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Constraint on $\bar{\rho}$, $\bar{\eta}$ from $B \to K^* \pi$

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A linear relation between Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing parameters, $\bar{\eta} = \tan\Phi_{3/2}(\bar{\rho} - 0.24 \pm 0.03)$, involving a 1σ range for $\Phi_{3/2}$, $20^\circ < \Phi_{3/2} < 115^\circ$, is obtained from $B^0 \to K^*\pi$ amplitudes measured recently in Dalitz plot analyses of $B^0 \to K^+\pi^-\pi^0$ and $B^0(t) \to K_S\pi^+\pi^-$. This relation is consistent within the large error on $\Phi_{3/2}$ with other CKM constraints. We discuss the high sensitivity of this method to a new physics contribution in the $\Delta S = \Delta I = 1$ amplitude.

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I. INTRODUCTION

Two anomalous features measured in $b \to s$ penguin-dominated processes have attracted substantial interest in recent years [1]: (i) CP asymmetries ΔS in $B^0 \to K_S X$ decays ($X = \pi^0, \phi, \eta', \rho^0, \omega, K_S K_S, \pi^0 K_S$) show a hint of systematic deviations from standard model predictions, and (ii) the pattern of direct CP asymmetries in $B \to K\pi$ decays is hard to explain using dynamical approaches based on $1/m_b$ expansion. Are these merely statistical fluctuations, a sign of our inabilities to reliably calculate the relevant observables, or are they first hints of new flavor-dependent CP-violating contributions from new physics at a TeV scale?

In order to answer this question it is important to obtain precise model-independent constraints on the CKM parameters $\bar{\rho}$ and $\bar{\eta}$ [2] using penguin-dominated $\Delta S = 1$ B decays. Comparing these constraints with CKM constraints which are not affected by new physics (NP) in $\Delta S = 1$ decays, e.g., the determination of γ from tree-dominated processes $B \to D^{(*)}K^{(*)}$ [3], may provide a test for the presence of NP in $b \to s$ penguin transitions.

In the present paper we study a linear constraint in the $(\bar{\rho}, \bar{\eta})$ plane following from a combination of $B^0 \to K^*\pi$ amplitudes. The method proposed in [4] and developed further in [5] will be summarized in Sec. II. The necessary observables required for applying the method have been measured recently in Dalitz plot analyses of $B^0 \to K^+\pi^-\pi^0$ [6] and $B^0 \to K_S\pi^+\pi^-$ [7]. They will be used in Sec. III to determine the slope of the linear constraint, comparing this constraint with other CKM constraints. Section IV discusses the sensitivity of this test to new physics effects, while Sec. V concludes.

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II. THE METHOD

The main idea of the method [4,5] is studying $\Delta I = 1$ combinations of $B \to K^*\pi$ amplitudes which do not receive dominant contributions from QCD penguin operators, and thus carry a weak phase γ in the absence of electroweak penguin (EWP) terms. In the present paper we focus our attention on the $I = 3/2$ final state,

$$3A_{3/2} = A(B^0 \to K^{*+}\pi^-) + \sqrt{2}A(B^0 \to K^{*0}\pi^0). \quad (1)$$

In the absence of EWP terms γ would be given by

$$\gamma = \Phi_{3/2} \equiv -\tfrac{1}{2}\arg(R_{3/2}), \qquad R_{3/2} \equiv \frac{\bar{A}_{3/2}}{A_{3/2}}, \quad (2)$$

where $\bar{A}_{3/2}$ is the amplitude for charge-conjugated states.

The phase $\Phi_{3/2}$ can be obtained by measuring magnitudes and relative phases of $B^0 \to K^{*+}\pi^-$ and $B^0 \to K^{*0}\pi^0$ amplitudes and their charge-conjugates. The advantage of $B \to K^*\pi$ over $B \to K\pi$ decays is that $K^*\pi$ quasi–two-body states occur in Dalitz plots of $B \to K\pi\pi$, where overlapping resonances permit determining both the magnitudes and relative phases of $B \to K^*\pi$ amplitudes. In contrast, the relative phases of $B \to K\pi$ amplitudes cannot be measured directly.

