ON THE PERFORMANCE CHARACTERISTICS OF ELECTRON RING ACCELERATORS†

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On the basis of our present understanding of the physical phenomena involved in an electron ring accelerator (ERA), a theoretical study is made of the performance which might be expected for an ERA. Rigorous upper bounds are obtained on the rate of energy gain, from which it is shown that, in order to prevent azimuthal instability, parameters must be selected such that (for reasonable fields, injector properties, etc., but with *no* safety factors) the proton energy gain is less than 80 MeV/m. Numerical examples and approximate formulas are given for the properties of rings satisfying the stability conditions for both azimuthal oscillations and ion–electron oscillations. It is found that for reasonable fields and injector properties, but *without* safety factors, the usable proton energy gain is less than 45 MeV/m.

1. INTRODUCTION

1.1. *History*

The motivation for the development of the electron ring accelerator (ERA) has been the expectation that an ERA would have exceptionally attractive performance characteristics. Thus, already in the first paper on the subject[1] estimates were given of the properties which one might some-day achieve in an ERA: namely, 1000 GeV protons in a 1500-meter accelerator (or an average gradient of 670 MeV/m). Subsequent papers have remarked on achieving, with an ERA, energy gradients in the range of 100 to 1000 MeV/m.[2,3]

An effort was made, by Bovet and Pellegrini,[4] to assess in as careful a manner as possible the performance characteristics of an ERA. · In their exercise they employed the then-current state of theoretical knowledge concerning the limits imposed by a diverse collection of physical phenomena; and in cases where theoretical understanding was incomplete they employed reasonable assumptions. They found, for example, that with present-day conventional technology one might expect an ERA to be able to accelerate protons to 100 GeV in a column of approximately 500 meters length; i.e.,

with an average energy gradient of 200 MeV/m.

More recently, Zenkevich and Koshkarev evaluated the limit imposed on electron rings by ion–electron instabilities.[5] They concluded that an energy gradient of 200 MeV/m was conceivable, but that 48 MeV/m was a more reasonable expectation. In a second report,[6] these authors included the effects of ring acceleration and concluded—without a careful attempt to optimize parameters—that a gradient in excess of (approximately) 24 MeV/m was unlikely to be achieved.

Stimulated by the work of Zenkevich and Koshkarev, and also by the realization that the stringent conditions which longitudinal stability requires had been underestimated in Ref. 4, we decided to re-do the work of Bovet and Pellegrini. This report summarizes our investigations (which have stretched over the last 9 months, and which are described in more detail in four unpublished reports), two containing analytic work[7,8] and two describing computational studies.[9,10]

1.2. *Aspirations and Actualities*

We wished to analyze the present conception of an ERA, with due regard to the limits imposed by technology and physics, and deduce, in a manner that would be generally acceptable, the performance that might (someday) be reasonably expected. Thus, we needed, firstly, to characterize concisely the relevant physics and technology, secondly, to establish ERA performance criteria and, thirdly, to optimize these criteria.

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The first task was relatively easy, and is accomplished in Sec. 2. Unfortunately, the restraints are often complicated—and, in some cases, our theoretical understanding is incomplete, with the result that there is a wide margin of uncertainty associated with the restraint—but the task is reasonably well done.

On the other hand, the ERA concept is so broad—ranging from high-flux, low-energy, heavy ion accelerators to extremely high-energy proton accelerators—and involves such diverse techniques—as magnetic expansion acceleration and/or electric acceleration—that we have been unable to find any single, and adequate, performance criterion.

At first[7] we thought holding power was an adequate performance criterion, and we simply optimized it, but soon we learned that we must also be concerned with (1) beam-loading in an electric acceleration column (which tends to limit the number of electrons in a ring), (2) ion number (that affects both the proton yield and the briskness with which a ring can be accelerated), and (3) ring radius (that relates to the electron energy—and hence, in combination with N_e, to the energy that in part may be transferred to the ions in a magnetic-expansion acceleration column—and that also may influence the over-all diameter and field-energy of the acceleration column).

