

BEAM-LOADING EFFECTS IN A STANDING-WAVE ACCELERATOR STRUCTURE

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The steady-state beam-loading effects on the accelerating field in the disk-loaded structure of a standing-wave cavity have been systematically studied. The electron bunch from a 15-MeV electron linac is injected into the test structure at arbitrary phase of the external driving field. The changes of the phase shift of the accelerating field and of the stored energy are measured as a function of the phase on which the bunch rides. The former shows drastic change when the bunch is near the crest of the driving field and when the beam loading is heavy, whereas the latter varies sinusoidally for any beam loading. The resonant frequency shift of the structure due to beam loading is estimated by using the measured results. All the experimental results are quantitatively explained by normal-mode analysis of microwave cavity theory.

1. INTRODUCTION

With the development of accelerator utilization, it is a contemporary requirement for the accelerator technology to increase the intensity, to reduce the energy spread and to improve the stability of the output beam. In achieving these objectives, one cannot neglect the beam-loading effect on the accelerating field. This effect is especially serious in a linear accelerator (linac) because it usually accelerates very intense beam.

The effect of steady-state beam loading can be regarded as caused by the electromagnetic field induced in the accelerator structure by the bunched beam. Among the Fourier components of the induced field, the fundamental mode gives rise to so-called detuning of the accelerator structure, which will affect the intensity and the energy spectrum of the beam. Instabilities such as beam break-up in large-scale electron linacs are known to originate from some higher modes of the induced field. Here we will treat with the problems related to the detuning effect both from the experimental and the theoretical aspect.

The detuning of the accelerator structure by beam loading has an effect, not only on the beam quality, but also on the microwave source. The effect on the beam is such that the amplitude reduction and phase shift of the accelerating field cause deterioration of the accelerating efficiency and of the energy spectrum. On the other hand,

reflections of the microwave power from the accelerator structure increase because of the impedance change of the structure, and these reflections make it difficult to operate the microwave source stably.

Methods and devices to compensate for these effects have been studied by many authors.¹⁻⁴ Through these studies, a general theory for beam-loading phenomena,⁵⁻⁸ based upon equivalent-circuit analysis or upon microwave cavity theory, has been developed. The experimental studies of the beam loading⁹⁻¹¹ have so far been limited to regions around the synchronous phase of the accelerating field or to measurements on simulated systems.

In the present experiment, the bunched beam from the electron linac is passed through an S-band disk-loaded accelerator structure of standing-wave type. The phase of the bunch with respect to the driving field in the structure is systematically varied. To separate the effect on the microwave source, the accelerator structure has been connected to it by a well-padded transmission line. Thus the measurements can be a direct test of the beam-loading theory.

The following subjects will be discussed in the subsequent sections:

- 1) Changes of amplitude and phase shift of the accelerating field as functions of the initial phase on which the bunch rides.

2) Relation between the phase shift of the accelerating field and the resonant-frequency shift of the accelerator structure.

3) Application of normal-mode analysis to the experimental results.

2. EXPERIMENTAL ARRANGEMENTS

A schematic diagram of the experimental setup and the microwave apparatus is shown in Fig. 1. The test cavity is a disk-loaded waveguide of standing-wave type, which consists of 6 disks, 5 cylinders, 2 half-cylinders and 2 end-plates, as seen in Fig. 1. The dimensions of the structure are as follows: the disk-hole diameter $2a$ is 22.255 mm, the cylinder diameter $2b$ is 85.056 mm, the periodic length d is 36.233 mm, the disk thickness t is 5.000 mm and the total length of the structure is 21.74 cm. The normalized phase velocity v_p/c and the normalized group velocity v_g/c of the structure are 1.00 and 1.13×10^{-2} , respectively. Each end-plate has a hole 3 mm in diameter at the center for the beam and a small antenna at a distance of 16 mm from the center for micro-

wave coupling; the coupling coefficient is 0.05. This structure is assembled on a V-block and is put into a vacuum chamber as shown in Fig. 2.

Microwave power for exciting the structure is supplied from a traveling-wave tube amplifier operating in a pulsed mode in the microwave system of the linac. The peak power and pulse width of the microwave pulse to the structure are 16 W and 5 μ sec, respectively.

The electron beam of the INS (Institute for Nuclear Study, University of Tokyo) linac, which has an energy of 15 MeV and a beam pulse width of 2 μ sec, is used as the bunched beam in the present experiment. The output beam from the linac is collimated to have a diameter of 2 mm and is transported to the test structure. The beam current is measured by a Faraday-cup located downstream of the structure.

