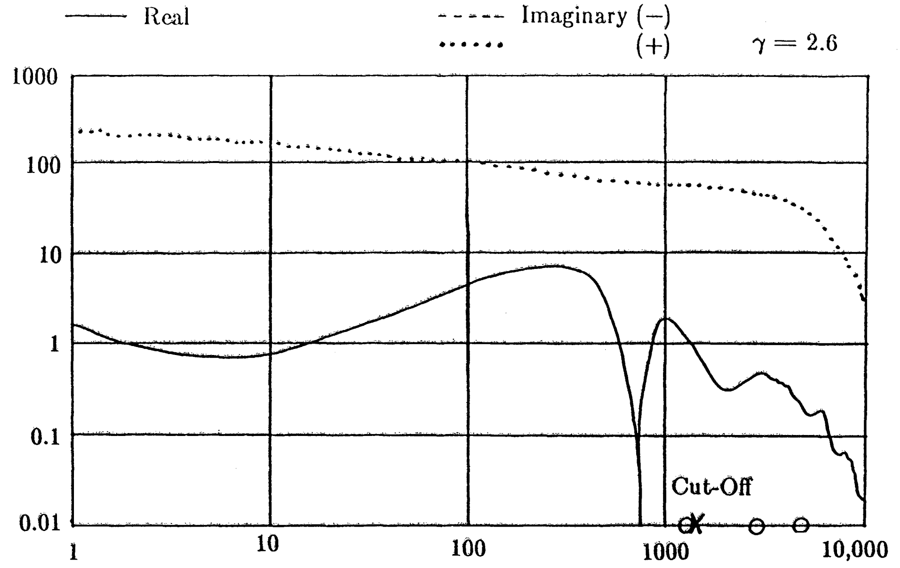

FIGURE 1 Z/n versus n, $\gamma = 2.6$.

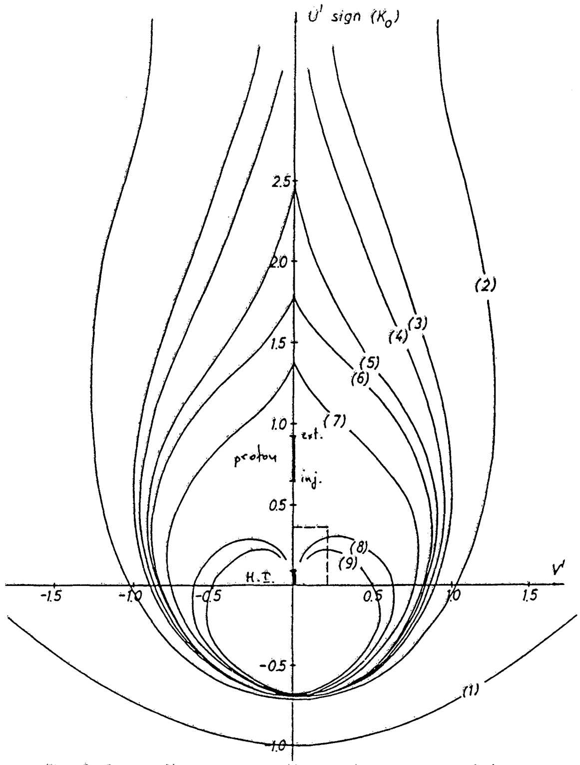

FIGURE 2 Stability diagram. Distributions: (1) Lorentzian, (2) Gaussian, (3) 5th-order parabolic, (4) 4th-order parabolic, (5) 3rd-order parabolic, (6) squared cosine, (7) 2nd-order parabolic, (8) truncated cosine, (9) 1st-order parabolic.