Thus, we had to reduce our aspirations, and content ourselves with considering electric acceleration separately from magnetic acceleration, and, furthermore, either (1) optimizing holding power, while imposing a number of somewhat arbitrarily formulated constraints (Sec. 3), or (2) not optimizing performance, but simply exploring representative examples (Sec. 4).

We wished also to avoid consideration of ring formation; that is we would have liked to assume that an injector-compressor can always be designed such as to produce any ring which in its compressed state is consistent with the laws of physics. This view is too extreme, however, for in some cases we find that rings with a very large energy spread are advantageous. But we believe that the injection process must put an upper limit on the energy spread in a ring—a limit which depends on the details of the injection process and depends on it in a manner which we are unable to characterize in general. We accordingly have incorporated this into our analysis, in a rather unsatisfactory way, by simply putting an upper bound on the energy spread of a ring.

1.3. *Program*

Finally, then, our analysis is concentrated upon the limits to, and nature of, a loaded electron ring in its compressed state—i.e., just prior to its release from the magnetic well. Ring formation problems are contained in a simple bound on energy spread; acceleration-column effects are contained in limits on electron number, magnetic field strength, ion loading, coupling impedance, and ring radius. In Sec. 2, we present these limits, as well as the relevant physics of compressed rings. In Sec. 3 we derive (analytically) upper bounds on the maximum rate of acceleration in an ERA, and also approximate formulas for the ring parameters of interesting devices. Section 4 has a number of representative examples, and Sec. 5 is devoted to a discussion of our results and the implications of our work.

2. PHYSICAL PHENOMENA AND FORMULAS

In this section we write down all the constraints on a ring at the end of compression (uniform external field). We include azimuthal instability, transverse instability, axial focusing, and ion–electron instability in the axial direction only. (The theory is not yet available for radial modes, and we optimistically assume no serious constraint will be imposed from this analysis.) The notation is that employed in Refs. 4 and 5 and references quoted therein. We introduce S_{FT}, S_{FA}, and S_{FX} as safety factors (> 1) that describe how far the ring is below the threshold for transverse, azimuthal, and axial instability.

We also summarize in this section the limits arising from consideration of ring formation and ring acceleration.

2.1. *Notation*

B—axial magnetic field, assumed uniform

γ—relativistic factor, assumed large so that we take $\beta \approx 1$

N_e—number of electrons

N_i—number of ions, taken as protons in all our examples

R—ring radius

$f \equiv N_i/N_e$—loading fraction

$r_c \equiv e^2/mc^2$—classical electron radius (2.82 $\times 10^{-13}$cm)

σ_a—ring rms minor radial radius

σ_b—ring rms minor axial radius

$r \equiv \sigma_b/\sigma_a$—aspect ratio of ring

ΔE—full width of ring energy spread, at half maximum

Z_0—impedance of space: $(\mu_0/\varepsilon_0)^{1/2}$ in mks, or $4\pi/c$ in cgs units

Z_n—azimuthal coupling impedance of mode n

Z_T—transverse coupling impedance

v_z—axial betatron oscillation tune

Q_e, Q_i—electron and ion tunes, as defined in Ref. 6

m—electron rest mass

M—ion mass, taken as the proton mass in all our examples

R_i—radius of an inner conducting cylinder

$\mu = N_e r_c/2\pi\gamma R$

$P = 2 \ln [16R/(\sqrt{2}(\sigma_b + \sigma_a)]$

$k = e/mc^2 = 0.587 \text{ kG}^{-1} \text{ cm}^{-1}$

S_{FT}, S_{FA}, S_{FX}—safety factors (≥ 1)

2.2. Major Radius

We have

$$\gamma = kBR \qquad (2.1)$$

with $k = 0.587 \text{ kG}^{-1} \text{ cm}^{-1}$.

