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On the Impact of Flavour Oscillations in Leptogenesis

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Abstract

When lepton flavour effects in thermal leptogenesis are active, they introduce important differences with respect to the case in which they are neglected, the so-called one-flavour approximation. We investigate analytically and numerically the transition from the one-flavour to the two-flavour case when the τ-lepton flavour becomes distinguishable from the other two flavours. We study the impact of the oscillations of the asymmetries in lepton flavour space on the final lepton asymmetries, for the hierarchical right-handed neutrino mass spectrum. Flavour oscillations project the lepton state on the flavour basis very efficiently. We conclude that flavour effects are relevant typically for $M_1 \lesssim 10^{12}$ GeV, where M_1 is the mass of the lightest right-handed neutrino.

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1 Introduction

Baryogenesis through Leptogenesis [1] is a simple mechanism to explain the observed baryon asymmetry of the Universe. A lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to $(B+L)$-violating sphaleron interactions [2] which exist within the Standard Model (SM). A simple scheme in which this mechanism can be implemented is the 'see-saw' (type I) model of neutrino mass generation [3]. In its minimal version it includes the SM plus two or three right-handed (RH) heavy Majorana neutrinos. Thermal leptogenesis [4, 5, 6] can take place, for instance, in the case of hierarchical spectrum of the heavy RH Majorana neutrinos. The lightest of the RH Majorana neutrinos is produced by thermal scattering after inflation. It subsequently decays out-of-equilibrium in a lepton number and Charge and Parity (CP) violating way, thus satisfying Sakharov's conditions [7]. On the other hand, the see-saw mechanism of neutrino mass generation [3] provides a natural explanation of the smallness of neutrino masses: integrating out the heavy RH Majorana neutrinos generates a mass term of Majorana type for the left-handed flavour neutrinos, which is inversely proportional to the large mass of the RH ones.

The importance of the lepton flavour effects in thermal leptogenesis has been recently realized in [8, 9, 10, 11]. The dynamics of leptogenesis was usually addressed within the 'one-flavour' approximation. In the latter, the Boltzmann equations are written for the abundance of the lightest RH Majorana neutrino, N_1, responsible for the out of equilibrium and CP-asymmetric decays, and for the *total lepton charge* asymmetry. However, this 'one-flavour' approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. Assuming for the moment that leptogenesis takes place at temperatures $T \sim M_1$, where M_1 is the mass of N_1, and that the RH spectrum is hierarchical, the 'one-flavour' approximation holds only for $T \sim M_1 \gtrsim 10^{12}$ GeV. For $M_1 \gtrsim 10^{12}$ GeV, all lepton flavours are not distinguishable. The lepton asymmetry generated in N_1 decays is effectively 'stored' in one lepton flavour. However, for $T \sim M_1 \lesssim 10^{12}$ GeV, the interactions mediated by the τ-lepton Yukawa couplings come into equilibrium, followed by those mediated by the muon Yukawa couplings at $T \sim M_1 \lesssim 10^9$ GeV, and the notion of lepton flavour becomes physical. Flavour effects are important because leptogenesis is a dynamical process, involving the production and destruction of the heavy RH Majorana neutrinos, and of a lepton asymmetry that is distributed among *distinguishable* flavours. Contrary to what is generically assumed in the one-flavour approximation, the $\Delta L = 1$ inverse decay processes which wash out the net lepton number are flavour dependent. The asymmetries in each lepton flavour, are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of washout in the one-flavour approximation, where the flavours are taken indistinguishable.

The impact of flavour in thermal leptogenesis has been recently investigated in detail in [9, 10, 11, 12, 13], including the quantum oscillations/correlations of the asymmetries in lepton flavour space [9]. The interactions related to the charged Yukawa couplings enter in the dynamics by inducing nonvanishing quantum oscillations among the lepton asymmetries in flavour space [9]. Therefore the lepton asymmetries must be represented as a matrix Y in flavour space, the diagonal elements are the flavour asymmetries, and the off-diagonals encode the quantum correlations. The off-diagonals should decay away when the charged Yukawa couplings mediate very fast processes. The Boltzmann equations therefore contain new terms encoding all the information about the action of the decoherent plasma onto the coherence of the flavour oscillations: if the damping rate is large, the quantum correlations among the flavours asymmetries are quickly damped away. If leptogenesis takes place when the charged Yukawa couplings do not mediate processes in thermal equilibrium, the quantum correlators play a crucial role to recover the one-flavour approximation. On the other hand, if leptogenesis takes place when the charged Yukawa couplings mediate processes well in thermal equilibrium quantum correlations play no role in the dynamics of leptogenesis.

The goal of this paper is to study the transition from the one-flavour to the two-flavour case. In the case of hierarchical RH mass spectrum, the baryon asymmetry is directly proportional to the mass M_1 of the lightest RH neutrino. A large enough baryon asymmetry is obtained only for a sufficiently large value of M_1. Therefore, we will restrict ourselves to the transition from the one-flavour state, to be identified with the total lepton number, to the two-flavour states, to be identified with the τ lepton doublet ℓ_τ and a linear combination of the μ and e doublets. The most interesting region is for values of masses of the lightest RH neutrino centered around $M_1 \sim 10^{12}$ GeV where we expect the quantum correlators to play a significant role in projecting the lepton state on the flavour basis and, eventually, in the generation of the baryon asymmetry. Studying the details of the transition is relevant to understand if it is a good approximation to compute the baryon asymmetry just solving the Boltzmann equations with only the diagonal entries of the matrix Y for the lepton asymmetries (as usually done in the recent literature for the flavoured leptogenesis [11, 12, 13]) and neglecting altogether the off-diagonal entries. We would like to see under which conditions on the leptogenesis parameters the full two-flavour regime is attained.

The paper is organized as follows. In Section 2 we summarize the general framework and the Boltzmann equations. In Section 3 we describe in detail the one-flavour limit, while the two-flavour limit is described in Section 4. Section 5 contains the main body of our results; we present both analytical and numerical results for the various regimes. Finally our conclusions are contained in Section 6 together with some comments.

