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DETUNING EFFECT IN A TRAVELLING-WAVE ACCELERATOR STRUCTURE DUE TO BEAM LOADING

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Operational experience on a 15-MeV conventional electron linac of the travelling-wave type shows that the operating microwave frequency should be varied according to the degree of beam loading to obtain the maximum energy gain or the minimum energy spread of the accelerated electron beam. In order to examine this phenomenon, a test-accelerator waveguide consisting of six cells has been made and the electron beam from a 15-MeV linac has been introduced into the waveguide. By varying the microwave phase for the electron bunch in the waveguide, the detuning effect of the waveguide due to beam loading has been studied. It is shown that the microwave phase shift produced by reactive beam loading can be approximately compensated by changing the external microwave frequency from the tuned frequency of the waveguide. This fact is consistent with operational experience on the 15-MeV linac.

A normal-mode analysis based upon microwave cavity theory is developed for interpreting the experimental results, and the detuning effect in a travelling-wave accelerator waveguide is theoretically defined.

1. INTRODUCTION

Electron linacs are widely used in the fields of radiation chemistry, medical diagnosis and therapy, and industry, as well as in nuclear and elementary-particle physics. It has been useful to produce very short pulses, of the order of picoseconds or, in contrast with this, pulses as long as possible, depending on the application. Another desirable beam property is a sharp energy spectrum of accelerated electrons, as is required for injection into an electron synchrotron.

A 15-MeV travelling-wave electron linac is at the Institute for Nuclear Study (INS), University of Tokyo, as an injector for a 1.3-GeV electron synchrotron.[1–2] Operational experience shows that the capture efficiency of the linac beam in the synchrotron is improved when the operating microwave frequency of the linac is set higher than the tuned frequency of the accelerator linac waveguide. This means that operation with higher frequency can provide better output beam in the sense of its energy spectrum. It has also been shown that the difference between the optimum operating frequency and the tuned frequency of the accelerator waveguide becomes larger as the accelerated beam intensity increases.[3] This suggests that beam loading affects the accelerating field and changes the acceleration characteristics of the linac.

In order to obtain a clear physical interpretation for the acceleration characteristics of the INS linac, the present study was planned and performed as a first step. The acceleration characteristics of the INS linac are studied in the companion paper "Detuning Effect in a Travelling-Wave Electron Linac".

Studies of beam-loading effects in standing-wave structures have been made by many authors. In these studies amplitude and phase changes in accelerating field due to beam loading and methods for compensating these effects have been discussed.[4–9] In travelling-wave structures, beam-loading effects in transient or stationary states have been studied up to now.[10–11] In these studies, beam-loading effects on the energy gain or energy spread of the accelerated beam and the problem of phasing for the optimum operation are mainly discussed. In particular, the behavior of electrons in a stationary state where the bunch is not synchronized with the microwave phase has been investigated in detail by G. A. Loew[11] and in relation to the phasing problem. He has analysed it by decomposing the accelerating field into an in-phase component and an out-of-phase component compared with the initial phase on which the bunch rides.

In our paper, beam-loading effects and the problem of compensation for them are investigated in relation to the resonant frequency and the operating frequency of the accelerator waveguide by means of the following viewpoint and method. A travelling-wave accelerator waveguide consists of unit cells, each of which is formed by the walls of two adjacent disks and a cylinder between the disks. These cells can be regarded as resonators that are strongly coupled with each other by the disk holes and which can be detuned by beam loading. With this point of view, one can deal with a travelling-wave structure by the methods developed for standing-wave structures.

We have studied the interaction of an electron beam with a travelling-wave accelerator structure from this standpoint. In this work, we have prepared a test-accelerator waveguide consisting of six constant-impedance cells. The electron beam from the INS linac has been passed through the test waveguide. The field induced by the beam and the amplitude and phase variations of the accelerating field have been measured. Compensation of the phase shift of the field in the structure by changing the external microwave frequency has also been examined.

These experiments have shown that the function of the accelerator waveguide depends strongly on the microwave phase at which the electron bunches ride. While a phase shift due to resistive beam loading has not been observed, the reactive loading has been shown to detune the accelerator waveguide and to produce a large phase shift.

These experimental results have been analyzed using the normal-mode analysis developed by J. C. Slater, T. Nishikawa and others.[12–15] Normal-mode analysis has so far been applied to standing-wave linacs or to the accelerating cavities of circular accelerators. In this paper we deal with travelling-wave structures for the first time by this method, in which the variation of the accelerating field and the detuning of the accelerator waveguide when electron bunches pass through the waveguide are discussed.

2. EXPERIMENTS WITH A TEST-ACCELERATOR WAVEGUIDE

2.1. Experimental Arrangements

A schematic diagram of experimental setup and a photograph of the test-accelerator waveguide are shown in Figs. 1 and 2, respectively. The test-accelerator waveguide is a constant-impedance travelling-wave structure consisting of six cells whose length is twice the guide wavelength. At both ends of the waveguide are mounted door knob microwave couplers as shown in Fig. 3. The tuned frequency of the test waveguide is the same as that of the INS linac since the test electron beam is provided by the INS linac.

