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## **TUNABLE ACHROMATS AND CLIC APPLICATIONS**

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It is imperative for linear colliders that the bunch length be adjustable. In most cases bunch compression is required, but recently, in the design of the Compact Linear Collider (CLIC) RF Power Source, it was shown that bunch stretching may also be necessary. In some situations, both modes may be needed, which implies the need for tunable magnetic insertions. This is even more essential in a test facility, to span a wide experimental range. In addition, flexible tuning provides a better control of the stability of an isochronous insertion. To start a numerical search for a tunable insertion from scratch is very uncertain because the related phase space is very uneven. However, a starting point obtained with an analytical approximation is often sufficient to ensure convergence. Another advantage of the analytical treatment described in this paper is that it sheds light on the shape of the entire phase space. To achieve this the isochronous achromat developed previously has been given tuning capabilities by expanding the expressions obtained for its main parameters. An application to the future CLIC Test Facility (CTF) is shown.

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# TUNABLE ACHROMATS AND CLIC APPLICATIONS

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## Abstract

It is imperative for linear colliders that the bunch length be adjustable. In most cases bunch compression is required, but recently, in the design of the Compact Linear Collider (CLIC) RF Power Source, it was shown that bunch stretching may also be necessary. In some situations, both modes may be needed, which implies the need for tunable magnetic insertions. This is even more essential in a test facility, to span a wide experimental range. In addition, flexible tuning provides a better control of the stability of an isochronous insertion. To start a numerical search for a tunable insertion from scratch is very uncertain because the related phase space is very uneven. However, a starting point obtained with an analytical approximation is often sufficient to ensure convergence. Another advantage of the analytical treatment described in this paper is that it sheds light on the shape of the entire phase space. To achieve this the isochronous achromat developed previously has been given tuning capabilities by expanding the expressions obtained for its main parameters. An application to the future CLIC Test Facility (CTF3) is shown.

## 1 INTRODUCTION

Lately the CLIC study [1], [2] made two very important advances. Firstly a consistent set of parameters for the main linac was found, which made the relative tolerances comparable with those of the other proposed electron-positron colliders [3]. Secondly an efficient RF Power Source was designed [4]. Among the many parameters, the length of the bunch is an essential quantity. It should be 30  $\mu\text{m}$  inside the main linac and carefully controlled in the bends of the injector complex. The isochronous rings and transfer lines of the RF power source also require that the bunch length of the drive beam be modified, either by stretching, in order to limit the coherent synchrotron radiation effects, or by compression, in order to optimise the power transfer to the main beam. In the first order approximation the bunch length is proportional to the  $R_{56}$  parameter which is defined by the following integral :

$$R_{56} = \int_{s_1}^{s_2} \frac{D_x}{\rho(s)} ds \quad (1)$$

where  $D_x$  is the horizontal dispersion,  $\rho(s)$  the radius of curvature, and  $s_1, s_2$  are the longitudinal coordinates of the beginning and end of the beamline considered. The  $R_{56}$  parameter is positive if high momentum particles of the bunch travel longer paths. Of course the values of the  $R_{56}$  parameter of the various insertions can be fixed at the design stage, but the operation of both the accelerator and

the decelerator are much easier if some flexibility is given to modify it in a given range. This flexibility becomes a feature in a test facility such as CTF3 [5], whose purpose is to validate most of the RF Power Source design and at the same time to study the behaviour of coherent synchrotron radiation for which the theory and the simulations remain to be confronted with experimental data. Thus a study was started to find an ensemble of several magnetic components (dipoles and quadrupoles) called an "insertion", which would be able to generate both a negative or a positive  $R_{56}$  parameter by only modifying the strength of the quadrupoles. Quite naturally the isochronous insertion developed five years ago [6] was chosen as a promising candidate. It turned out that it was possible to obtain the expression for the absolute values of the focal lengths as a function of the  $R_{56}$  parameter in the thin lens approximation. This will be shown in the next section. It demanded much more algebra to derive the conditions on the minimum and maximum values of the  $R_{56}$  parameter and on the drift lengths, such that the absolute values of the focal lengths were positive. Actually sixteen different sets of conditions exist [7]. It is impossible to decide analytically which one is best. This depends on the geometry and on the constraints imposed on the Twiss parameters at the entrance and exit of the insertion. A simple interactive program guides the user towards the best choice. The last section shows an application to the transfer line between the Delay Loop and the Isochronous Ring of CTF3.

