

## MEASUREMENT OF THE COHERENT BEAM-BEAM TUNE SHIFT IN THE TRISTAN ACCUMULATION RING

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**Abstract** In the TRISTAN accumulation ring, the coherent beam-beam tune shift was measured at 5.0 GeV in various optics. The emittance and the luminosity were derived from the observed tune shifts. They were compared with the design value of the emittance and with the measured value of the luminosity obtained by a luminosity monitor, respectively. Based on these comparisons, the relation between the incoherent beam-beam parameter and the coherent beam-beam tune shift is discussed.

### INTRODUCTION

It is well known that the beam-beam interaction is one of the most important factors that limit the luminosity of a colliding machine. The effect of the interaction is expressed by the beam-beam parameter  $\xi$ ;

$$\xi_{x,y} = r_e I \beta_{x,y} / 2\pi e f \gamma \sigma_{x,y} (\sigma_x + \sigma_y), \quad (1)$$

where  $r_e$  is the classical electron radius,  $I$  the bunch current,  $\beta_{x,y}$  the beta function,  $e$  the elementary charge,  $f$  the revolution frequency,  $\gamma$  the beam energy in units of the electron rest mass and  $\sigma_{x,y}$  the r.m.s. beam size. The beam-beam parameter is not able to be measured directly. What we can observe is the tune difference between the coherent 0-mode and  $\pi$ -mode oscillations,  $\Delta\nu$ .

Before our colliding experiment, there had been provided a few formulae which relate  $\Delta\nu$  to  $\xi$ . The typical ones are;

$$\cos 2\pi(\nu_0 + \Delta\nu) = \cos 2\pi\nu_0 - 4\pi\xi \sin 2\pi\nu_0 \quad (\text{by Piwinski}^1) \quad (2)$$

$$\cos 2\pi(\nu_0 + \Delta\nu) = \cos 2\pi\nu_0 - 2\pi\xi \sin 2\pi\nu_0 \quad (\text{by Hirata}^2), \quad (3)$$

where  $\nu_0$  is the unperturbed tune between collision points. For small  $\Delta\nu$  and  $\xi$ , these formulae are reduced to

$$\Delta\nu = 2\xi \quad (4)$$

$$\Delta\nu = \xi, \quad (5)$$

respectively. The latter was believed to be more reliable and actually gave acceptable estimation in the measurement of the tune shift in the TRISTAN main ring.<sup>3</sup> The estimated value of the emittance using Hirata's formula agreed with its design value within 15 % accuracy. However, in our measurements in the TRISTAN accumulation ring (AR), the discrepancy amounted to larger than 30 %. In order to settle the problem, more elaborated formula was devised by Yokoya.<sup>4</sup> The relation between  $\xi$  and  $\Delta\nu$  is given as

$$\Delta\nu_x/\xi_x = \Lambda(r) = 1.330 - 0.370r + 0.279r^2 \quad (6)$$

$$\Delta\nu_y/\xi_y = \Lambda(1-r), \quad (7)$$

where  $r$  is  $\sigma_y/(\sigma_x + \sigma_y)$ . In the following sections, we describe results the measurement of the coherent beam-beam tune shift and compare the experimental results with theoretical estimation by the above formulae and with luminosity measurements. Based on this comparison, the relation between  $\xi$  and  $\Delta\nu$  is discussed.

#### EXPERIMENTAL CONDITIONS

AR is a single ring collider with two interaction regions (IR's), the north IR and the south. A pair of electro-static separators are installed in each IR in order to separate electron and positron beams during injection at 2.5 GeV and acceleration up to the colliding energy. The locations of quadrupole magnets in the IR's are different from each other so that the beta function at the south interaction point (IP) can be squeezed to 3 cm or less. A luminosity monitor which counts Bhabha events is placed only at the south IP.<sup>5</sup>

The colliding experiment was performed at 5.0 GeV with a single bunch per beam. The coherent beam-beam tune shift and the luminosity were measured under five optics whose parameters are summarized in Table 1.

Four of the optics listed in Table 1 were designed for the double-IP operation and had the same  $\beta_y/\beta_x$  ratio at the both IP's. The optic D was used for single-IP operation where two beams were separated vertically at the north IP. To localize a bump orbit made by the separators, the vertical phase advance between the separators was adjusted to  $\pi$ . The distance between the beams was 3.4 mm at 5.0 GeV at the north IP and the long-range beam-beam tune shift was

estimated to be less than .001 in the vertical direction.

TABLE 1 Parameters of optics used in the experiment.

