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EFFECT OF SHIELDING OF ELECTRON RING MAGNETIC FIELD ON THE RING MOTION IN THE VICINITY OF THIN METAL SCREENS

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An analytical method for the determination of shielded field of current, moving in the vicinity of thin metal screens is used for finding the retarding force and analysis of the electron ring stability in ERA.

1. INTRODUCTION

In an analysis of the electron ring stability during compression inside the compressor chamber[1] as well as in an analysis of the electron–ion ring acceleration along a coaxial conducting cylinder with it,[2] the problem arises of finding the magnetic field of image currents induced in thin metal screens by a moving electron ring.

Shielding of the magnetic field of a moving current was studied in the case of planar geometry in papers[3-5] and in paper[6] in cylindrical geometry. In all these studies, it was assumed that the electron ring (or straight beam) was moving at constant speed parallel to the screen surface. However, it is necessary to know the magnetic field of image currents in the case when the beam velocity is an arbitrary function of time and is not parallel to the screen.†

A computational procedure for the numerical solution of this problem has been proposed in Ref. 8. It is of advantage, however, to have an approximate analytical solution, permitting a qualitative study of the processes of interest and a determination of the validity limits to the basic integral equation in Ref. 8. Such a solution for the current-carrying filament moving arbitrarily near a thin plane screen is obtained in Section II and is used to calculate the retarding forces (Section III). In Section IV the analytical method is extended to the case of shielding by two plane-parallel screens of equal thickness and conductivity. The solution obtained is used in the electron-ring stability analysis (Section V).

† The problem of the magnetic field shielding for the current moving parallel to the screen with the arbitrarily changing velocity was treated earlier by Merkel.[7]

It is assumed throughout that the velocity of transverse motion is nonrelativistic.

2. SHIELDING BY A SINGLE SCREEN

Let an infinitely long filament carrying the current with density

$$j_y^{(0)}\ (x,z,t) = I(t)\ \delta(x - x_0\ (t))\ \delta\ (z - z_0\ (t))$$

be parallel to the infinite plane metal sheet ($x = 0$) of thickness h and conductivity σ (Fig. 1).

The density of the currents induced in the metal is given by

$$j_y^{\text{ind}} = \sigma E_y = -\frac{\sigma}{c}\frac{\partial A_y}{\partial t}, \tag{1}$$

where $A_y = A_y^{(0)} + A_y^{\text{ind}}$ is the vector potential of the total magnetic field of the filament and induced currents. In the nonrelativistic limit ($v^2 = \dot{x}_0^2 + \dot{z}_0^2 \ll c^2$, where c is the velocity of light), the stationary

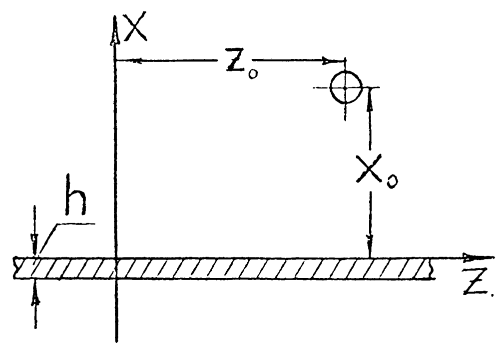

FIGURE 1 Schematic of configuration.

Green's function can be used to calculate the vector potential

$$G(x,z;x',z') = -\ln[(x-x')^2 + (z-z')^2]. \quad (2)$$

Using Eq. (1), one can present A_y in the form

$$A_y(x,z,t) = A_y^{(0)}(x,z,t) - \frac{\sigma}{c^2}\cdot\frac{\partial}{\partial t}\int G(x,z;x'z')A_y(x',z',t')dx'dz', \quad (3)$$

where the integration is performed over the cross section of the screen with the plane $y = \text{const}$. This relation can be regarded as an equation for the vector potential of the total magnetic field in metal.[8]

If the screen thickness h is substantially smaller than i) the skin-depth δ_m at the characteristic frequency $v/|x_0|$

$$\left(\frac{h}{\delta_m}\right)^2 = \frac{v}{u}\cdot\frac{h}{|x_0|} \ll 1,$$

where $u = c^2/2\pi\sigma h$, and ii) the distance over which $A_y^{(0)}$ changes significantly on the metal

$$\frac{h}{|x_0|} \ll 1,$$

then the induced currents density and the vector potential may be assumed constant over thickness of the screen. Then Eq. (3) may be reduced to the one-dimensional form[8]

$$A_y(z,t) = A_y^{(0)}(z',t) - \frac{\sigma h}{c^2}\frac{\partial}{\partial t}\int_{-\infty}^{\infty} G(z,z')A_y(z',t)dz' \quad (4)$$

with the Green's function $G(z,z') = -2\ln|z-z'|$.