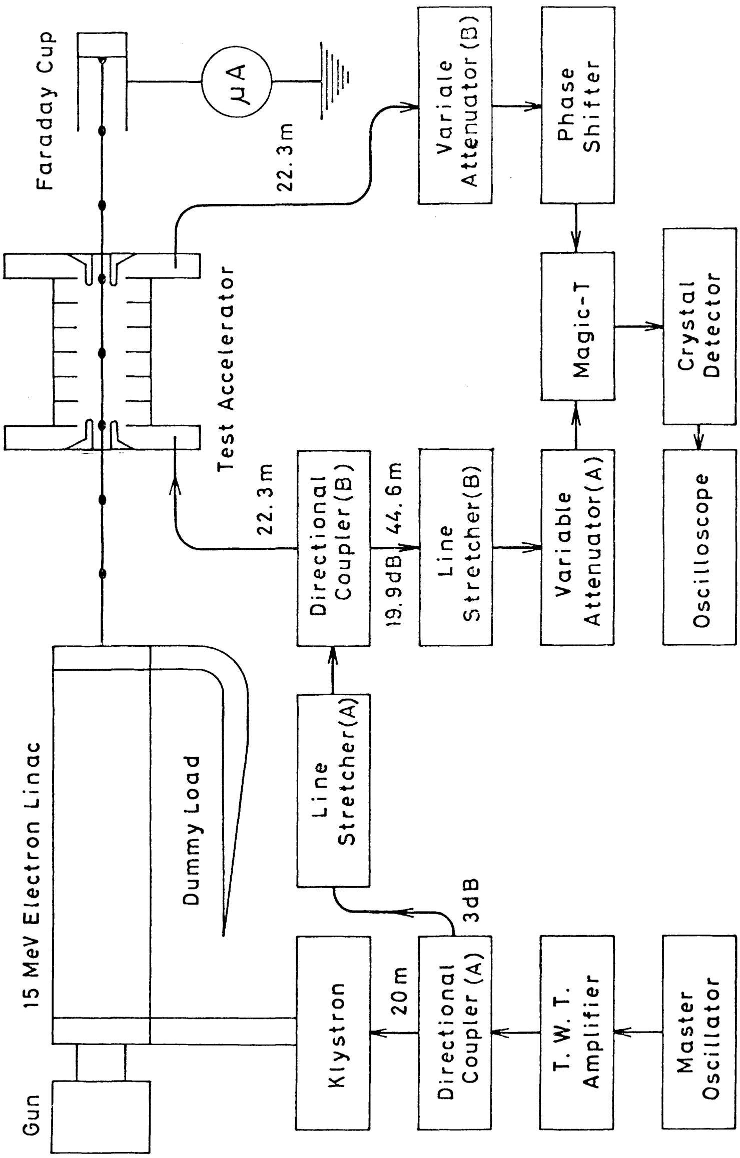

FIGURE 1 Schematic diagram of the experimental setup.

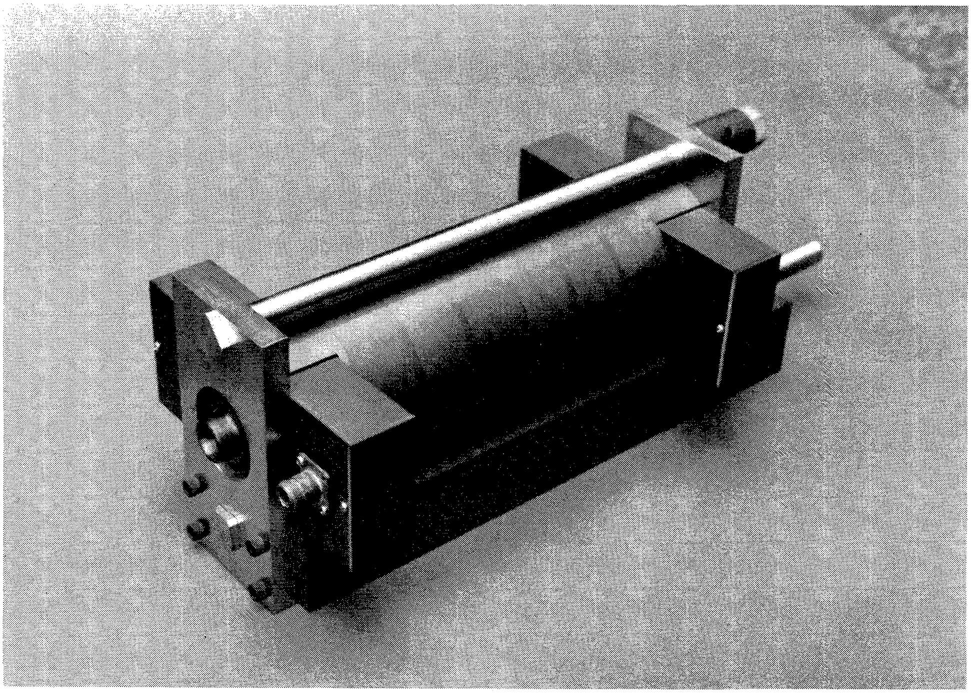

FIGURE 2 Photograph of the test accelerator waveguide assembled on a *V*-block.

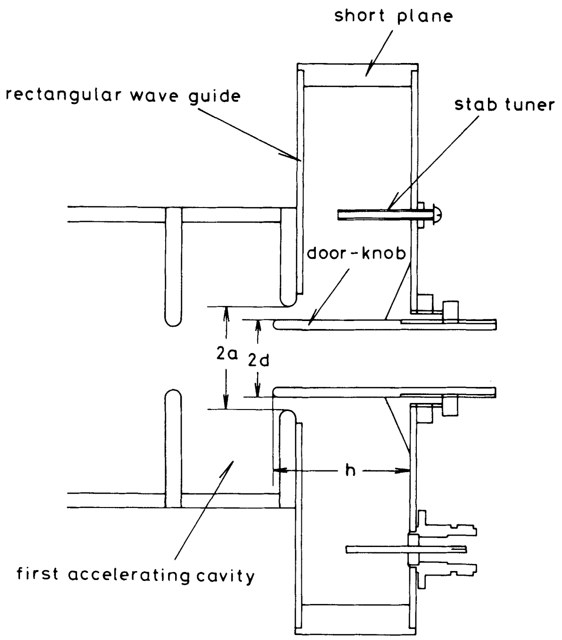

FIGURE 3 Cross-sectional view of the "door-knob" coupler.

TABLE I
Parameters of Test Accelerator Waveguide

Type of construction	constant impedance
Number of cavities	6
Diameter of disk hole	19.893 mm
Diameter of cylinder	84.587 mm
Distance between disks	36.233 mm
Disk thickness	5.000 mm
Phase accuracy	$120° \pm 2.5°$
Frequency	2758.00 MHz (at 30°C)
Normalized group velocity v_g/c	7.97×10^{-3}
Normalized phase velocity v_p/c	1.00
Unloaded Q	11400
Peak shunt impedance	58.0 MΩ/m
Microwave coupler	"door-knob" type
Diameter of coupling disk-hole ($2a$)	32.0 mm
Diameter of door-knob cylinder ($2d$)	24.0 mm
Height of door-knob cylinder (h)	44.0 mm
Voltage standing-wave ratios	below 1.1 in ± 1 MHz

The energy of the INS linac, 15 MeV, is sufficiently high to make the wave velocity in the structure equal to the light velocity. The dimensions and the microwave characteristics of the test waveguide are summarized in Table I.