2 Two-flavour Boltzmann equations

The lagrangian we consider consists of the SM one plus three RH neutrinos N_i ($i = 1, 2, 3$), with Majorana masses M_i. Such RH neutrinos are assumed to be heavy (i.e. with masses well above the weak scale) and hierarchical ($M_1 \ll M_{2,3}$), so that we can safely focus our attention on the dynamics of N_1 only. The interactions among RH neutrinos, Higgs doublets H, lepton doublets ℓ_α and singlets e_α ($\alpha = e, \mu, \tau$) are described by the lagrangian

$$\mathscr{L}_{\rm int} = \lambda_{i\alpha} N_i \ell_\alpha H + h_\alpha \bar{e}_\alpha \ell_\alpha H^c + \frac{1}{2} M_i N_i N_i + {\rm h.c.}\,, \tag{1}$$

with summation over repeated indeces. The lagrangian is written in the mass eigenstate basis of RH neutrinos and charged leptons. The interactions mediated by the charged lepton Yukawa couplings are out of equilibrium for $T \sim M_1 \gtrsim 10^{12}$ GeV. In this regime, flavours are indistinguishable and one can perform a rotation in flavour space to store all the asymmetry in a single flavour. At smaller temperatures, though, this operation is not possible. The τ flavour becomes distinguishable for $T \sim M_1 \lesssim 10^{12}$ GeV. As we already discussed in the Introduction, we will restrict ourselves to the study of the transition occuring around $T \sim M_1 \sim 10^{12}$ GeV. This choice is motived by the following considerations. In the case of hierarchical RH mass spectrum, the baryon asymmetry is directly proportional to the mass M_1 of the lightest RH neutrino. Therefore, a large enough baryon asymmetry is obtained only for a sufficiently large value of M_1. Since the transition which makes the μ flavour distinguishable occurs at $T \sim M_1 \sim 10^9$ GeV, the corresponding value of M_1 is generically too small to provide a baryon asymmetry in the observed range. Therefore, we will study the transition from the one-flavour state, to be identified with the total lepton number stored in the lepton doublets, to the two-flavour states, to be identified with the τ lepton doublet ℓ_τ and a linear combination of the μ and e doublets (which at temperatures between 10^9 and 10^{12} GeV are indistinguishable), $\hat{\ell}_2 = (\lambda_{1e}\ell_e + \lambda_{1\mu}\ell_\mu) \,/\, \left(|\lambda_{1e}|^2 + |\lambda_{1\mu}|^2\right)^{1/2}$.

Having therefore in mind the transition between a one-flavour and a two-flavour system, we study a toy model with two lepton doublets $\alpha = 1, 2$ and generically represent the lepton asymmetry matrix by a 2×2 density matrix Y given by the difference of the density matrices for the lepton and anti-lepton number densities (normalized to the entropy density s). The diagonal elements are the lepton asymmetries stored in each flavour while the off-diagonal elements describe the quantum correlations between different flavours. The total lepton asymmetry is given by the trace of this matrix.

In order to follow the evolution of the lepton asymmetry, one needs to write down the equations of motion for the matrix Y. The proper evolution equations for the matrix Y has been found and discussed in [9], neglecting the transformations to bring the asymmetries in the lepton doublets to the the SM conserved charges $\Delta_\alpha = (B/3 - L_\alpha)$, where L_α is the total lepton number in a single flavour. Including these transformations only change the final result by a factor of order unity and therefore we will also neglect them for the sake of presentation. The interactions mediated by the Yukawa couplings h_α are also taken into account. We will assume a large hierarchy between the Yukawa couplings (which holds for the realistic case, since $h_\tau \gg h_{\mu,e}$).

The system of Boltzmann equations for the generic components $Y_{\alpha\beta}$ of the density matrix, as a function of the variable $z = M_1/T$, read [3]

$$
\begin{aligned}
\frac{\mathrm{d}Y_{\alpha\beta}}{\mathrm{d}z} &= \frac{1}{szH(z)}\left[(\gamma_D + \gamma_{\Delta L=1})\left(\frac{Y_{N_1}}{Y^{\rm eq}_{N_1}} - 1\right)\epsilon_{\alpha\beta} - \frac{1}{2Y^{\rm eq}_\ell}\left\{\gamma_D + \gamma_{\Delta L=1}, Y\right\}_{\alpha\beta}\right] \\
&\quad -\Big[\sigma_2 \mathrm{Re}(\Lambda) + \sigma_1 |\mathrm{Im}(\Lambda)|\Big] Y_{\alpha\beta}\,, \qquad Y_{\alpha\beta} = Y^*_{\beta\alpha}\,,
\end{aligned} \tag{2}
$$

while the Boltzmann equation for the N_1 abundance (Y_{N_1}) is

$$
\frac{\mathrm{d}Y_{N_1}}{\mathrm{d}z} = -\frac{1}{szH(z)}(\gamma_D + \gamma_{\Delta L=1})\left(\frac{Y_{N_1}}{Y^{\rm eq}_{N_1}} - 1\right)\,, \tag{3}
$$

where the equilibrium N_1 abundance is given by $Y^{\rm eq}_{N_1}(z) = \frac{1}{4g_*}z^2\mathrm{K}_2(z)$, and g_* is the number of effective degrees of freedom in the thermal bath. Notice that we have included the contribution to the CP asymmetry from the $\Delta L = 1$ scatterings [11].

We remark that to obtain Eq. (2) we have assumed that the lepton asymmetries oscillate with an approximately momentum-independent frequency. The oscillation frequency in flavour space depends on the energy (momentum) of the leptons and, within one oscillation timescale, leptons are involved in many momemtum-changing interactions caused by the fast, but flavour-blind, gauge interactions. Our assumption amounts to adopting the thermally averaged energy $\langle E\rangle$ to estimate the oscillation frequency. In other words, we have approximated the integral $\int iEdt$ with $i\langle E\rangle \int dt$ along the path from one lepton number violating interaction to the next. This approximation is well justified in [14], where it has been shown that fast gauge interactions do not affect the coherence of the flavour oscillations.

Before discussing the Eqs. (2) and (3), we explain the various quantities appearing in them. The matrix $(\gamma_D)_{\alpha\beta}$ represents the thermally averaged N_1-decay rates ant it is given by

$$
(\gamma_D)_{\alpha\beta} = \gamma_D \frac{\lambda_{1\alpha}\lambda^*_{1\beta}}{[\lambda\lambda^\dagger]_{11}} = \gamma_D \frac{\lambda_{1\alpha}\lambda^*_{1\beta}}{\sum_\gamma |\lambda_{1\gamma}|^2}\,, \tag{4}
$$

[3]As usual, $\{,\}$ stands for anti-commutator while the σ's are Pauli matrices.

normalized in such a way that the total decay rate γ_D is the trace of the matrix. The $\Delta L = 1$ scatterings were also included in the equations (see [11] for a discussion about this point). The thermally averaged interaction rate matrix $(\gamma_{\Delta L=1})_{\alpha\beta}$ has the same form as $(\gamma_D)_{\alpha\beta}$ in (4) with γ_D replaced by the total scattering rate $\gamma_{\Delta L=1}$. The explicit expressions for the total rates γ_D and $\gamma_{\Delta L=1}$ can be found in the literature (see e.g. [4]).