OPTICS	A	B	C	D	E
$\nu_x$	10.17	10.13	9.15	10.13	10.87
$\nu_y$	10.25	9.24	9.25	10.27	10.78
$\beta_{xS}$ (m)	2.0	1.5	1.5	1.4	2.0
$\beta_{yS}$ (m)	0.1	0.03	0.03	0.03	0.1
$\beta_{xN}$ (m)	5.0	12.5	10.0	(8.9)	12.5
$\beta_{yN}$ (m)	0.25	0.25	0.20	(1.5)	0.25
$\epsilon_{x0}$ ( $10^{-7}$ m rad)	1.66	2.24	2.65	1.71	1.64
# of IP's	2	2	2	1	2

### HORIZONTAL TUNE SHIFT

The beam-beam parameter of very flat beams,  $\xi_x$ , is related to the emittance  $\epsilon_x$  as

$$\xi_x \cong r_e I / 2\pi e f \gamma \epsilon_x, \quad (8)$$

and is independent of the beta function. We may normally expect the horizontal emittance  $\epsilon_x$  to be close to its design value. We can then estimate  $\xi_x$  by this expression.

The horizontal beam-beam tune shift per revolution was measured with the accuracy of 0.001 under the optics A, B, C and E in double-IP operation. The tune shift per collision,  $\Delta\nu_x$ , was obtained by dividing the observed values by 2 since it was expected to be almost equal at the both IP's. Results were plotted by full circles in Figure 1. Throughout the measurement, the currents of the electron and positron beams were equal to each other within 5 % and the vertical-to-horizontal emittance ratio was ascertained to be less than .025 by measuring the vertical and horizontal tune shifts simultaneously.

As is evident in Figure 1, the horizontal tune shift per collision  $\Delta\nu_x$  is proportional to the beam current, which suggests that the value of the emittance is approximately constant for each optics. Three solid lines in the figure show the expected values of the tune shift calculated by the three formulae (4), (5) and (6) with  $r=0$ , respectively, on the assumption that the emittance is kept to be its design value. Among the three formulae, eq.(6) reproduces excellently the behavior of the horizontal tune shift for all the four optics. In other words, estimated values of the emittance agrees very well with their design values using eq.(6), while they are smaller by more than 30 % using eq.(5).

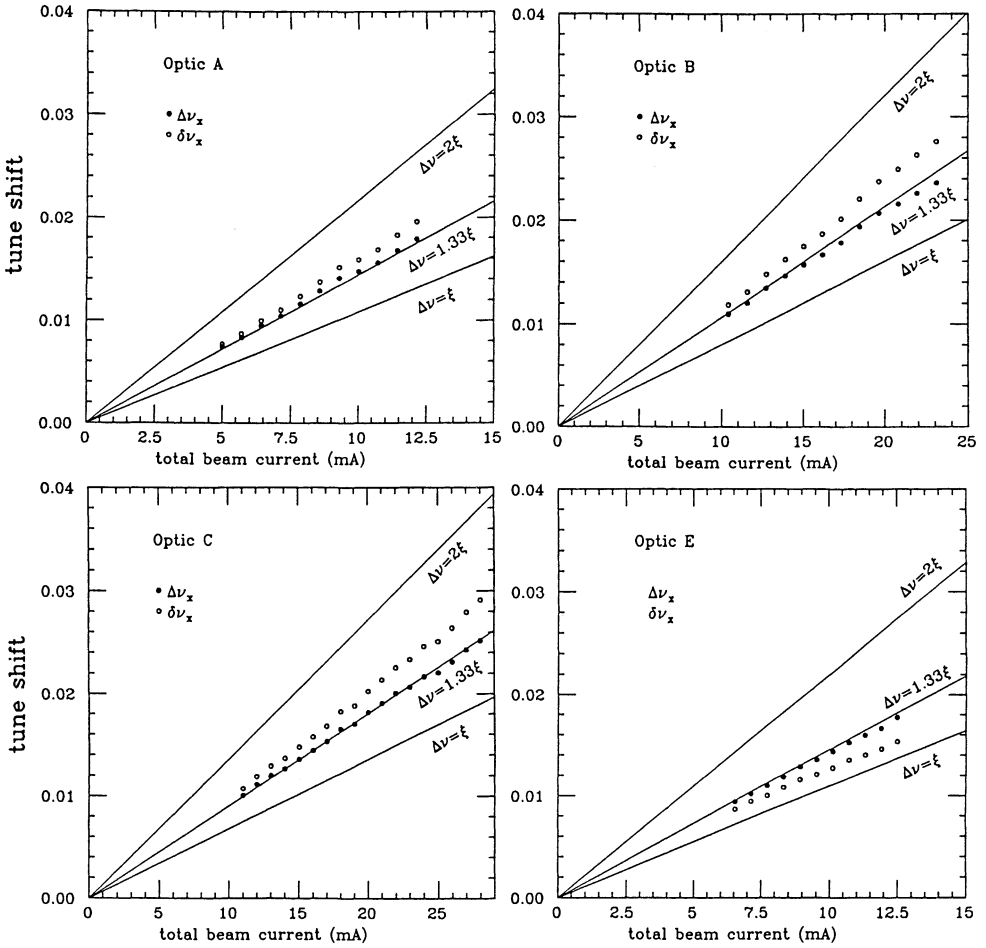


FIGURE 1 Coherent beam-beam tune shift per collision vs. total beam current.

