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$|V_{us}|$ and m_s from hadronic tau decays

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Recent progress in the determination of $|V_{us}|$ employing strange hadronic τ-decay data are reported. This includes using the recent OPAL update of the strange spectral function, as well as augmenting the dimension-two perturbative contribution with the recently calculated order α_s^3 term on the theory side. These updates result in $|V_{us}| = 0.2220 \pm 0.0033$, with the uncertainty presently being dominated by experiment, and already being competitive with the standard extraction from K_{e3} decays and other new proposals to determine $|V_{us}|$. In view of the ongoing work to analyse τ-decay data at the B-factories BaBar and Belle, as well as future results from BESIII, the error on $|V_{us}|$ from τ decays is expected to be much reduced in the near future.

1. INTRODUCTION

In the past decade hadronic τ decays have been an extremely fruitful laboratory for the study of low-energy QCD. Detailed investigations of the τ hadronic width

$$R_\tau \equiv \frac{\Gamma[\tau^- \to \text{hadrons}\,\nu_\tau(\gamma)]}{\Gamma[\tau^- \to e^-\overline{\nu}_e\nu_\tau(\gamma)]}\,, \tag{1}$$

as well as invariant mass distributions, have served to determine the QCD coupling α_s to a precision competitive with the current world average [1–5]. The experimental separation of the Cabibbo-allowed decays and Cabibbo-suppressed modes into strange particles [6–10] also opened a means to determine the mass of the strange quark [11–17], one of the fundamental QCD parameters within the Standard Model.

These determinations suffer from large QCD corrections to the contributions of scalar and pseudoscalar correlation functions [1,11,18,19] which are additionally amplified by the particular weight functions which appear in the τ sum rule. A natural remedy to circumvent this problem is to replace the QCD expressions of scalar and pseudoscalar correlators by corresponding phenomenological hadronic parametrisations [6,13,15,20,17], which turn out to be more precise than their QCD counterparts, since the dominant contribution stems from the well known kaon pole.

Additional suppressed contributions to the pseudoscalar correlators come from the pion pole as well as higher exited pseudoscalar states whose parameters have recently been estimated [21]. The remaining strangeness-changing scalar spectral function has been extracted from a study of S-wave $K\pi$ scattering [22,23] in the framework of resonance chiral perturbation theory [25]. The resulting scalar spectral function was also em-

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ployed to directly determine m_s from a purely scalar QCD sum rule [24].

Nevertheless, as was already realised in the first works on strange mass determinations from the Cabibbo-suppressed τ decays, m_s turns out to depend sensitively on the element $|V_{us}|$ of the quark-mixing (CKM) matrix. With the theoretical improvements in the τ sum rule mentioned above, in fact $|V_{us}|$ represents one of the dominant uncertainties for m_s. Thus it appears natural to actually determine $|V_{us}|$ with an input for m_s as obtained from other sources [17,26,27].

Succeeding the high-precision status on τ-decay observables already attained by by ALEPH and OPAL at LEP and CLEO at CESR, now the B-factories BaBar and Belle are starting to produce their first results on hadronic τ decays, and in particular on Cabibbo-suppressed modes [28–30]. These two facts make the strange hadronic τ decay data an ideal place for determining SU(3) breaking parameters such as $|V_{us}|$ and/or m_s. The obvious advantage of this procedure is that the experimental uncertainty will eventually be reduced at the B-factories and at future facilities like the τ-charm factory BEPCII