For the Fourier transform

$$a_k(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-ikz)\,A_y(z,t)dz,$$

a differential equation follows from Eq. (4)

$$\tau_k\frac{da_k}{dt} + a_k = a_k^{(0)}, \tau_k = \frac{1}{u|k|}, \quad (5)$$

the solution to which is

$$a_k = \exp\left(-\frac{t}{\tau_k}\right)\cdot [a_k(0). + \frac{1}{\tau_k}\int_0^t a_k^{(0)}(t')\exp\left(\frac{t'}{\tau_k}\right)dt'], \quad (6)$$

where†

$$a_k^{(0)} = \frac{I(t)}{c|k|}\exp[-ikz_0(t) - |kx_0(t)|].$$

Calculating the image field at observation points at distances $|x| \sim |x_0| \gg h$ from the screen one can use the surface density of induced currents $i_y^{\text{ind}} = hj_y^{\text{ind}}$, the Fourier transform of which, according to (1) and (5), is

$$i_{yk}^{\text{ind}} = \frac{c|k|}{2\pi}(a_k - a_k^{(0)}).$$

With $A_y|_{t=0} = \lambda A_y^{(0)}|_{t=0}$ as the initial condition for the vector potential of the total magnetic field in the metal ($\lambda = 0$ corresponds to complete shielding of the filament field, $\lambda = 1$ corresponds to absence of shielding) the vector potential of the image field is presented in the form

$$A_y^{\text{ind}}(x,z,t) = -\lambda\frac{I(0)}{c}\ln\{[z - z_0(0)]^2 + [|x| + |x_0(0)| + ut]^2\} + \frac{I(t)}{c}\ln\{[z - z_0(t)]^2 + [|x| + |x_0(t)|]^2\} + 2u\int_0^t dt'\frac{I(t')}{c}\cdot\frac{|x| + |x_0(t')| + u(t-t')}{[z - z_0(t')]^2 + [|x| + |x_0(t')| + u(t-t')]^2}. \quad (7)$$

We shall first consider the decay of image currents. Let us assume that at point $x_0 > 0, z_0$ at time t_0

† Performing the integration we ignore the logarithmic divergence of the two-dimensional vector potential at infinity. The correctness of the results obtained is confirmed by the solution of the problem for a cylindrical geometry with subsequent limiting transition to a planar geometry.

> 0 a current I appeared instantaneously. Inserting $z_0(t) = z_0, x_0(t) = x_0$ into Eq. (7) and integrating one finds

$$A_y^{\text{ind}} = \frac{I}{c} \ln\{(z - z_0)^2 + [|x| + x_0 + u(t - t_0)]^2\}. \tag{8}$$

The field (8) in the region $x > 0$ is adequate to describe the field of current $-I$ with the coordinates $-[x_0 + u(t - t_0)], z_0$; therefore, the velocity u has the physical meaning of the image "run-away" velocity.[9] The characteristic time of the induced currents decay is $\tau \simeq x_0/u$, hence for the processes with duration $\Delta t \ll x_0/u$ the screen may be considered as perfectly conducting.

For a filament moving from time $t = 0$ at constant speed ($z_0 = v_z t$, $x_0 = x_0^0 + v_x t$, $x_0^0 > 0$, $I =$ const.) without crossing the screen ($x_0(t) > 0$) the vector potential of the image field in the upper half-space ($x > 0$) is given by

$$\frac{c}{I} A_y^{\text{ind}}(x,z,t) = -\left[\lambda + \frac{u(v_x - u)}{v_z^{02} + (v_x - u)^2}\right] \ln[z^2 + (x + x_0^0 + ut)^2] + \frac{v_z^2 + v_x(v_x - u)}{v_z^2 + (v_x - u)^2} \ln[(z - v_z t)^2 + (x + x_0(t))^2] + \frac{2uv_z}{v_z^2 + (v_x - u)^2} \left\{ \arctan \frac{(v_x - u)(x + x_0)t)) - v_z(z - v_z t)}{v_z(x + x_0(t)) + (v_x - u)(z - v_z t)} - \arctan \frac{(v_x - u)(x + x_0^0 + ut) - v_z \cdot z}{v_z(x + x_0^0 + ut) + (v_x - u) \cdot z} \right\}.$$