2.3. Axial Focussing

The axial betatron oscillation tune is taken to be

$$v_z^2 = \left[\frac{2R^2[f - (1/\gamma^2)]}{\sigma_b(\sigma_a + \sigma_b)} - (1-f)\frac{P}{2}\right]\mu. \qquad (2.2)$$

This formula is obtained from that derived for a ring of uniform density in cross section,[11] by replacing the ring radii a, b by $\sqrt{2}\sigma_a$ and $\sqrt{2}\sigma_b$ (see Ref. 10 for details). Image effects have been ignored. We require that v_z^2 at least be positive. (This might be considered overly conservative, but in view of the size of the two terms in v_z^2 and the small amount of image focussing which seems to be achievable in practical configurations, we believe it a fair criterion.)

2.4. Transverse Collective Modes

We have the formula

$$N_e \leq \frac{2\sqrt{2\pi}\gamma\sigma_a}{S_{FT} r_c(Z_T/Z_0)}, \qquad (2.3)$$

where S_{FT} is a safety factor (≥ 1) and the transverse coupling impedance is, in the absence of radiative correction, given by:[12]

$$\frac{Z_T}{Z_0} = \frac{R^2}{2\gamma^2\sigma_b(\sigma_a + \sigma_b)} + \frac{P}{8}. \qquad (2.4)$$

Recently, radiative contributions to Z_T have been studied.[13] Convenient formulas are not available for a ring in a tube, but fortunately we find that in all our studies, even for $Z_T = Z_0$, the transverse limit is not reached. Presumably an acceleration column can be designed such that $Z_T \leq Z_0$.

2.5. Electron–Ion Collective Modes

From the work of Zenkevich and Koshkarev[6] we evaluate

$$Q_e^2 = \frac{N_i r_c R}{\pi\gamma\sigma_b(\sigma_a + \sigma_b)}, \qquad (2.5)$$

$$Q_i^2 = \frac{N_e r_c R}{\pi(M/m)\sigma_b(\sigma_a + \sigma_b)}, \qquad (2.6)$$

which differ from Ref. 6 because of the replacement described in Sec. 2.3. The excluded regions for Q_e and Q_i are indicated in Fig. 1.

2.6. Azimuthal Instability

We have the limit

$$N_e \leq \frac{\gamma R(\Delta E/E)^2}{2S_{FA} r_c |Z_n/nZ_0|}, \qquad (2.7)$$

where $S_{FA}(\geq 1)$ is a safety factor. The coupling impedance $|Z_n/n|$ is strongly dependent upon the ring surroundings, and, since this instability is always of importance, its value is crucial to our analysis. The coupling impedance has been studied by many workers. In particular it forms the subject of another contribution to this Journal.[14] In that contribution the coupling impedance, Z_n, was evaluated as a function of n for $n = 1, 2, \ldots 40$. The quality factor Q was allowed to be a function of n (if desirable) and was so selected as to make $|Z_n|/n$ as small as possible. In general Q was taken large for small n and small for large n, so that
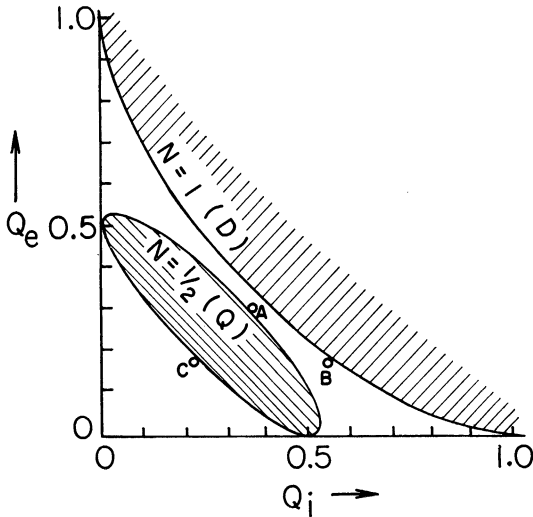
FIG. 1. Excluded regions (shaded) of the Q_i-Q_e plane due to the $N = 1$ (dipole) mode and the $N = 1/2$ (quadrupole) mode of ion–electron oscillation. The examples of Sec. 4 are indicated on the figure by the circles A, B, C.

