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PHASE DEBUNCHING BY FOCUSSING-FOILS IN A PROTON LINEAR ACCELERATOR

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BY

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SUMMARY.

The statistical variation of energy loss of particles traversing focussing-foils in a Berkeley type accelerator leads to phase debunching. A treatment of the effect is developed which is based on an analysis by Landau of straggling in single foils. The method is valid for small bunches. Calculations are done for a 47-470 MeV. machine with operating wavelength 1.5 metres, energy gradient 2.3 MeV. per metre, and synchronous phase 20°. With 3×10^{-4} inch beryllium foils the final bunch width is quite tolerable, being of the order of a few degrees. The foils may be somewhat thicker without affecting the overall bunch width, but it varies inversely with the square root of the wavelength.

INTRODUCTION

A previous report[1] has considered the radial spreading of the proton accelerator beam by scattering in focussing foils. We are here concerned with the effect on the axial motion of the particles of the energy loss in traversing foils. If a particle lost nearly the same amount of energy in successive foils it would simply tend to lag behind the original synchronous phase. However, there is a considerable statistical variation in the energy loss and this initiates phase oscillations. In the cases of interest these have amplitudes large compared with the average phase lag; the latter merely shifts the centre of the bunch a distance small compared with its width.

MEAN LOSS OF ENERGY.

The mean loss of energy of protons in beryllium has been tabulated conveniently by Siri[2]. Below we give the loss at various energies in 3×10^{-4} inch foils, and the corresponding loss per metre if they were used in the Berkeley type accelerator[1] with operating wavelength 1.5 metres.

Energy in MeV.	Loss in MeV. per foil	Loss in MeV. per metre
1	.37	5.4
5	.11	.71
50	.016	.035
500	.0035	.0031

The figures in the third column are to be compared with the 2.3 MeV/metre accelerating rate in the existing machine. It is plain that at 50 MeV. and above the loss is unimportant; even if it were ignored in the design it would cause a phase lag of only 2.5° from a phase stable angle of 20° at 50 MeV., decreasing to about .2 degrees at 500 MeV. At 5 MeV., on the other hand, the loss is a substantial fraction of the available acceleration and the machine would have to be designed accordingly. However, at such low energies the coulomb scattering becomes excessive for foils much thicker than 3×10^{-5} inch, if this thickness could be used the energy loss would be small above 5 MeV.

STRAGGLING IN A SINGLE FOIL.

The work of Landau[3] on straggling in single foils will now be described briefly. We are not very interested in applying this to the individual foils of the accelerator, but it will be seen later how we can use Landau's results in discussing the phase distribution of particles emerging from the machine as a whole.

A proton traversing a foil loses energy in collision with electrons. The individual collisions cause energy losses which range from the minimum excitation potential of the atom to the energy which a proton can deliver to an electron in a "head-on" collision. If the foil is sufficiently thick for even large losses to occur many times then the distribution of total-loss about the mean becomes the normal gaussian. For thinner foils the smaller losses may still occur often enough to set up a roughly gaussian distribution, but when a particle does happen to suffer a heavier collision the latter may contribute the major part of its total energy-loss. The probability distribution for total loss is then roughly gaussian for small values, but the probability of a larger loss is nearly equal to the probability of a single correspondingly heavy collision. This situation has been investigated analytically by Landau. His work rests on the formula

$$W(\varepsilon)\ d\varepsilon = \xi\ d\varepsilon/\varepsilon^2$$

$$\xi = 2m_0c^2\ \pi n z v_0^2\ \beta^{-2}$$

for the average number of collisions in which energy between ε and $\varepsilon + d\varepsilon$ is lost.

m_0c^2 = electron rest energy

v_0 = e^2/m_0c^2 = 2.8×10^{-13} cm.

e = electron charge

β = ratio of particle velocity to velocity of light

z = atomic number of material

n = number of atoms per sq. cm. of foil.

The formula is valid when ε is large compared with ε_0, the mean binding energy of the electrons, and small compared with ε_m, the maximum energy loss possible. If the total energy loss is denoted by $\alpha\xi$, and its most probable value by $\alpha_0\xi$, Landau finds that when $\varepsilon_0 << \xi << \varepsilon_m$ the probability distribution for $\alpha - \alpha_0$ depends only on the region of $W(\varepsilon)$ where the formula is true and that it is given by the universal curve in fig. 1.