Since the test structure has the same dimensions as a regular section of the INS linac,¹² its resonant frequency is equal to the operation frequency of the linac if each structure has the same temperature as the other. Then the linac is operated with the same frequency as the resonant frequency of the structure by temperature control

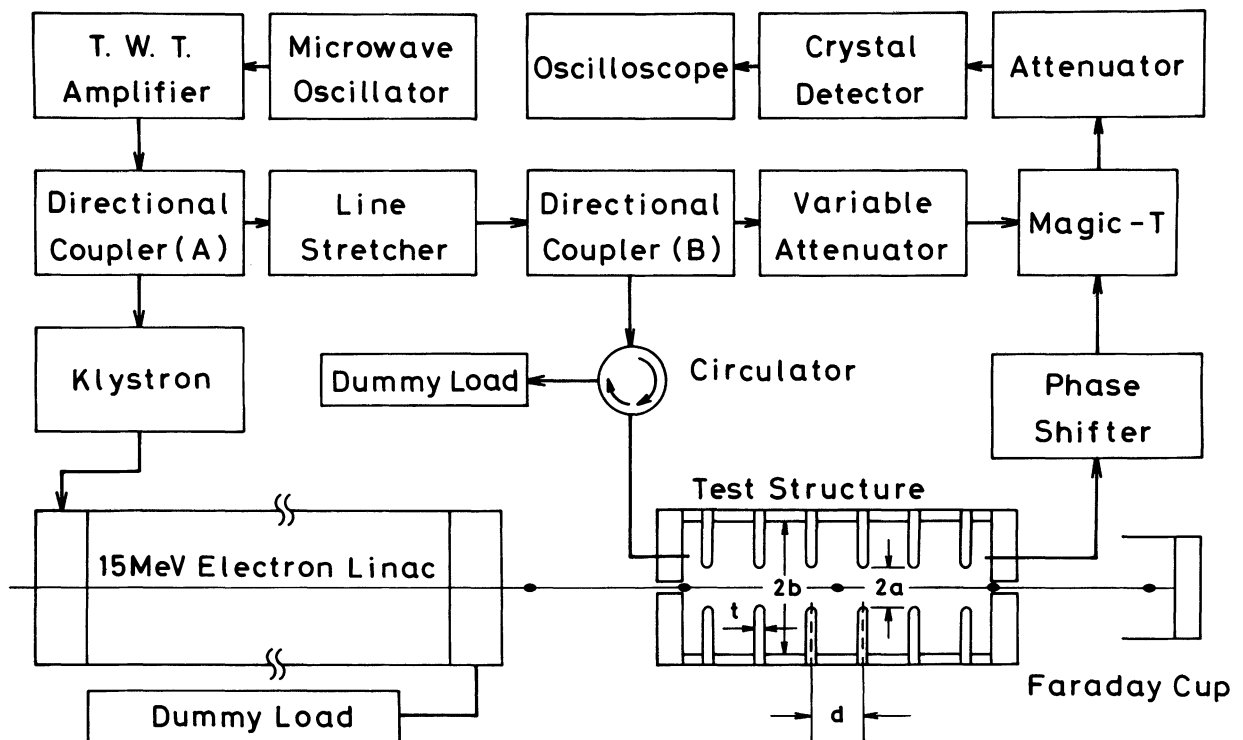


FIGURE 1 Schematic diagram of the experimental set-up, and the microwave apparatus.

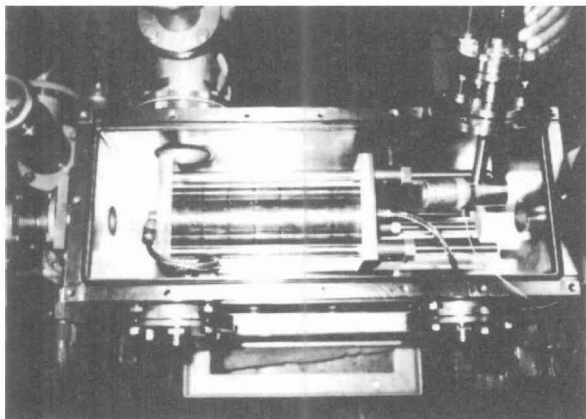


FIGURE 2 Photograph of the accelerator structure installed in the vacuum chamber.

in order to synchronize the rf field in the structure with the incident electron bunch.

3. MEASUREMENT METHOD AND EXPERIMENTAL RESULTS

The changes of stored energy and phase shift of the microwave field in the structure due to beam loading are measured as functions of the phase of the external driving field for the beam bunch.