It is possible to generalize the usual decay parameter to the two-flavour case. The natural definition is a 2×2 matrix

$$K_{\alpha\beta} = \left.\frac{\Gamma_{\alpha\beta}}{H}\right|_{z=1} , \tag{5}$$

where

$$\Gamma_{\alpha\beta} = \frac{(\gamma_D)_{\alpha\beta}}{sY_{N_1}^{\rm eq}\frac{\mathrm{K}_1(z)}{\mathrm{K}_2(z)}} , \tag{6}$$

and $\mathrm{K}_i(z)$ are modified Bessel function of the second kind. The trace of $K_{\alpha\beta}$ will be denoted by $K = \sum_\alpha K_{\alpha\alpha}$.

The CP-asymmetry matrix is given by [9]:

$$\epsilon_{\alpha\beta} = \frac{1}{16\pi}\frac{1}{[\lambda\lambda^\dagger]_{11}}\sum_{j\neq 1}\mathrm{Im}\left\{\lambda_{1\alpha}[\lambda\lambda^\dagger]_{1j}\lambda^*_{j\beta} - \lambda^*_{1\beta}[\lambda^*\lambda^T]_{1j}\lambda_{j\alpha}\right\} f\left(\frac{M_j^2}{M_1^2}\right) , \tag{7}$$

where the loop function f is [15]

$$f(x) = \sqrt{x}\left[1 - (1+x)\log\left(1+\frac{1}{x}\right) + \frac{1}{1-x}\right] \stackrel{x\gg 1}{\longrightarrow} -\frac{3}{2\sqrt{x}} . \tag{8}$$

Notice that

$$\epsilon_{\alpha\beta} = \epsilon_{\beta\alpha} \tag{9}$$

and the normalization is such that the trace of the CP asymmetries reproduces the total CP asymmetry produced by the decays of the lightest RH neutrino N_1, in the single-flavour approximation

$$\epsilon_1 \equiv \sum_\alpha \epsilon_{\alpha\alpha} = \frac{1}{8\pi}\frac{1}{[\lambda\lambda^\dagger]_{11}}\sum_{j\neq 1}\mathrm{Im}\left([\lambda\lambda^\dagger]^2_{1j}\right) f\left(\frac{M_j^2}{M_1^2}\right) . \tag{10}$$

If $\overline{m}$ denotes the heaviest light neutrino mass ($= m_{\rm atm}$ for the non-degenerate case) then the entries of the CP-asymmetry matrix are subject to the bounds [9]

$$\epsilon_{\alpha\alpha} \leq \frac{3M_1\overline{m}}{8\pi v^2}\sqrt{\frac{K_{\alpha\alpha}}{K}} , \qquad \epsilon_{12}, \epsilon_{21} \leq \frac{3M_1\overline{m}}{16\pi v^2}\left(\sqrt{\frac{K_{11}}{K}} + \sqrt{\frac{K_{22}}{K}}\right) , \tag{11}$$

where v is the vacuum expectation value of the Higgs doublet.

The Λ parameter accounts for interactions mediated by the dominant Yukawa coupling, which from now on we denote by h_1. It is given by

$$\Lambda = \left. \frac{\omega_1 - i\Gamma_1}{H(M_1)} \right|_{T=M_1} , \tag{12}$$

having defined the thermal mass $\omega_1 \simeq h_1^2 T/16$ and the interaction rate $\Gamma_1 \simeq 8 \times 10^{-3} h_1^2 T$ [16]. The dependence on M_1 is easily made explicit:

$$\mathrm{Re}(\Lambda) \simeq 4 \times 10^{-3} h_1^2 \frac{M_P}{M_1} , \ \mathrm{Im}(\Lambda) \simeq -5 \times 10^{-4} h_1^2 \frac{M_P}{M_1} , \ \mathrm{Re}(\Lambda) \simeq 10 \, |\mathrm{Im}(\Lambda)| , \tag{13}$$

where $M_P = 1.2 \times 10^{19}$ GeV is the Planck mass. In the realistic case, one should identify h_1 with h_τ. The flavour 1 will therefore become distinguishable when $M_1 \lesssim 10^{12} (h_1/h_\tau)^2$ GeV.

The parameter Λ will play a crucial role in what follows. It contains all the informations about the action of the decoherent plasma onto the coherence of the flavour oscillations. Changing the parameter Λ, that is changing the value of the mass M_1, and assuming that leptogenesis takes place at a temperature $T \sim M_1$, one can analyze the various regimes: for $|\Lambda| \ll 1$, the Yukawa coupling h_1 does not mediate processes in thermal equilibrium and one expects therefore that the one-flavour approximation holds. In this regime the off-diagonal entries $Y_{\alpha\beta}$ are expected to be nonvanishing. For $|\Lambda| \sim 1$ the transition between the one-flavour and the two-flavour states takes place. For $|\Lambda| \gg 1$ the transition is occured, there are two flavours in the system and one expects the off-diagonal entries in the matrix Y to be decaying very fast since the quantum correlations among the flavours is efficiently damped away by the decoherent interactions with the plasma.

It is simpler to work with the Boltzmann equations obtained from (2)-(3) by eliminating the thermally averaged rates in favor of the decay parameter matrix $K_{\alpha\beta}$ and two functions, $f_1(z)$ and $f_2(z)$, which account for the $\Delta L = 1$ scatterings in the N_1 thermalization and in the wash-out of the asymmetry, respectively (see [11, 4]). Their asymptotic behaviours are

$$f_1(z) \simeq \begin{cases} 1 & \text{for } z \gg 1 \\ \frac{N_c^2 m_t^2}{4\pi^2 v^2 z^2} & \text{for } z \lesssim 1 , \end{cases} \tag{14}$$

and

$$f_2(z) \simeq \begin{cases} 1 & \text{for } z \gg 1 \\ \frac{a_K N_c^2 m_t^2}{8\pi^2 v^2 z^2} & \text{for } z \lesssim 1 , \end{cases} \tag{15}$$

where $\frac{N_c^2 m_t^2}{8\pi^2 v^2} \sim 0.1$ parametrizes the strength of the $\Delta L = 1$ scatterings and $a_K = 4/3\,(2)$ for the weak (strong) wash out case. A good approximation to the total wash-out term (inverse decays and $\Delta L = 1$ scatterings) at small z is given by $\sim 10^{-1} a_K K$.