The component of the magnetic field $B_x^{\text{ind}} = -\partial A_y^{\text{ind}}/\partial z$ retarding the filament in z-direction, for $t \gg x_0^0/u$ and $v_x < 0$ is of the form

$$B_x^{\text{ind}} = \frac{2I}{c[v_z^2 + (u + |v_x|)^2]} \cdot \frac{uv_z(x + x_0(t)) - [v_z^2 + |v_x|(u + |v_x|)](z - v_z t)}{(x + x_0(t))^2 + (z - v_z t)^2} \tag{9}$$

If the filament moves parallel to the screen ($v_x = 0$), the Eq. (9) is reduced to

$$B_x^{\text{ind}} = -\frac{2I}{c} \cdot \frac{\kappa}{1 + \kappa^2} \cdot \frac{\kappa(z - v_z t) - (x + x_0)}{(z - v_z t)^2 + (x + x_0)^2}, \tag{10}$$

where $\kappa = v_z/u$. An analysis of the nonrelativistic limit of the strict solution obtained in Ref. 4 shows that formula (10) is valid under conditions $h/x_0 \ll 1$, $|\kappa|\, h/x_0 \ll 1$ which are the same with the assumptions made when deriving Eq. (4).

At velocities $|v_z| \gg u$, the exact solution[4] and the approximate solution (10) as well tend asymptotically to the solution for a perfectly conducting screen. For this reason, at sufficiently high ratios x_0/h, when the ranges of values of κ, given by the inequalities $|\kappa| \ll x_0/h$, $|\kappa| \gg 1$, overlap, formula (10) and consequently Eq. (4) are approximately valid for all nonrelativistic velocities v_z.

Analogously, when the currents in the screen are induced due to variation of the current in the immobile filament, Eq. (4) under condition $h/x_0 \ll 1$ is valid (approximately) for all frequencies $\omega \ll c/x_0$.

3. RETARDING FORCE

The retarding magnetic field B_x^{ind} for the parameters of interest to electron–ion ring acceleration $I = 1$–10kA, $x_0 = 0.5$–1 cm and velocity $v_z \sim u$, may reach a high value (10^2–10^3 gauss), exceeding the magnitude of the external accelerating field at which ions can be held inside the electron ring.† This leads to a "run-away situation" discussed by Herrmann[10] in detail.

If the filament velocity changes little during time $\tau = x_0/u$:

$$\left|\frac{\tau}{v_z}\frac{dv_z}{dt}\right| = \left|\tau \frac{dv_z}{dz}\right| \ll 1, \tag{11}$$

then Eq. (10) can be used to determine the magnetic field acting on the filament

$$B_x^{\text{ind}}\bigg|_{\substack{x = x_0 \\ z = v_z t}} = \frac{I}{cx_0} \cdot \frac{\kappa}{1+\kappa^2}. \tag{12}$$

† The contribution of the image charge electric field to the retarding force may be disregarded[6] since we have assumed the filament velocity to be nonrelativistic.

This result is essentially the same as derived in Refs. 3–6. The maximum values of the field ($B_{x\ \max}^{\mathrm{ind}} = I/2cx_0$) and the retarding force are reached at $v_z = u = c^2/2\pi\sigma h$. For the stainless-steel screen of thickness $h = 0.1$ cm the velocity $u \simeq 4 \cdot 10^{-6}c$.