First, the stored energy W in the structure can be obtained by measuring the radiated power from the structure, P_{rad} , and by using the relation

$$P_{rad} = \frac{\beta\omega W}{Q_o}, \quad (3.1)$$

where Q_o is the unloaded Q of the structure, ω is the angular frequency of the wave field, and β is the coupling coefficient of the antenna, which can in turn be determined from the relation between the input power P_o and the transmitted power P_t of the structure

$$\frac{P_t}{P_o} = \frac{4\beta^2}{(1 + 2\beta)^2}. \quad (3.2)$$

The stored energy due purely to the beam-induced field, W_b , can be measured by the experimental arrangement of Fig. 1, but without feeding any external wave. An example of the waveform of the induced field is shown in Fig. 3. It is seen in the figure that the field is built up in the beam-pulse duration (2μ second) and then

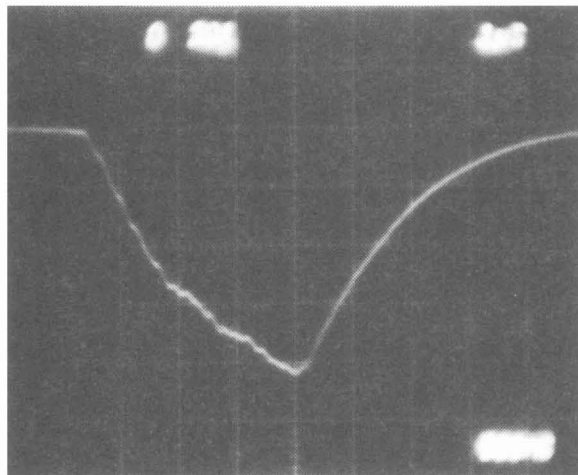


FIGURE 3 The waveform of the beam-induced field. Time scale = $0.5 \mu\text{sec}$, amplitude scale = 10 mV/div .

decays gradually. The peak of the waveform corresponds to P_{rad} at the pulse end and to a certain peak beam intensity that can be found from the average current measured by the Faraday cup and from the duty factor of the beam. Figure 4 shows the peak stored energy as a function of the beam current thus measured. The solid line in the figure is a calculation with the same parameters as the experiment (see Appendix A). Fairly good agreement of both results confirms the validity of the present experimental procedure.

The waveforms of energy stored in the structure excited by the external microwave are shown in Figs. 5(a) through 5(c). Figure 5(a) rep-

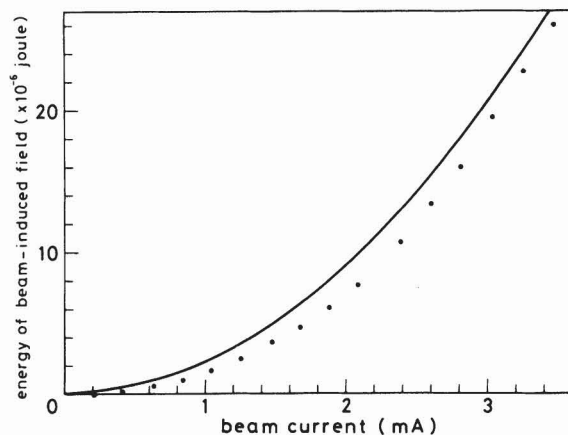
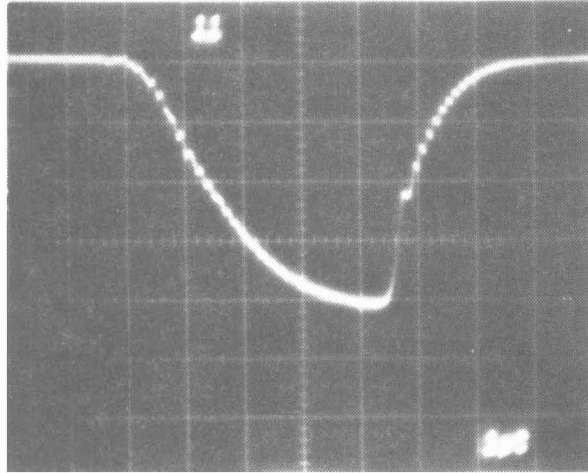
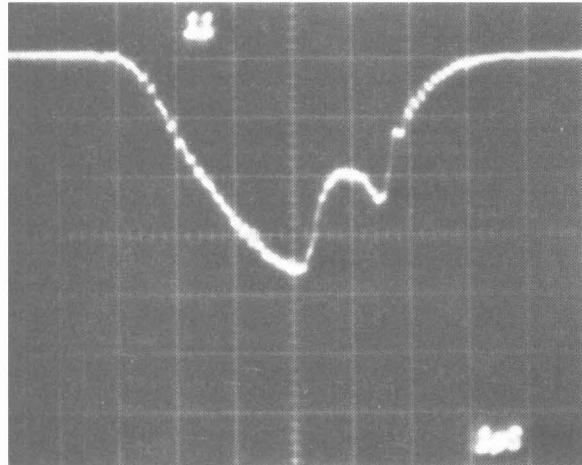