The test-accelerator waveguide is set in a vacuum chamber on the beam line from the linac. Microwave power for the test waveguide is distributed from the travelling-wave tube amplifier in the microwave system of the linac so that the electron bunch and the microwave in the test waveguide are exactly synchronous with each other. The peak power and the pulse width of input microwave are 8.2 W and 5 μsec, respectively. The electron beam of the linac, which has a beam pulse width of 2 μsec, is collimated to have a diameter of 4.6 mm and transported to the test waveguide. In order to simulate beam-loading effects in a practical linac, the beam intensity has been determined so that the ratio of the beam current to the square root of input microwave power in the test waveguide is approximately the same as that for the usual linac. The beam current passing through the test waveguide is measured by a Faraday cup.

As described above, the present experiment is based on the condition that the test waveguide is working exactly like a travelling-wave accelerator. To confirm this condition, the test waveguide has been checked with the beam as follows: microwave power is dissipated by the beam when it is supplied to the input coupler, but there is no coupling to the beam when power is fed into the output coupler. Another proof that the test waveguide works as a travelling-wave structure is that the beam-induced power is emitted for the most part from the output coupler, the portion from the input coupler being weaker by 22 db.

2.2 Measurement Method and Experimental Results

The magnitude of the beam-induced field of the test waveguide was first measured as a function of the beam current without the external microwave. The electron bunch, whose frequency is closely adjusted to be the same as the resonant frequency of the test waveguide, is passed through the test waveguide and the induced power from the exit coupler is measured by using a calibrated crystal detector. The result is shown in Fig. 4.

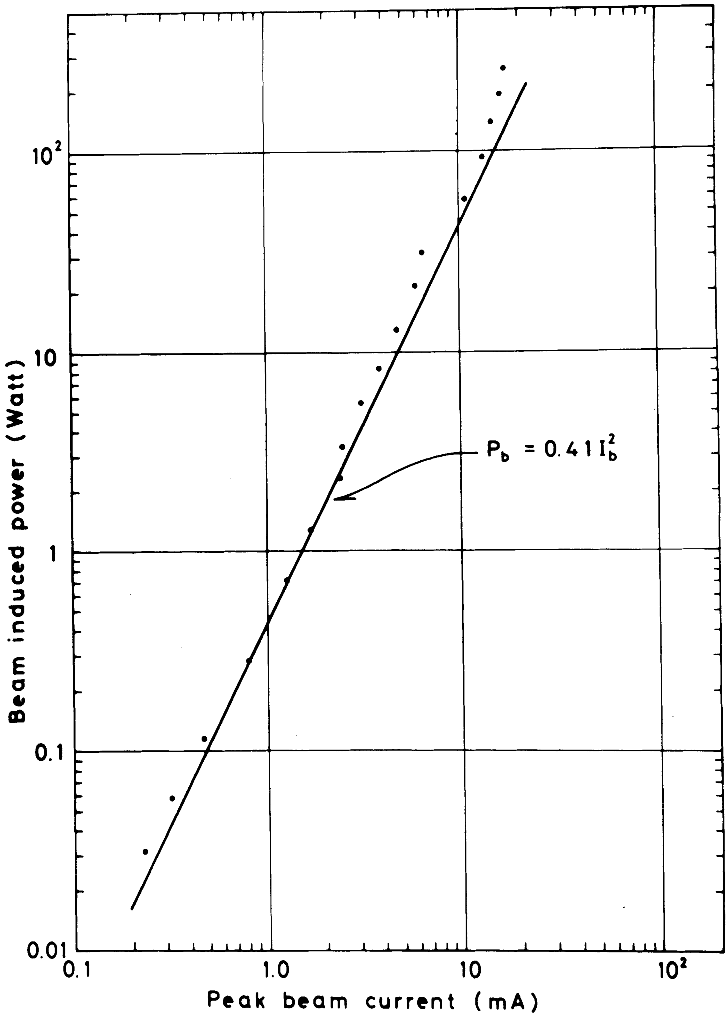

FIGURE 4 Beam-induced power measured as a function of the beam current. Solid line represents the calculated result.

The beam-induced power is seen to be proportional to the square of the beam current.

Next, the phase shift and the power variation of the accelerating field due to beam loading were measured by letting the bunch ride at varying phases with respect to the external driving field. In addition, in this measurement the bunch frequency is made to coincide with the resonant frequency of the test waveguide without beam loading. The measurement was performed with two different beam currents, 1.09 mA and 2.00 mA, in order to investigate the beam-intensity dependence. The phase shift of the microwave is measured in the following way (see Fig. 1): the microwave divided by a directional coupler (*A*) is supplied to the test waveguide through a line stretcher (*A*) and a directional coupler (*B*). The line stretcher (*A*) determines the phase of the accelerating microwave field for the bunch. A part of the input microwave to the test waveguide divided by the directional coupler (*B*) is sent through a line stretcher (*B*) and a variable attenuator (A) into a magic-T. At the same time, the output microwave from the test waveguide is transmitted through an attenuator (B) and a phase shifter to the magic-T, where the phase of the input microwave is compared with that of the output microwave from the test waveguide. The vector sum of the two microwaves is observed by a crystal detector on a oscilloscope. The phase shifter is always adjusted so as to minimize the vector sum of the two signals arriving at the magic-T. The amplitudes of the two microwaves are kept approximately equal by adjusting the attenuators (A) and (B). The difference of the readings of the phase shifter when the beam is on or off gives the phase shift of the accelerating field due to beam loading. The power variation can also be obtained by measuring the output power from the test waveguide. This result is shown in Fig. 5.

