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Impact picture for the analyzing power A_N in very forward pp elastic scattering

Claude Bourrely*

Centre de Physique Théorique, CNRS Luminy, Case 907 F-13288 Marseille Cedex 09, France

Jacques Soffer†

Department of Physics, Temple University, Philadelphia, Pennsylvania 19122-6082, USA

Tai Tsun Wu‡

Harvard University, Cambridge, Massachusetts 02138, USA,
and Theoretical Physics Division, CERN, 1211 Geneva 23, Switzerland

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In the framework of the impact picture we compute the analyzing power A_N for pp elastic scattering at high energy and in the very forward direction. We consider the full set of Coulomb amplitudes and show that the interference between the hadronic nonflip amplitude and the single-flip Coulomb amplitude is sufficient to obtain a good agreement with the present experimental data. This leads us to conclude that the single-flip hadronic amplitude is small in this low momentum transfer region and it strongly suggests that this process can be used as an absolute polarimeter at the Relativistic Heavy Ions Collider, at Brookhaven National Laboratory, working as a polarized pp collider.

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I. INTRODUCTION

The measurement of spin observables in hadronic exclusive processes is the only way to obtain full knowledge of the corresponding set of scattering amplitudes, and, in particular, their relative size and phase difference. Taking the specific case of proton-proton elastic scattering, a reconstruction of the five amplitudes has been worked out in the low-energy domain [1]. This situation is very different at high energy; due to the lack of data, in the range $p_{\text{lab}} \simeq 100$–$300$ GeV, besides the nonflip hadronic amplitude ϕ_1^h, only the hadronic helicity-flip amplitude ϕ_5^h is known and to a rather poor level of accuracy. The advent of the Brookhaven National Laboratory Relativistic Heavy Ions Collider (BNL-RHIC) pp collider, where the two proton beams can be polarized, longitudinally and transversely, up to an energy $\sqrt{s} = 500$ GeV, offers a unique opportunity to measure single- and double-spin observables, and thus to provide a determination of the spin-dependent amplitudes, which remain unknown so far.

For instance, for an elastic collision of transversely polarized protons, the differential cross section as a function of the momentum transfer t and the azimuthal angle ϕ reads

$$2\pi \frac{d^2\sigma}{dt d\phi} = \frac{d\sigma}{dt}[1 + (P_B + P_Y)A_N \cos\phi + P_B P_Y(A_{NN}\cos^2\phi + A_{SS}\sin^2\phi)], \tag{1}$$

where P_B and P_Y are the beam polarizations, A_N is the analyzing power, and A_{NN}, A_{SS} are double-spin asymmetries (see Ref. [2] for definitions). In this expression, the values of the beam polarizations have to be known accurately in order to reduce the errors on the spin asymmetries. So new measurements are indeed required to achieve an amplitude analysis of pp elastic scattering at high energy, and the success of the vast BNL-RHIC spin program [3] also relies heavily on the precise determination of the beam polarizations. One possibility for an absolute polarimeter[1] is provided by the measurement of the analyzing power A_N in the very forward $|t|$ region, where significant Coulomb nuclear interference (CNI) occurs [5–7].

In the calculation of the analyzing power an important question arises: is the interference fully dominated by the hadronic nonflip amplitude with the one-photon exchange helicity-flip amplitude, or must one also take into account the contribution of the hadronic helicity-flip amplitude ϕ_5^h mentioned above? Several arguments concerning the magnitude and phase of ϕ_5^h in the small t-region have been discussed in great detail in Ref. [6], and it was concluded that the measurement of A_N in the CNI region was badly needed to get the answer. The purpose of this paper is to study this problem in the framework of the impact picture developed almost three decades ago [8], which has led to a very successful phenomenology that has been repeatedly verified by high-energy experiments, including near the forward direction.[2]

*Claude.Bourrely@cpt.univ-mrs.fr
†jsoffer@temple.edu
‡ttwu@seas.harvard.edu

[1]Proton-helium elastic scattering has been also considered as a possible high-energy polarimeter [4].
[2]An accurate measurement for the real part of the pp forward scattering amplitude is a real challenge for the LHC [9].