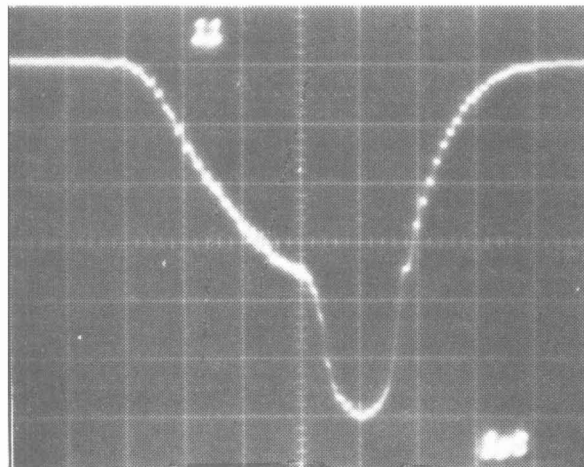
FIGURE 4 The stored energy due to the induced field in the structure is shown as a function of the beam current.



(a)



(b)



(c)

FIGURE 5 The waveforms observed a) without bunch; b), c) with bunch on the accelerating phase and the decelerating phase, respectively. Time scale = 1 μ sec/div., amplitude scale = 10 mV/div.

resents P_{rad} without the electron beam. Figure 5(b) shows the buildup of the field when the electron bunches are injected into the structure at the accelerating phase and are taking the energy from the driving field. Figure 5(c) corresponds to the case when the bunches are at the decelerating phase and are giving energy to the structure.

The phase of the driving field for the electron bunch can be adjusted by use of the line stretcher located between the structure and the external microwave source, as seen in Fig. 1. One can know the scale of the line stretcher at which the electron bunch is riding just on the crest of the accelerating or of the decelerating phase by making the stored energy minimum or maximum, and hence one can realize any phase of the driving field for the electron bunch.

The phase shift of the field in the structure (superposition of the driving field and the induced field) from the original driving field is measured in the following way: the driving microwave power divided from the source is further divided by another directional coupler. One of the branches is used for exciting the structure, and the other is connected to a magic-T for comparing the phase with that of the microwave from the structure (see Fig. 1). At the start of the measurement, the electron beam is turned off and the phase shifter between the structure and the magic-T is adjusted so that the vector summation of the signals to the magic-T is minimized. By this adjustment, the phase difference between two signals at the magic-T is confirmed to be 180° . Then, passing the electron beam, the phase shifter is adjusted again to minimize the vector summation. The scale difference of the phase shifter between the first and the second measurements is the phase shift from the beam loading for given phase of the bunch and intensity of the beam.

Following the method above, the stored energy and phase shift of the field in the structure have been measured systematically by varying the phase of the driving field relative to the bunch and the degree of beam loading. We have defined the beam loading in terms of the ratio of the beam induced field E_b to the external driving field E_o , which is given by

$$\frac{E_b}{E_o} = \sqrt{\frac{W_b}{W_o}} \quad (3.3)$$

Here W_b and W_o are the stored energies due to induced field and due to driving field, respectively, as discussed above.

The measurement has been performed for E_b/E_o of 0.30 and 0.62. The results are shown in Fig. 6 by open circles for stored energy and by solid circles for phase shift, as functions of the relative phase of the electron bunch to the driving field. The absolute value of the phase of the bunch for the driving field is so defined that it is zero and 180° when the bunch is on the crest of the accelerating and of the decelerating phase, respectively. The upper diagram in the figure represents the small beam-loading case, $E_b/E_o = 0.30$ and the lower one is the heavy beam-loading case, $E_b/E_o = 0.62$.

