Particle Accelerators, 1990, Vol. 27, pp. 21–26
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DAMPING THE TRANSVERSE RESISTIVE WALL INSTABILITY IN THE AGS BOOSTER*

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Abstract A transverse bunch-to-bunch feedback system will be installed on the AGS Booster to suppress coherent coupled bunch oscillations driven by the resistive wall impedance of the vacuum chamber when accelerating high intensity proton beams (circulating currents > 3 amp). It is possible to estimate the expected growth rate of the instability by scaling from the machine parameters and measurements made on the AGS. An upper limit of 1.5×10^3 sec^{-1} at 1.5 GeV kinetic energy with $\chi = 0$ and 0.5×10^{13} protons/bunch is obtained for the vertical plane.

The position signal of each bunch (h=3), at two locations separated by one quarter of a betatron wavelength, will be integrated, digitized and stored in a memory whose clock is synchronized with the acceleration frequency. This pair of inputs will be used in conjunction with look up tables, which will be related to a given range of the tune Q, to generate the proper correction amplitude. This will then be D/A converted, amplified and applied to 50Ω traveling wave kickers. The overall time delay will be adjusted to equal four revolution periods in order to permit the necessary digital signal processing ($T_o = 0.73$ μs at $\gamma = 2.6$). An analysis of the performance of this and similar systems for within the bunch modes m=0, 1 and both zero and non-zero head tail phase shift χ will be presented.

GROWTH RATE SCALING (AGS, BOOSTER)

We use the following expression due to Sacherer[1] for the growth rate due to the resistive wall impedance.

$$\Delta\omega_m = \frac{j}{(1+m)} \frac{e\beta I}{2Q\omega_o\gamma m_o 2\pi R} \left[\sqrt{\frac{\pi}{MB}}\, Z_\perp(\omega_o)\, F_m(\chi) + Z_\perp(\omega_p)\, F_m'(\chi-\omega_p\tau_\ell) \right] \quad (1)$$

Here I = total current = Nef_o; ω_o the rotation (angular) frequency; N the number of protons; M the number of bunches; $B = \ell M/2\pi R$ with ℓ the bunch length and R the machine radius; $\chi = \xi Q\omega_o\tau_\ell/\eta$ with τ_ℓ the bunch length, $\xi=\Delta Q/Q/\Delta p/p$ the chromaticity and $\omega_\xi=\chi/\tau_\ell$. $Z_\perp$ is the resistive wall impedance in ohm/meter which is $\sim KR\sqrt{|\omega|}/\omega$ where R is the

* Work performed under the auspices of the U.S. Dept. of Energy.

machine radius and K depends upon the vacuum chamber composition and dimensions. The form factors $F'(\chi)$, $F(\chi)$ are given in Reference 1. There it is assumed that the output of a position sensitive detector is $\Delta y \propto p_m(t) \exp j(\omega_\xi t + 2\pi kQ)$ for the kth revolution. Also, $p_m(t)$, the oscillating part of the charge distribution are assumed to be approximately sines or cosines.

We assume that the real part of the resistive wall impedance is the sole source of growth and note that for $\chi = 0$ the real part of $F_m(0) = 0$ so only the m = 0 mode is unstable. Now $\omega_p = (p+Q)\,\omega_o$ where p=n+kM, n=0, 1, --M and $-\alpha < k < \alpha$. The coupled bunch mode giving rise to the smallest negative value of ω_p will have the largest growth rate. In the Booster this will be $\omega_p = (-5+Q)\omega_o$ i.e. n=1, and since positive frequencies contribute to damping the mode giving rise to an $\omega_p = (-4+Q)\omega_o$, n=2, is stable. The other unstable mode n=0, will have a reduced growth rate since its smallest $\omega_p = (-6+Q)\omega_o$.

Now the Booster vacuum chamber is made of the same type of stainless steel and has essentially the same cross-section as the AGS so we shall assume the K is also the same. Thus, for $Q_B = 4.8$ and $Q_{AGS} = 8.8$ we have the smallest $\omega_p = -0.2\omega_o$ for both machines. For the same line charge density in both rings we find that $\Delta\omega_o$(Booster)= $(8.8\ F'_B/4\text{x}9.6\ F'_{AGS})\ \Delta\omega_o$(AGS) since $R_B = R_{AGS}/4$. Assuming the same τ_ℓ in both machines we note that $F'_B\ (.2\omega_o) < F'_{AGS}(.2\omega_o)$ therefore, the maximum growth rate for the Booster should be <1/4 that of the AGS for the same number of particles per bunch N_B. In the AGS the growth rate for 9 x 10^{12} protons on a 1.5 GeV kinetic energy flattop is 900 sec^{-1} in the vertical plane where $\xi \cong 0$. Thus, the maximum growth rate in the Booster for 1.5 x 10^{13} protons should be <1500 sec^{-1} at $\chi = 0$ and the same τ_ℓ.