The curve must be in error for $\alpha - \alpha_0 < - \alpha_0$, where it would give a small probability for energy losses which are in fact negative. However, if $\xi >> \varepsilon_0$ the curve is anyway very near the axis in this region.

It must again be in error for $\varepsilon \sim \varepsilon_m$ where the formula for $W(\varepsilon)$ is modified. According to Rossi and Greisen[4]

$$\varepsilon_m = 2 m_0 c^2 \quad \frac{\beta^2}{1 - \beta^2}$$

$$W(\varepsilon)\, d\varepsilon = \xi \frac{d\varepsilon}{\varepsilon^2} \left\{ 1 - \beta^2 \; \varepsilon/\varepsilon_m \right\}, \quad \varepsilon_0 << \varepsilon < \varepsilon_m$$

$$W(\varepsilon) = 0 \qquad\qquad \varepsilon > \varepsilon_m$$

Since total energy losses of order ε_m are due mainly to single heavy collisions the probability distribution for α should stop near ε_m/ξ and for relativistic particles will fall below the Landau curve at this point is approached.

In Landau's work collisions with different electrons are treated as independent. For very swift particles this is not in fact correct. and it is found that the degree of condensation of the material affects the process of energy loss. However, according to Rossi and Greisen this effect is unimportant as long as the kinetic energy is not large compared with the rest energy; therefore it need not be considered at the energies in the proposed accelerator.

FINAL PHASE DISTRIBUTION IN ACCELERATOR.

If a particle travelling along the accelerator with the synchronous phase and velocity loses some energy in a collision it will be subsequently oscillate in phase. Small oscillations may be superposed, so that at the end of the machine the phase of a particle is the sum of the contributions from the phase oscillations set up by all the individual collisions. There will also be other contributions for particles not injected with the synchronous phase and velocity; we shall refer briefly to these later on. It turns out that over a considerable range the average number of collisions which contribute a final phase deviation in the region $d\varphi$ is inversely proportional to φ^2. Apart from the substitution of phase deviation for energy loss there is only this difference from the situation analysed by Landau – that while ε was essentially positive both negative and positive φ's occur. Thus one obtains two Landau curves representing respectively the probability distributions for the sum of all positive deviations and for the sum of all negative deviations, and a convolution of the two gives the probability distribution for the resultant final phase.

The fact that all phase oscillations start by lagging behind the synchronous phase (corresponding to energy loss) leads to a certain asymmetry in the final distribution. The centre of the bunch tends to lag behind the synchronous phase by an amount corresponding to the mean energy loss. In the cases of interest, as we have already seen, this is not a large effect, and in obtaining the curve of fig. 2 it has been neglected. Fig. 2 is in fact obtained by a (rough) convolution of a pair of mirror-image Landau curves; it represents the probability distribution for φ/σ, where σ here plays a role analogous to that of ξ for single foils.

Fig. 2 overestimates the probability of large total deviations. As with the Landau curve the error is mainly in overestimating the probability of single collisions with large energy loss. Although considerable computation would be involved in exactly prescribing the falling off at all angles, it is not difficult to give an angle φ_m by which it is nearly complete – that is to say beyond which there are almost no deviations. It is a condition of validity of the approach used that φ_m should be large compared to σ.

RESULTS.

We shall quote here the results of the calculations, leaving the mathematical development to an Appendix.

With 3×10^{-4} inch beryllium foils on the 47–470 MeV accelerator described at the beginning of the report, σ is found to be .16 degrees. Referring to fig. 2 it is seen that the distribution has a width at half-height of about $\pm 3\sigma$, or roughly plus or minus half a degree. In this case fig. 2 overestimates the probability of deviations greater than about a degree, and the correct distribution will not extend at all much beyond $\pm \varphi_m \sim \pm 5^\circ$. σ is proportional to foil thickness, but φ_m is independent of this, so the foils may be rather thicker than 3×10^{-4} inch without increasing the overall width of the distribution as distinct from its width at half-height. Both σ and φ_m increase with decreasing wavelength, σ as $\lambda^{-3/2}$ and φ_m as $\lambda^{-1/2}$; the former increases more than the latter because it involves the increased number of foils per unit length as well as the decreased effectiveness of the phase bunching forces.

Calculations were done also for a 4.7–47 MeV machine otherwise similar to that just discussed. With 3×10^{-5} inch foils it is found that both the overall bunch width and the width at half-height are comparable with those obtained for the thicker foils on the higher energy accelerator.

