Back reaction and the local cosmological expansion rate

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We calculate the back reaction of cosmological perturbations on a general relativistic variable which measures the local expansion rate of the Universe. Specifically, we consider a cosmological model in which matter is described by a single field. We analyze back reaction both in a matter-dominated Universe and in a phase of scalar field-driven chaotic inflation. In both cases, we find that the leading infrared terms contributing to the back reaction vanish when the local expansion rate is measured at a fixed value of the matter field which is used as a clock, whereas they do not appear to vanish if the expansion rate is evaluated at a fixed value of the background time. We discuss possible implications for more realistic models with a more complicated matter sector.

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I. INTRODUCTION

Because of the nonlinear nature of the Einstein equations, linear fluctuations about a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) cosmology will back react on the background on which they live. The back reaction of short wavelength gravitational waves was studied a long time ago by Brill, Hartle and Isaacson [1]. In this approach, the back reaction of fluctuations on the spatially averaged metric can be described by an effective energy-momentum tensor which contains the spatially averaged terms in the Einstein tensor which are second order in the amplitude of the fluctuations. More recently, Tsamis and Woodard [2] have initiated a detailed study of the back reaction of long wavelength gravitational waves in a de Sitter background. Scalar-type metric fluctuations (called ''cosmological fluctuations'' in what follows) are also expected to back react on the background space-time. Since in the context of inflationary and post-inflationary cosmology the scalar metric fluctuations are believed to dominate over the effects of gravitational waves, the back reaction of these cosmological perturbations is expected to give the dominating back reaction effect.

In [3,4], the formalism of [1] was generalized to describe the back reaction of cosmological perturbations on the spatially averaged metric. On this basis, it was argued [3–5] that gravitational back reaction in scalar field-driven inflationary models, calculated up to quadratic order in perturbations and to leading order in the long wavelength expansion and in the slow roll approximation could decrease the expansion rate of the universe and potentially solve the cosmological constant problem [6] (see also [7] for similar results in the context of back-reaction studies of gravitational waves). The formalism of [3,4] is covariant under first order space-time diffeomorphisms.[1]

However, as emphasized by Unruh [21],[2] the approach of Refs. [2–4] is deficient in several respects. First of all, due to the nonlinear nature of the Einstein equations, calculating an ''observable'' from the spatially averaged metric will not in general give the same result as calculating the spatially averaged value of the observable. More importantly, the spatially averaged metric is not a local physical observable. Thus, to take into account the deficiencies of the previous work on gravitational back reaction, we must identify a local physical variable which describes the expansion rate of the Universe, calculate the back reaction of cosmological perturbations on this quantity, and then take the spatially averaged value of the result. It is important to fix the hypersurface of averaging by a clear physical prescription in order to remove the possibility of being misled by effects which are second order gauge artifacts.

In this paper we propose an implementation of this approach. We focus on the variable which yields the general relativistic definition of the local expansion rate and calculate this quantity to second order in the amplitude of the cosmological fluctuations in terms of a time variable defined

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[1]The back reaction of small-scale (i.e. smaller than the Hubble radius) cosmological perturbations has been considered (without taking into account the issues of gauge freedom) in [8–15]. This problem has also been considered in the context of Newtonian cosmological perturbation theory in [16,17]. More recently, Nambu [18–20] has initiated a program to compute back-reaction effects on the spatially averaged metric using the renormalization group method.

[2]We are also grateful to Andrei Linde and Alan Guth for detailed private discussions on these points.

by an unambiguous physical prescription. For simplicity, we assume that matter is described by a single field (either a hydrodynamical field or a single scalar field).

We study two examples, first a matter-dominated Universe, and second the inflationary phase of a cosmology dominated by a single scalar field. In both cases we find that the leading infrared contributions to the back reaction on the local expansion rate of the Universe vanish, in contrast to the findings of the initial work on gravitational back-reaction of cosmological fluctuations [3–5], and confirming the analysis of [21]. We thus confirm the conclusions reached recently in [22,23] where a different variable related to the local expansion rate is proposed, and different techniques to evaluate this variable are used.