DESCRIPTION OF THE DAMPING SYSTEM

The individual bunch difference signals from two pair of pickup electrodes located 90^o apart at the nominal tune of Q = 4.8, will be integrated on a turn-by-turn basis, digitized, normalized and stored in a serial memory. Thus, only the net dipole motion of the entire bunch will be sensed.

The combined correction signals from each bunch will be delayed

4 revolution periods (T_o) in order to permit digital signal processing, before being applied to a 50Ω travelling wave deflector located just upstream of one of the vertical pickups. For a fixed tune the required phase of the correction signal also remains a constant. Due to the long delay between measurement and kick small changes in tune call for large changes in the correction signal phase which the 90° electrode spacing can generate. Since the rotation period changes with energy it is necessary to vary the time delay between pickup and kicker. In a pure analogue feedback system this is done by switching cable lengths in the signal loop. In a digital system the correction signal is stored in a FIFO memory for a period of time T. It is planned to make $T = 10\ \tau_{rf} + \delta\tau$ in the Booster system ($T_o = 3T_{rf}$). $\delta\tau$ will be determined by measuring T_{rf} with a 250 MHz clock and using the result to control an eight bit programmable delay.

Damping Rate Calculations

For ideal damping one has $a = a_o e^{-\epsilon f_o t/2}$ where $\epsilon = \sqrt{\beta_k/\beta_p}(\Delta p/p)_\perp/(\Delta y/\beta_p)$ is a measure of the open loop (linear) gain of the feedback system. Here $f_o = \beta\ f_\infty$ is the rotation frequency and β_k, β_p are the beta functions at the kicker and pickup. For the $\Delta p_\perp$ produced by a pair of stripline kicker plates we can write

$$\Delta p_\perp(\omega) = \frac{(1+\beta)}{\beta}\ e\ \sqrt{Z_o\hat{P}}\ \frac{\ell k}{c}\ \frac{\sin\Theta}{\Theta}\ ;\ k = \frac{1}{\pi b}\sqrt{\frac{Z_o}{Z_c}}\ \frac{(1-b^2)}{a^2}\ \frac{\sin\phi}{\phi} \qquad (2,3)$$

where $Z_o = 377\Omega$, $\hat{P}$ is the peak power at a frequency ω delivered to the 50 impedance of the plate(s), ℓ their length, $\theta = \omega\ell/c$ and k is a geometrical factor that includes the effect of image currents in the vacuum chamber.[2] Here $Z_c = 50\Omega$, a is the outer radius of the kicker chamber b the radius of the deflection plates and ϕ their azimathal extent. We shall assume that $k = 4m^{-1}$, with β_p=13.5 m, β_k=11m, f_o=1.367 MHz. Then for $\omega = 0.25\omega_o$, $(\sin\Theta/\Theta) \approx 1$ and we obtain at 1.5 GeV with ℓ=1m a $(\Delta p/p)_\perp = 1\text{x}10^{-6}$ for $\bar{P}$=100W so that a $\Delta y = 1\text{x}10^{-6}$ x $12.2/2.2$ x 10^{-3} = 5.65mm should produce full power out of the amplifier ($\hat{P} = 2\ \bar{P}$ the average power) in order to obtain a damping rate of $1.5 \text{ x } 10^{-3}\text{sec}^{-1}$. Now we can write $\Delta p_\perp$ amd $Z_\perp$ in the following forms

$$\Delta p_\perp = \frac{e}{\beta c}\int_o^{2\pi R}(E + v \times B)_\perp ds;\ Z_\perp = \frac{j\int_o^{2\pi R}[E+vxB]_\perp ds}{\beta I\Delta y} \qquad (4,5)$$

where E and B are respectively the deflection fields of the kicker and those due to wall currents induced by the displacement of the current I by an amount Δy[1]. Hence, for the deflector we have $Z_{\perp} = \beta c \Delta p_{\perp}(\omega)/\beta I \Delta y(\omega)$ where the j drops out since we assume a 90^{o} phase shift between the measured displacement Δy and the kick $\Delta p_{\perp}$. Now one can also express $\Delta\omega_m$ as[1], (where $h_m(\omega)=|\tilde{p}_m(\omega)|^2$),

$$\Delta\omega_m = \frac{j}{(m+1)} \quad \frac{eI\beta}{\gamma m_o 2Q\omega_o 2\pi R} \quad \frac{\sum_p Z_{\perp}\,(\omega_p)\ h_m\,(\omega_p - \omega_{\xi})}{B\ \sum_p h_m\,(\omega_p - \omega_{\xi})} \tag{6}$$