The inclusion of EWP contributions modifies the expression for $R_{3/2}$ which becomes [5]

$$R_{3/2} = e^{-2i[\gamma + \arg(1+\kappa)]}\frac{1 + c_\kappa^* r_{3/2}}{1 + c_\kappa r_{3/2}}, \quad (4)$$

$$\kappa \equiv -\frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2}\frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}}, \qquad c_\kappa \equiv \frac{1-\kappa}{1+\kappa}, \quad (5)$$

$$r_{3/2} \equiv \frac{(C_1 - C_2)\langle (K^*\pi)_{I=3/2}|\mathcal{O}_1 - \mathcal{O}_2|B^0\rangle}{(C_1 + C_2)\langle (K^*\pi)_{I=3/2}|\mathcal{O}_1 + \mathcal{O}_2|B^0\rangle}. \quad (6)$$

Here $\mathcal{O}_1 \equiv (\bar{b}s)_{\text{V-A}}(\bar{u}u)_{\text{V-A}}$ and $\mathcal{O}_2 \equiv (\bar{b}u)_{\text{V-A}}(\bar{u}s)_{\text{V-A}}$ are the V-A current-current operators.

The straight line $\bar{\eta} = \bar{\rho}\tan\Phi_{3/2}$, in the absence of EWP terms, is shifted by these contributions along the $\bar{\rho}$ axis by a calculable finite amount. The actual constraint becomes [5]

$$\bar{\eta} = \tan\Phi_{3/2}[\bar{\rho} + C[1\text{–}2\,\mathrm{Re}(r_{3/2})] + \mathcal{O}(r_{3/2}^2)], \quad (7)$$

where ($\lambda = 0.227$)

$$C \equiv \frac{3}{2}\frac{C_9 + C_{10}}{C_1 + C_2}\frac{1 - \lambda^2/2}{\lambda^2} = -0.27. \quad (8)$$

A finite positive shift of the straight line (7) along the $\bar{\rho}$ axis, given by $-C = 0.27$, is obtained using next to leading order values of Wilson coefficients C_i at $\mu = m_b$ [8]. The theoretical error in this parameter is smaller than 1%. The complex parameter $r_{3/2}$ was calculated in factorization, which gives a real result of the order of several percent, $r_{3/2} \leq 0.05$ [4].

A similar but more conservative result is obtained for $r_{3/2}$ by applying flavor SU(3) to corresponding $\Delta S = 0$ decay amplitudes. Noting that the operators in the numerator and denominator in (6) transform as **6** and $\overline{\mathbf{15}}$ of SU(3), one finds [5],

$$r_{3/2} = \frac{|\sqrt{\mathcal{B}(\rho^+\pi^0)} - \sqrt{\mathcal{B}(\rho^0\pi^+)}|}{\sqrt{\mathcal{B}(\rho^+\pi^0)} + \sqrt{\mathcal{B}(\rho^0\pi^+)}} = 0.054 \pm 0.045 \pm 0.023. \quad (9)$$

The first error is experimental. The second error is due to SU(3) breaking, small $\Delta S = 0$ penguin amplitudes and small strong phase difference between $B \to \rho\pi$ decay amplitudes which are neglected.

We have assumed that SU(3) breaking in ratios of $\Delta S = 1$ amplitudes and corresponding $\Delta S = 0$ amplitudes introduces an uncertainty of 30% in these ratios. The $B \to \rho\pi$ phase difference is expected to be suppressed by $1/m_b$ and $\alpha_s(m_b)$ [9,10]. Indeed, evidence for a small phase difference is provided by an isospin pentagon relation obeyed by measured $B \to \rho\pi$ amplitudes [5]. The error in (7) from neglecting this small strong phase difference is negligible because $\mathrm{Re}(r_{3/2})$ depends quadratically on this phase. We will use the calculation (9) for $r_{3/2}$ which is more conservative than the one using factorization. Combining in quadrature the two errors in $r_{3/2}$, the constraint (7) becomes

$$\bar{\eta} = \tan\Phi_{3/2}[\bar{\rho} - 0.24 \pm 0.03]. \quad (10)$$

The dominant uncertainty in this linear constraint originates in $r_{3/2}$.