Note that when evaluated at a fixed background time, our leading infrared back-reaction terms give a non-vanishing contribution. This leads us to the conjecture that in more realistic models in which a second field is present to determine time (e.g. the microwave background), the leading infrared back-reaction terms will not vanish.

II. A LOCAL OBSERVABLE

For a general perfect fluid flow in a curved space-time we consider the velocity four-vector field u^α tangential to a family of world lines. In the context of cosmology, we can always define a preferred family of world lines representing the motion of a set of comoving observers. In the case of hydrodynamical matter, this is easy since the energy-momentum tensor is already defined in terms of a velocity four-vector field. Also in the case of scalar field matter, a corresponding four-vector field can be defined, although a bit more care is required to obtain a consistent definition. In both cases we have

$$u^\alpha u_\alpha = 1, \tag{1}$$

where α runs over the space-time indices. The projection tensor onto tangential three-surfaces orthogonal to u^α is

$$h_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta . \tag{2}$$

The first covariant derivative of the four-velocity can be decomposed as (see, e.g. [24] for details)

$$u_{\alpha;\beta} = \omega_{\alpha\beta} + \sigma_{\alpha\beta} + \frac{1}{3}\Theta h_{\alpha\beta} - \dot{u}_\alpha u_\beta . \tag{3}$$

Here

$$\Theta \equiv u^\alpha_{;\alpha} \tag{4}$$

is the local expansion rate of the tangential surfaces orthogonal to the fluid flow, ω is the vorticity tensor (with $\omega_{\alpha\beta}u^\beta = 0$), and σ is the shear tensor (satisfying $\sigma_{\alpha\beta}u^\beta = 0$, $\sigma^\alpha_\alpha = 0$).

For a homogeneous Universe with scale factor $a(t)$ the Hubble expansion rate H is

$$H \equiv \dot{a}/a = \frac{1}{3}\Theta . \tag{5}$$

For a cosmological model with fluctuations, Θ is local in space and time. We will use Θ to define the local expansion rate $\dot{S}/S$ via the local analog of Eq. (5), namely via the equation

$$\dot{S}/S \equiv \frac{1}{3}\Theta . \tag{6}$$

The quantity $\dot{S}/S$ is a much better measure of the locally measured expansion rate in a Universe with fluctuations than the Hubble expansion rate used in [3,4] determined via the spatially averaged metric, including back reaction. It is a mathematically simpler object than the variable recently introduced in [22] which involves the integral along the past light cone of the observation point. If we are interested in evaluating the expansion rate for a typical observer, we propose to take the spatial average of the local expansion rate defined via Eq. (6).

Now that we have defined the observable we are interested in, the procedure will be as follows. First, we must determine the velocity four-vector field u^α for the models we are interested in. Then, we use the Einstein equations to express u^α in terms of the metric perturbation. Taking the relative amplitude of the metric fluctuations as the expansion parameter, we then calculate Θ, our local measure of the Hubble expansion rate, to second order. After evaluating the result on a physically determined hypersurface we can then study the back reaction of cosmological fluctuations on the locally measured Hubble expansion rate. In this paper we will focus on the leading infrared contributions to back reaction, the terms found to dominate the back-reaction effects in [3–5].

III. DERIVING THE EXPANSION RATE FOR SCALAR METRIC PERTURBATIONS

In this section we consider a model with hydrodynamical matter. Starting from the expression for the metric to linear order in the fluctuations Φ (see [25] for a detailed review), we determine the velocity four-vector field u^α to the order required to analyze the leading infrared terms in the back reaction to quadratic order. To obtain the full back reaction terms (including terms which dominate in the ultraviolet but are negligible in the infrared) we should calculate u^α consistently up to second order. However, if we are only interested in the leading infrared terms, it is sufficient to keep all the terms quadratic in Φ but not containing any spatial gradients. Note that in a consistent second order perturbative formalism, one also needs to include second order metric perturbations. However, since by definition the averages of linear functions of fluctuation variables cancels, the contribution of second order metric fluctuation to average values of back-reaction quantities calculated to second order vanishes. Thus, in this work we will not take into account second order metric fluctuations.