resonant responses are suppressed and values of $|Z_n|/n$ at large n may approach, but not significantly exceed, the values that would apply in free space. The maximum value of $|Z_n|/n$, with Q selected as a function of n so as to minimize the maximum value of $|Z_n|/n$, was then examined as a function of ring radius for n between 1 and 40. For an electron ring situated a small distance outside an inner conducting tube, it was found that, approximately,

$$\frac{|Z_n|}{n} = 300\frac{R - R_i}{R_i} \text{ ohms.} \qquad (2.8)$$

In an electric acceleration column, if an intense electron ring is to be accelerated without excessive cavity radiation, the bore of the acceleration tube must be large (see Sec. 3.9, below). For modest intensities, this restriction can become less severe. Also, for moderate electric acceleration, it may be possible to design a continuous inner cylinder that is axially non-conducting and azimuthally conducting; and, in this manner, maintain the impedance of (2.8), but this seems to be a non-trivial electrical and mechanical design problem.

In the absence of such a cylinder there are resonances and $|Z_n/nZ_0|$ can easily exceed unity.[15] We optimistically believe that in a careful design resonances can be avoided, and that $|Z_n/n|$ can be

kept below its value in free space, which, ignoring terms of order γ^{-2}, is (see papers cited in Ref. 14),

$$Z_n = 354i^{1/3}n^{1/3}(\text{ohms}).$$

Thus we might take, for the case of no inner cylinder, the maximum of $|Z_n/nZ_0| \approx 0.85$.

2.7. Ring Dimensions

The full energy spread ΔE creates a synchrotron width

$$\sigma_{a,\text{syn}} = \frac{1}{2.36}R\left(\frac{\Delta E}{E}\right) \qquad (2.9)$$

and consequently

$$\sigma_a{}^2 = \sigma_{a,\text{syn}}^2 + \sigma_{a,\text{bet}}^2, \qquad (2.10)$$

where $\sigma_{a,\text{bet}}$ is the betatron contribution to the ring width.

2.8. Injection Limit

As discussed in Sec. 1, we believe the injection process limits $(\Delta E/E)$. Typically, this must be not more than of the order of 10 per cent.

2.9. Electric Acceleration

Beam loading, by an intense ring, in an electric acceleration column has been studied by many workers, but most extensively by Keil.[16] The net energy gain, per unit length, dU/dz, of a ring of charge eN_e may be written in the form

$$\frac{dU}{dz} = \mathcal{E}eN_e - \Lambda(eN_e)^2, \qquad (2.11)$$

where $\mathcal{E}$ is the average applied field. Numerical studies[16] have shown that Λ is a strong function of the acceleration column bore: Even for a bore as large as 20 cm, and $\mathcal{E}$ as large as 5 MV/m, the beam loading is 50 per cent at $N_e = 3 \times 10^{13}$. Clearly N_e cannot greatly exceed this value. More importantly, since the column bore cannot be small, the acceleration column cannot supply sufficient image focussing (see Sec. 2.11) and by itself cannot provide a low coupling impedance (see Sec. 2.6).

2.10. Magnetic Acceleration

It has been emphasized by Lewis,[17] that a ring with a large value of f, although it can initially be accelerated rapidly, must, after a while, be acceler-

ated considerably more slowly than a ring with small f. On the other hand, to achieve axial stability (see Sec. 2.3) one is inclined to make f large. And a high flux of ions, clearly a desirable feature, is an additional pressure towards large f.

We have not been able to combine these conflicting features into one convenient criterion, although generally we find $f \gtrsim 2$ per cent is not desirable for magnetic acceleration.