If the external accelerating field rises steeply in time or in space, then the condition (11) of a low acceleration may not be satisfied and the vector potential (7) must be used for calculation of the drag force. Assuming $x = x_0 =$ const., $z_0(0) = 0$ and $I =$ const., one obtains the retarding field in the form

$$\frac{c}{2I} B_x^{\mathrm{ind}} = \lambda \frac{z}{z^2 + (2x_0 + ut)^2}$$

$$+2u \int_0^t dt' \frac{[z - z_0(t')] \cdot [2x_0 + u(t - t')]}{\{[z - z_0(t')]^2 + [2x_0 + u(t - t')]^2\}^2} . \tag{13}$$

We assume that the external accelerating field B_x^{ext} has negligibly short rise time,† the initial conditions for the filament velocity and self-field being $\dot{z}_0(0) = z_0(0) = 0, \lambda = 1$ (absence of shielding at the start). During short time periods $t \ll 2x_0/u$, the integral in the right-hand part of Eq. (13) may be disregarded, the remaining term corresponds to a self-field "frozen" in the metal. Then, for the force acting on the unit length of the filament one obtains

$$F_z = \frac{I}{c} B_{x\ \max}^{\mathrm{ind}} \cdot \left(b - \frac{4x_0 z}{z^2 + 4x_0^2}\right),$$

where $b = - B_x^{\mathrm{ext}}/B_{x\ \max}^{\mathrm{ind}}$. In the case of an external field which does not depend on z, the potential energy per unit length defined by the relations $F_z = - dU/dz$, $U(0) = 0$, is of the form

$$U = - \left(\frac{I}{c}\right)^2 [b\zeta - \ln(1 + \zeta^2)],$$

where $\zeta = z/2x_0$. The condition of infinite motion $U < 0$ is satisfied for $b > b_0$, where $b_0 \simeq 0.8$ is the root of the equation

$$1 + \sqrt{1 - b^2} = \ln \frac{2[1 + \sqrt{1 - b^2}]}{b^2}.$$

At lower accelerating field strength ($b < b_0$), reflection from the potential barrier occurs, and the filament exhibits z-oscillation of amplitude $\lesssim x_0$. The velocity of the slow motion averaged over this oscillation may be found by equating the retarding field (12) to the applied accelerating field.

Thus, acceleration of the initially rested filament to velocity $v_z > u$ is possible in the case $|B_x^{\mathrm{ext}}| \gtrsim |B_{x\ \max}^{\mathrm{ind}}|$ only.

In a real system,[2] however, the roll-out of the electron–ion ring starts in the end of the compression phase, when the radial velocity is still rather high: $|v_x| \sim 10^{-6}c$. The magnetic field of currents induced on the internal conducting tube[2] by the compressing ring, according to (9) is equal to

$$B_x^{\mathrm{ind}}\Big|_{\substack{x=0 \\ z=z_0(t)}} = \frac{I}{cx_0(t)} \cdot \frac{uv_z}{v_z^2 + (u + |v_x|)^2},$$

where $x_0 = R - R_t \ll R$. R and R_t are the radii of ring and tube, respectively.

The maximum retarding field

$$B_{x\ \max}^{\mathrm{ind}} = \frac{I}{2cx_0} \cdot \frac{u}{u + |v_x|}$$

may be diminished to an acceptable value by appropriate selection of conductivity and thickness of the tube ($u \ll |v_x|$). Hence the run-away situation can be avoided if the inner tube conductivity is sufficiently high.

4. SHIELDING BY TWO PARALLEL SCREENS

If the filament moves between two metal screens of equal thickness h and conductivity σ, coincident with planes $x = \pm d$, then for the vector potential of the total magnetic field in the metal there are equations analogous to Eq. (4)

$$A_\pm(z,t) = A_\pm^{(0)}(z,t) - \frac{\sigma h}{c^2} \cdot \frac{\partial}{\partial t} \int_{-\infty}^{\infty} G_\pm(z - z') A_\pm(z',t) dz', \tag{14}$$

where $A_\pm = \tfrac{1}{2}(A_y|_{x=+d} \pm A_y|_{x=-d})$, $G_\pm = - [2\ln|z - z'| \pm \ln(|z - z'|^2 + 4d^2)]$.