Equation (4) and a real value of $r_{3/2}$ imply $|R_{3/2}| = 1$. The strong phase of $r_{3/2}$ is expected to be suppressed by $1/m_b$ and $\alpha_s(m_b)$ [9,10]. Using (9) we take

$$|r_{3/2}| < 0.11, |\arg(r_{3/2})| < 30°, \quad (11)$$

leading to the bounds

$$0.8 < |R_{3/2}| < 1.2. \quad (12)$$

III. DETERMINING $\Phi_{3/2}$

The phase $\Phi_{3/2}$ can be determined by measuring the magnitudes and relative phases of the $B^0 \to K^{*+}\pi^-$, $B^0 \to K^{*0}\pi^0$ amplitudes and their charge-conjugates. A graphical representation of the triangle relation equation (1) and its charge-conjugate is given in Fig. 1.

The above four magnitudes of amplitudes and the two relative phases, $\phi \equiv \arg[A(B^0 \to K^{*0}\pi^0)/A(B^0 \to K^{*+}\pi^-)]$ and $\bar{\phi} \equiv \arg[A(\bar{B}^0 \to \bar{K}^{*0}\pi^0)/A(\bar{B}^0 \to K^{*-}\pi^+)]$, determine the two triangles separately. These quantities have been measured recently in a Dalitz plot analysis of $B^0 \to K^+\pi^-\pi^0$ and its charge-conjugate [6]. The relative phase $\Delta\phi \equiv \arg[A(B^0 \to K^{*+}\pi^-)/A(\bar{B}^0 \to K^{*-}\pi^+)]$, which fixes the relative orientation of the two triangles, has been measured in a time-dependent Dalitz plot analysis of $B^0 \to K_S\pi^+\pi^-$ [7].

Table I quotes CP-averaged branching ratios and CP asymmetries for $B^0 \to K^{*+}\pi^-, B^0 \to K^{*0}\pi^0$ using Refs. [6,11]. A value $\Delta\phi = (-164 \pm 30.7)°$ was measured in $B^0(t) \to K_S\pi^+\pi^-$ [7]. The experimental situation is less clear for the phases ϕ and $\bar{\phi}$, measured recently in an amplitude analysis performed for $B^0 \to K^+\pi^-\pi^0$ and its charge-conjugate [6].

In order to calculate the χ^2 dependence on $\Phi_{3/2}$ we use the χ^2 dependence on ϕ and $\bar{\phi}$ given in Ref. [6], assuming Gaussian errors for $\Delta\phi$ and for branching ratios and CP asymmetries in $B^0 \to K^{*+}\pi^-$ and $B^0 \to K^{*0}\pi^0$. Potential correlations between ϕ, $\bar{\phi}$ and branching ratios and asymmetries are neglected. Two resulting χ^2 plots as function of $\Phi_{3/2}$ are shown in Fig. 2. The broken purple curve corresponds to an unconstrained $|R_{3/2}|$, while the solid blue curve is obtained by imposing the bounds (12), expected

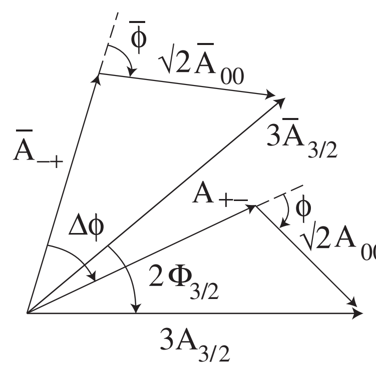

FIG. 1. Geometry for Eq. (1) and its charge-conjugate, using notations $A_{+-} \equiv A(B^0 \to K^{*+}\pi^-)$, $A_{00} = A(B^0 \to K^{*0}\pi^0)$ and similar notations for charge-conjugated modes.

TABLE I. Branching ratios in units of 10^{-6} and CP asymmetries in $B^0 \to K^*\pi$ [6,11].