After a short manipulation the Boltzmann equations read

$$Y'_{\alpha\beta} = -Y'_{N_1}\epsilon_{\alpha\beta} - \frac{1}{2}h(z)\{K,Y\}_{\alpha\beta} - \Big[\sigma_2\mathrm{Re}(\Lambda) + \sigma_1|\mathrm{Im}(\Lambda)|\Big]Y_{\alpha\beta}\,, \tag{16}$$

$$Y'_{N_1} = -zK\frac{\mathrm{K}_1(z)}{\mathrm{K}_2(z)}f_1(z)(Y_{N_1} - Y^{\rm eq}_{N_1})\,, \tag{17}$$

where primes denote derivatives with respect to z and $h(z) \equiv \frac{1}{2}z^3\mathrm{K}_1(z)f_2(z)$. These equations are the starting point of our analysis. Although they are just classical equations, they reproduce the correct expected limits (as shown in the next two Sections) and also have the virtue of providing information on the transition between the one-flavour and the two-flavour regimes.

3 The one-flavour limit

In this section we deal with the one-flavour limit, corresponding to $|\Lambda| \ll 1$. More precisely, inspecting Eq. (16), one learns that the quantum correlators need to be accounted for if[4]

$$|\Lambda| \ll \frac{1}{2}h(z)K\,. \tag{18}$$

which implies

$$\left(\frac{M_1}{10^{12}\,\mathrm{GeV}}\right) \gg \frac{2}{Kh(z)}\,. \tag{19}$$

This condition has to be satisfied at the time when the asymmetry is generated. In the weak wash-out regime, $K \lesssim 1$, and supposing that the initial abundance of RH neutrinos is vanishing, the production of the baryon asymmetry takes place at some $\bar{z} \gtrsim 1$. Since the wash-out term for $K \lesssim 1$ is always smaller than unity, we conclude that in the weak wash-out regime the one-flavour limit is reached for $M_1 \gtrsim 10^{12}$ GeV.

In the strong wash-out regime, $K \gg 1$, the baryon asymmetry is generated at some $\bar{z} \sim \ln K + (5/2)\ln\bar{z} \gtrsim 1$ when $Kh(\bar{z})/2 \simeq 1$. Since the wash-out function $Kh(z)/2$ is larger than unity for $z \lesssim \bar{z}$, we conclude that in the strong-wash out regime the condition (18) implies $|\Lambda| \ll (1/2)Kh(\bar{z}) \sim 1$, that is $M_1 \gg 10^{12}$ GeV.

Under the conditions that the Λ-terms may be dropped in Eq. (16), the latter reads

$$Y'_{\alpha\alpha} = -Y'_{N_1}\epsilon_{\alpha\alpha} - \frac{1}{2}h(z)\left[K_{\alpha\beta}Y_{\beta\alpha} + K_{\beta\alpha}Y_{\alpha\beta}\right] - h(z)K_{\alpha\alpha}Y_{\alpha\alpha}\,, \tag{20}$$

$$Y'_{\alpha\beta} = -Y'_{N_1}\epsilon_{\alpha\beta} - \frac{1}{2}h(z)\mathrm{Tr}(Y)K_{\alpha\beta} - \frac{1}{2}h(z)KY_{\alpha\beta}\,, \tag{21}$$

[4] We thank P. Di Bari for sharing with us prior to publication his paper in collaboration with Blanchet and Raffelt [17] where similar considerations have been presented.

with $\alpha \neq \beta$ and no summation over repeated indices. Notice that these equations are implicit, since the trace of Y appears in the right hand side. Now, we perform an ad hoc rotation in the flavour space. The quantities referred to the new basis will be denoted by a 'hat'. In general, we are free to rotate the lepton doublets by a unitary matrix A:

$$\hat{\ell}_\alpha = A_{\alpha\beta}\ell_\beta \tag{22}$$

($AA^\dagger = \mathbf{1}$) and this is equivalent to a basis change in the flavour space. A useful choice for A is

$$A = \frac{1}{\sqrt{[\lambda\lambda^\dagger]_{11}}} \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ -(\lambda_{12})^* & (\lambda_{11})^* \end{pmatrix} , \tag{23}$$

where $[\lambda\lambda^\dagger]_{11} = |\lambda_{11}|^2 + |\lambda_{11}|^2 = [\hat{\lambda}\hat{\lambda}^\dagger]_{11}$ by the unitarity of A, which leads to the rotated Yukawa couplings:

$$\hat{\lambda} = \frac{1}{\sqrt{[\lambda\lambda^\dagger]_{11}}} \begin{pmatrix} |\lambda_{11}|^2 + |\lambda_{12}|^2 & 0 \\ (\lambda_{11})^*\lambda_{21} + (\lambda_{12})^*\lambda_{22} & \det[\lambda] \end{pmatrix} , \tag{24}$$

with $\det[\lambda] = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{22}$. The zero entry makes manifest that N_1 is coupled only to $\hat{\ell}_1 = \sum_{\alpha=1,2}\lambda_{1\alpha}\ell_\alpha/\sqrt{[\lambda\lambda^\dagger]_{11}}$.

The matrices $K_{\alpha\beta}$ and $\epsilon_{\alpha\beta}$ in the new basis are obtained by replacing $\lambda \to \hat{\lambda}$; in particular, one finds

$$\hat{K}_{11} = K\,, \qquad \hat{K}_{12} = \hat{K}_{21} = \hat{K}_{22} = 0 \tag{25}$$

$$\hat{\epsilon}_{11} = \epsilon_1\,, \qquad \hat{\epsilon}_{22} = 0\,. \tag{26}$$

Thanks to these relations, the equations for the diagonal components (20) give $\hat{Y}_{22} = 0$, so the lepton asymmetry is concentrated on the lepton $\hat{\ell}_1$ only and it evolves according to the equation

$$\hat{Y}'_{\hat{\ell}_1} = -Y'_{N_1}\epsilon_1 - h(z)K\,\hat{Y}_{\hat{\ell}_1}\,, \tag{27}$$

which exactly reproduces the Boltzmann equation for the one single flavour. The latter can be identified with the total lepton asymmetry, that is with the trace of the lepton asymmetries. The total lepton asymmetry in the lepton doublets is indeed the only quantity which treats indistinguishably all the flavours.