Assuming χ=0 and inserting our expression for $Z_{\perp}$we obtain with $\bar{\beta}$=R/Q

$$\Delta\omega_m = \frac{j}{(m+1)}\ \frac{\beta c}{2\pi R}\ \sum_p \frac{\Delta p_{\perp} h_m}{2p\Delta y/\bar{\beta}}\ \frac{1}{B\sum_p h_m} = \frac{j}{(1+m)}\ \frac{fo}{2}\ \sum_p \varepsilon\ (\omega_p)\ F'_m(\omega_p) \tag{7}$$

Given the transfer function between $\Delta p_{\perp}(\omega)$ and $\Delta y(\omega)$ we can calculate the net damping rate by summing the terms $\epsilon(\omega_p)\ F'_m(\omega_p)$. In the case of pure analog feedback and for m = 0 one can generally have $\epsilon(\omega)$ a constant for all ω_p up to where $F'_o(\omega_p)\approx 0$. Then we obtain $\Delta\omega_o = jf_o\epsilon/2B$ for the damping rate.

Now $p_o(t)=\cos \pi\tau/\tau_{\ell}$ for m = 0 and it can be shown that for χ=0, $\Delta y_o \sim \cos\phi \cos \pi t/\tau_{\ell}$ where $\phi=2\pi kQ$. For m=1 we have $p_1=\sin 2\ \pi t/\tau_{\ell}$ and for χ=0 we obtain[3] $\Delta y_1 \sim (\sin 2\ \pi t/\tau_{\ell})\cos\phi$. For m=0, χ=0, integration of Δy_o gives $\delta y_o \sim (2\tau_{\ell}/\pi\tau_{rf})\cos\phi$. We now shall assume that $\tau_{\ell} = \tau_{rf}/2$ or $\phi_{\ell} = \pi$ which is approximately true in the Booster at 1.5 GeV. Then the voltage that is applied to the deflectors is a series of pulses of duration τ_{rf} whose amplitude is $\sim\delta y_o$ as shown in Fig. 1. We must now find the transfer function for this process. If we assume that the coherent mode (-5+Q) is present in a <u>continuous</u> beam then one would see a signal at $(5\text{-}Q)f_o$ when measuring Δy (t) at a position sensitive pickup. If we were to sample that signal at f_{rf} and locked to the bunch center in phase then one would obtain a similar $\delta y_o(t)$.

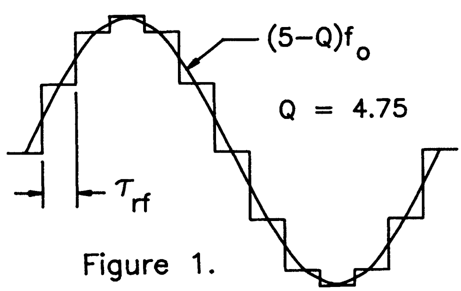

Figure 1.

Hence one can consider that the bunches constitute a sampling of the coherent signal $(5-Q)f_o$ and that the sampling function is $p_o(t)$. For the case m=0, χ=0 the δy_o signal is equivalent to sampling the signal with a δ function since the integral is always proportional to the peak amplitude of $\Delta y(t)$. Thus, the output pulse can be thought of as the "impulse" response of a "sampled data system", containing a zero order data hold, that is used to reconstruct the signal $(5-Q)f_o t$.[4] It can be written as $[1-\exp(-s\tau_{rf})]/s$ which by definition is the transfer function from $\Delta y(\omega)$ to $\Delta p_\perp(\omega)$. The full transfer function would be, $(\cos\phi/\pi)(\sin x/x)\exp[-j\omega(4T_o+\tau_{rf}/2)]$ for $s=j\omega$, $x=(\omega\tau_\ell)=(\omega\tau_{rf}/2)$, when the output voltage level is changed at the center of the bunch. If the loop delay is reduced by $\tau_{rf}/2$ (as shown in Figure 1) then one should multiply by $\exp(j\omega\tau_{rf}/2)$. Then $\epsilon(\omega)$ becomes

$$\epsilon(\omega) \sim \frac{A\beta_{max}}{p} \quad \tau_{rf} \quad \frac{\cos\phi}{\pi} \quad \frac{\sin x}{x} \quad \frac{e^{-j\omega 4T_o}}{\Delta y_o(\omega)\cos\phi}$$