A measurement was also performed to see if the phase variation due to the above-mentioned reactive beam loading can be compensated by changing the frequency of the external driving field. This experiment is performed as follows: the phase shifter is adjusted without beam so as to minimize the vector sum of the two microwaves to the magic-T. When the beam is switched on, the phase of the accelerating field shifts due to reactive beam loading and the vector sum of the two microwaves deviates from the minimum value. The frequency of the external driving field is again adjusted to

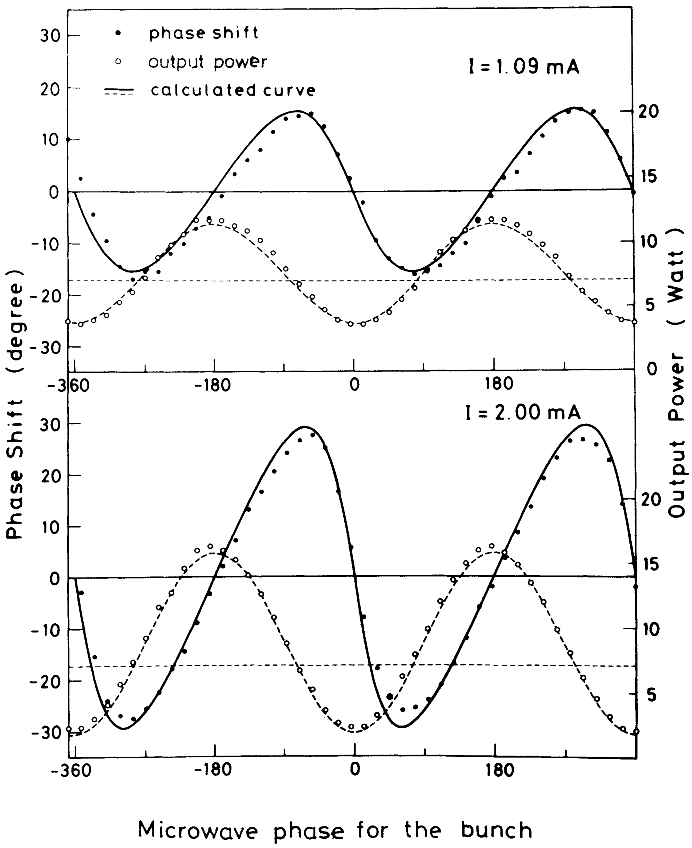

FIGURE 5 The phase shift and the output power of the microwave when the bunch rides on the various phase of the driving microwave. Input power of the driving microwave is 8.2 Watt.

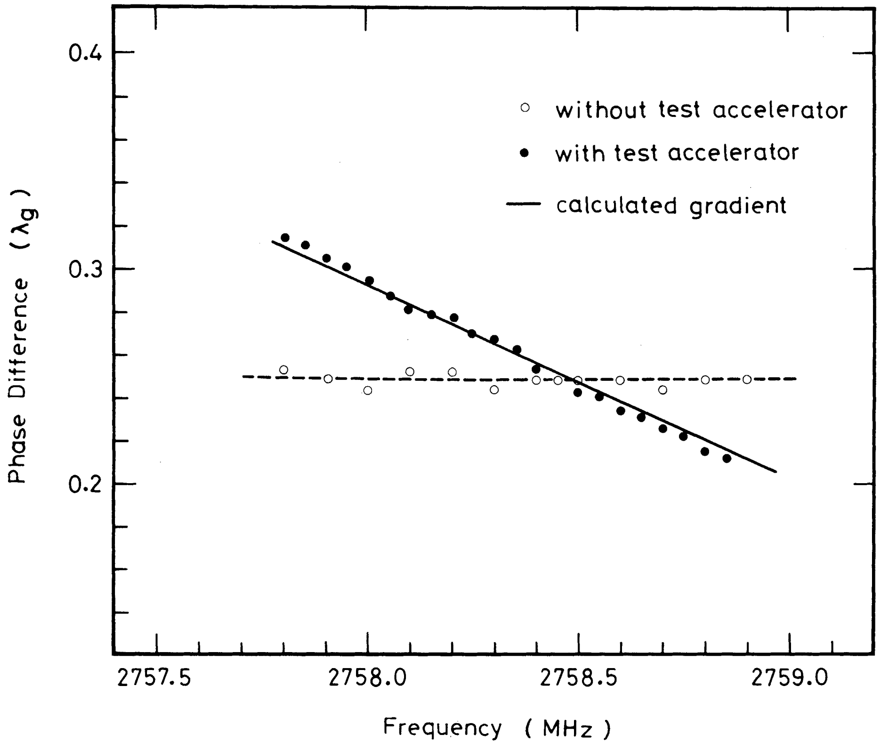

FIGURE 6 Frequency dependence of the phase difference of two microwaves coming into the magic-T

minimize the vector sum and the amount of frequency change is recorded. In this experiment, the following two problems were taken into account. First, the phase difference of the two microwaves input at the magic-T should depend only on phase shifts in the test waveguide when the frequency is changed. To avoid variation of the phase difference with line length, the lengths of the coaxial cables in both lines from the directional coupler (B) to magic-T are made equal with each other. The result is satisfactory, as shown in Fig. 6.

Another problem is that the phase of the driving field for the bunch at the entrance of the test waveguide, once set by the line stretcher (A), can vary when the frequency is changed. This variation of bunch phase caused by frequency change is examined as follows: the line stretcher (A) is adjusted to make the beam loading maximum as the frequency is changed. By this adjustment, the bunches are, on the average, situated on the crests of the external field. The shift of the bunch phase during the passage of the bunch through the test waveguide can be calculated. Thus the variation of the bunch phase in the compensation experiment can be estimated. The result is shown in Fig. 7. It is seen that the variation is within 21° when the frequency change is limited to ± 500 kHz. This variation is included in the experimental result as a systematic error.

The result of the compensating experiment is shown in Fig. 8. It is seen that the frequency must be changed to lower or higher values according to the phase on which the bunch initially rides.