2. THEORETICAL FRAMEWORK

Employing the analytic properties of two-point correlation functions for vector ($\mathcal{J} = V$) and axial-vector ($\mathcal{J} = A$) two quark-currents,

$$\begin{aligned}\Pi^{\mu\nu}_{\mathcal{J},ij}(q) &\equiv i \int \mathrm{d}^4x\, e^{iq\cdot x} \langle 0|T[\mathcal{J}^{\mu\dagger}_{ij}(x)\mathcal{J}^{\nu}_{ij}(0)]|0\rangle \\ &\equiv \left[q^\mu q^\nu - q^2 g^{\mu\nu}\right] \Pi^T_{\mathcal{J},ij}(q^2) + q^\mu q^\nu \Pi^L_{\mathcal{J},ij}(q^2)\,, \end{aligned} \tag{2}$$

one can express R_τ as a contour integral running counter-clockwise around the circle $|s| = M_\tau^2$ in the complex s-plane:

$$\begin{aligned} R_\tau &\equiv -i\pi \oint_{|s|=M_\tau^2} \frac{\mathrm{d}s}{s} \left[1 - \frac{s}{M_\tau^2}\right]^3 \\ &\quad\times \left\{3\left[1 + \frac{s}{M_\tau^2}\right] D^{L+T}(s) + 4\, D^L(s)\right\}. \end{aligned} \tag{3}$$

Here, we have used integration by parts to rewrite R_τ in terms of the logarithmic derivatives

$$D^{L+T}(s) \equiv -s\, \frac{\mathrm{d}}{\mathrm{d}s}\, \Pi^{L+T}(s)\,,$$

$$D^L(s) \equiv \frac{s}{M_\tau^2} \frac{\mathrm{d}}{\mathrm{d}s} \left[s\Pi^L(s)\right]\,. \tag{4}$$

Moreover, experimentally R_τ can be decomposed into the following three contributions

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}\,, \tag{5}$$

according to the quark content

$$\begin{aligned}\Pi^J(s) &= |V_{ud}|^2 \left\{\Pi^J_{V,ud}(s) + \Pi^J_{A,ud}(s)\right\} \\ &+ |V_{us}|^2 \left\{\Pi^J_{V,us}(s) + \Pi^J_{A,us}(s)\right\}\,, \end{aligned} \tag{6}$$

where $R_{\tau,V}$ and $R_{\tau,A}$ correspond to the first two terms in the first line and $R_{\tau,S}$ to the second line, respectively.

Additional information can be inferred from the measured invariant-mass distribution of the final hadrons, which defines the moments

$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} \mathrm{d}s \left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{\mathrm{d}R_\tau}{\mathrm{d}s}\,. \tag{7}$$

At large enough Euclidean $Q^2 \equiv -s$, both $\Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ can be organised in a dimensional operator series using well established QCD operator product expansion (OPE) techniques. One then obtains

$$\begin{aligned} R_\tau^{kl} &= N_c\, S_{\mathrm{EW}} \Big\{ (|V_{ud}|^2 + |V_{us}|^2) \left[1 + \delta^{kl(0)}\right] \\ &+ \sum_{D\geq 2} \left[|V_{ud}|^2 \delta^{kl(D)}_{ud} + |V_{us}|^2 \delta^{kl(D)}_{us}\right] \Big\}\,. \end{aligned} \tag{8}$$

The electroweak radiative correction $S_{\mathrm{EW}} = 1.0201 \pm 0.0003$ [31] has been pulled out explicitly and $\delta^{kl(0)}$ denote the purely perturbative dimension-zero contributions. The symbols $\delta^{kl(D)}_{ij}$ stand for higher dimensional corrections in the OPE from dimension $D \geq 2$ operators, which contain implicit $1/M_\tau^D$ suppression factors [1,11,13,18]. The most important being the operators m_s^2 with $D = 2$ and $m_s\langle \overline{q}q\rangle$ with $D = 4$.

In addition, the flavour-SU(3) breaking quantity

$$\begin{aligned} \delta R_\tau^{kl} &\equiv \frac{R^{kl}_{\tau,V+A}}{|V_{ud}|^2} - \frac{R^{kl}_{\tau,S}}{|V_{us}|^2} \\ &= N_c\, S_{EW} \sum_{D\geq 2} \left[\delta^{kl(D)}_{ud} - \delta^{kl(D)}_{us}\right] \end{aligned} \tag{9}$$

enhances the sensitivity to the strange quark mass. The dimension-two correction $\delta_{ij}^{kl(2)}$ is known to order α_s^3 for both correlators, $J = L$ and $J = L + T$ [11,13,32].