In order to obtain the complete result for gravitational

back reaction we would have to look at the Einstein equations for a perfect fluid with energy density ρ and pressure P,

$$G_{\mu\nu}=(P+\rho)u_\mu u_\nu - P g_{\mu\nu} \tag{7}$$

(in units in which $8\pi G=1$), which, since $G^\mu_\mu = R$, will yield

$$-R=\rho-3P \tag{8}$$

$$\rho = u^\mu G_{\mu\nu} u^\nu, \tag{9}$$

and lead to an equation that can be solved perturbatively to any desired order for u_i:

$$G^0_i = \frac{4}{3} u^\mu G_{\mu\nu} u^\nu u^0 u_i + R u^0 u_i . \tag{10}$$

However, as mentioned above, here we just use the results for u^i which are of linear order. Since we will calculate the divergence of u^μ, our prescription implies that we are ignoring some of the extra second order gradient terms. The result can also be used for scalar fields if we define the u vector field in a proper way.

For an unperturbed Robertson-Walker metric, the four-velocity field u in comoving coordinates would be

$$u^\mu = (1,0,0,0). \tag{11}$$

In linear perturbation theory, and in the case of simple forms of matter (such as a single fluid or a single scalar field) for which there is to linear order no anisotropic stress, the metric (in longitudinal gauge) can be written as

$$ds^2 = a(\eta)^2[(1+2\Phi)d\eta^2 - (1-2\Psi)\gamma_{ij}dx^i dx^j], \tag{12}$$

$$\gamma_{ij} = \delta_{ij}\left[1+\frac{1}{4}\mathcal{K}(x^2+y^2+z^2)\right] \tag{13}$$

where $\mathcal{K}=0,1,-1$ depending on whether the three-dimensional space corresponding to the hypersurface $t=$const is flat, closed or open. In this paper we will take it to be zero in order to simplify the calculations. The time variable η appearing in Eq. (12) is conformal time and is related to the physical time t via $d\eta = a^{-1}dt$. For the forms of matter considered here, $\Psi=\Phi$ at linear order.[3] As discussed e.g. in Ref. [25], in longitudinal gauge the spatial components of the four-velocity vector field are related to Φ via

$$\delta u_i = -a^{-2}(\mathcal{H}^2 - \mathcal{H}' + \mathcal{K})^{-1}(a\Phi)'_{,i} \tag{14}$$

where a prime denotes differentiation with respect to η and $\mathcal{H}=a'/a$. Using Eq. (1) we can derive the expression for the time component of u^α in terms of Φ:

$$u^0(\eta) = a^{-1}\left(1-\Phi+\frac{3}{2}\Phi^2\right). \tag{15}$$

[3]Even if we did not make the assumption $\Psi=\Phi$, it turns out that at second order all infrared terms depending on $\Psi-\Phi$ will drop out.

Now that we have all components of u^α, we take the covariant derivative of it and retain all Φ dependence up to second order.[4] These second order terms principally come from the Christoffel symbols. Other second order terms (which as mentioned before are gradient terms) could be added to the ones computed here if we were to solve the Einstein equations in the form (10) beyond linear order. A straightforward calculation yields

$$\Theta = 3\frac{a'}{a^2}\left(1-\Phi+\frac{3}{2}\Phi^2\right) - \frac{3}{a}(\Phi' + \Phi\Phi') + \frac{(a\partial_i\Phi)'(\partial_i\Phi)+(a\partial_i^2\Phi)'}{a^2(\mathcal{H}'-\mathcal{H}^2)}. \tag{16}$$

Since $\dot{a}/a = a'/a^2$, we can immediately read off the extra terms contributing to the local expansion rate which result from the presence of cosmological fluctuations. Upon spatial averaging at a fixed conformal time, the terms linear in Φ drop out. Hence, it follows that if evaluated at a fixed conformal time, infrared modes on average lead to an increase in the expansion rate compared to what would be obtained at the same conformal time in the absence of metric fluctuations. Whether this is a physically measurable effect from an observational point of view will be discussed in more depth in following sections.