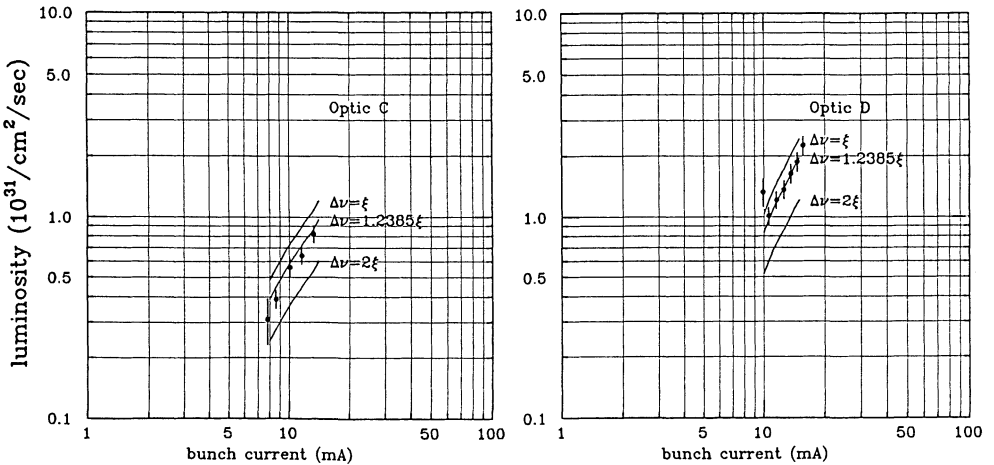


FIGURE 2 Luminosity vs. bunch current.

Because the formulae (6) and (7) were obtained considering only the linear term in the beam-beam parameter, they are correct only for small  $\Delta\nu$  and  $\xi$ . However, our data suggests that the formula (6) is valid up to the region where  $\Delta\nu_x$  is larger than 0.02.

In Yokoya's theory, the beam-beam force has been averaged over the ring. Accordingly, it has been ignored that the beam-beam interaction occurs only at the interaction point. In his paper, it has been suggested that the localization effects can be taken into account to some extent by using the formula like eqs.(2) and(3). Then we have introduced a parameter,  $\delta\nu$ , defined as

$$\cos 2\pi(\nu_0 + \Delta\nu) = \cos 2\pi\nu_0 - 2\pi\delta\nu \sin 2\pi\nu_0. \quad (9)$$

If this method is adequate, the following equation is more accurate than eq.(6);

$$\delta\nu_x / \xi_x = \Lambda(r). \quad (10)$$

The calculated values of  $\delta\nu_x$  from the measured tune shift  $\Delta\nu_x$  are plotted by open circles in Figure 1. Because  $\nu_0$  was close to an integer or a half-integer, the difference between  $\Delta\nu_x$  and  $\delta\nu_x$  was amount to more than 10 % in the higher current region. As seen in the figure,  $\delta\nu_x$  is systematically larger than  $\Delta\nu_x$  under the optics A, B, and C, while it is smaller under the optic E, since the fractional part of  $\nu_0$  is between 0.25 and 0.5 in the optic E. We may regard  $\delta\nu_x$  as depending linearly on the beam current. Thus we can estimate the emittance from  $\delta\nu_x$ . The estimated values using eq.(10) are different from their design values by 7~10 %. However, at this stage, we cannot determine which is better between eqs.(6) and (10), considering the expected accuracy of the measurements and the theory.

### VERTICAL TUNE SHIFT

In the vertical case, we compared the luminosity estimated from the vertical tune shift with the measured one by the luminosity monitor. In the case of very flat beams, the luminosity depends on the vertical beam-beam parameter  $\xi_y$  as

$$L \cong \gamma \xi_y I / 2e r_e \beta_y. \quad (11)$$

The vertical beam-beam tune shift and the luminosity were measured in both single-IP and double-IP operation under the optics C and D. Results are shown in Figure 2. Three solid lines in the figure correspond to the estimated values from the tune shift measurement by

correspond to the estimated values from the tune shift measurement by the three formulae (4), (5) and (7) with  $r=0$ . The error of the tune shift measurement is within  $\pm 5\%$ . The data obtained by the luminosity monitor is plotted with bars which show the statistical error only. The systematic error is estimated to be  $\pm 10\%$ .

As shown in the figure, only the formula (7) agrees with the luminosity measurement within the errors. However, the superiority of the formula over the other two is not so clear as in the horizontal case because of possible large errors in the luminosity measurement.

#### SUMMARY

The coherent beam-beam tune shift was measured at 5.0 GeV in the TRISTAN accumulation ring. Yokoya's formula explains well the experimental results. In the horizontal case, the estimated values of the emittance using eq.(6) agree very well with their design values in four different optics. In the vertical case, the estimated values of the luminosity using eq.(7) are also consistent with those obtained by the luminosity monitor within the errors.

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