4 The two-flavour limit

Let us now turn to the opposite regime where the Λ terms are important, i.e. we are in the full two-flavour regime. Again, we split (16) in equations for the diagonal and off-diagonal components of Y:

$$Y'_{\alpha\alpha} = -Y'_{N_1}\epsilon_{\alpha\alpha} - \frac{1}{2}h(z)\left[K_{\alpha\beta}Y_{\beta\alpha} + K_{\beta\alpha}Y_{\alpha\beta}\right] - h(z)K_{\alpha\alpha}Y_{\alpha\alpha}\,, \tag{28}$$

$$Y'_{\alpha\beta} = -Y'_{N_1}\epsilon_{\alpha\beta} - \frac{1}{2}h(z)\mathrm{Tr}(Y)K_{\alpha\beta} - \left[\frac{1}{2}h(z)K + \Big(|\mathrm{Im}(\Lambda)| + (\sigma_2)_{\alpha\beta}\mathrm{Re}(\Lambda)\Big)\right]Y_{\alpha\beta}\,, \tag{29}$$

with $\alpha \neq \beta$ and no summation over repeated indices. The Λ terms appear in the wash-out of the off-diagonal elements. Therefore, the solutions of (29) will contain exponential factors of the form $e^{i\Lambda z}$. The real part of Λ leads to oscillating behaviours, while the imaginary part controls the damping. The latter is originated by the decoherence effect of the high temperature plasma on the flavour oscillations: if Yukawa coupling h_1 mediates processes which are fast enough, the correlations between different flavours are rapidly lost. Such correlations are encoded in the off-diagonal components of the lepton asymmetry density matrix Y. As long as the off-diagonal entries become negligibly small, Eq. (28) reduces to that studied in [11], where the flavours are considered as completely decoupled and the system of equations reduces to two equations for the diagonal entries of the Y matrix. More in detail, we can say that the two-flavour state is reached when the oscillations are efficiently damped, i.e when the following condition holds

$$|\mathrm{Im}(\Lambda)| \gtrsim \frac{1}{2}h(z)K \tag{30}$$

or

$$\left(\frac{M_1}{10^{12}\,\mathrm{GeV}}\right) \lesssim \frac{2}{Kh(z)}\,, \tag{31}$$

around the point when the baryon asymmetry in a given flavour α is generated. In the weak wash-out regime for all flavours, $K_{\alpha\alpha}, K \lesssim 1$, the flavour asymmetry is generated at $\bar{z}_\alpha \gtrsim 1$ and the function $(1/2)Kh(z)$ is always smaller than unity. Therefore, we obtain that the two flavour regime is dynamically relevant for $M_1 \lesssim 10^{12}$ GeV.

In the strong wash-out regime for all flavours, $K_{\alpha\alpha}, K \gg 1$, the condition (31) on the mass of the RH neutrino is $M_1 \lesssim (K_{\alpha\alpha}/K)\,10^{12}$ GeV for $\bar{z}_\alpha \sim \ln K_{\alpha\alpha} + (5/2)\ln \bar{z}_\alpha \gtrsim 1$. The most stringent bound is obtained for the smallest $K_{\alpha\alpha}$, which corresponds to the smallest wash-out. Of course the bound should be applied only if the same flavour gives also the largest asymmetry. This depends upon the CP asymmetry $\epsilon_{\alpha\alpha}$. In particular, if $K_{\alpha\alpha}$ takes the smallest value compatible with the strong wash-out, $K_{\alpha\alpha} \sim 3$ and if the CP asymmetry $\epsilon_{\alpha\alpha}$ is the largest, then one obtains the most stringent bound, $M_1 \lesssim (3/K)10^{12}$ GeV.

In the case of strong wash-out for some flavour α, $K_{\alpha\alpha} \gtrsim 1$, but weak wash-out for some other flavour β, $K_{\beta\beta} \lesssim 1$, the asymmetry in the flavour β is generated at $z = \mathcal{O}(5)$ [11] and the condition on the mass of the lightest RH neutrino is given by $M_1 \lesssim (10/K)10^{12}$ GeV, provided that the final baryon asymmetry is mainly generated by the flavour β. If this is not the case, one should apply the condition $M_1 \lesssim (K_{\alpha\alpha}/K)\,10^{12}$ GeV $\sim 10^{12}$ GeV.

Let us close this section with a comment. We expect the bounds obtained in this section comparing rates to be in fact too restrictive. They have been derived just comparing the rate of the $\Delta L = 1$ inverse decays and scatterings with the rate of damping of the flavour oscillations. However, the real dynamics is more involved. For instance, the flavour oscillations are characterized by a rapidly oscillating behaviour. The oscillation rate is dictated by $|\mathrm{Re}(\Lambda)|$ which is a factor about ten larger than the damping rate of the flavour oscillations, $\mathrm{Re}(\Lambda) \sim 10|\,\mathrm{Im}(\Lambda)|$. This is relevant because computing the flavour asymmetries involves integrals over time. Since the flavour oscillations decay and also have an oscillatory behaviour, this restricts the range of time integration, thus leading to a suppression of the contribution from the flavour oscillations. We therefore expect the influence of the the flavour oscillations to disappear even in the vicinity of $M_1 \sim 10^{12}$ GeV. Our numerical results support this expectation.

5 The transition between the one- and the two-flavour case

Having elaborated about the two extreme regimes, we now investigate what happens in the intermediate region where the one flavour – two flavours transition takes place. To achieve this, we perform an analytical study of the solutions of (28) and (29), in two representative regimes of K's, showing also some numerical simulations to enforce our findings. In the figures we will present two different quantities which may serve as indicators of the transition. The first quantity is $Y_{\alpha\alpha}/(Y_{\alpha\alpha})_{\rm dec}$ which is the ratio between the flavour asymmetry $Y_{\alpha\alpha}$ in the flavour α computed solving the full system of Boltzmann equations (28) and (29) over the same asymmetry $(Y_{\alpha\alpha})_{\rm dec}$ computed neglecting the off-diagonal terms in the same equations. This ratio should tend to unity in the full two-flavour regime because the off diagonal correlators have been efficiently damped out. The second indicator is the ratio of the the trace of the 2×2 matrix Y, $\mathrm{Tr}[Y]$ computed solving Eqs. (28) and (29) and the asymmetry computed in the one-flavour approximation, $Y_{1-\rm flavour}$, assuming a single flavour with CP asymmetry ϵ_1 and wash-out parameter $K = K_{11} + K_{22}$. This ratio should tend to unity in the one-flavour regime, when the off-diagonal terms are not damped.