Mode	Branching ratio	A_{CP}
$K^{*+}\pi^-$	10.4 ± 0.9	-0.14 ± 0.12
$K^{*0}\pi^0$	3.6 ± 0.9	-0.09 ± 0.24

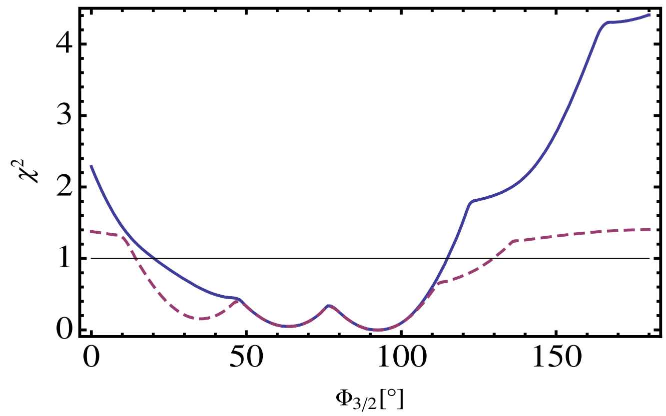

FIG. 2 (color online). χ^2 dependence on $\Phi_{3/2}$ for unconstrained $|R_{3/2}|$ (broken purple line) and for $0.8 < |R_{3/2}| < 1.2$ (solid blue line). A black horizontal line at $\chi^2 = 1$ defines 1σ ranges for $\Phi_{3/2}$.

to hold in the standard model. The latter curve defines a 1σ range,

$$20° < \Phi_{3/2} < 115°. \quad (13)$$

Figure 3 shows the linear constraint (10) with the large range of slopes (13) overlaid on CKMFitter results following from [11,12] $|V_{ub}|/|V_{cb}| = 0.086 \pm 0.009$, obtained in semileptonic B decays, and values $\beta = (21.5 \pm 1.0)°$, $\alpha = (88 \pm 6)°$ and $\gamma = (53^{+15}_{-18} \pm 3 \pm 9)°$ [13], obtained in $B \to J/\psi K_S$, $B \to \pi\pi, \rho\rho, \rho\pi$, and $B^+ \to D^{(*)}K^{(*)+}$, respectively. The small theoretical error in the $B \to K^*\pi$ constraint [± 0.03 in Eq. (10)] is described by the difference between dark and light shaded regions in Fig. 3. The large experimental error in $\Phi_{3/2}$ originates to a large extent in ambiguities in ϕ and $\bar{\phi}$ measured in $B^0 \to K^+\pi^-\pi^0$, using an integrated luminosity on the $\Upsilon(4S)$ of only about 200 fb^{-1} [6]. This error is expected to be reduced considerably by analyses based on higher up-to-date and future luminosities.

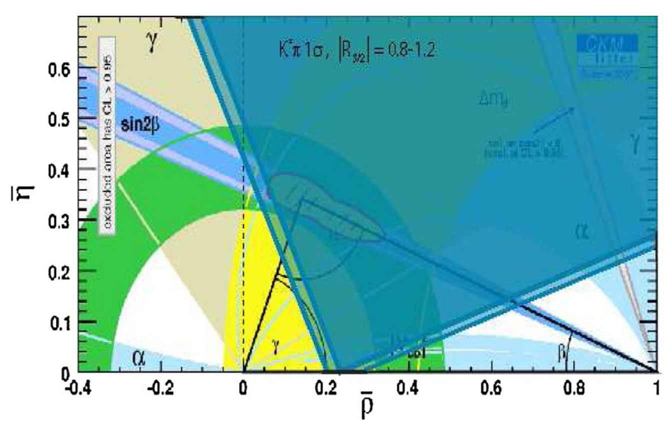

FIG. 3 (color online). Constraint in the $\bar{\rho} - \bar{\eta}$ plane following from Eqs. (10) and (13). The dark shaded region marked $K^*\pi 1\sigma$ corresponds to the experimental error on $\Phi_{3/2}$ given by the 1σ range (13), while the light shaded region includes also the error on $r_{3/2}$ (9). Also shown are CKMfitter constraints obtained using $|V_{ub}|/|V_{cb}|$, β, α, γ and Δm_d [12].