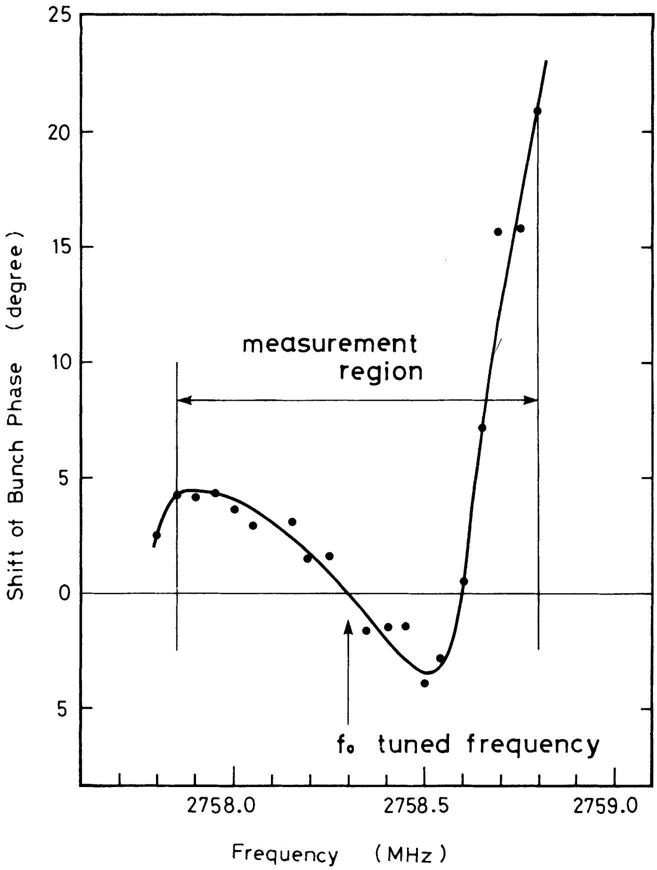

FIGURE 7 Displacement of microwave phase for the bunch at the entrance of the test waveguide caused by the frequency effect of the INS linac.

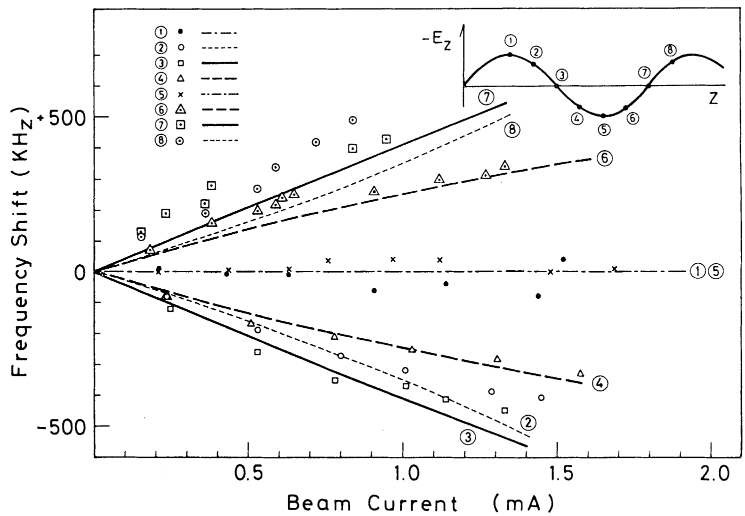

FIGURE 8 The amount of frequency change needed in compensating the phase shift due to beam loading. Numbers 1, 2, 3, ... indicate the phase of the driving microwave on which the bunch rides.

3. THEORETICAL ANALYSIS

3.1. Variation of the Accelerating Field due to Beam Loading

In this section, we will theoretically treat the accelerator waveguide by regarding it as a chain of unit resonant cavities, as described above. The wave equation in a unit cavity is obtained from Maxwell's equations and the orthogonality relations of the normal-field as[12–15]

$$\frac{d^2}{dt^2}\int EE_a{}^* \, dv + \frac{\omega_a'}{Q_L}\frac{d}{dt}\int EE_a{}^* \, dv + \omega_a'^2 \int EE_a{}^* \, dv = jU_0 e^{j\omega t} - \frac{1}{\epsilon_0}\frac{d}{dt}\int JE_a{}^* \, dv, \tag{3-1}$$

where U_0 is the magnitude of the driving microwave, J is the current density of the electron beam, E is the electric field. E_a is the ath normal-mode field, ω_a' is the resonant frequency without beam loading, Q_L is the loaded Q and ϵ_0 is the dieletric constant of the vacuum. E_a is represented as $E_a = E_{a_0} \exp(-jk_a z)$ in terms of the propagation constant of the microwave k_a and the amplitude of the normal-mode field, E_{a_0}.

The relation between the amplitude of the normal-mode field E_{a_0} and the peak shunt impedance per unit length r is

$$r = \frac{2Q_0 E_{a_0}{}^2 \, \delta z}{\epsilon_0 \omega_a'}, \tag{3-2}$$

where δz is the length of the unit cavity. The loaded Q of the unit cavity is

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}\cdot 1}} + \frac{1}{Q_{\text{ext}\cdot 2}},$$

where Q_0 is the unloaded Q and $Q_{\text{ext}\cdot 1}$, $Q_{\text{ext}\cdot 2}$ are the external Q's determined by the disk holes. $Q_{\text{ext}\cdot 1}$ and $Q_{\text{ext}\cdot 2}$ are equal to each other in this case. The external Q is given by the microwave group velocity v_g as

$$Q_{\text{ext}\cdot 1} = Q_{\text{ext}\cdot 2} = \frac{\omega}{v_g}\,\delta z.$$

Since the unloaded Q is sufficiently high, the loaded Q is approximately

$$Q_L = \omega \, \delta z / 2v_g.$$

The right-hand side of Eq. (3-1) represents the oscillatory force of the driving microwave and of the bunched beam current. The beam-force term is

$$-\frac{1}{\epsilon_0}\cdot\frac{d}{dt}\int JE_a{}^* \, dv = -j\frac{2FI_0 E_{a_0}\,\delta z\omega}{\epsilon_0}\, e^{j(\omega t + \phi_b)}, \tag{3-3}$$

where ϕ_b is the phase of the external field driving the bunch, F is the form factor of the bunch and I_0 is the mean beam current during one pulse.

Substituting Eq. (3-3) into Eq. (3-1) and taking the initial condition $\int EE_a^*\,dv = 0$ at $t = 0$, the solution for $\int EE_a^*\,dv$ of Eq. (3-1) is

$$\int EE_a^*\,dv = \frac{1}{\Omega}\left(jU_0 - \frac{j\omega 2FI_0E_{a0}\,\delta z}{\epsilon_0}\,e^{j\phi_b}\right)\left(1 - e^{-(\omega_a'/2Q_L)t}\,e^{j(\omega_a'-\omega)t}\right)e^{j\omega t}, \tag{3-4}$$

where

$$\frac{1}{\Omega} \equiv \frac{\dfrac{Q_L}{j\omega\omega_a'}}{\sqrt{\left\{\dfrac{2(\omega_a'-\omega)}{\omega}Q_L\right\}^2 + 1}}\,e^{j\psi} \doteqdot \frac{Q_L}{j\omega\omega_a'}\,e^{j\psi}, \tag{3-5}$$

$$\psi \equiv \tan^{-1}\left\{\frac{2(\omega_a'-\omega)}{\omega}Q_L\right\} = \tan^{-1}\left\{\frac{\omega_a'-\omega}{v_g}\,\delta z\right\} \doteqdot \frac{\omega_a'-\omega}{v_g}\,\delta z. \tag{3-6}$$