2.11. Energy Gradient

The peak field inside a ring has been studied by Bovet, and can be expressed as[18]

$$\mathscr{E}_H = \frac{\bar{\beta} N_e e}{\pi R (\sigma_a + \sigma_b)}, \qquad (2.12)$$

where the coefficient $\bar{\beta}$ depends upon the ratio σ_a/σ_b. For $\sigma_a \gg \sigma_b$, $\bar{\beta} = 1.15$, while for $\sigma_a \ll \sigma_b$, $\bar{\beta} = 0.8$ and for $\sigma_a = \sigma_b$, $\bar{\beta} = 0.9$, we adopt the value $\bar{\beta} = 1.0$ (corresponding to $\sigma_a \approx 2\sigma_b$) in part because our best rings are in this range, but primarily because the variation of 0.8 to 1.15 is negligible compared to other errors.

The actual rate at which a ring can be accelerated is less than $\mathscr{E}_H$ because of shear effects.[6,19] We express the energy gradient as

$$e\mathscr{E}_{\text{acc}} = \frac{e\mathscr{E}_H}{S_{FX} \eta}, \qquad (2.13)$$

with S_{FX} an axial safety factor ($\geqq 1$) that indicates the degree to which a ring is removed from being axially unstable. The factor η depends on the degree of image focussing; it is 2.0 when images dominate (which we shall assume to be the case in magnetic acceleration columns) and 4.0 when images are negligible (which we shall assume to be the case, in order to control beam loading, in electric acceleration columns).

3. THE ENERGY GRADIENT

In this section we, firstly, obtain upper-bound formulas for the energy gradient achievable in an ERA. Secondly, we obtain an upper bound on the energy gradient under the assumption that azimuthal instability problems can be ignored. Thirdly, we obtain non-rigorous estimation formulas for the parameters of rings satisfying the

criteria of stability of azimuthal and ion–electron oscillations.

3.1. Rigorous Upper Bound

We start with the very severe requirement for azimuthal stability, (2.7), which with (2.1) may be written as

$$N_e \leqq \frac{kBR^2 (\Delta E/E)^2}{2 S_{FA} r_c \left| Z_n/n Z_0 \right|}. \qquad (3.1)$$

Inserting this into (2.12), and using (2.13) yields:

$$e\mathscr{E}_{\text{acc}} \leqq \frac{kBR(\Delta E/E)^2 mc^2}{2 S_{FX} S_{FA} \pi \eta \left| Z_n/n Z_0 \right| (\sigma_a + \sigma_b)}. \qquad (3.2)$$

Now we can obtain an upper limit by neglecting σ_b and $\sigma_{a,\text{bet}}$ in comparison with σ_a, and then, from (2.9):

$$e\mathscr{E}_{\text{acc}} \leqq \frac{1.2 kB(\Delta E/E) mc^2}{\pi S_{FX} S_{FA} \eta \left| Z_n/n Z_0 \right|}. \qquad (3.3)$$

For numerical evaluation we take $B = 20 \text{ kG}$, $(\Delta E/E) = 10$ per cent (as discussed in Sec. 2.8), and $S_{FX} = S_{FA} = 1$, with the result

$$e\mathscr{E}_{\text{acc}} \leqq \frac{22.4}{\eta \left| Z_n/n Z_0 \right|} \left(\frac{\text{MeV}}{\text{m}} \right), \qquad (3.4)$$

For an electric acceleration column $\eta = 4$, and taking $\left| Z_n \right|/n Z_0 = 0.85$ (see Sec. 2.6) for no close inner wall, we have $e\mathscr{E}_{\text{acc}} \leqq 6.4 \text{ MeV/m}$. If, on the other hand, a structure interior to the ring could be devised that would reduce $\left| Z_n \right|/n$ to values comparable to those achievable in a magnetic-acceleration column, $e\mathscr{E}_{\text{acc}}$ could become of the order 1.0 to 0.5 times the magnetic-column values discussed below.