IV. SENSITIVITY TO NEW PHYSICS

As has already been stressed, new physics (NP) $\Delta S = 1$ contributions may lead to an inconsistency between the linear constraint (7) in penguin-dominated $B \to K^*\pi$ decays and values of $|V_{ub}|/|V_{cb}|$, β, α and γ obtained in the above-mentioned processes. The constraint (7) is affected by $\Delta I = 1$ NP operators, while NP contributions from potential $\Delta I = 0$ operators drop out. A general discussion of ways for distinguishing between NP in $\Delta I = 0$ and $\Delta I = 1$ $b \to s$ transitions can be found in Ref. [14].

The $I = 3/2$ amplitude consists of complex tree and EWP terms, T and P_{EW}, both of which involve strong phases,

$$A_{3/2} = Te^{i\gamma} - P_{EW}. \quad (14)$$

The ratio [5]

$$\frac{P_{EW}}{T} = |\kappa| \frac{1 - r_{3/2}}{1 + r_{3/2}} \quad (15)$$

involves the parameter κ defined in (5), which has some dependence on CKM matrix elements whose central values correspond to $|\kappa| \simeq 0.66$.

Allowing for a NP term $A_{NP}\exp(i\psi)$, where A_{NP} involves a CP conserving strong phase while ψ is a new CP-violating phase, the $\Delta I = 1$ amplitude becomes

$$A_{3/2} = Te^{i\gamma} - P_{EW} + A_{NP}e^{i\psi}. \quad (16)$$

The NP term can be reabsorbed quite generally in redefined tree and electroweak penguinlike contributions, $\bar{T}$ and $\bar{P}_{EW}$, without changing the structure (14) [15],

$$A_{3/2} = \bar{T}e^{i\gamma} - \bar{P}_{EW}. \quad (17)$$

Here

$$\bar{T} = T + A_{NP}\frac{\sin\psi}{\sin\gamma}, \qquad \bar{P}_{EW} = P_{EW} + A_{NP}\frac{\sin(\psi - \gamma)}{\sin\gamma}. \quad (18)$$

The amplitudes $\bar{T}$ and $\bar{P}_{EW}$ can be used to define a complex parameter $\bar{r}$ in analogy to Eq. (15),

$$\frac{\bar{P}_{EW}}{\bar{T}} = |\kappa| \frac{1 - \bar{r}}{1 + \bar{r}}. \quad (19)$$

Thus, the parameter $\bar{r}$ replaces $r_{3/2}$ in the expression (4) for $R_{3/2}$. Values of $\bar{r}$ outside the range (11) lead for most such values (unless $\arg(\bar{r})$ is small) to a violation of the bounds (12). *This would be likely evidence for new physics.*

A criterion for the sensitivity of the method to observing a NP amplitude is provided by requiring that $\bar{r}$ lies outside the range of values (11) allowed for $r_{3/2}$. Because of these small values this criterion is expected to hold also for values of A_{NP} which are small relative to T and P_{EW}. An exception is a singular case where the weak phases ψ and γ

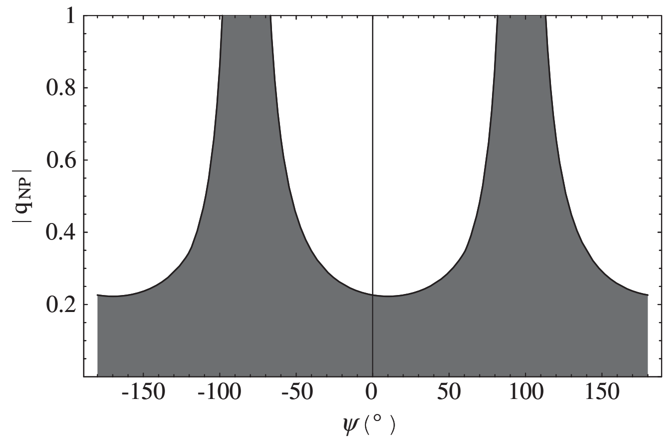

FIG. 4. Values of $|q_{NP}|$ and ψ providing a signal for NP (at $\gamma = 60°$) are given by points outside the dark area, which is obtained by requiring values of $r_{3/2}$ and $\bar{r}$ in the range (11).

are related by

$$\frac{\sin(\psi - \gamma)}{\sin\psi} = \frac{P_{EW}}{T}, \tag{20}$$

for which $\bar{P}_{EW}/\bar{T} = P_{EW}/T$ is independent of A_{NP}. In the following discussion we will assume a value $\gamma = 60°$.