## 2 THE TUNABLE ACHROMAT

Let us consider a module consisting of three bending magnets, geometrically and magnetically symmetric around the median plane of the second magnet [6]. To simplify the algebra, these magnets are treated as sector magnets of the same length  $l_m$  but of different deflection angles  $\phi_1$  and  $\phi_2$  for the first and second dipole respectively. The space between the first two magnets is filled by a drift length  $L_1$ , by a focusing quadrupole of length  $l_q$  and normalised gradient  $k_1$ , by a second drift length  $L_2$ , by a defocusing quadrupole of length  $l_q$  and normalised gradient  $k_2$ , and finally by a third drift length  $L_3$  [6]. Assuming that the dispersion and its derivative are zero at the entrance of the first dipole, the contributions of the first dipole and of half the second dipole to the integral (1) are [6] :

$$\begin{aligned} & \rho_1 (\phi_1 - \sin \phi_1) \\ & \text{and} \\ & D_j \sin (\phi_2/2) - \rho_2 D'_j [\cos (\phi_2/2) - 1] + \\ & \rho_2 [\phi_2/2 - \sin (\phi_2/2)] \end{aligned} \quad (2)$$

respectively, where  $\rho_1$  and  $\rho_2$  are the curvature radii of the first and of the second dipole respectively and  $D_j$  and  $D'_j$  are the dispersion and its derivative at the entrance to the second dipole. Adding the two contributions, the  $R_{56}$  parameter for half the insertion is given by :

$$\frac{R_{56}}{2} = \rho_1 (\phi_1 - \sin \phi_1) + D_j \sin (\phi_2/2) - \rho_2 D'_j [\cos (\phi_2/2) - 1] + \rho_2 [\phi_2/2 - \sin (\phi_2/2)] \quad (3)$$

In order to obtain a nondispersive module, the derivative of the dispersion at the point of symmetry should be zero, providing a second equation :

$$-\frac{\sin (\phi_2/2)}{\rho_2} D_j + D'_j \cos (\phi_2/2) + \sin (\phi_2/2) = 0 \quad (4)$$

From these two equations it is easy to obtain :

$$\begin{aligned} D'_j &= \frac{x}{\rho_2} \\ D_j &= \rho_2 \left[ 1 + D'_j \cot (\phi_2/2) \right] \\ x &= \frac{R_{56}}{2} - l_m \left( \frac{3}{2} - \frac{\sin \phi_1}{\phi_1} \right) \end{aligned} \quad (5)$$

It is possible to obtain, in the same way as in reference [6] the following expressions for the first two drift lengths as functions of  $k_1, k_2$  and of the third drift length  $L_3$  :

$$\begin{aligned} L_1 &= a \frac{C_2 q_1}{C_1 q_2} (\widetilde{L}_3 + q_2) - l + q_1 \\ L_2 &= q_1 - q_2 + \frac{b}{\widetilde{L}_3 + q_2} \\ \widetilde{L}_3 &= L_3 - \frac{D_j}{D'_j} \end{aligned} \quad (6)$$

where

$$l = \rho_1 \tan (\phi_1/2) \quad a = -\frac{x}{\rho_2 \sin \phi_1} \quad (7)$$

$$b = \frac{q_2}{C_2} \left( \frac{q_2}{C_2} + \frac{q_1}{a C_1} \right) \quad q_i = \frac{C_i}{S_i \sqrt{k_i}} \quad (8)$$

$$C_1 = \cos (l_q \sqrt{k_1}) \quad S_1 = \sin (l_q \sqrt{k_1}) \quad (9)$$

$$C_2 = \cosh (l_q \sqrt{k_2}) \quad S_2 = \sinh (l_q \sqrt{k_2}) \quad (10)$$

These drift lengths depend on the parameter  $R_{56}$  through the quantities  $D_j$  and  $D'_j$ . The aim of the study is to achieve  $R_{56}$  tuning i.e. the ability to vary this parameter between a minimum value (negative)  $R_{56, \min}$  and a maximum value (positive)  $R_{56, \max}$  without of course displacing the quadrupoles. Thus  $L_1$ ,  $L_2$  and  $L_3$  are fixed and the normalized strengths  $k_1$  and  $k_2$  should be expressed as functions of  $R_{56}$ , which implies inverting the two equations (6). Unfortunately these are transcendental equations and