The remarkable features of the results are

- 1) The stored energy changes sinusoidally with the phase of the electron bunch.
- 2) The phase shift is negligible for resistive beam loading, that is for the phases of the bunch of 0 or 180° , at least for the beam loading factor E_b/E_o under 0.62 .
- 3) The phase of the field changes drastically for reactive beam loading, that is for any phase of the bunch except 0° and 180° . The phase shift

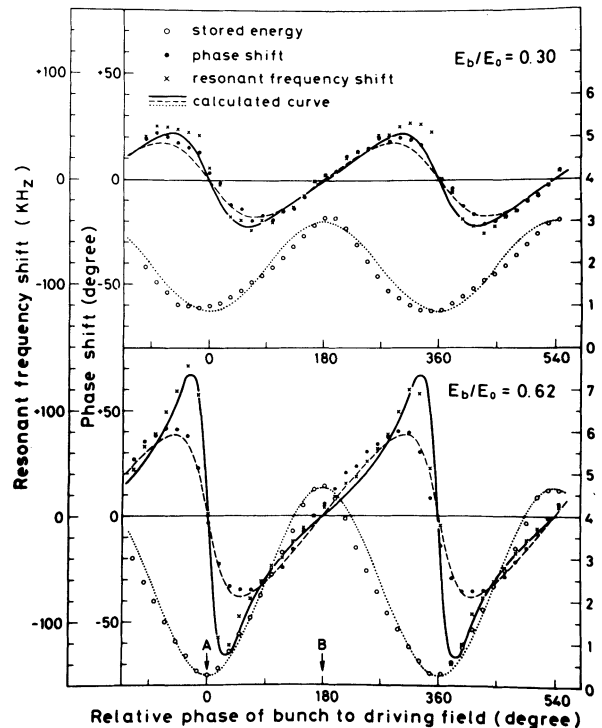


FIGURE 6 The variation of the stored energy, the phase shift, and the resonant frequency shift due to the beam loading. The theoretical results are shown by dotted line, dashed line, and solid line.

varies steeply near 0° , especially when the beam loading is heavy.

4. THEORETICAL ANALYSIS AND DISCUSSION

The experimental results presented in the preceding section are analyzed by normal-mode analysis of microwave-cavity theory.^{6,7,13,14} In the analysis, modes other than the fundamental are neglected. This is justified by spectrum measurements of the induced field, in which the fundamental mode is found to be more intense by 30 db compared to any other mode (See Appendix B).

Now we will treat the phenomena expected in the accelerator structure into which the external field is fed and the electron bunches are injected. The stored energy can be expressed as

$$W = \frac{4P_o}{\omega Q_{\text{ext}\cdot 1}} \times \frac{1}{\left(\frac{1}{Q_{\text{ext}\cdot 1}} + \frac{1}{Q_{\text{ext}\cdot 2}} + \frac{1}{Q_o} + \frac{1}{Q_b}\right) + \left(\frac{2\Delta\omega}{\omega_a'}\right)^2}, \quad (4.1)$$

where P_o is the input microwave to the structure, $Q_{\text{ext}\cdot 1}$ and $Q_{\text{ext}\cdot 2}$ are the external Q for the input and the output coupler, respectively, Q_b is the beam Q , ω_a' is the resonant frequency of the structure without beam and $\Delta\omega$ is the resonant frequency shift due to reactive beam loading (See Appendix C). Q_b and $\Delta\omega = \omega - \omega_a'$ are given by the following equations⁶

$$\frac{1}{Q_b} = \frac{1}{Q_L} \frac{B \cos \phi_b - B^2}{1 + B^2 - 2B \cos \phi_b}, \quad (4.2)$$

$$\frac{2\Delta\omega}{\omega_a'} = \frac{1}{Q_L} \frac{-B \sin \phi_b}{1 + B^2 - 2B \cos \phi_b}, \quad (4.3)$$

where Q_L is the loaded Q , B is the ratio of the beam-induced field E_b to the driving field E_o , and ϕ_b is the relative phase of the bunch to the driving field as defined in the preceding section and shown in Fig. 7.

Substituting Eqs. (4.2) and (4.3) into Eq. (4.1), we obtain an expression for the stored energy W

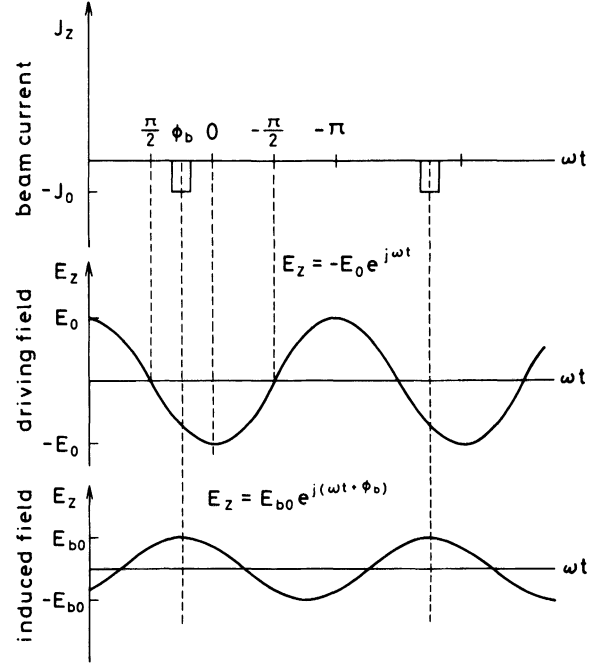


FIGURE 7 The diagram showing the phase relation of the bunches, the driving field and beam-induced field.