II. THE IMPACT-PICTURE APPROACH

In the impact picture, the spin-independent hadronic amplitude $\phi_1^h = \phi_3^h$ for pp and $\bar{p}p$ elastic scattering reads as [8]

$$\phi_{1,3}^h(s,t) = \frac{is}{2\pi}\int e^{-i\mathbf{q}\cdot\mathbf{b}}(1 - e^{-\Omega_0(s,\mathbf{b})})d\mathbf{b}, \tag{2}$$

where $\mathbf{q}$ is the momentum transfer ($t = -\mathbf{q}^2$) and $\Omega_0(s,\mathbf{b})$ is the opaqueness at impact parameter $\mathbf{b}$ and at a given energy s. We take

$$\Omega_0(s,\mathbf{b}) = S_0(s)F(\mathbf{b}^2) + R_0(s,\mathbf{b}). \tag{3}$$

Here the first term is associated with the Pomeron exchange, which generates the diffractive component of the scattering, and the second term is the Regge background. The Pomeron energy dependence is given by the crossing symmetric expression [10,11]

$$S_0(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}}, \tag{4}$$

where u is the third Mandelstam variable. The choice one makes for $F(\mathbf{b}^2)$ is crucial, and, as explained in Ref. [8], we take the Bessel transform of

$$\tilde{F}(t) = f[G(t)]^2\frac{a^2 + t}{a^2 - t}. \tag{5}$$

Here $G(t)$ stands for the proton electromagnetic form factor, parametrized as

$$G(t) = \frac{1}{(1 - t/m_1^2)(1 - t/m_2^2)}. \tag{6}$$

The slowly varying function occurring in Eq. (5) reflects the approximate proportionality between the charge density and the hadronic matter distribution inside a proton [12]. So the Pomeron part of the amplitude depends on only *six* parameters: c, c', m_1, m_2, f, and a. The asymptotic energy regime of hadronic interactions are controlled by c and c', which will be kept, for all elastic reactions, at the values obtained in 1984 [13], namely

$$c = 0.167 \quad \text{and} \quad c' = 0.748. \tag{7}$$

The remaining four parameters are related more specifically to the reaction pp ($\bar{p}p$), and they have been fitted in [14] by the use of a large set of elastic data.

We now turn to the Regge background. A generic Regge exchange amplitude has an expression of the form

$$\tilde{R}_i(s,t) = C_i e^{b_i t}[1 \pm e^{-i\pi\alpha_i(t)}]\left[\frac{s}{s_0}\right]^{\alpha_i(t)}, \tag{8}$$

where $C_i e^{b_i t}$ is the Regge residue, $\pm$ refers to an even- or odd-signature exchange, $\alpha_i(t) = \alpha_{0i} + \alpha_i' t$ is a standard linear Regge trajectory, and $s_0 = 1\ \text{GeV}^2$. If $\tilde{R}_0(s,t) = \sum_i \tilde{R}_i(s,t)$ is the sum over all the allowed Regge trajectories, the Regge background $R_0(s,\mathbf{b})$ in Eq. (3) is the Bessel transform of $\tilde{R}_0(s,t)$. In pp ($\bar{p}p$) elastic scattering, the allowed Regge exchanges are A_2, ρ, ω, so the Regge background involves several additional parameters, which are given in Ref. [14].

In earlier work, spin-dependent hadronic amplitudes were implemented [8,15,16] using the notion of rotating matter inside the proton, which allowed us to describe the polarizations and spin correlation parameters, but for the present purpose hadronic spin-dependent amplitudes will be ignored. In order to describe the very small t-region we are interested in, one adds to the hadronic amplitude considered above the full set of Coulomb amplitudes $\phi_i^C(s,t)$, whose expressions are given in Ref. [17], and the Coulomb phase in Ref. [18].

The two observables of interest are the unpolarized cross section $d\sigma/dt$ and the analyzing power A_N, whose expressions in terms of the hadronic and Coulomb amplitudes are, respectively,

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{s^2}\sum_{i=1,\cdots,5}|\phi_i^h(s,t) + \phi_i^C(s,t)|^2 \tag{9}$$

and

$$A_N(s,t) = \frac{4\,\mathrm{Im}((\phi_1^h(s,t))^*\phi_5^C(s,t))}{\sum_{i=1,\cdots,5}|\phi_i^h(s,t) + \phi_i^C(s,t)|^2}. \tag{10}$$

The numerator of this last expression is not fully general because we have assumed that $\phi_1^h = \phi_3^h$ and $\phi_{2,4,5}^h = 0$.