The solutions to Eqs. (14) are of the form (6) with

$$\tau_\pm = \frac{1 \pm \exp(-2|k|d)}{u|k|},$$

† This field can be generated by a rectangular current pulse in additional coils.

$$a_{\pm}^{(0)}(t) = \frac{I(t)}{c|k|} \exp[-ikz_0(t)$$

$$-|k|d] \cdot \begin{Bmatrix} \cosh[|k|x_0(t)] \\ \sinh[|k|x_0(t)] \end{Bmatrix}, \ (|x_0| < d).$$

The vector potential of the image field can be expressed from the Fourier transform of the surface density of induced currents

$$i_k^{\pm} = \frac{c|k|(a_{\pm} - a_{\pm}^{(0)})}{2\pi(1 \pm \exp(-2|k|d))}$$

in the following way

$$A_y^{\text{ind}}(x,z,t) = \frac{4\pi}{c} \int_{-\infty}^{\infty} \exp(ikz$$

$$-|k|d)(i_k^{+}\cosh|k|x + i_k^{-}\sinh|k|x)\frac{dk}{|k|}.$$

The magnetic field, retarding the uniformly moving filament ($v_z = \text{const.}, v_x = 0, x_0 = 0, I = \text{const.}$) for $t \gg d/u$ is given by

$$B_x^{\text{ind}}\Big|_{\substack{x=0 \\ z=z_0t}} = \frac{2I}{cd} \arctan \frac{\kappa}{1 + 2\kappa^2} \qquad (15)$$

If $\kappa \ll 1$, then the field (15) is twice as large as the field (12) reflected from a single screen; if $\kappa \gg 1$, then Eqs. (12) and (15) yield approximately equal values. The maximum retarding field

$$B_{x\max}^{\text{ind}} \simeq \frac{I}{\sqrt{2}\, cd}$$

is reached at $\kappa = 1/\sqrt{2}$.

5. ELECTRON-RING STABILITY WITH RESPECT TO COHERENT SHIFT

In the compressor of the heavy ion collective accelerator,[1] the electron ring is compressed inside a narrow metal chamber. The electric charges induced by the ring on the conducting walls close to it essentially affect the axial (towards the wall) motion of the electrons.

The electron motion in the ring may be considered as a super-position of single-particle betatron oscillations about the cross-section center and coherent motion of the ring as a whole. It was shown by Laslett[11] that the destabilizing effect of image charges on these two types of motion differs: in the first case, defocusing is determined by the gradient of the electric field of immobile images, whereas during the coherent shift, the change of force acting on the ring electrons is also connected with a shift of images, which cause stronger defocusing.

The currents induced in the chamber walls by radially compressing ring exert stabilizing effect. Assuming that

i) the chamber is formed by two infinite plane-parallel metal sheets at distance $2d$ from each other;

ii) the curvature of the ring and its images is not essential ($R \gg d$, where R is the ring major radius), we shall use the results obtained in Section IV in order to find the induced currents. In this Section we use cylindrical coordinates r, θ, z, with planes at $z = \pm d$ corresponding to conducting surfaces (see Fig. 2). The transition from Cartesian coordinates x,y,z to the cylindric coordinates in the formulae of the preceding Section is accomplished by substitution $x \to z$. $y \to r\theta$, $(-z) \to r$.

If the ring with charge of line density $q = I/v_\theta$ is shifted from the median plane at distance z_0, then the electric field of image charges is given by[12]

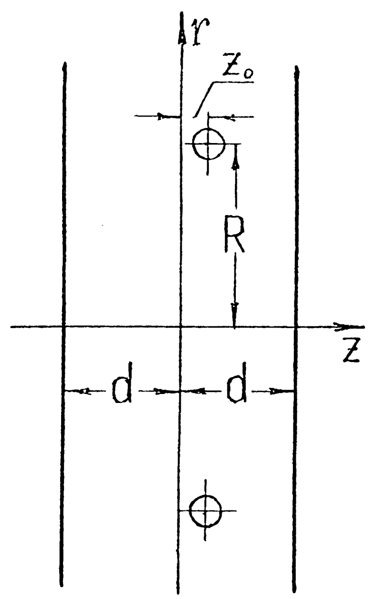

FIGURE 2 Geometry of ring between conducting side-walls.

$$E_z^{\mathrm{ind}}\big|_{r=R} = \frac{\pi q}{2d}\left[\tan\frac{\pi(z+z_0)}{4d} + \cot\frac{\pi(z-z_0)}{4d} - \frac{4d}{\pi(z-z_0)}\right]. \tag{16}$$

If $|z|, |z_0| \ll d$, then Eq. (16) can be written in the form

$$E_z^{\mathrm{ind}} = \frac{q}{R^2}\,(\phi_+ \cdot z + \phi_- \cdot z_0) \tag{17}$$

with

$$\phi_+ = \frac{\pi^2}{12}\left(\frac{R}{d}\right)^2, \phi_- = 2 \cdot \phi_+ .$$