no closed form may be obtained for  $k_1$  and  $k_2$ . However it can be shown that it is possible in the thin lens approximation, that is for such a small  $l_q$  that the assumptions  $C_1 = C_2 = 1$ ,  $S_1 = l_q \sqrt{k_1}$  and  $S_2 = l_q \sqrt{k_2}$  hold to a very good accuracy. Then the absolute values of the focal lengths  $f_1 = l_q k_1$  and  $f_2 = l_q k_2$  replace  $q_1$  and  $q_2$  respectively and the set of equations (6) becomes :

$$\begin{aligned} L_1 &= a \frac{f_1}{f_2} (\widetilde{L}_3 + f_2) - l + f_1 \\ L_2 &= f_1 - f_2 + \frac{f_2 (f_2 + f_1/a)}{\widetilde{L}_3 + f_2} \end{aligned} \quad (11)$$

which can also be expanded in the form :

$$\begin{aligned} \frac{a+1}{a} f_1 f_2 + \widetilde{L}_3 f_1 - \frac{L_1 + l}{a} f_2 &= 0 \\ \frac{a+1}{a} f_1 f_2 + \widetilde{L}_3 f_1 - (\widetilde{L}_3 + L_2) f_2 - L_2 \widetilde{L}_3 &= 0 \end{aligned} \quad (12)$$

Subtracting the two equations,  $f_2$  can be obtained :

$$f_2 = \frac{a L_2 \widetilde{L}_3}{L_1 + l - a (\widetilde{L}_2 + \widetilde{L}_3)} \quad (13)$$

and by replacing this value in the first equation,  $f_1$  is also obtained :

$$f_1 = \frac{L_2 (L_1 + l)}{L_2 + L_1 + l - a \widetilde{L}_3} \quad (14)$$

By using the expressions (5) and (7) the quantity  $a \frac{D_j}{D'_j}$  which enters in  $\widetilde{L}_3$ , becomes :

$$a \frac{D_j}{D'_j} = -\frac{1}{\sin \phi_1} [x \cot (\phi_2/2) + \rho_2] \quad (15)$$

Using this expression and the definition of  $a$ ,  $f_1$  and  $f_2$  can be expressed as functions of  $x$ , which is linearly related to the  $R_{56}$  parameter as shown by equation (5) :

$$\begin{aligned} f_1 &= L_2 \frac{\rho_2 (\mathcal{L}_1 + \rho_2)}{x \mathcal{L}_3 + \rho_2 (\mathcal{L}_1 + L_2 \sin \phi_1)} \\ f_2 &= L_2 \frac{-x \mathcal{L}_3 + \rho_2^2}{x (L_2 + \mathcal{L}_3) + \rho_2 \mathcal{L}_1} \end{aligned} \quad (16)$$

where :

$$\begin{aligned} \mathcal{L}_1 &= (L_1 + l) \sin \phi_1 - \rho_2 \\ \mathcal{L}_3 &= L_3 - \rho_2 \cot (\phi_2/2) \end{aligned} \quad (17)$$

In order to design a  $R_{56}$  tunable module, it is necessary to find the intervals of  $L_1, L_2, L_3$  such that the absolute values of the focal lengths remain positive when  $R_{56}$  varies in the interval  $R_{56, \min} < R_{56} < R_{56, \max}$  with  $R_{56, \min} < 0$  and  $R_{56, \max} > 0$ . The algebra is very tedious and can be found in [7]. Sixteen different sets of conditions satisfy

the imposed constraints. A simple interactive program permits to choose the best one according to the geometry and Twiss parameter requirements. A standard program for accelerator design, such as MAD can then be used to derive the thick lens solutions using the thin lens results as a very effective starting point.