IV. EXPANSION RATE FOR A MATTER-DOMINATED UNIVERSE

Now let us use the result of the preceding section to (as an example) calculate the local Hubble expansion rate for a matter-dominated Universe. In this case, the scale factor a and the scalar metric perturbation Φ have the following dependence on the conformal time η:

$$a(\eta) = a_m \eta^2/2 \tag{17}$$

$$\Phi(\eta,x) = C_1(x) + C_2(x)\eta^{-5}, \tag{18}$$

where a_m is a constant, and C_1 and C_2 are time independent. The second equation is valid in the long wavelength (super-Hubble-scale) limit and has been explicitly derived in [25]. Now using Eqs. (16), (17) and (18) we obtain Θ in terms of conformal time:

[4]This is in the philosophy of the general back-reaction approach in which it is assumed that the fluctuations of the metric and matter satisfy the linear perturbation equations, and we compute their back reaction on physical quantities to second order. It is not a consistent second order perturbative formalism.

$$\Theta = \frac{12}{a_m}\eta^{-3}\left(1 - C_1(x) + \frac{3}{2}C_1^2\right) - \frac{2}{3}\frac{1}{a_m}\eta^{-1}[(\partial_i C_1(x))^2 + \partial_i^2 C_1(x)] + \frac{1}{a_m}\eta^{-6}\left(\frac{1}{3}(\partial_i C_1(x))(\partial_i C_2(x)) + \partial_i^2 C_2(x)\right) + \frac{1}{a_m}\eta^{-8}[18C_2(x) + 66C_1(x)C_2(x)] + \frac{1}{a_m}\eta^{-11}(\partial_i C_2(x))^2 + \frac{48}{a_m}\eta^{-13}(C_2(x))^2. \quad (19)$$

Some of the terms are decreasing very fast as a function of time and thus we can ignore them. If we just keep the terms with the powers -3 and -1 of η then we get:

$$\Theta = \frac{12}{a_m}\eta^{-3}\left(1 - C_1(x) + \frac{3}{2}C_1^2\right) - \frac{2}{3}\frac{1}{a_m}\eta^{-1}[(\partial_i C_1(x))^2 + \partial_i^2 C_1(x)]. \quad (20)$$

If we take the average of Θ on a constant η hypersurface, only the terms quadratic in the fluctuation variables survive. Thus, considering large values of η and focusing on the second order terms, it appears from Eq. (20) that infrared modes give a positive contribution to Θ and thus lead to a speeding-up of the expansion, whereas ultraviolet modes enter with a negative sign and thus yield a slowing effect, the latter becoming more significant (relative to the unperturbed expansion rate) for larger values of η.

However, before drawing definite physical conclusions from our analysis, we must take into account that the background time η is not an observable quantity. To obtain results for back reaction of any real physical significance we have to find an observable variable like proper time and evaluate the expansion rate in terms of this variable, so that we can discuss its evolution from an observer's point of view.

If we use Eq. (12) for the metric, Eq. (18) for Φ, and Eq. (17) for the scale factor, we can find the expression for the proper time τ in terms of conformal time. Since

$$d\tau^2 = a(\eta)^2(1 + 2\Phi)d\eta^2, \quad (21)$$

a simple integration yields

$$\tau = \frac{a_m}{6}\left(1 + C_1 - \frac{1}{2}C_1^2\right)\eta^3 - \frac{a_m}{4}(C_2 - C_1C_2)\eta^{-2} + \frac{a_m}{28}C_2^2\eta^{-7}. \quad (22)$$

In the approximation of large values of η we can ignore the second and the third term of this equation and thus obtain

$$\eta^{-3} = \frac{a_m}{6}\left(1 + C_1 - \frac{1}{2}C_1^2\right)\tau^{-1}. \quad (23)$$

Now we can use this relation and substitute into the first term of Eq. (20). We see the effects of the dominant infrared terms on the local expansion rate cancel exactly up to second order in perturbations, when evaluating Θ at a fixed proper time:

$$\Theta_{IR} = 2\tau^{-1}. \quad (24)$$

This implies that at least in the approximation of keeping only the leading infrared terms, there is no local gravitational back reaction of cosmological fluctuations on the local Hubble expansion rate in this matter-dominated universe.