For the magnetic field of currents induced by the ring compressing with constant radial velocity ($v = \dot{R} =$ const.), after some manipulations one finds ($t \gg d/u$)

$$B_r^{\mathrm{ind}}(z,t) = \frac{I}{cR^2} \cdot \Big\{\phi_+ \cdot f_+ \cdot z + \phi_- \cdot \Big[f_- \cdot z_0(t) + \int_0^t K_\kappa\Big[\frac{u(t-t')}{2d}\Big]\,\dot{z}_0(t')\,dt'\Big]\Big\}, \tag{18}$$

with

$$f_+(\kappa) = \frac{12\kappa^2}{\pi^2}\int_0^\infty \frac{e^{-x}(1+e^{-x})\,x\,dx}{1+\kappa^2(1+e^{-x})^2},$$

$$f_-(\kappa) = \frac{6\kappa^2}{\pi^2}\int_0^\infty \frac{e^{-x}(1-e^{-x})\,x\,dx}{1+\kappa^2(1-e^{-x})^2},$$

$$K_\kappa(\eta) = \frac{6}{\pi^2}\int_0^\infty \frac{\cos(\kappa\eta x) - \kappa(1-e^{-x})\sin(\kappa\eta x)}{(1-e^{-x})[1+\kappa^2(1-e^{-x})^2]} \exp\left[-x\left(1+\frac{\eta}{1-e^{-x}}\right)\right] x\,dx$$

where $\kappa = |v_r|/u$. In the limiting cases $\kappa \ll 1$ and $\kappa \gg 1$ the following approximations are obtained

1) $f_+ \simeq \dfrac{15}{\pi^2}\kappa^2, f_- \simeq \dfrac{9}{2\pi^2}\kappa^2$ for $\kappa \ll 1$;

2) $f_+ \simeq 1 - \dfrac{3(1+2\ln 2)}{\pi^2\kappa^2}, f_- \simeq 1 - \dfrac{6}{\pi^2\kappa}$ for $\kappa \gg 1$.

The dependence of factors $f_\pm$ on parameter κ, as well as the dependence of the shielding factor for the filament, moving along single screen

$$f_1 = \frac{\kappa^2}{1+\kappa^2}$$

is shown in Fig. 3.

Taking into account the image fields (17) and (18), the frequency of the single-particle axial oscillation (in units of gyrofrequency $\omega_0 = v_\theta/R$) is given by

$$\nu_z^2 = n - \Delta n_b - \frac{\nu}{\beta^2\gamma}\,\phi_+(1-\beta^2 f_+), \tag{19}$$

where $\beta = v_\theta/c$, $\gamma = (1-\beta^2)^{-1/2}$, v_θ is the azimuthal velocity,

$$n = -\frac{r}{B_z^{\mathrm{ext}}} \cdot \frac{\partial B_z^{\mathrm{ext}}}{\partial r}\bigg|_{r=R}$$

is the external magnetic field index, $\nu = eq/mc^2$ is the Budker parameter with $e < 0$ and m being the electron charge and rest mass, and Δn_b is the frequency shift due to the unshielded ring self-fields (for a ring with a circular cross section of radius a

$$\Delta n_b = \frac{2\nu}{\beta^2\gamma^3}\left(\frac{R}{a}\right)^2$$

At high radial compression speed ($|v_r| \gtrsim u$), the

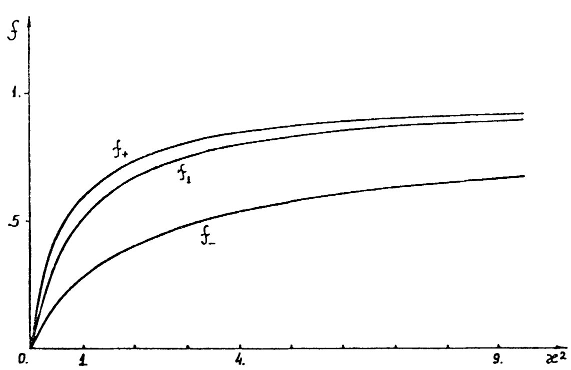

FIGURE 3 Magnetic field shielding factors as functions of κ^2 for the filament moving between two plane-parallel screens ($f_\pm$) and along a single screen (f_1).

defocusing effect of the image charges is substantially weakened ($f_+ \simeq 1$). In practice, however, the requirement of a low level of distortion of the compressing magnetic field by the chamber walls[1] has the result that $|v_r| \ll u$ and, consequently $f_\pm \ll 1$. Therefore, the frequency shift due to the image fields must be taken into account.