as a linear function of $\cos \phi_b$

$$W = \frac{4P_o Q_L^2}{\omega Q_{\text{ext}\cdot 1}} (1 + B^2 - 2B \cos \phi_b), \quad (4.4)$$

where $4P_o Q_L^2 / \omega Q_{\text{ext}\cdot 1}$ is the stored energy without beam loading.

This equation explains well the experimental result that the stored energy changes sinusoidally for the variation of ϕ_b . The curves in Fig. 6 are the numerical results of Eq. (4.4) for $B = 0.30$ and 0.62 .

The field in the structure is the superposition of the driving field and the beam-induced field

$$Ee^{j(\omega t + \psi)} = E_o e^{j\omega t} - E_b e^{j(\omega t + \phi_b)}, \quad (4.5)$$

where ψ is the phase shift between the resultant field and the driving field and can be expressed as $\tan^{-1}[(-B \sin \phi_b)/(1 - B \cos \phi_b)]$,

$$\psi = \tan^{-1} \left(\frac{-B \sin \phi_b}{1 - B \cos \phi_b} \right), \quad (4.6)$$

The results calculated for $B = 0.30$ and 0.62 are shown in Fig. 6.

We are interested in the resonant frequency shift due to beam loading. The pulse length of the driving field and of the electron beam, however, is not sufficiently long for us to measure the frequency shift directly and it is therefore estimated from the measured quantities.

One can calculate the resonant frequency shift from the beam-loading factor B by using Eq. (4.3). The result is shown in Fig. 6 by a solid line. On the other hand, it is possible to find the frequency shift from the measured phase shift. Combining Eqs. (4.2), (4.3) and (4.6), one obtains the relation

$$\psi = \tan^{-1} \left(\frac{2\Delta\omega}{\omega_a'} Q_T \right), \quad (4.7)$$

where

$$\frac{1}{Q_T} = \frac{1}{Q_L} + \frac{1}{Q_b}.$$

This is a familiar equation for estimating the phase shift in the case when the operating frequency is different from the resonant frequency of the cavity. Since the beam $Q(Q_b$ in Eq. (4.7)) cannot be measured, it is calculated using Eq. (4.2). The Q_T thus obtained is shown in Fig. 8. The resonant frequency shift given by this method is shown in Fig. 6 by cross marks. It is seen that the results estimated in different ways agree with each other.

From the experimental results and the theoretical analysis so far discussed, we can conclude:

1) The resonant frequency shifts to higher frequency when the electron bunch is in a phase where the field is decreasing with time and vice versa. This agrees with the well-known fact that the rf acceleration system of an electron synchrotron, in which the stable phase is behind the crest of the field, is operated with higher frequency than the resonant frequency without the beam, and that the resonant frequency of an Alvarez proton linac, which accelerate low velocity protons and whose stable phase is ahead of the field crest, shifts to lower frequency.

2) The variation of the phase shift or the resonant frequency shift is steepest around the crest of the field, especially in the case of heavy beam loading. This means that the acceleration of the particle which is riding just on the field crest will