III. NUMERICAL RESULTS

The analyzing power A_N has been measured at high energy for $\sqrt{s} = 13.7$, 19.4, and 200 GeV, but before turning to the calculation of this quantity, it is necessary to look at the predictions for the differential cross section at the corresponding energies. They are given in the upper plot in Fig. 1 and compared with the available experimental results at $\sqrt{s} = 13.7$ and 19.4 GeV. We underestimate a bit the data for high t values at $\sqrt{s} = 13.7$ GeV, which might indicate the presence of a small hadronic spin-dependent amplitude. However, this is not the case at $\sqrt{s} = 44$ GeV, where the agreement is excellent, as shown in the lower plot in Fig. 1. Note that the momentum transfer runs over four decades and the cross section over 11 orders of magnitude, which is a good illustration of the validity of the impact picture. Concerning the energy $\sqrt{s} = 200$ GeV, we cannot make a detailed comparison with the data. The pp2pp experiment [19] has only determined the slope of the cross section for $0.01 < |t| < 0.019\ \text{GeV}^2$, which is $b = 16.3 \pm 1.6(\text{stat}) \pm 0.9(\text{syst})\ \text{GeV}^{-2}$, consistent with the average value obtained in the impact picture, namely, $b = 16.25\ \text{GeV}^{-2}$.

In Fig. 2, we compare the predictions with the data for A_N in the CNI region versus $|t|$ for three different energies, and let us make the following remarks. First, there is

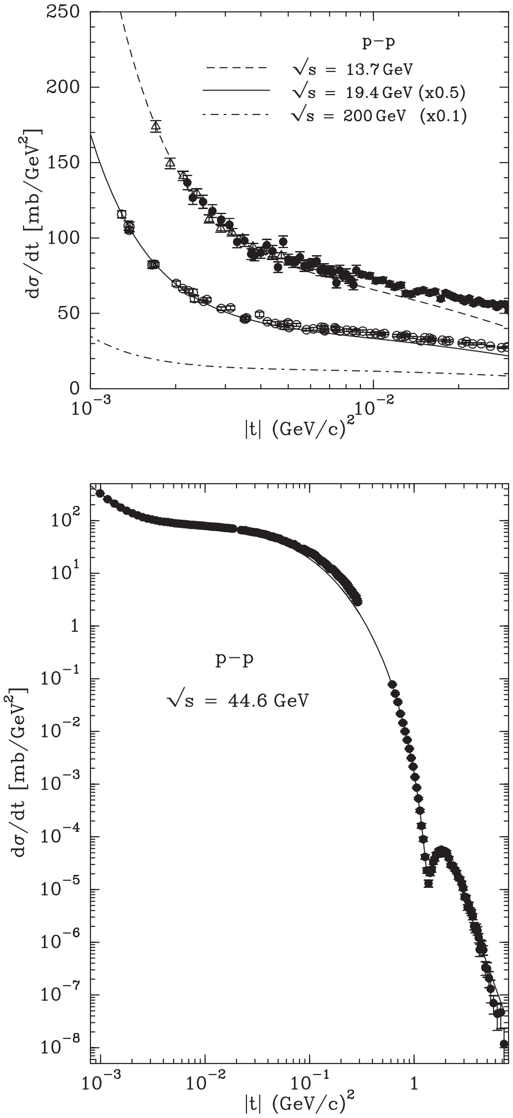

FIG. 1. The differential cross section versus the momentum transfer t for different energies. Data from Refs. [24–29].

almost no energy dependence between $\sqrt{s} = 13.7$ and 19.4 GeV, but the curve has a slightly different shape at $\sqrt{s} = 200$ GeV. Second, although this is not obvious from the plot, A_N does not vanish for $|t| > 0.1$ GeV^2, and we would like to stress that for pp elastic scattering at high energy in the dip region, the hadronic and the Coulomb amplitudes are of the same order of magnitude [20], so the behavior of spin observables is sensitive to this interference. Finally, the predictions agree well indeed with the present experimental data, and in view of future data taking in the BNL-RHIC spin program, we display in Fig. 3 some predictions at $\sqrt{s} = 62.4$ GeV and $\sqrt{s} = 500$ GeV. When the energy increases, the maximum of A_N decreases and occurs at a lower t value, which clearly reflects the rise of the total cross section [6]. The above discussion shows that the hadronic spin-flip amplitude is not necessary to describe the analyzing power, at least when compared with