Proceeding to the coherent motion analysis we first introduce the condition of static equilibrium, i.e., the requirement that the total force acting on the shifted ring from the external field $B_r^{\text{ext}} = (mc^2/e)\,(\beta\gamma/R^2)/nz$ and image fields at $\dot{z}_0 = 0$ be restoring. Calculating the fields (17) and (18) at the ring position $z = z_0$ one obtains the stability criterion in the form

$$Q_{st}^2 = n - \frac{\nu}{\beta^2\gamma}\,[\phi_+(1-\beta^2 f_+) + \phi_-(1-\beta^2 f_-)] > 0 \tag{20}$$

or, in the case of $f_\pm \ll 1$, in the form[11]

$$n > \frac{\nu}{\beta^2\gamma}\,\phi_c,$$

where $\phi_c = \phi_+ + \phi_-$. For the chamber geometry considered, $\phi_c = 3\phi_+$.

Comparison of this result with Eq. (19) shows that in ultra-relativistic limit $\gamma \gg d/a$ the coherent equilibrium is stable for the ring current which is three times lower than the value compatible with the requirement of the ring integrity ($v_z^2 > 0$).

At $R = 20$ cm, $2d = 5$ cm, $\gamma = 7$, and $n = 0.33$, for example, the equilibrium is stable at ring current $I <$ 250 A.

As the ring nears the wall ($|z_0| \to d$), the force of electrostatic attraction, according to (16), rises to infinity. Therefore, in a real compressor where the mid-planes of the chamber and the external magnetic field may not coincide, the intensity limitation imposed by the requirement of the coherent stability is even more stringent that it follows from (20).

In order to satisfy the equilibrium condition at high ring current, it is necessary to increase the external field index n and/or the aperture of the chamber ($2d$). At a substantial separation of the walls, the ring is subjected to a rapidly developing negative mass instability.[13] The possibility of increasing the field index is also limited: the coherent radial motion becomes unstable at $n > 1$. A substantial increase of n is also undesirable because of inevitable (in this case) crossing of a large number of single-particle betatron resonances during the ring compression.

For this reason we shall make a further study of the coherent axial motion at field index values assuring the integrity of the ring, but not necessarily its equilibrium in the mid-plane of the chamber.

Substitution of fields (17) and (18) into the equation of motion of the ring center of mass

$$\ddot{z}_0 + n\omega_0^2 z_0 = \frac{e}{m\gamma}\,(E_z^{\text{ind}} - \beta B_r^{\text{ind}})\Big|_{z=z_0}$$

and subsequent Laplace transformation

$$z_0(p) = \int_0^\infty e^{-pt} z_0(t)\,dt$$

yield a dispersion equation

$$F(\tilde{p}) = -\frac{Q_{st}^2 + \tilde{p}^2\left(\dfrac{u}{v_\theta}\cdot\dfrac{R}{2d}\right)^2}{\dfrac{\nu}{\gamma}\,\phi_-}, \tag{21}$$

with

$$F(\tilde{p}) = \frac{6\tilde{p}}{\pi^2}\int_0^\infty dx$$

$$\cdot x\,\frac{e^{-x}\,[x + \tilde{p}(1-e^{-x}) - \kappa^2 x(1-e^{-x})^2]}{[1+\kappa^2(1-e^{-x})^2]\{[x+\tilde{p}(1-e^{-x})]^2 + \kappa^2 x^2(1-e^{-x})^2\}}$$

where $\tilde{p} = \dfrac{2dp}{u}$ The roots of Eq. (21) with $Re\tilde{p} > 0$ correspond to unstable motion. Figure 4 shows $F(\tilde{p})$ as a function of $\tilde{p}$ for several values of κ.