V. EXPANSION RATE IN TERMS OF SCALAR FIELD AS AN OBSERVABLE

We now move on to the example more relevant to the work of [3–5], namely a Universe dominated by a single real scalar field φ, which is a toy model for inflationary cosmology. During inflation, fluctuations which are generated on sub-Hubble scales early on during the inflationary phase are redshifted to scales much larger than the Hubble radius. Thus, in this context it is of great interest to consider the back reaction of infrared modes.

In the following we will generalize the previous analysis to be applicable to matter consisting of a single scalar field. In this case one can treat the scalar field as a perfect fluid and derive the velocity four-vector field. To do this, we need to write the energy-momentum tensor $T_{\mu\nu}$ of the scalar field (right-hand side of the following equation) in the form of an energy-momentum tensor for a perfect fluid (left-hand side of the following equation):

$$(\rho + P)u_\mu u_\nu - P g_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \mathcal{L}\cdot g_{\mu\nu}. \quad (25)$$

At the level of the background fields, the two expressions are identical if we take $P = \mathcal{L}$ and $u_\mu = A\partial_\mu\varphi$ with

$$A = (\partial^\nu\varphi\partial_\nu\varphi)^{-1/2}. \quad (26)$$

Now that we have shown that the energy-momentum tensor of a scalar field can be written in the form of that of a perfect fluid, we can use the expression (16) which gives Θ in terms of the metric fluctuations to evaluate the local effect of gravitational back reaction of cosmological fluctuations. Let us first for convenience rewrite Θ of Eq. (16) in terms of the physical time t:

$$\Theta = 3\frac{\dot{a}}{a}\left(1 - \Phi + \frac{3}{2}\Phi^2\right) - 3(\dot{\Phi} + \dot{\Phi}\Phi) + \frac{(\partial_i\dot{\Phi})(\partial_i\Phi) + \partial_i^2\dot{\Phi}}{a\ddot{a} - \dot{a}^2}. \quad (27)$$

Theoretically, the scalar field φ is an observable. In fact, in a system with a single matter field φ, it is this field which must be used as a clock. Hence, to obtain results with physical meaning, we must evaluate Θ on a surface of constant φ and not constant t.

As discussed above, in the context of inflationary cosmology it is important to study the effects of infrared modes up to second order. We will also assume that we are in the slow rolling regime of inflation. In our case (as well as for a more general case), the prescription is to calculate t, Φ, a and $\partial/\partial t$

in terms of φ, and to insert the results into the general expression (27) for Θ. The relation between t and φ can be derived starting from

$$\varphi(t) = \varphi_0(t) + \delta\varphi_1(t), \tag{28}$$

which can be written as

$$t = \varphi_0^{-1}[\varphi - \delta\varphi_1(t)]. \tag{29}$$

Thus, the equation

$$t = \varphi_0^{-1}(\varphi) - \frac{\partial\varphi_0^{-1}(\varphi)}{\partial\varphi}\delta\varphi_1(\varphi_0^{-1}(\varphi)) + \left(\frac{\partial\varphi_0^{-1}(\varphi)}{\partial\varphi}\right)^2 \frac{\partial\delta\varphi_1(\varphi_0^{-1}(\varphi))}{\partial t}\delta\varphi_1(\varphi_0^{-1}(\varphi)) + \frac{1}{2}\frac{\partial^2\varphi_0^{-1}(\varphi)}{\partial\varphi^2}\delta\varphi_1^2(\varphi_0^{-1}(\varphi)) \tag{30}$$

relates t and φ.

We now wish to express the value of Θ in terms of φ (note that we are considering the local value of Θ in this analysis, and there is no need to perform a spatial averaging). In order to relate the metric, its fluctuations and the scale factor to the scalar field and its fluctuations we need to take an explicit form for the potential and make use of the Einstein constraint equations. The G^{0i} and G^{00} equations relate Φ to φ (at the level of the first order fluctuations) and the Hubble parameter of the background to the background scalar field φ_0 (at the level of the unperturbed Friedmann equations). In our simple system we do not need to go through these calculations explicitly (as long as we are interested only in the leading infrared terms) since by dropping the gradient terms from the G^{0i} and G^{00} equations, it can be shown that [26]