In a magnetic-acceleration column the situation is better, as $\eta = 2$ and the coupling impedance can be rather small. However, as is clear from (2.8), small $\left| Z_n/n \right|$ requires the ring to be very close to the wall, which is inconsistent with a large value of $(\Delta E/E)$. The ring width is $4\sigma_a$ (90 per cent of the beam) and we believe the clearance to the wall should be at least $2\sigma_a$. Thus we employ (2.8) with $R - R_i = 4\sigma_a$. From (2.9) and (2.8) Eq. (3.3) becomes

$$e\mathscr{E}_{\text{acc}} \leqq \frac{0.286 \, kBmc^2}{S_{FX} S_{FA} \eta}. \qquad (3.5)$$

Taking $B = 20 \, \text{kG}$, $S_{FX} = S_{FA} = 1$, and $\eta = 2$, we deduce $e\mathscr{E}_{\text{acc}} \leqq 82 \, \text{MeV/m}$. Note that for an electric column with a close inner wall, $e\mathscr{E}_{\text{acc}} \leqq 41 \, \text{MeV/m}$ if $\eta = 4$ (no appreciable image effects).

Finally, it should be noted that we have obtained *upper limits* on $\mathscr{E}_{\text{acc}}$. Taking into account various other physical phenomena can only (as will be seen in the next section) reduce $\mathscr{E}_{\text{acc}}$. In particular, ion–electron instabilities will make it impossible to attain this value. Of course, as discussed in Sec. 1, an over-all better accelerator may result from choosing parameters which don't optimize $\mathscr{E}_{\text{acc}}$. But such an accelerator must, necessarily, have a smaller energy gradient than the upper bounds just derived. Finally, the reader should note that we have taken $S_{FX} = S_{FA} = 1$; safety factors larger than unity will correspondingly reduce $e\mathscr{E}_{\text{acc}}$.

3.2. Upper Bound Without Concern for Azimuthal Stability

Although we have no basis for believing that our understanding of azimuthal instabilities is greatly in error, it is interesting to explore the upper bound on accelerating field coming exclusively from other phenomena.

From (2.5) and (2.6):

$$Q_e{}^2 = f \frac{M}{m} \frac{1}{\gamma} Q_i{}^2, \qquad (3.6)$$

where we have introduced $f = N_i/N_e$. Letting $r = \sigma_b/\sigma_a$, we write (2.6) as:

$$Q_i{}^2 = \frac{N_e r_c R}{\pi (M/m) \sigma_a{}^2 r (1+r)}. \qquad (3.7)$$

The focussing condition (2.2) is (for $f \ll 1$):

$$f \gtrsim \frac{1}{\gamma^2} + \frac{Pr(1+r)\sigma_a{}^2}{4R^2}; \qquad (3.8)$$

taking $f = \Gamma/\gamma^2$, where $\Gamma > 1$ and in practice may be expected to be 2 or greater, yields, from (3.6),

$$R = \frac{1}{kB} \left(\frac{MQ_i{}^2\Gamma}{mQ_e{}^2} \right)^{1/3}. \qquad (3.9)$$

Hence, from (3.7),

$$\sigma_a{}^2 = \frac{N_e r_c [M/m (Q_i{}^2\Gamma/Q_e{}^2)]^{1/3}}{\pi kB (M/m) Q_i{}^2 r (1+r)}. \qquad (3.10)$$

Now, (2.12) and (2.13), along with (3.10), yield:

$$e\mathscr{E}_{\text{acc}} = \left(\frac{N_e r_c}{\pi \Gamma} \right)^{1/2} \frac{(kB)^{3/2}}{S_{FX}\eta} Q_e \left(\frac{r}{1+r} \right)^{1/2} mc^2. \qquad (3.11)$$

For given N_e, B, Q_e (all as large as possible), this formula peaks at $r \gg 1$ and gives an upper bound:

$$e\mathscr{E}_{\text{acc}} \leqq \left(\frac{N_e r_c}{\pi \Gamma} \right)^{1/2} \frac{(kB)^{3/2} Q_e mc^2}{S_{FX}\eta}. \qquad (3.12)$$