Equation (3-4) is rewritten, using the approximation of Eqs. (3-5) and (3-6) as

$$\begin{aligned}\int EE_a^*\,dv = {} & \left(\frac{Q_L}{\omega\omega_a'}U_0 - \frac{Q_L}{Q_0}\frac{2FI_0E_{a0}Q_0\,\delta z}{\epsilon_0\omega_a'}\,e^{j\phi_b}\right)\\ & \times\left(1 - e^{-(a'/2Q_L)t}\,e^{j(\omega_a'-\omega)t}\right)e^{j(\omega t+\psi)},\end{aligned} \tag{3-7}$$

where Q_L/Q_0 is approximately $(1 - e^{-(\omega/2v_gQ_0)\,\delta z})$.

Therefore the electric field in the unit cavity, $E = E_a\int EE_a^*\,dv$, is

$$\begin{aligned}E = {} & \left\{\frac{Q_LE_{a0}}{\omega\omega_a'}U_0 - FI_0r\,e^{j\phi_b}\left(1 - e^{-(\omega/2v_gQ_0)\,\delta z}\right)\right\}\\ & \times\left(1 - e^{-(\omega_a'/2Q_L)t}\,e^{j(\omega_a'-\omega)t}\right)e^{j(\omega t - k_az+\psi)}\end{aligned} \tag{3-8}$$

The term $(1 - e^{-(\omega_a'/2Q_L)t}\,e^{j(\omega_a'-\omega)t})$ in the equation represents the transient state. Since the steady state is of interest in the present analysis, this term is set equal to unity. When the frequency of the driving microwave is different from the resonant frequency of the cavity, the phase shift ψ given by Eq. (3-6) appears in the unit cavity.

In the following, the accelerating field after propagating through n unit cavities is found. The microwave phase for the bunch ϕ_b changes along the waveguide from the initial value ϕ_{b0} according to

$$\phi_b = (\omega_bt - k_bz) - (\omega t - k_az) + \phi_{b0}, \tag{3-9}$$

where ω_b and $k_b = \omega/v_e$ are the angular frequency and propagation constant defined for the bunch. Since ω_b is naturally equal to ω, $\phi_b = (k_a - k_b)z + \phi_{b0}$.

When ψ and the initial value of the driving field are expressed as $\psi_0\,\delta z$ and $E_0\exp(-(\omega/2v_gQ_0)\,\delta z)$, respectively, and the attenuation in the unit cavity, $\exp(-(\omega/2v_gQ_0)\,\delta z)$, is taken into consideration, the accelerating field E_n in the nth unit cavity is given by the geometrical progression

$$E_n = \Big([\{E_0 e^{-(\omega/2v_gQ_0)\delta z} - FI_0 r\, e^{j\phi_{b0}}(1 - e^{-(\omega/2v_gQ_0)\delta z})\} e^{-(\omega/2v_gQ_0)\delta z + j\psi_0 \delta z}$$

$$- FI_0 r\, e^{j\phi_{b0}}(1 - e^{-(\omega/2v_gQ_0)\delta z}) e^{j(k_a - k_b)\delta z}] e^{-(\omega/2v_gQ_0)\delta z + j\psi_0 \delta z}$$

$$- \cdots\cdots\cdots\cdots\cdots\cdots \Big) e^{j\psi_0 \delta z}\, e^{j(\omega t - k_a z)}$$

$$= \Big\{E_0 e^{-(\omega/2v_gQ_0)n\delta z + jn\psi_0\delta z} - FI_0 r\, e^{j\phi_{b0}}(1 - e^{-(\omega/2v_gQ_0)\delta z})$$

$$\times \sum_{m=1}^{n} e^{j(k_a - k_b)(m-1)\delta z}\, e^{-(\omega/2v_gQ_0)(n-m)\delta z + j(n+1-m)\psi_0\delta z}\Big\} e^{j(\omega t - k_a z)}$$

$$= \Big\{E_0 e^{-(\omega/2v_gQ_0)n\delta z + jn\psi_0\delta z} - FI_0 r\, e^{j\phi_{b0}}(1 - e^{-(\omega/2v_gQ_0)\delta z})$$

$$\times \frac{e^{j(k_a - k_b)n\delta z} - e^{-(\omega/2v_gQ_0)n\delta z + jn\psi_0\delta z}}{e^{j(k_a - k_b)\delta z - j\psi_0\delta z} - e^{-(\omega/2v_gQ_0)\delta z}}\Big\} e^{j(\omega t - k_a z)} \tag{3-10}$$

Since the denominator of the second term can be approximated by $(1 - \exp(-\omega/2v_gQ_0)\delta z)$, E_n can be simplified as

$$E_n = \{E_0 e^{-(\omega/2v_gQ_0)n\delta z + jn\psi_0\delta z} - FI_0 r\, e^{j\phi_{b0}}(e^{j(k_a - k_b)n\delta z}$$

$$- e^{-(\omega/2v_gQ_0)n\delta z + jn\psi_0\delta z})\} e^{j(\omega t - k_a z)}$$

$$= \{E_0 e^{-(\omega/2v_gQ_0)n\delta z} - FI_0 r\, e^{j\phi_{b0}}(e^{j(k_a - k_b - \psi_0)n\delta z}$$

$$- e^{-(\omega/2v_gQ_0)n\delta z})\}\, e^{jn\psi_0\delta z}\, e^{j(\omega t - k_a 3)} \tag{3-11}$$

Replacing $n\,\delta z$ by z and E_n by $E(z)$,

$$E(z) = \{E_0 e^{-(\omega/2v_gQ_0)z} - FI_0 r\, e^{j\phi_{b0}}(e^{j(k_a - k_b - \psi_0)z} - e^{-(\omega/2v_gQ_0)z})\}$$

$$\times e^{j(\omega t - (k_a - \psi_0)z)} \tag{3-12}$$

The first and the second terms of the right-hand side of Eq. (3-12) represent the attenuation of the external driving field and the buildup of the beam-induced field, respectively. The term $e^{j(k_a - k_b - \psi_0)z}$ represents the oscillation due to the velocity difference between the bunch and the accelerating field.