5.1 Strong wash-out regime for all flavours

In this case $K_{11}, K_{22} \gg 1$. This implies that the N_1 abundance closely follow the equilibrium abundance, $Y'_{N_1} \simeq (Y^{\rm eq}_{N_1})' = -\frac{1}{2g_*}h(z)/z$. The integrals giving the lepton asymmetries are evaluated by using the steepest descent method twice. One finds the following analytical estimates

$$
\begin{aligned}
Y_{\alpha\alpha} &\simeq \frac{1}{K_{\alpha\alpha}}\left[\frac{\epsilon_{\alpha\alpha}}{2g_*\bar{z}_\alpha} - \frac{1}{2}\Big(K_{\alpha\beta}Y_{\beta\alpha}(\bar{z}_\alpha) + K_{\beta\alpha}Y_{\alpha\beta}(\bar{z}_\alpha)\Big)\right] && (32)\\
Y_{12}(z > z_\Lambda) &\simeq \frac{2}{K}\left(\frac{\epsilon_{12}}{2g_* z_\Lambda} - \frac{1}{2}K_{12}{\rm Tr}[Y(z_\Lambda)]\right)\times \\
&\quad e^{i(z-z_\Lambda){\rm Re}(\Lambda)}e^{-(z-z_\Lambda)|{\rm Im}(\Lambda)|}\,, && (33)\\
Y_{21}(z > z_\Lambda) &\simeq \frac{2}{K}\left(\frac{\epsilon_{21}}{2g_* z_\Lambda} - \frac{1}{2}K_{21}{\rm Tr}[Y(z_\Lambda)]\right)\times \\
&\quad e^{-i(z-z_\Lambda){\rm Re}(\Lambda)}e^{-(z-z_\Lambda)|{\rm Im}(\Lambda)|}\,, && (34)
\end{aligned}
$$

where $z_\Lambda \sim 1/{\rm Im}(\Lambda)$ and the $\bar{z}_\alpha$'s are implicitly defined by $K_{\alpha\alpha}h(\bar{z}_\alpha) \simeq 1$. We remark that the relation $Y_{\beta\alpha} = (Y_{\alpha\beta})^*$ holds, this assures that the diagonal asymmetries are real. To a first approximation we can take $\bar{z}_1 \approx \bar{z}_2 \equiv \bar{z}$, which is true up to logarithmic corrections. From Eqs. (32)-(34) it is possible to find an expression for the trace of Y, which allows us to write the diagonal asymmetries explicitly:

$$
\begin{aligned}
Y_{11} &\simeq \frac{1}{2g_*\bar{z}}\left\{\frac{\epsilon_{11}}{K_{11}} + \frac{e^{-(\bar{z}-z_\Lambda)|{\rm Im}(\Lambda)|}}{K_{11}K\left(K_{11}K_{22} - K_{12}K_{21}\cos\left[(\bar{z}-z_\Lambda){\rm Re}(\Lambda)\right]e^{-(\bar{z}-z_\Lambda)|{\rm Im}(\Lambda)|}\right)}\times\right.\\
&\left.\left[(\epsilon_{11}K_{22} + \epsilon_{22}K_{11})K_{12}K_{21}\cos\left[(\bar{z}-z_\Lambda){\rm Re}(\Lambda)\right] - \frac{\bar{z}}{z_\Lambda}K_{11}K_{22}\left(K_{21}\epsilon_{12}e^{i(\bar{z}-z_\Lambda){\rm Re}(\Lambda)} + {\rm c.c.}\right)\right]\right\}, \qquad (35)\\
Y_{22} &\simeq \frac{1}{2g_*\bar{z}}\left\{\frac{\epsilon_{22}}{K_{22}} + \frac{e^{-(\bar{z}-z_\Lambda)|{\rm Im}(\Lambda)|}}{K_{22}K\left(K_{11}K_{22} - K_{12}K_{21}\cos\left[(\bar{z}-z_\Lambda){\rm Re}(\Lambda)\right]e^{-(\bar{z}-z_\Lambda)|{\rm Im}(\Lambda)|}\right)}\times\right.\\
&\left.\left[(\epsilon_{11}K_{22} + \epsilon_{22}K_{11})K_{12}K_{21}\cos\left[(\bar{z}-z_\Lambda){\rm Re}(\Lambda)\right] - \frac{\bar{z}}{z_\Lambda}K_{11}K_{22}\left(K_{21}\epsilon_{12}e^{i(\bar{z}-z_\Lambda){\rm Re}(\Lambda)} + {\rm c.c.}\right)\right]\right\}. \qquad (36)
\end{aligned}
$$

The terms proportional to $\epsilon_{\alpha\alpha}/K_{\alpha\alpha}$ are the familiar asymmetries in the strong wash-out regime, while the remaining terms are the corrections due to the correlation between flavours. Such corrections are quickly damped by the imaginary part of Λ, and this behaviour is also confirmed by numerical simulations. In the limit $\Lambda \to \infty$ we recover the total lepton asymmetry of two decoupled flavours:

$$\mathrm{Tr}[Y] = Y_{11} + Y_{22} \stackrel{\Lambda\to\infty}{\longrightarrow} \frac{1}{2g_*\bar{z}} \left(\frac{\epsilon_{11}}{K_{11}} + \frac{\epsilon_{22}}{K_{22}} \right) , \tag{37}$$

as expected. On the other hand, the limit $\Lambda \to 0$ leads to

$$\mathrm{Tr}[Y] \stackrel{\Lambda\to 0}{\longrightarrow} \frac{1}{2g_*\bar{z}} \frac{K_{11}\epsilon_{22} + K_{22}\epsilon_{11} - (K_{21}\epsilon_{12} + K_{12}\epsilon_{21})}{K_{11}K_{22} - K_{12}K_{21}} . \tag{38}$$