As an example, take $B = 20 \, \text{kG}$, $\eta = 4$ (electric acceleration), $\Gamma = 2$, and $S_{FX} = 1$. Now, N_e is limited by cavity radiation (see Sec. 2.9) and we take $N_e = 3 \times 10^{13}$ (50 per cent efficiency). The value of Q_e is limited by unity (see Fig. 1); we take $Q_e = 0.5$ to give some safety, and obtain $e\mathscr{E}_{\text{acc}} \lesssim 285 \, \text{MeV/m}$.

3.3. Estimation Formulas

In this section we consider B, R, Q_i, Q_e, S_{FA}, S_{FX}, η and $(\Delta E/E)$ to be inputs. We use the formulas of Sec. 2 to determine the (approximate) nature of rings consistent with the physical limits described in that section. Attention is confined to magnetic columns, so we employ (2.8) for the coupling impedance.

From (3.6) and (2.1):

$$f = \left(\frac{Q_e}{Q_i} \right)^2 \frac{m}{M} kBR. \qquad (3.13)$$

From (2.9)

$$\sigma_a = \frac{1}{2.36} \left(\frac{\Delta E}{E} \right) R. \qquad (3.14)$$

From (3.1) and (2.8), taking $R - R_i = 4\sigma_a$ and employing (3.14):

$$N_e \approx \left(\frac{2.36}{8} \right) \left(\frac{377}{300} \right) \frac{kBR^2(\Delta E/E)}{S_{FA} r_c}. \qquad (3.15)$$

From (3.7), and (3.14):

$$r(1+r) = \frac{(2.36)^2 r_c N_e}{\pi (M/m) Q_i{}^2 R(\Delta E/E)^2}. \qquad (3.16)$$

Finally, (2.13) and (2.12) along with (3.14) and (3.15), yield

$$e\mathscr{E}_{\text{acc}} \approx \frac{8.2B}{S_{FA} S_{FX}\eta(1+r)} \left(\frac{\text{MeV}}{m} \right). \qquad (3.17)$$

In using these estimation formulas, $\Delta E/E$ and R must be selected. Generally, large values of $\Delta E/E$ give greater accelerating fields, but $\Delta E/E$ may be limited by injection, as discussed in Sec. 2.8, The radius R has a lower limit imposed by the requirement of sufficient γ to allow effective magnetic acceleration. An upper limit to R (or γ) could arise from considerations relating to compressor design, and possibly also from desired limits on the magnetic energy stored in the compressor and magnetic-acceleration column. Clearly a large field strength (B) and large Q_i are dynamically advantageous.

Numerical comparison of formulas (3.13) through (3.17) with the results for the examples of Sec. 4 has shown the formulas to be valid to an accuracy of 10 per cent.

4. NUMERICAL EXAMPLES

In this section we augment the analytic work of Sec. 3 with some careful numerical examples. We restrict ourselves to magnetic-column acceleration. Examining Fig. 1, one sees three regions of interest. If the quadrupole resonance is not serious, due to sufficient Landau damping (a moot point), then the region near point A may be available. Alternatively a loading procedure may be devised which allows one to reach the region near point B. Finally, a conservative viewpoint is that only the region up to point C is available.

We present examples of parameters corresponding to these points in Table I. We have taken $\eta = 2$ and for each point consider $S_{FA} = 1$ and $S_{FA} = 2$.

We take $S_{FX} = 1$, since the degradation from larger values can be readily evaluated by the reader. In all cases $B = 20$ kG. It can be seen that the cases with $S_{FA} = 2$ all have $e\mathscr{E}_{\text{acc}}$ less than 30 MeV/m; if a similar factor, S_{FX}, is required to avoid axial instability, then $e\mathscr{E}_{\text{acc}}$ will be reduced to values less than 15 MeV/m. On the other hand, if the stability limits can actually be attained, or even surpassed, then a reasonably good value of $e\mathscr{E}_{\text{acc}}$ can be achieved.