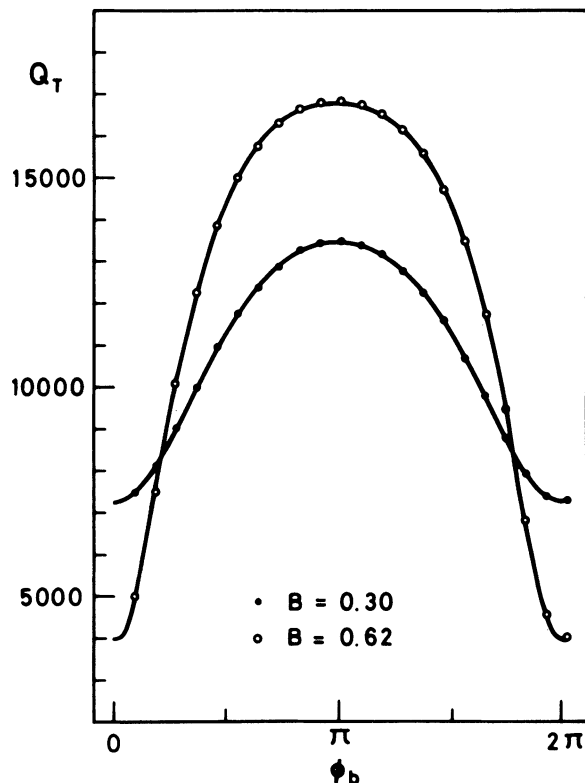


FIGURE 8 The calculated values of the total loaded Q , Q_T , as a function of the relative phase of the bunch to the driving field, ϕ_b , where the loaded Q without beam loading is 10400.

suffer from the instability of the tuning frequency.

3) The detuning of the structure due to the resistive beam loading has not been observed with beam loading limited below $B = 0.62$.

4) All the experimental results can be explained by normal-mode analysis including only the fundamental mode. As is evident from the deduction procedure, the results are valid for steady-state beam loading in any cavity of standing-wave type.

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ment. They are indebted to the members of INS mechanical shop for their skillful construction of the experimental apparatus. The numerical analysis has been carried out with the computer TOS-BAC-3400 of the Electronic Computer Section at INS.

APPENDIX A: Beam-Induced Field

According to J. C. Slater's theory of the hollow cavity, the equation of forced oscillation due to the bunched beam is

$$\begin{aligned} \frac{d^2}{dt^2} \int \mathbf{E} \cdot \mathbf{E}_a \, dv + \frac{\omega_a'}{Q_L} \frac{d}{dt} \int \mathbf{E} \cdot \mathbf{E}_a \, dv \\ + \omega_a'^2 \int \mathbf{E} \cdot \mathbf{E}_a \, dv \\ = - \frac{1}{\epsilon_o} \frac{d}{dt} \int \mathbf{J} \cdot \mathbf{E}_a \, dv, \quad (\text{A.1}) \end{aligned}$$

where E is the electric field, E_a is the a -th normal-mode field, J is the current density of the bunched beam and ω_a' is the angular resonant frequency of the cavity without the beam. The beam-cavity coupling integral on the right-hand side is given approximately⁶ by

$$\int \mathbf{J} \cdot \mathbf{E}_a \, dv = 2I_o E_{ao} L e^{j\omega_a' t}, \quad (\text{A.2})$$

if the bunch is composed of a rectangular pulse whose width is relatively short compared with the period of the rf pulse and the bunch is synchronous with the a -th mode. In Eq. (A.2), I_o is the beam current averaged over the bunches, E_{ao} is the amplitude of the a -th normal mode field and L is the cavity length.

If the beam is expressed by a step function ($I_o = 0$ when $t \leq 0$ and $I_o = \text{const}$ when $t > 0$) the solution of Eq. (A.1) is given by

$$\begin{aligned} \int \mathbf{E} \cdot \mathbf{E}_a \, dv = \frac{2Q_L I_o E_{ao} L}{\epsilon_o \omega_a'} \\ \times [1 - e^{-(\omega_a'/2Q_L)t}] e^{j\omega_a' t}, \quad (\text{A.3}) \end{aligned}$$

The stored energy is defined by

$$W = \frac{1}{2} \epsilon_o \left| \int \mathbf{E} \cdot \mathbf{E}_a \, dv \right|^2. \quad (\text{A.4})$$

Substituting Eq. (A.3) into Eq. (A.4), we have

$$W = \frac{2Q_L^2 I_o^2 E_{ao}^2 L^2}{\epsilon_o \omega_a'^2} [1 - e^{-(\omega_a'/2Q_L)t}]^2. \quad (\text{A.5})$$

On the other hand, the relation between E_{ao} and the effective shunt impedance, r_s , is given by

$$E_{ao}^2 = \frac{\epsilon_o \omega_a' r_s}{2Q_o L}. \quad (\text{A.6})$$

From Eqs. (A.5) and (A.6), the energy of the beam-induced field stored in the structure is expressed by

$$W = \frac{Q_L^2 I_o^2 L r_s}{Q_o \omega_a'} [1 - e^{-(\omega_a'/2Q_L)t}]^2. \quad (\text{A.7})$$

This equation represents the buildup of the beam-induced field that is seen in Fig. 3. For the present experiment, the values of the parameters in Eq. (A.7) are: $Q_L = 10400$, $Q_o = 11400$, $L = 21.74$ cm, $\omega_a' = 1.733 \times 10^{10}$, $r_s/Q_o = 24 \Omega/\text{cm}$ and $t = 2.2 \mu\text{sec}$.