It is easy to see that the quantity on the right hand side is left invariant by a transformation of the matrices K, ϵ of the form

$$K \to MKN\,, \quad \epsilon \to M\epsilon N\,, \tag{39}$$

where M, N are two generic 2×2 non-singular matrices. In fact, the denominator in (38) is just the determinant of K which simply transforms as: $\det(K) \to \det(M)\det(N)\det(K)$. On the other hand, the numerator may be written as $\varepsilon_{ij}\varepsilon_{mn}K_{im}\epsilon_{jn}$, where ε is the antisymmetric Levi-Civita symbol in two dimensions and summation over repeated indices is assumed. So, the numerator in (38) transforms as:

$$\begin{aligned} \varepsilon_{ij}\varepsilon_{mn}K_{im}\epsilon_{jn} \;\; &\to \;\; \varepsilon_{ij}\varepsilon_{mn}(MKN)_{im}(M\epsilon N)_{jn} = \\ &= \;\; \varepsilon_{ij}\varepsilon_{mn}(M_{ia}K_{ab}N_{bm})(M_{jp}\epsilon_{pq}N_{qn}) = \\ &= \;\; (\varepsilon_{ij}M_{ia}M_{jp})(\varepsilon_{mn}N_{bm}N_{qn})K_{ab}\epsilon_{pq} = \\ &= \;\; \det(M)\det(N)\varepsilon_{ap}\varepsilon_{bq}K_{ab}\epsilon_{pq} \\ &= \;\; \det(M)\det(N)\varepsilon_{ij}\varepsilon_{mn}K_{im}\epsilon_{jn} \end{aligned} \tag{40}$$

under (39). Therefore the numerator picks up an extra factor, namely $\det(M)\det(N)$, which exactly cancels that in the denominator and the invariance of (38) is proved. This fact means that a transformation of K and ϵ matrices does not affect the trace in (38). In particular, we can evaluate it in the rotated flavour basis defined in Section 3, and obtain

$$\mathrm{Tr}[Y] \sim \frac{\hat{\epsilon}_{11}\hat{K}_{22} + \hat{\epsilon}_{22}\hat{K}_{11}}{\hat{K}_{11}\hat{K}_{22}} = \frac{\epsilon_1}{K} , \tag{41}$$

which is the single-flavour result, as expected. In the one-flavour limit, $M_1 \gg 10^{12}$ GeV, the efficiency factor $\eta(K)$ for the final baryon asymmetry depends only upon K. In the opposite limit, $M_1 \ll 10^{12}$ GeV, the final baryon asymmetry depends upon two different efficiency factors, one for each $K_{\alpha\alpha}$. As discussed in [18], $K_{\alpha\alpha}/K \lesssim 2$ for large mixing angles and therefore the efficiency is enhanced by $\mathcal{O}(2)$ when going from $M_1 \gg 10^{12}$ GeV to $M_1 \ll 10^{12}$ GeV.

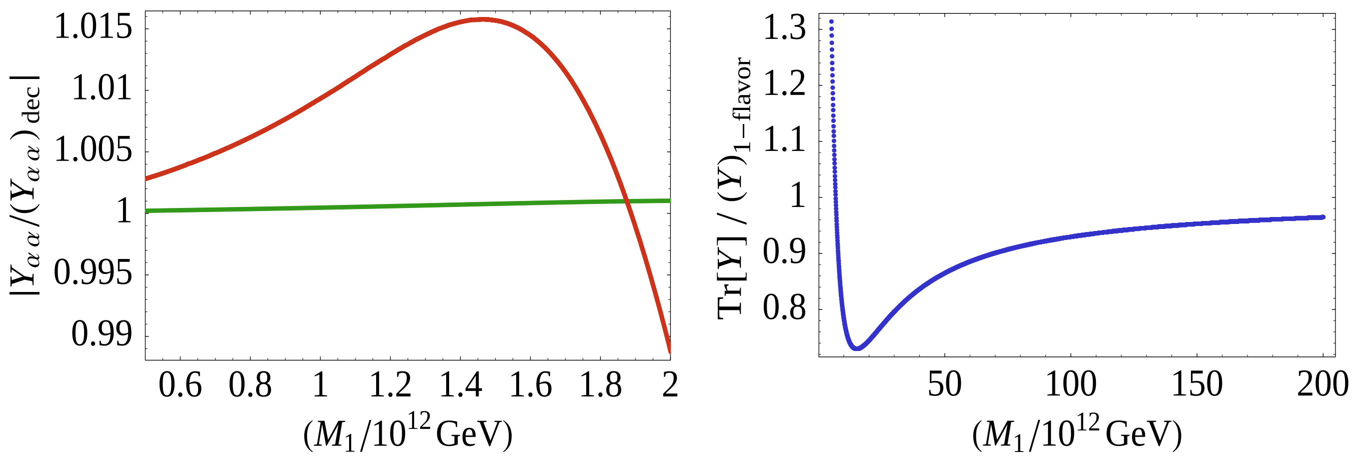

Figure 1: The ratio between the lepton asymmetries Y_{11} (green) and Y_{22} (red) computed including the off-diagonal terms of Eqs. (28) and (29) and the ones neglecting them (see text) as a function of M_1 *(left)*. The trace of the lepton asymmetry divided by the same trace computed in the single-flavour approximation (see text) as a function of M_1 *(right)*. The parameters are $K = 50$, $K_{11} = 40$, $K_{22} = 10$, $K_{12} = K_{21} = 20$, $\epsilon_{11} = 0.4$, $\epsilon_{22} = 0.1$, $\epsilon_{12} = \epsilon_{21} = 0.2$. Here and in the following, the relative magnitudes of the ϵ entries are chosen consistent with the bounds (11).

Figure 1 on the left shows $Y_{\alpha\alpha}/(Y_{\alpha\alpha})_{\rm dec}$, the diagonal lepton asymmetries $Y_{\alpha\alpha}$, as functions of M_1. In this figure, as well as in all others, we have chosen compatible values for the parameters $K_{\alpha\beta}$ by fixing the Yukawa couplings $\lambda_{i\alpha}$. The analytical results reproduce the numerical ones within 10%. On the right we show $\text{Tr}[Y]/Y_{1-\text{flavour}}$ as a function of M_1. We see that the ratio tends to unity for $M_1 \gtrsim 2 \times 10^{12}$ GeV in agreement with our previous findings. In our numerical example, the two flavours give rise to the same asymmetries, and for the bound discussed in Section 4 to be in the full two-flavour state would require $M_1 \lesssim (K_{22}/K)10^{12}$ GeV $\sim 2 \times 10^{11}$ GeV. However, we see from our numerical results that the two-flavour state is reached for larger values of M_1. To our understanding this is due to the rapidly oscillating behaviour of the off-diagonal terms. As we already mentioned, computing the flavour asymmetries involves an integral over time (or, better, over the parameter z). Since the quantum correlators not only decay, but also have a rapid oscillatory behaviour, this restricts the range of time integration, thus leading to a suppression of the contribution from the flavour oscillations. This effect is magnified by the fact that the oscillations have a time scales which is about a factor of ten smaller than the damping timescale. We deduce from our results that even for values of $M_1 \sim 10^{12}$ GeV the full two-flavour regime is attained.