PHASE OSCILLATIONS.

As in the investigation of scattering(1) the presence of large phase oscillations has not been allowed for. Strictly speaking the theory is sufficient only if the particles are fairly well bunched on entering the foil focussed section. The phase oscillations set up by straggling are simply linearly superposed on any other small oscillations present.

CONCLUSION.

We have found for the cases considered that with small phase oscillations the foil thickness is limited by scattering rather than by mean energy loss or straggling.

ACKNOWLEDGEMENT.

I would like to thank Mr. D. West for drawing my attention to Landau's paper.

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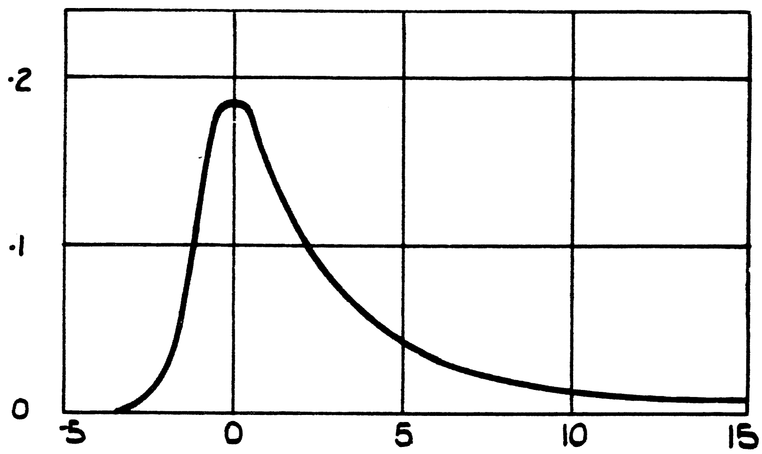

FIG. 1. THE LANDAU CURVE.

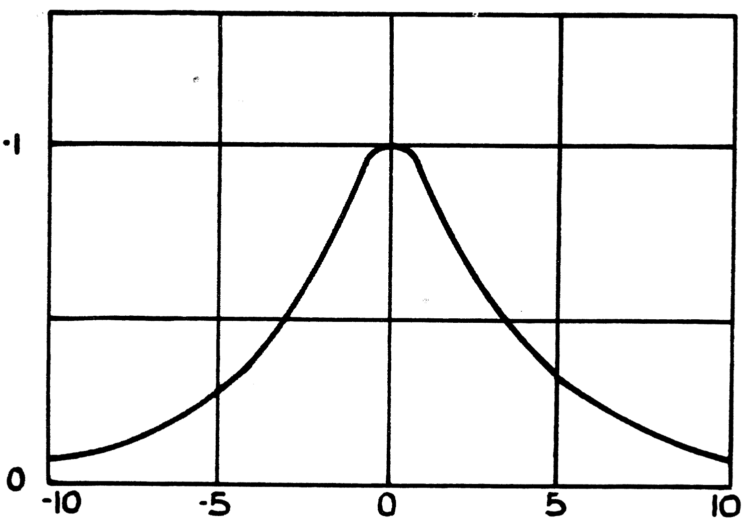

FIG. 2. DISTRIBUTION OF $\alpha = \varphi/\sigma$

APPENDIX

PHASE OSCILLATIONS

The axial electric field in a particular section of accelerator may be analysed into components varying sinusoidally in space. The most important of these for the motion of the proton is the one whose phase velocity is nearly equal to the particle velocity. It is not correct(5) to neglect the others entirely, but their effect is small and will not be discussed here. The equation of motion is then

$$\frac{d}{dt}(p) = eE \cos(\varphi_0 + \varphi)$$

where p is the relativistic momentum, φ_0 the synchronous phase, and $\varphi_0 + \varphi$ the actual phase of the particle considered. If p_0 is the synchronous momentum

$$\frac{d}{dt}(p - p_0) = eE \{\cos(\varphi_0 + \varphi) - \cos \varphi_0\}$$

For small φ the right hand side can be written

$$- \frac{2\pi A}{\beta\lambda} \tan \varphi_0 \,(z - z_0)$$

where $A = eE \cos \varphi_0$ and $z - z_0 = \frac{\beta\lambda}{2\pi}\varphi$ is the distance of the particle from the synchronous particle. The difference of momenta is approximately

$$\frac{dp}{d\dot{z}} (\dot{z} - \dot{z}_0) = M_L (\dot{z} - \dot{z}_0)$$

with M_L being the longitudinal mass of the synchronous particle, equal to the rest mass M_0 divided by $(1-\beta^2)^{3/2}$. We then have

$$\frac{d}{dt} \left\{M_L (\dot{z} - \dot{z}_0)\right\} = - \frac{2\pi A}{\beta\lambda} \tan \varphi_0 \,(z - z_0)$$