Where $\Delta y_o(\omega) = \tilde{p}_o(\omega)$ and A is a gain factor. Now it can be shown that $\sum_p h_m(\omega_p)=\tau_\ell^2/2B$ so we can write $\epsilon_{eff} = \sum_p \epsilon(\omega_p)\, F'_m(\omega_p)$ as

$$\frac{8}{\tau_{rf}^2}\ \frac{\tau_{rf}}{\pi}\ \frac{A}{\pi}\ \frac{\beta_{max}}{p}\ \tau_{rf} \sum_p \frac{\sin x}{x}\ \frac{\cos(x/2)}{\left[1-\left(\frac{x}{\pi}\right)^2\right]}\ e^{-j\omega_p 4T_o}$$

where $\tilde{p}_o(\omega)=(2\tau_\ell/\pi)\cos(x/2)/[1-(x/\pi)^2]$. The summation out to $3\omega_p \cong 3\omega_{rf}$ gives ≈ 1 so that we obtain $\epsilon_{eff}=(8A/\pi^2)\ (\beta_{max}/p)\ (\Delta p_\perp(\omega)/\Delta y(\omega))$. Relative to an ideal analogue system giving the same damping rate the gain A would have to be 2/.834 = 2.4 times greater. Hence, the gain should be such that a displacement of .834 x 5.65 = 4.7mm peak will produce full output power since our initial calculation of the damping rate and hence ϵ did not include the 1/B factor. Now the $e^{-j\omega_p 4T_o}$ phase factor should really be written as $\exp j(\psi-4\omega_p T_o)$ where ψ is the phase of the correction signal. It can be written as $(\psi-2\pi\delta Q f_o 4T_o) = (\psi-2\pi 4\delta Q) = (2n+1)\pi/2$ where $\delta Q=(5-Q)$ or $(6-Q)$, since ψ will be made to track any changes in tune, i.e., δQ.

Next we consider the other potentially unstable coupled bunch mode (-6+Q) and evaluate the summation for m=0, χ=0. It turns out to be 0.975 for Q=4.75. Hence, the damping rate would be essentially the same but the growth rate for this mode would be only 0.12 times that of the (-5+Q) mode.

Finally, let us consider what happens for $\chi=\pi$ both for the m-0 and the m=1 modes since now the latter is potentially unstable. Then $\delta y_0 \sim (\cos \phi)/4$ due to the integration and $\delta y_1 \sim -2 \sin \phi/3\pi$. Now $\tilde{p}_1(\omega)=(4\tau_\ell/\pi)\sin(x/2)/[4-(x/\pi)^2]$ and in order to perform the summation over ω_p one must replace x by $(x-\pi)$ in $\tilde{p}_0(\omega)$ and $\tilde{p}_1(\omega)$. We obtain 0.71 for the $\chi=\pi$, m=0 case and 0.67 for the m=1, $\chi=\pi$ case. Thus, if A remains the same the damping rate for the m=0, $\chi=\pi$ case becomes $(\pi/4 \times 0.71)=0.56$ of the $\chi=0$ rate. However, the growth rate for this mode decreases by a factor of ≈ 10. This result can be obtained by using either equation 1 or 6. Hence, a finite amount of negative chromaticity is desirable to control the growth rate of this mode.

Now for the m=1 mode with $\chi=\pi$ the growth rate would be $\approx 0.5/(1+1)$ or one quarter of the m=0, $\chi=0$ value if it is unstable. On the other hand, the damping rate would be $(\pi \text{x} 2/3\pi)\text{x}.67\div(1+1)=.222$ of the $\chi=0$ m=0 value if the loop parameters were unchanged. For $\chi=\pi/2$ we find also that the loop gain is still less than the growth rate for the m=1 case. Here the m=0 growth rate is 58% of the $\chi=0$ value. We conclude that operating at small values of negative chromaticity would be helpful if the m=1 mode is near the intensity threshold for instability. This is because the growth rate would be less than the values calculated by equation 1 which is only valid well above the intensity threshold.

ACKNOWLEDGEMENT: The possibility of using a digital system with a four turn delay in the Booster was suggested by Y.Y.Lee and W.T.Weng.

REFERENCES:

1. F. Sacherer, "Transverse Bunched Beam Instabilities", CERN 77-13, 19 Jul '77, pg. 198-218 or CERN/PS/BR 76-21 1 Nov 76.
2. J.L. Pellegrin, "Design of an Electrode System for Beam Transverse Excitation", SPEAR-19, PEP-120, August 1975.
3. E. Raka, "Damping the Resistive Wall Instability in the AGS Booster", Booster Tech. Note #137, Accelerator Development Department, BNL.
4. J. Ragazzini, G. Franklin, Sampled Data Control Systems, McGraw-Hill, New York, 1958 p.34-37.