$$\frac{H}{\sqrt{1+2\Phi}} = \frac{1}{\sqrt{3}}\sqrt{V(\varphi)}. \tag{31}$$

If we expand the right-hand side in terms of Φ we get the result

$$H\left(1 - \Phi + \frac{3}{2}\Phi^2\right) = \frac{1}{\sqrt{3}}\sqrt{V(\varphi)}, \tag{32}$$

and substituting this result into Eq. (27) and neglecting terms containing $\dot{\Phi}$ and $\partial_i\Phi$ (which are sub-dominant compared to other terms in the infrared and slow roll limit) leads to the final result for the local expansion rate:

$$\Theta = \sqrt{3}\sqrt{V(\varphi)} \tag{33}$$

which as a function of φ is the same as the relation for an unperturbed background.

Thus, again we do not see any back reaction of cosmological perturbations on the local expansion rate in this approximation. In retrospect it is easy to understand this result, since [26] by neglecting gradient terms we have a Friedmann Universe whose expansion rate satisfies Eq. (33).

VI. CONCLUSIONS

In this paper we have studied the back-reaction effects on a local observable which measures the local expansion rate of the Universe. The observable gives the rate at which neighboring comoving observers separate and coincides with the usual definition of expansion in the context of the fluid approach to cosmology. In order to obtain a physical quantity, we evaluated the observable at a fixed value of the scalar field.

We evaluated our observable, the *local physical expansion rate*, in a simple toy model of chaotic inflation consisting of a single scalar matter field coupled to gravity. We found that the leading infrared terms, the terms which dominate the effects discussed in [3] and [5], cancel if we evaluate the observable at a fixed value of the scalar field, whereas they do not vanish if we evaluate them at a fixed value of the background time. The former result is a physical result since it corresponds to a physical observable evaluated at a spacetime point specified by a physical prescription, whereas the latter result (obtained by evaluating at a fixed background time) does not have a diffeomorphism-invariant meaning. Our analysis thus confirms the concern of [21] that the results obtained in [3] and [5] are not invariant under second order gauge transformations. Our results confirm the conclusions of [23] reached by means of a different method of analysis applied to a different physical observable.

Our result does *not* imply that there is no back reaction of the infrared modes of cosmological perturbations. There is no reason to expect that the next to leading infrared terms in our result will cancel (they do not cancel in the analysis of [23]). One of the advantages of our technique is that they can be evaluated without too much trouble. This is left to a future publication. So, even in single field models of inflation there might be some non-vanishing back reaction of infrared modes.

We expect that back reaction of infrared modes will be much more important in two field models of inflation. Let us assume that the matter sector of the theory contains both an inflation field φ and a regular matter field χ (with a nonvanishing and time-dependent spatial average) which, for example, could represent the cosmic microwave background. In this case, it is no longer true that long wavelength fluctuations have no physical effects on local observables. If the measurement point is (in an unambiguous physical way) determined by a fixed value of the field χ, then the local expansion rate may sensitively depend on the amplitude of the long wavelength fluctuations in φ. Thus, the leading infrared terms may *not* cancel when evaluated according to the above-mentioned physical prescription in the same way that they do not cancel in the analysis of this paper when the observable is evaluated at a fixed value of the background time.

There is a close analogy with the analysis of the parametric amplification of super-Hubble-scale cosmological fluctuations during inflationary reheating. From the point of view

of the background space-time coordinates, it appears [27] that the parametric amplification of matter fluctuations on super-Hubble scales in an unperturbed cosmological background (see, e.g. [28,29] for a discussion of parametric resonance during reheating) would imply the parametric amplification of the cosmological fluctuations on these scales. However, it can be shown that in single field models physical observables measuring the amplitude of cosmological fluctuations do not feel any resonance [26,30–32]. In contrast, in two field models of inflation there is [33,34] parametric amplification of super-Hubble-scale cosmological fluctuations. In this case, there is a fluctuation mode corresponding to entropy fluctuations which cannot locally be gauged away. This mode is (in certain theories) parametrically amplified during reheating, and in turn drives the parametric resonance of the super-Hubble scale curvature fluctuations.

Our techniques allow us to calculate the back reaction of cosmological fluctuations in two field models in a very similar way to what is presented here. Results will be presented in a future publication [35].

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