In Ref. [11], an extensive analysis of this $D = 2$ correction was performed and it was shown that the perturbative $J = L$ correlator behaves very badly. The $J = L+T$ correlator was also analysed there to order α_s^2 and showed a relatively good convergence. In the following, we have included the recently calculated $O(\alpha_s^3)$ correction for $J = L + T$ [32]. One can see that the $J = L + T$ series starts to show its asymptotic character at this order, though it is still much better behaved than the $J = L$ component. Due to the asymptotic behaviour, it does not make much sense to sum all known orders of the series. This question will be investigated in more detail by us in the future.

3. DETERMINATION OF $|V_{us}|$ WITH FIXED m_s

One can now use the relation

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\dfrac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\rm th}^{00}}\,, \tag{10}$$

and analogous relations for other moments, to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{us}|$. Notice that on the right-hand side of (10) the only theoretical input is $\delta R_{\tau,\rm th}^{00}$, which is around 0.24 and should be compared to the experimental quantity $R_{\tau,V+A}^{00}/|V_{ud}|^2$ which is around 3.7. Therefore, with a not so precise theoretical prediction for $\delta R_{\tau,\rm th}^{00}$ one can get a quite accurate value for $|V_{us}|$, depending on the uncertainty provided by experiment.

The very bad QCD behaviour of the $J = L$ component in $\delta R_{\tau,\rm th}^{kl}$ induces a large theoretical uncertainty, which can be reduced considerably using phenomenology for the scalar and pseudo-scalar correlators [17,26,27]. In particular, the pseudo-scalar spectral functions are dominated by far by the well-known kaon pole, to which we add suppressed contributions from the pion pole, as well as higher excited pseudo-scalar states whose parameters have been estimated in Ref. [21]. For the strange scalar spectral function, we employ the result [22], obtained from a study of S-wave $K\pi$ scattering within resonance chiral perturbation theory [25], which has been recently updated in Ref. [33].

The smallest theoretical uncertainty arises for the $kl = 00$ moment, for which we get

$$\begin{aligned}\delta R_{\tau,\rm th}^{00} \;&=\; 0.1544\,(37) + 9.3\,(3.4)\,m_s^2 \\ &+\; 0.0034\,(28) \;=\; 0.240\,(32)\,,\end{aligned} \tag{11}$$

where m_s denotes the strange quark mass in units of GeV, and in the $\overline{\rm MS}$ scheme at a renormalisation scale of $\mu = 2$ GeV. The first term contains the phenomenological scalar and pseudo-scalar contributions, the second term contains the rest of the perturbative $D = 2$ contribution, while the last term stands for the rest of the contributions. Notice that the phenomenological contribution is more than 64% of the total, while the rest comes almost from the perturbative $D = 2$ contribution. Here, we update $\delta R_{\tau,\rm th}^{00}$ of Refs. [17,26,27] in various respects. Firstly, we use the recently updated scalar spectral function [33]; secondly, we include the α_s^3 corrections to the $J = L + T$ correlator as calculated in Ref. [32]; and finally, we use an average of contour improved [34] and fixed order perturbation results for the asymptotically summed series, in order to have a more conservative account of uncertainties resulting from unknown higher orders. A detailed discussion of this contribution will be presented elsewhere [35].

For the m_s input value, we use the recent average $m_s(2\,{\rm GeV}) = (94 \pm 6)$ MeV [33], which includes the most recent determinations of m_s from QCD sum rules and lattice QCD. The strange quark mass uncertainty corresponds to the most precise determination from the lattice.