Next, the microwave power variation in the accelerator waveguide due to beam loading is estimated. The electric field amplitude is given, using Eq. (3-12), as

$$E_p = [\{E_0 e^{-(\omega/2v_gQ_0)z} - E_b(\cos(\phi_{b_0} + k'z) - \cos\phi_{b_0}\, e^{-(\omega/2v_gQ_0)z})\}^2$$
$$+ E_b{}^2\{\sin(\phi_{b_0} + k'z) - \sin\phi_{b_0}\, e^{-(\omega/2v_gQ_0)z}\}^2]^{1/2}, \tag{3-14}$$

where $k' = k_a - k_b - \psi_0$ and $E_b = FI_0 r$. The power flow in the accelerator waveguide is the product of the stored energy per unit length W and the group velocity v_g and is expressed in terms of the electric-field amplitude as

$$p = v_g W = \frac{v_g Q_0 E_p{}^2}{r\omega}. \tag{3-15}$$

By substituting Eq. (3-14) into Eq. (3-15), we obtain the microwave power flow at position z as

$$P(z) = P_0(z)\left[1 + \frac{E_b^2(1 - 2\cos k'z\, e^{-(\omega/2v_gQ_0)z} + e^{-(\omega/v_gQ_0)z}}{E_0^2 e^{-(\omega/v_gQ_0)z}} - \frac{2E_b(\cos(\phi_{b_0} + k'z) - \cos\phi_{b_0} e^{-(\omega/2v_gQ_0)z})}{E_0 e^{-(\omega/2v_gQ_0)z}}\right] \tag{3-16}$$

$$P_0(z) = \frac{v_g Q_0}{r\omega} E_0^2 e^{-(\omega/v_gQ_0)z} = P_{\text{in}} e^{-(\omega/v_gQ_0)z}, \tag{3-17}$$

where $P_0(z)$ represents the power flow at z when beam loading is not present and P_{in} is the input power to the accelerator waveguide.

3.2. Resonant Frequency Shift

The effect of beam loading on the resonant frequency of the unit cavity can be generally expressed[13] as

$$j\left(\frac{\omega}{\omega_a'} - \frac{\omega_a'}{\omega}\right) + \frac{1}{Q_L} = \frac{j}{\epsilon_0\omega\omega_a'}\frac{\frac{d}{dt}\int JE_a^*\,dv}{\int EE_a^*\,dv}. \tag{3-18}$$

The value of ω that satisfies Eq. (3-18) is the resonant frequency of the unit cavity with beam loading. Substituting Eqs. (3-3) and (3-12) into the above equation, we find

$$j\left(\frac{\omega}{\omega_a'} - \frac{\omega_a'}{\omega}\right) + \frac{1}{Q_L} = \frac{-2FI_0E_{a_0}^2\,\delta z}{\epsilon_0\omega_{a'}}\frac{e^{j(\omega t - k_bz + \phi_{b_0})}}{\{E_0e^{-(\omega/2v_gQ_0)z} - FI_0r\,e^{j\phi_{b_0}}(e^{jk'z} - e^{-(\omega/2v_gQ_0)z})\}e^{j(\omega t - (k_a - \psi_0)z)}}. \tag{3-19}$$

When E_{a_0} is eliminated by using Eq. (3-2) and FI_0r is replaced by E_b, Eq. (3-19) can be rewritten as

$$j\left(\frac{\omega}{\omega_a'} - \frac{\omega_a'}{\omega}\right) + \frac{1}{Q_L} = \frac{1}{Q_0}\frac{-E_be^{j\phi_{b_0}}\,e^{jk'z}}{E_0e^{-(\omega/2v_gQ_0)z} - E_be^{j\phi_{b_0}}(e^{jk'z} - e^{-(\omega/2v_gQ_0)z})}. \tag{3-20}$$

The imaginary part of the right-hand side of Eq. (3-20) represents the reactive component of the beam loading and its contribution to the resonant frequency shift $\Delta\omega \equiv \omega - \omega_a'$ is

$$\Delta\omega = \frac{\omega_a'}{2Q_0}\frac{-E_b^2T\sin k'z - E_0E_bT\sin(\phi_{b_0} + k'z)}{E_0^2T^2 - 2E_0E_bT\{\cos(\phi_{b_0} + k'z) - T\cos\phi_{b_0}\} + E_b^2(1 + T^2 - 2T\cos k'z)}, \tag{3-21}$$

where T is the attenuation factor of the electromagnetic field in the accelerator waveguide, $\exp(-\omega/2v_gQ_0 z)$. The real part represents the resistive component and gives the beam Q, Q_b, which is determined by the microwave power dissipation by the beam. That is

$$\frac{1}{Q_b}=\frac{1}{Q_0}\frac{E_0E_bT\cos(\phi_{b_0}+k'z)-E_b^2(1-T\cos k'z)}{E_0^2T^2-2E_0E_bT\{\cos(\phi_{b_0}+k'z)-T\cos\phi_{b_0}\}+E_b^2(1+T^2-2T\cos k'z)}. \tag{3-22}$$

The resonant frequency shift due to the real part is

$$\Delta\omega = \omega_a'\left\{\sqrt{1-\left(\frac{1}{2Q_L}+\frac{1}{2Q_b}\right)^2}-\sqrt{1-\left(\frac{1}{2Q_L}\right)^2}\right\}.$$

The phase shift of the accelerating field in the unit cavity can subsequently be estimated from the resonant frequency shift as follows: When the resonant frequency of the accelerator wave-guide changes by Δf, the phase velocity of the accelerating field is, assuming the group velocity to be constant around the resonant frequency, given in the form

$$v_p = \frac{v_g\lambda_g f_0}{v_g-\lambda_g\Delta f}, \tag{3-23}$$

where λ_g and f_0 are the guide wavelength of the accelerating field and the original resonant frequency before its change, respectively.