5. DISCUSSION

The important conclusions of this paper are Eqs. (3.4) and (3.5) and the numerical examples of Sec. 4.

Examination of the magnetic-acceleration column cases shows that, consistent with our present understanding of the instabilities limiting the density of electron rings, one should be able to obtain rings with a holding power in the range of 30 MeV/m while observing reasonable safety factors (≈ 1.4) with respect to the azimuthal and axial instability thresholds.

Rings of this quality should be adequate for use in the acceleration of heavy ions; and might even be expected to reduce the expense of a heavy ion accelerator. Examining the electric acceleration situation one is struck with the small holding power values we obtain. These small values are a result of the large-bore acceleration column (itself, the only way presently-conceived to control the phenomenon of cavity radiation). These holding powers—at best less than 40 MeV/m and most

TABLE I

Numerical examples

Case	Q_e	Q_i	R(cm)	$N_e \times 10^{-13}$	$N_i \times 10^{-11}$	σ_a(cm)	σ_b(cm)	$\Delta E/E$(%)	f(%)	(Z_n/n)(ohms)	γ	$e\mathscr{E}_{\text{acc}}$(MeV/m)
$A(S_{FA} = 1)$	0.291	0.356	3.5	1.6	2.4	0.15	0.09	10.0	1.5	62	41	43
$A(S_{FA} = 2)$	0.291	0.356	3.5	0.78	1.17	0.15	0.053	10.0	1.5	62	41	25
$B(S_{FA} = 1)$	0.18	0.55	6.0	3.5	1.44	0.18	0.11	7.2	0.41	42	70	45
$B(S_{FA} = 2)$	0.18	0.55	6.0	2.25	0.92	0.25	0.069	9.7	0.41	59	70	27
$C(S_{FA} = 1)$	0.185	0.225	3.5	1.6	2.4	0.15	0.17	10.0	1.5	63	41	32
$C(S_{FA} = 2)$	0.185	0.225	3.5	0.8	1.2	0.15	0.11	10.0	1.5	63	41	20

$B = 20$ kG, $\eta = 2$, $M/m = 1836$.

likely in the 10 MeV/m range —are dictated by the requirement of azimuthal stability despite the presence of a large coupling impedance.

Hence, it seems, ways have to be found to circumvent the phenomenon of cavity radiation or to overcome the azimuthal instability, in order to arrive at holding powers much higher than the 40 MeV per meter circumference obtained in present day proton synchrotrons (CERN PS, Brookhaven AGS) or the 80 MeV per meter circumference to be obtainable at 500 GeV in the NAL synchrotron.

One possibility is that the azimuthal instability growth rate is slow enough that one can operate above threshold. A study[8] shows that this isn't possible until the ring is moving at extreme speeds ($\gamma_\parallel \gtrsim 10$). Conceivably one could use close walls (accepting the large cavity radiation, or using magnetic acceleration with flux bars[20]), until an adequately large $\gamma_\parallel$ is achieved, and then start a large-bore, efficient acceleration column.

Finally, it must be noted that even if the azimuthal instability can be overcome, the ion–electron instability is still present and provides a serious— although much higher—limit (Sec. 3.2). No method is presently known for circumventing this limit: recent work on B_ϕ-focussing[21] has shown that a very large field is required to change the threshold, but that there is any change at all has even been questioned;[22] furthermore, it has been shown[23] that neither Landau damping nor image focussing (in reasonable amounts) significantly alters the limit.

In summary, on the basis of the analysis which we have presented, the performance characteristics which one can anticipate for an ERA appear to be less exciting than once was believed. It would be extremely useful to obtain experimental information which could be used to judge the validity of the theoretical formulas we have employed and to guide the choice of safety factors. It is gratifying to note that several laboratories have now achieved compressed rings of sufficient quality to permit progress along such lines.

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