Writing $u = z - z_0$, $\alpha = \frac{2\pi A}{\beta\lambda} \tan \varphi_0$

$$\frac{d}{dt} M_L \frac{du}{dt} = - \alpha u$$

which has the adiabatic solution

$$u \sim (\alpha M_L)^{-1/4} \sin \left(\gamma + \int \left(\frac{\alpha}{M_L}\right)^{1/2} dt\right)$$

where γ is arbitrary. With

$$\varphi = \frac{2\pi}{\beta\lambda} u \quad \text{and} \quad \chi = \beta(1-\beta^2)^{-1/2} \quad \text{we have}$$

$$\varphi \sim \chi^{-3/4} \sin \left(\gamma + \int \left(\frac{\alpha}{M_L}\right)^{1/2} dt \right)$$

COLLISIONS.

Consider now a particle which travels initially with the synchronous phase and velocity but suddenly loses an energy ε with which is associated a small velocity decrease $c\delta\beta$. It is easily seen that

$$\varepsilon = M_L c^2 \beta \delta\beta$$

An oscillation is set up with initial amplitude in terms of u

$$c\delta\beta \left(\frac{\alpha}{M_L}\right)^{-1/2} = \frac{c}{\beta} \frac{\varepsilon}{M_L c^2} \left(\frac{M_L}{\alpha}\right)^{1/2}$$

or in terms of φ

$$\frac{2\pi}{\beta\lambda} \frac{c}{\beta} \frac{\varepsilon}{M_L c^2} \left(\frac{M_L}{\alpha}\right)^{1/2}$$

$$= \frac{\varepsilon}{M_o c^2} \left\{ \frac{2\pi M_o c^2}{A\lambda \tan \varphi_o} \right\}^{1/2} \chi^{-3/2}$$

In proceeding to the end of the accelerator at χ_2, the amplitude changes by a factor $\left(\chi_2/\chi\right)^{-3/4}$ and is finally

$$\frac{\varepsilon}{M_o c^2} \left\{ \frac{2\pi M_o c^2}{A\lambda \tan \varphi_o} \right\}^{1/2} \chi_2^{-3/4} \chi^{-3/4}$$

which we will call εf. At the end of the accelerator

$$\varphi = \varepsilon f \sin \psi$$

where

$$\psi = \int \left(\frac{\alpha}{M_L}\right)^{1/2} dt$$

the limits of integration being the time of occurrence of the collision and the time of exit from the machine.

LARGEST AND SMALLEST COLLISIONS.

The energy loss in a single collision ranges from the maximum

$$\varepsilon_m = 2m_o c^2 \beta^2/1-\beta^2$$

$$= 2m_o c^2 \chi^2$$

down to energies less than ε_0 – the mean binding energy of the atom. For a 5 MeV. proton – the least energetic we shall consider – $\varepsilon_m \sim 10^4$ eV. The greatest binding energies in beryllium are of order 100 eV., so $\varepsilon_m >> \varepsilon_0$. If φ_m and φ_0 denote the corresponding final amplitudes of phase oscillation $\varphi_m >> \varphi_0$. Substituting the formula for ε_m in that for φ we get

$$\varphi_m = 2 \frac{m_0}{M_0} \left\{ \frac{2\pi M_0 c^2}{A\lambda \tan\varphi_0} \right\}^{\frac{1}{2}} \chi_2^{-3/4} \chi^{5/4}$$

It is seen that the largest φ_m originates at the high energy end of the machine. In the 47–470 MeV. machine already described φ_m varies from 1° to 4.5°, while in the similar 4.7–47 MeV. accelerator φ_m varies from $.6^\circ$ to 2.4°.

FINAL PHASE DISTRIBUTION.