When the phase velocity of the accelerating field before the change of resonant frequency is represented by v_{p_0}, the phase difference $\Delta\phi$ between the accelerating fields having the phase velocity of v_p and v_{p_0} can be expressed as

$$\Delta\phi = 2\pi f_0\,\delta z\left(\frac{1}{v_{p_0}}-\frac{1}{v_p}\right). \tag{3-24}$$

By using Eq. (3-23), $\Delta\phi$ is given by

$$\Delta\phi = 2\pi f_0\,\delta z\left(\frac{1}{v_{p_0}}-\frac{v_g-\lambda_g\Delta f}{v_g\lambda_g f_0}\right) = \delta z\,\frac{\Delta\omega}{v_g}. \tag{3-25}$$

4. DISCUSSION AND CONCLUSION

First, the dependence of the beam-induced microwave power P on the beam intensity I_0 is examined. The power P as a function of the I_0 is given by

$$P = \frac{v_gQ_0F^2I_0^2r}{\omega}(1-e^{-(\omega/2v_gQ_0)z})^2. \tag{4-1}$$

This expression can be deduced from general considerations of bunch-cavity interaction and is not inherent in the theory developed in the preceding section. The aim here is to find if we can approximate the bunch form factor F by 1 by comparing the expression (4-1) with the experimental result. Overall consistency of the experimental procedure can also be inspected by the comparison. Substituting the parameters of the test accelerator waveguide, $v_g = 23.9 \times 10^7$ cm/sec, $Q_0 = 11400$, $\omega = 2\pi \times 2758 \times 10^6$, $r = 58 \times 10^6\ \Omega$/m and $z = 21.7$ cm, into Eq. (4-1), we find

$$P = 4.1 \times 10^5 \times I_0^2 \text{ (Watt)},$$

where I_0 is expressed in units of Amperes. The variation of P with I_0 is shown by the solid line in Fig. 4, together with the experimental results. The calculation agrees well with the experiment and the assumptions for the succeeding discussion can therefore be said to be justified.

Next, effects of beam loading on the microwave phase and power flow observed at the exit of the test waveguide are discussed. Theoretical estimations for these effects are given by Eqs. (3-17) and (3-25) in the preceding section. Numerical results for the test waveguide are shown in Fig. 5 as a function of the microwave phase for the bunch and of the accelerated beam current. It is seen that both the phase and the power of the microwave output agree well with the theoretical calculations of the normal-mode analysis developed for the microwave behaviour in a travelling-wave accelerating waveguide. It is also seen that the electron beam detunes the waveguide and produces a phase shift analogous to that usually observed in standing-wave accelerating cavities.

Among the results of the experiment, the behaviour of the microwave phase is of special interest. As seen in Fig. 5, the variation of the amount of phase shift is steepest for a bunch phase around 0° and the value of the phase shift reaches maximum at a bunch phase just below 90°. The polaritys of phase shift are opposite for bunch positions before and after the accelerating-field crest. When the electron bunch rides just on the field crest, namely, the beam loading is resistive, no phase shift is observed under the present experimental conditions.

As the degree of beam loading is increased, the amount of phase shift as well as its rate of variation with input phase become larger. For a beam current of 2mA, the phase shift amounts to 30° at maximum. These microwave phase shifts can produce a large effect in the acceleration characteristics of the waveguide. The microwave power from the exit coupler varies sinusoidally according to the phase on which the bunch rides. These results are similar to beam-loading effects in a standing-wave accelerator.[16]

We next discuss an experiment to compensate the microwave phase shift at the exit of the test waveguide by changing the operating microwave frequency. The amount of frequency change needed for the compensation as obtained by experiment is shown in Fig. 8. Theoretical estimates for this change are given by Eq. (3-25) in the preceding section, the result of which is also plotted in Fig. 8. Though the experimental values are somewhat scattered, the calculation is consistent with the experimental results.

For a single-cell cavity, the phase distortion of the accelerating field that is produced by the detuning due to beam loading can be compensated perfectly by changing the operating frequency to a new resonant frequency. For a travelling-wave structure, we cannot say simply that the frequency that is determined so as to compensate the phase shift is equal to a new resonant frequency of the structure, because the amount of the resonant-frequency shift is not constant, but varies along the accelerator structure, as seen from Eq. (3-21), and the compensation of the phase shift which appears at the exit of the structure corrects the detunings of the cells of the structure only on the average. However, this phase-shift compensation has a large effect in improving the acceleration efficiency in the case that the beam is accelerated with reactive phase.

From the above considerations, one can explain certain beam-loading phenomena observed on the INS 15-MeV linac. A detailed description of the work on the INS linac is found in the next paper "Detuning Effect in a Travelling-Wave Electron Linac".[3]

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REFERENCES

1. Annual Report, Institute for Nuclear Study, University of Tokyo, 57 (1974).
2. T. Katayama, S. Arai, K. Yoshida, and J. Tanaka, INS-Report 240 (1975).
3. S. Arai, K. Kobayashi, E. Tojo, and K. Toshida, *Particle Accelerators*, **15**, 99 (1984).
4. R. L. Gluckstern, Minutes of the Conference on Proton Linear Accelerators at Yale University, 95 (1963).
5. T. Katayama, T. Fukushima, and T. Yamakawa, *Jpn. J. Appl. Phys.*, **19**, 11, 2229 (1980).
6. T. Nishikawa, North-Holland Publishing Company, Amsterdam, 809 (1970).
7. J. S. Fraser, J. Mckeown, G. E. Mcmichael, and W. T. Diamond, Proc. of the 1976 Proton Linear Accelerator Conference, ed. S. O. Schriber (AECL-5677, Ontario, 1976) 166.
8. F. Pedersen, *IEEE Trans. Nucl. Sci.*, **NS-22**, 1906 (1975).
9. J. E. Griffin, *IEEE Trans. Nucl. Sci.*, **NS-22**, 1910 (1975).
10. J. E. Leiss, *IEEE Trans. Nucl. Sci.*, **NS-12**, 566 (1965).
11. G. A. Loew, M. L. Report No. 740, 1960.
12. J. C. Slater, Dover Publication, Inc., New York, 1950.
13. T. Nishikawa, Minutes of the 1964 Conference on Proton Linear Accelerators, (MURA Report 714) 214.
14. T. Nishikawa, BNL Accelerator Dept. Internal Report AADD-87 (1965).
15. M. Bell, CERN/MSP/DL 75–3 (1975).
16. S. Arai, T. Katayama, E. Tojyo, and K. Yoshida, *Particle Accelerators*, **11**, 103 (1980).