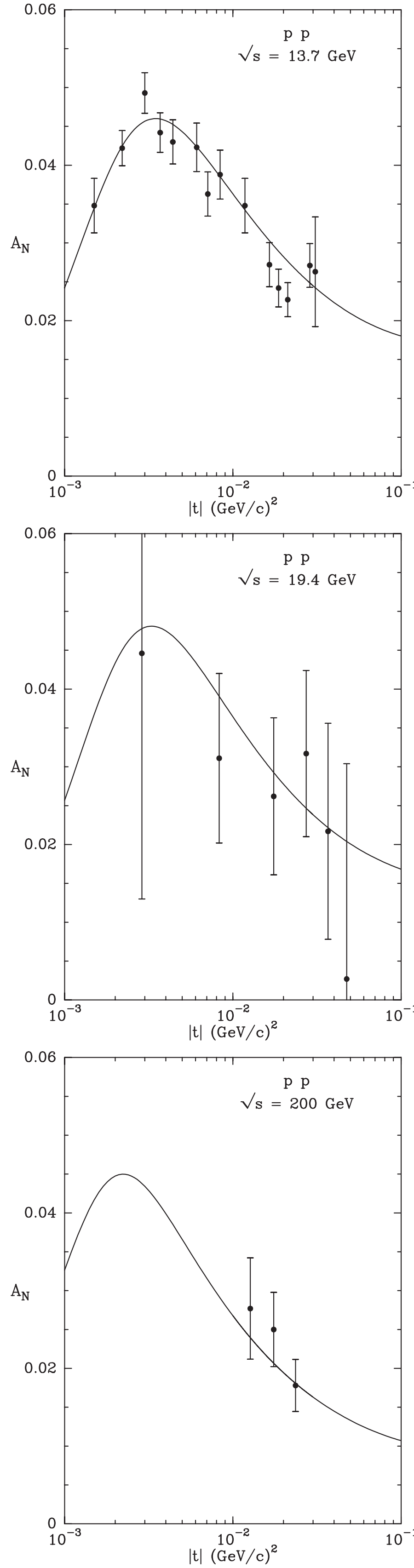

FIG. 2. The analyzing power A_N versus the momentum transfer t for different energies. Data from Refs. [21,30,31].

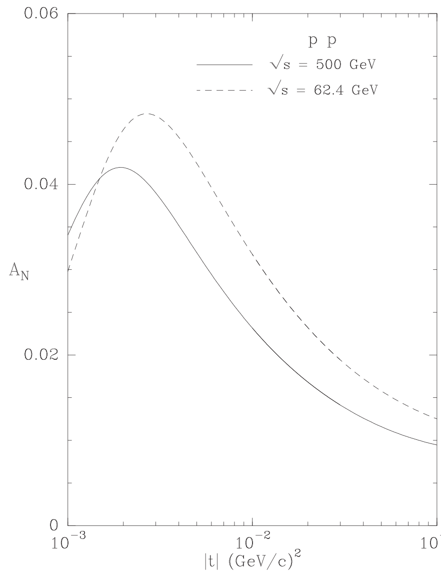

FIG. 3. Predictions for the analyzing power A_N versus the momentum transfer t for two energies.

the data with the present day accuracy. A similar conclusion was obtained in Ref. [21], which contains the best data sample so far. Note that analysis of these data, based on Ref. [6], was done using a simple model for ϕ_1^h, and they did not introduce the full expressions for the Coulomb amplitudes ϕ_i^C, as we do here for consistency.

Before going to the conclusion, it is worth mentioning some very recent data at $\sqrt{s} = 6.7$ GeV [22] with a statistically limited accuracy might indicate the existence of a nonzero ϕ_5^h. However, this energy is too low to allow a simple theoretical interpretation of ϕ_5^h in terms of a nonzero Pomeron flip coupling and would require a more elaborated phenomenological analysis, including dominant Regge contributions.

IV. CONCLUSION

We have shown, in the context of the impact picture, that the analyzing power A_N can be described in the CNI region by the interference between the nonflip hadronic amplitude and the single-flip Coulomb amplitude. Unfortunately, the data set at $\sqrt{s} = 200$ GeV is too limited to confirm the predicted trend. It should be extended to make sure this method is a reliable high-energy polarimeter. The RHIC machine offers a unique opportunity to measure single- and double-spin observables with both longitudinal and transverse spin directions, and we believe it worthwhile to improve such measurements, particularly in the small momentum transfer region, as discussed in Ref. [6]. So far A_{NN} was found consistent with zero within 1.5σ [22,23]. It is a trivial statement to say that at the moment we know almost nothing on the pp spin-flip amplitudes at high energy, due to the scarcity of previous experiments performed at CERN and Fermilab. This scarcity does not allow us to make a reliable amplitude analysis, which requires these new measured observables in a significant range of momentum transfer. This will be important for our understanding of spin-dependent scattering dynamics.

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