Recently, Maltman and Wolfe have criticised the theory error we previously employed for the $D = 2$ OPE coefficient [38]. Awaiting a more detailed study [35], in our updated estimate (11), we have decided to include a more conservative estimate of unknown higher-order corrections by using an average of contour improved and fixed-order perturbation theory. Still, we do not think that artificially doubling the perturbative uncertainty, as was done in [38], represents an error estimate which is better founded. Notice furthermore, that $\delta R^{00}_{\tau,\mathrm{th}}$ is dominated by the scalar and pseudoscalar contributions which are rather well known from phenomenology, and that the larger perturbative uncertainty is compensated by the smaller m_s error, so that our final theoretical uncertainty is practically the same as in previous works [17,26,27].

In order to finally determine $|V_{us}|$, we employ the following updates of the remaining input parameters: $|V_{ud}| = 0.97377 \pm 0.00027$ [36], the non-strange branching fraction $R^{00}_{\tau,V+A} = 3.471 \pm 0.011$, ,[10] as well as the strange branching fraction $R^{00}_{\tau,S} = 0.1686 \pm 0.0047$ [10] (see also Refs. [6] and [8]), which includes the theoretical prediction for the decay $B[\tau \to K\nu_\tau(\gamma)] = 0.715 \pm 0.003$ which is based on the better known $K \to \mu\nu_\mu(\gamma)$ decay rate. For $|V_{us}|$, we then obtain

$$|V_{us}| = 0.2220 \pm 0.0031_{\mathrm{exp}} \pm 0.0011_{\mathrm{th}} \,. \qquad (12)$$

The experimental uncertainty includes a small component from the error in $|V_{ud}|$, but it is dominated by the uncertainty in $R^{00}_{\tau,S}$, while the theoretical error is dominated by the uncertainty in the perturbative expansion of the $D = 2$ contribution.

4. SIMULTANEOUS FIT OF $|V_{us}|$ AND m_s

In principle, it is also possible to perform a simultaneous fit to $|V_{us}|$ and m_s from a certain set of (k, l) moments. As soon as more precise data are available, this will be the ultimate approach to determine $|V_{us}|$ and m_s from hadronic τ decays. With the current uncertainties in the data and a persistent question about a monotonous k-dependence of m_s [17,27], a bias could be present in the method. Furthermore, the correlations between different moments are rather strong and also have to be properly included on the theory side.

Here, we shall restrict ourselves to a simplified approach where all correlations are neglected. For the simultaneous fit of $|V_{us}|$ and m_s, we employ the five R^{kl}_τ moments $(0, 0)$ to $(4, 0)$ which have also been used in our previous analyses [17,27]. Performing this exercise, for the central values we find:

$$|V_{us}| = 0.2196 \,, \qquad m_s(2\,\mathrm{GeV}) = 76\,\mathrm{MeV} \,. \qquad (13)$$

The expected uncertainties on these results should be smaller than the individual error given in eq. (12) and the one for m_s presented in Ref. [27], but only slightly since the correlations between different moments are rather strong.

The general trend of the fit result can be understood easily. m_s from the simultaneous fit turned out lower than the global average $m_s(2\,\mathrm{GeV}) = 94 \pm 6\,\mathrm{MeV}$ considered above. Thus, also the corresponding $\delta R_{\tau,\mathrm{th}}$ is lower, resulting in a reduction of $|V_{us}|$. Furthermore, the moment-dependence of m_s is reduced as compared to our analysis [17] on the basis of the ALEPH data alone. Nevertheless, we shall leave a detailed error analysis for a future publication.

5. CONCLUSIONS

High precision Cabibbo-suppressed hadronic τ data from ALEPH and OPAL at LEP and CLEO at CESR already provide a competitive result for $|V_{us}|$. As presented above and in Refs. [17,26,27], the final uncertainty in the τ determination of $|V_{us}|$ becomes an experimental issue and will eventually be much reduced with the new B-factory data [28–30], and further reduced at future τ facilities. A combined fit to determine both $|V_{us}|$ and m_s will then be possible. Hadronic τ decays have the potential to provide the most accurate measurement of $|V_{us}|$ and a very competitive m_s determination.

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