If $\Phi(\varphi)\, d\varphi$ denotes the average number of collisions in the accelerator which result in oscillations whose final phase is in the region $d\varphi$, then

$$\Phi(\varphi)d\varphi = \Sigma\, W(\varepsilon) \left| \frac{d\varepsilon}{d\varphi} \right| d\varphi$$

the summation being over all foils. With

$$\varphi = \varepsilon f \sin\psi$$

we have

$$\Phi(\varphi) = \Sigma \left| f \sin\psi \right|^{-1} W(\varepsilon \left| f \sin\psi \right|^{-1})$$

Now for $\varepsilon_m > \varepsilon >> \varepsilon_0$

$$W(\varepsilon) = 2m_0c^2 \pi n z v_0^2 (1 - \beta^2 \,{}^{\varepsilon}/\varepsilon_m) \frac{1}{\beta^2 \varepsilon^2}$$

and

$$W(\varepsilon) = 0 \quad \text{if } \varepsilon < 0 \quad \text{or} \quad \varepsilon > \varepsilon_m .$$

Thus for $\varphi >> \varphi_0$

$$\Phi(\varphi) = \Sigma\, 2m_0c^2 \frac{\pi n z v_0^2}{\beta^2} \frac{\left| f \sin\psi \right|}{\varphi^2} \left\{ 1 - \beta^2 \,{}^{\varphi}/\varphi_m \right\}$$

provided that those foils be omitted from the summation for which φ and $\sin\psi$ are of opposite sign or for which $|\varphi_m \sin\psi| < \varphi$. The latter omission will not be very important provided $\varphi << \varphi_m$ and we may then neglect also the term $\beta^2\, {}^{\varphi}/\varphi_m$, and thus for $\varphi_0 << \varphi << \varphi_m$

$$\Phi(\varphi) = 1/\varphi^2 \quad \Sigma \quad 2m_oc^2 \; \frac{\pi n z v_o^2}{\beta^2} \; |f \sin \psi|$$

Considering positive and negative φ's separately we see that the distribution of φ's is the same as the distribution of ε's assumed in Landau's work and provided that

$$\varphi_o \ll \left| \Sigma \; 2m_oc^2 \; \frac{\pi n z v_o^2}{\beta^2} \; |f \sin \psi| \right| \ll \varphi_m$$

the separate probability distribution for their sums will be the appropriate Landau curves. The probability distribution for the resultant φ will be a convolution of the two. As in the single foil problem one should calculate separately the position of the centre of the bunch, for this, as distinct from the distribution within the bunch, depends on the small losses of energy ε_o for which the ε^{-2} and φ^{-2} formulae are not valid. It can easily be seen that

$$\varphi_{av} = \Sigma \; \delta \; f \sin \Psi$$

the summation being all foils and δ being the mean total energy loss in one. In the cases we shall consider (see section on mean energy loss) φ_{av} will be small compared with the bunch width and we shall ignore it.

Now in an accelerator with a fair number of phase oscillations sin ψ will be nearly as often positive as negative and we will not make a great error in either case if we replace it by an average value π^{-1}. Then the distribution of φ/σ where

$$\sigma = \frac{1}{\pi} \; \Sigma \; 2m_oc^2 \; \frac{\pi n z v_o^2}{\beta^2} \; f$$

becomes the convolution of two mirror-image Landau curves shown roughly in fig. 2. Substituting the formula for f and approximating the summation by an integration over χ, remambering(1) that the change of χ between successive foils is

$$\delta\chi = \frac{A\lambda}{M_oc^2}$$

we have finally

$$\sigma = 2nzv_o^2 \; \frac{m_oc^2}{A\lambda} \left\{ \frac{2\pi \; M_oc^2}{A\lambda \; \tan \varphi_o} \right\}^{1/2} \int_{\chi_1}^{\chi_2} \chi_2^{-3/4} \; \chi^{-3/4} \; \beta^{-2} \; d\chi$$

where χ_1 and χ_2 are the values of this quantity at the ends of the machine.

With 3 x 10^{-4} inch foils on the 47–470 MeV. machine $\sigma = .16°$. Referring to fig. 2, it is seen that the width of the distribution at half height is roughly plus and minus 3σ or in the present case ± .5°. It is apparent that ξ is not as small compared with the φ_m's calculated in the last section (1° – 4.5°) as would be required for the curve of fig. 2. to be really valid. It is no doubt a fair approximation to the actual distribution near the middle, but is increasingly an overestimate of the actual probability for angles of order a degree and larger, and the actual bunch will not extend much further than say, 5°.

With 3 x 10^{-5} inch foils on a similar 4.7–47 MeV. machine the situation is much the same; $\sigma = .24°$ and the bunch extends to some ± 3°.

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