Denoting $q_{NP} = A_{NP}/P_{EW}$, we plot in the dark area in Fig. 4 points corresponding to values of $|q_{NP}|$ and ψ, for which both $r_{3/2}$ and $\bar{r}$ are in the range (11). The region outside this area, including for most values of ψ rather small values of $|q_{NP}|$, $|q_{NP}| \sim 0.3$, implies a high sensitivity to an observable NP amplitude. The spikes around $\psi \sim \pm 90°$, implying very low sensitivity, correspond to solutions of (20) and nearby lying values of ψ.

V. CONCLUSION

Magnitudes and phases of $B^0 \to K^*\pi$ decay amplitudes, extracted in Dalitz plot analyses for $B^0 \to K^+\pi^-\pi^0$ and $B^0 \to K_S\pi^+\pi^-$, are used for obtaining the linear constraint (10) in the $\bar{\rho}$, $\bar{\eta}$ plane, where $\Phi_{3/2}$ lies in a 1σ range (13). This constraint is consistent with other CKM constraints which are unaffected by NP $\Delta S = 1$ operators. The dominant error in the slope of the straight line is purely experimental, while a much smaller theoretical uncertainty occurs in a parallel shift along the $\bar{\rho}$ axis. This small theoretical uncertainty is shown to imply in principle a high sensitivity to a new physics $\Delta S = 1$, $\Delta I = 1$ amplitude.

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[1] M. Gronau, Int. J. Mod. Phys. A **22**, 1953 (2007); E. Lunghi and A. Soni, J. High Energy Phys. 09 (2007) 053; J. Zupan, Proceedings of 5th Flavor Physics and *CP* Violation Conference (FPCP 2007), Bled, Slovenia, 2007, p. 012.
[2] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[3] M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991); M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991); D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997); A. Giri, Y. Grossman, A. Soffer, and J. Zupan, Phys. Rev. D **68**, 054018 (2003); A. Bondar, Proceedings of BINP Special Analysis Meeting on Dalitz Analysis, 2002 (unpublished).
[4] M. Ciuchini, M. Pierini, and L. Silvestrini, Phys. Rev. D **74**, 051301(R) (2006); Phys. Lett. B **645**, 201 (2007).
[5] M. Gronau, D. Pirjol, A. Soni, and J. Zupan, Phys. Rev. D **75**, 014002 (2007).
[6] B. Aubert *et al.* (*BABAR* Collaboration), arXiv:0711.4417.
[7] B. Aubert *et al.* (*BABAR* Collaboration), arXiv:0708.2097.
[8] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[9] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000); **B606**, 245 (2001); M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).
[10] C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004); C. W. Bauer and D. Pirjol, Phys. Lett. B **604**, 183 (2004); C. W. Bauer, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **74**, 034010 (2006); A. R. Williamson and J. Zupan, Phys. Rev. D **74**, 014003 (2006).
[11] E. Barberio *et al.* (Heavy Flavor Averaging Group (HFAG) Collaboration), arXiv:0704.3575, regularly updated in http://www.slac.stanford.edu/xorg/hfag.
[12] J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005), regularly updated in http://ckmfitter.in2p3.fr.
[13] A. Poluektov *et al.* (Belle Collaboration), Phys. Rev. D **73**, 112009 (2006).
[14] M. Gronau and J. L. Rosner, Phys. Rev. D **75**, 094006 (2007).
[15] See e.g. F. J. Botella and J. P. Silva, Phys. Rev. D **71**, 094008 (2005).