Then the peak stored energy W (joule) can be expressed as a function of current I_o (ampere) by

$$W = 2.29 I_o^2.$$

This relation between I_o and W is shown in Fig. 4 with a solid line.

As seen in the figure, there is slight difference between the calculation and the experiment. This is due to the approximations that the beam pulse, which looks like a trapezium with a half width of approximately $2.2 \mu\text{sec}$, is taken to be a $2.2 \mu\text{sec}$ rectangular pulse and the shape of the beam bunch is approximated by a very short rectangular pulse.

APPENDIX B: Frequency Components of the Beam-Induced Field

The frequency components of the beam-induced field have been measured by a spectrum analyzer. Four peaks have been observed, as seen in Fig. B(1). The highest peak represents the resonant frequency of the $2\pi/3$ mode. The two peaks on both sides of the highest peak represent the $\pi/2$ mode and $5\pi/6$ mode, respectively, and the smallest peak at the left-hand side corresponds to the $\pi/6$ mode. Each peak is accompanied by many sidebands, which are due to the pulsed change of the beam-induced field.

It is found that the fundamental mode is more intensive than any other mode at least by 30 dB. The frequency components of the higher-order modes were not found in this measurement.

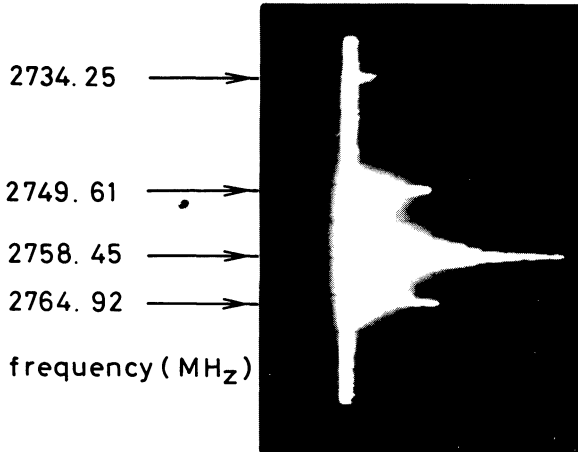


FIGURE B(1) The frequency component of the beam-induced field observed by spectrum analyzer. Horizontal scale = 5 MHz/div., vertical scale = 10 dB/div.

APPENDIX C: Change of the Stored Energy

The input admittance, Y_{in} , looking from the detuned short-circuit plane into a cavity with beam loading can be written as

$$\frac{Y_{in}}{Y_o} = jQ_{ext\cdot 1} \left(\frac{\omega}{\omega_{a'}} - \frac{\omega_{a'}}{\omega} \right) + Q_{ext\cdot 1} \left(\frac{1}{Q_o} + \frac{1}{Q_{ext\cdot 2}} + \frac{1}{Q_b} \right), \quad (C.1)$$

where Y_o is the characteristic admittance of the wave guide and the output wave guide of the cavity is assumed to be terminated by a matched load.

We write the input admittance as $Y_{in} = G + jB$; then the power flow into the cavity, P_{in} is expressed in the form

$$\frac{P_{in}}{P_o} = \frac{4(G/Y_o)}{[1 + (G/Y_o)]^2 + (B/Y_o)^2}, \quad (C.2)$$

where P_o is the input power.

Substituting the real and imaginary part of Eq. (C.1) into Eq. (C.2), we have

$$\frac{P_{in}}{P_o} = \frac{4}{\left(\frac{1}{Q_{ext\cdot 1}} + \frac{1}{Q_{L'}} \right)^2 + \left(\frac{2\Delta\omega}{\omega_{a'}} \right)^2}, \quad (C.3)$$

where

$$\frac{1}{Q_{L'}} = \frac{1}{Q_o} + \frac{1}{Q_{ext\cdot 2}} + \frac{1}{Q_b}.$$

From the conservation law of energy in the cavity, the stored energy is given by

$$W = \frac{Q_{L'}}{\omega} P_{in}. \quad (C.4)$$

Thus from Eqs. (C.3) and (C.4), the following equation for the stored energy is obtained

$$W = \frac{4P_o}{\omega Q_{ext\cdot 1}} \frac{1}{\left(\frac{1}{Q_{ext\cdot 1}} + \frac{1}{Q_{ext\cdot 2}} + \frac{1}{Q_o} + \frac{1}{Q_b} \right)^2 + \left(\frac{2\Delta\omega}{\omega_{a'}} \right)^2}, \quad (C.5)$$

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