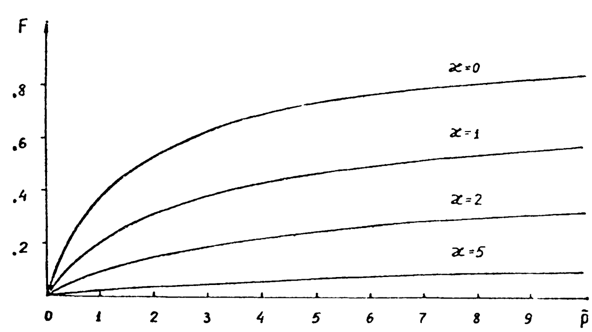

FIGURE 4 $F(\tilde{p})$ as a function of real positive $\tilde{p}$ for a number of κ values.

In the limiting cases the following approximations are valid:

1) $F(\tilde{p}) \simeq 1 - f_- - \dfrac{2}{\tilde{p}}$ for $|\tilde{p}| \gg \sqrt{1+\kappa^2}$ (22a)

2) $F(\tilde{p}) \simeq \tilde{p} F'$, where $F' = \dfrac{6}{\pi^2 \kappa} \displaystyle\int_0^{\kappa} \dfrac{1-y^2}{(1+y^2)^2}\, dy$ for $|\tilde{p}| \ll 1$. (22b)

The coefficient F' may be regarded as the magnetic friction coefficient. It is straighforward to show that it is positive for all values of the parameter κ.

For practical values of the velocity u, the ratio u/v_θ is very small (from 10^{-6} to 10^{-4}), so that Eq. (21) has roots differing strongly in absolute value. According to this the coherent motion of the ring is a superposition of fast $|\tilde{p}| \sim \dfrac{v_\theta}{u} \cdot \dfrac{2d}{R}$ and slow ($|\tilde{p}| \sim 1$) motion. The roots of Eq. (21), corresponding to fast motion are

$$\tilde{p} = \pm i\omega_0 Q_z - \frac{u}{2d} \cdot \frac{\nu}{\gamma} \cdot \frac{\phi_-}{Q_z^2},$$

where Q_z is given by

$$Q_z^2 = n - \frac{\nu}{\beta^2\gamma}\,\phi_+(1-\beta^2 f_+) - \frac{\nu}{\beta^2\gamma^3}\,\phi_- \simeq \nu_z^2 + \Delta n_b. \tag{23}$$

Relation (23) shows that the stability of the single-particle motion ($\nu_z^2 > 0$) guarantees the stability of the high-frequency coherent oscillation† independent of whether the condition of static equilibrium (20) has been satisfied or not. A fast ring movement into the wall is therefore impossible: the currents induced in the walls during an abrupt ring deflection compensate for the charge attraction. As a consequence, the beam performs damped oscillation of frequency $\omega_0 Q_z$ in respect to the mean position. The variation of the ring mean position in time is described by small-value roots of Eq. (21).

In the range $|\tilde{p}| \ll \dfrac{v_\theta}{u}\,\dfrac{2d}{R}$ Eq. (21) may be simplified to

$$F(\tilde{p}) = -\frac{Q_{st}^2}{\dfrac{\nu}{\gamma}\,\phi_-}. \tag{24}$$

At field-index values sufficient for single-particle stability the following inequality is valid (taking into account that $a < d$)

$$Q_{st}^2 > -\frac{\nu}{\gamma}\,\phi_-(1-f_-). \tag{25}$$

If $Q_{st}^2 < 0$, then, as it follows from (22) and (25), Eq. (24) has a real positive root. For values of n not too close to the single-particle stability limit $\tilde{p} \lesssim 4$.

If $Q_{st}^2 > 0$, then Eq. (24) has no root in the positive half-plane,† $Re\,\tilde{p} > 0$.

Thus, if the condition (20) is not satisfied, the ring moves slowly towards the wall. The ring displacement from the mid-plane increases by a factor of e in the time $\tau = \dfrac{2d}{u\tilde{p}} \gtrsim \dfrac{d}{2u}$. In a real compressor[1] this time is sufficiently long $\tau \gtrsim 5$ μsec. Such a slow motion may be stabilized by supplementary measures.

It should be noted that under the condition

$$1 < n < 1 + \frac{\nu}{\beta^2\gamma}\,\phi_+(1-\beta^2 f_+),$$

the coherent radial motion has qualitatively the same character.

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† We consider the azimuthally independent coherent motion only.

† In contradistinction to this, the movement of the electron beam near single screen bay be unstable in the case $Q_{st}^2 > 0$ as well, if $\kappa > 1$ (the negative friction effect[14]).

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