### 3 APPLICATION TO A CTF3 TRANSFER LINE

The CTF3 transfer line between the Delay Loop and the Combiner Ring should be able to increase or decrease the bunch length by 1.6 mm. Given the  $\Delta p/p$  of the order of 1 %, the range of  $R_{56}$  is between -0.16 m and 0.16 m. To accommodate this transfer line in a 'S' shape inside the available space, it is made of two insertions, one bending the beam by  $75^\circ$  and the other bending it back by  $-75^\circ$ . The analytical approach has permitted an identification of the ranges of possible solutions without using numerical searches which are very unstable in this specific problem. Thus the insertion could be optimised to find a compromise between the overall length imposed by the building dimensions, and the optics (Twiss parameters). The most useful condition in the design of this CTF3 transfer line has been the fifteenth set [6]. The three dipoles of the selected insertion have the same length (0.4 m) and generate the same beam deflection ( $25^\circ$ ). The drift lengths are  $L_1 = 1.2$  m,  $L_2 = 0.6$  m and  $L_3 = 1.55$  m. All the quadrupoles have the same length of 0.2 m. For a beam energy of 400 MeV, the gradients of the first and second quadrupoles vary between 12.04 T/m and 7.81 T/m, and between 12.13 T/m and 1.29 T/m respectively. The Figures 1, 2 and 3 show the optical functions of the full insertion when the  $R_{56}$  parameter of half one single insertion is -0.04 m, 0 m, 0.04 m respectively.

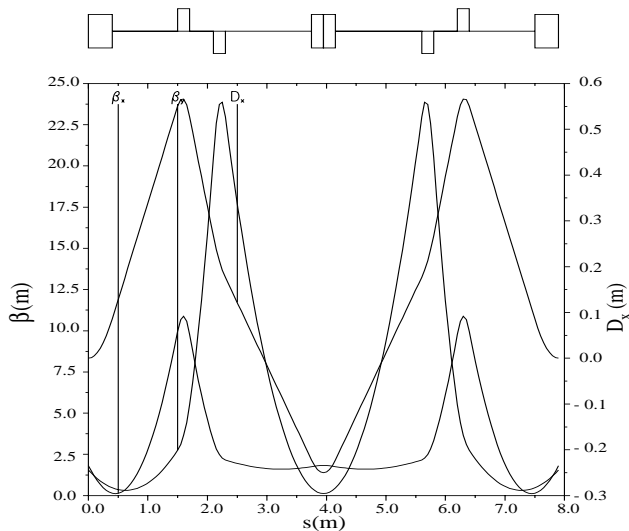


Figure 1: Optical functions for  $R_{56} = -0.04$  m.

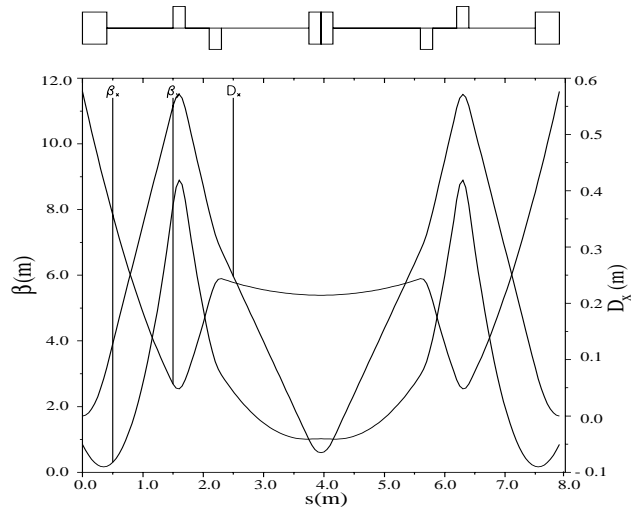


Figure 2: Optical functions for  $R_{56} = 0$  m.

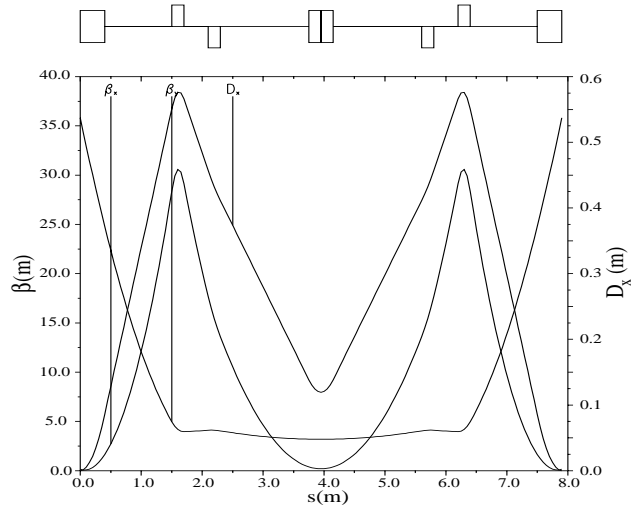


Figure 3: Optical functions for  $R_{56} = 0.04$  m.

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