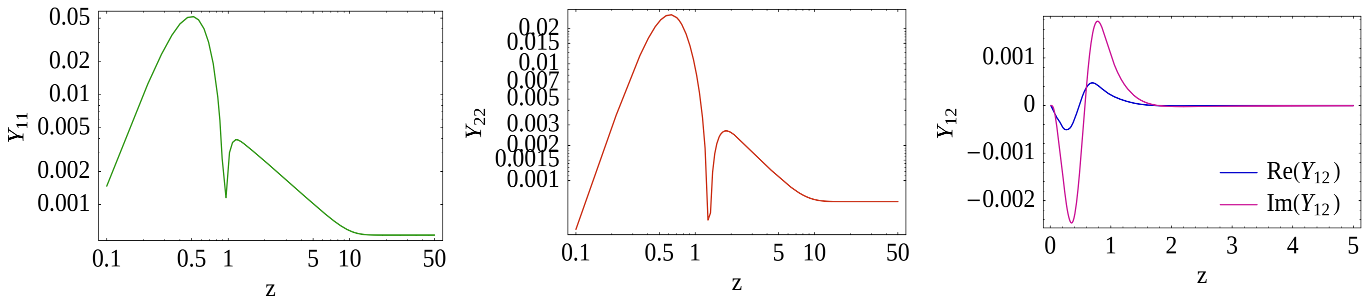

Figure 2: The time evolution of the asymmetries for $M_1 = 2 \times 10^{11}$ GeV, $K = 50$, $K_{11} = 40$, $K_{22} = 10$, $K_{12} = K_{21} = 20$, $\epsilon_{11} = 0.4$, $\epsilon_{22} = 0.1$, $\epsilon_{12} = \epsilon_{21} = 0.2$.

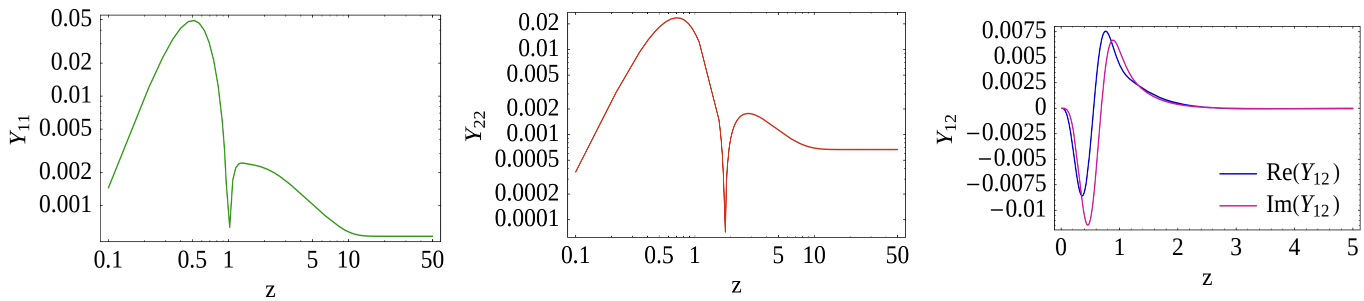

Figure 3: The time evolution of the asymmetries for $M_1 = 10^{12}$ GeV,, $K = 50$, $K_{11} = 40$, $K_{22} = 10$, $K_{12} = K_{21} = 20$, $\epsilon_{11} = 0.4$, $\epsilon_{22} = 0.1$, $\epsilon_{12} = \epsilon_{21} = 0.2$.

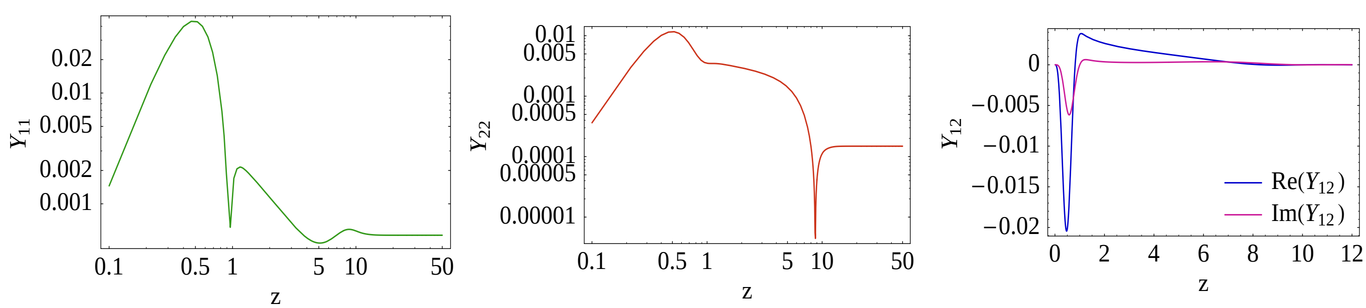

Figure 4: The time evolution of the asymmetries for $M_1 = 5 \times 10^{12}$ GeV,, $K = 50$, $K_{11} = 40$, $K_{22} = 10$, $K_{12} = K_{21} = 20$, $\epsilon_{11} = 0.4$, $\epsilon_{22} = 0.1$, $\epsilon_{12} = \epsilon_{21} = 0.2$.

In Figs. 2, 3 and 4 we present the evolution of the asymmetries for a given choice of the parameters. As expected, for smaller values of M_1 the off-diagonal terms die out for larger values of z. However, by the time the asymmetries stored in the diagonal terms are frozen out, the flavour oscillations have already been wiped out.

5.2 Strong wash-out for one flavour and weak wash-out for the other one

This regime is characterized by $K_{22} \ll 1 \ll K_{11}$. The main contribution to the total decay parameter comes from the strongly interacting flavour $K \simeq K_{11} \gg 1$, which means that N_1's are almost in equilibrium, as in the previous case. Since the damping of the off-diagonal terms is sensitive to K, it is still possible to perform the integrals for Y_{11} and Y_{22} by means of the steepest descent method, getting the same estimates as in the previous regime. We find

$$
\begin{aligned}
Y_{11} &\simeq \frac{1}{K_{11}}\left[\frac{\epsilon_{11}}{2g_*\bar{z}_1} - \frac{1}{2}\Big(K_{12}Y_{21}(\bar{z}_1) + K_{21}Y_{12}(\bar{z}_1)\Big)\right], && (42)\\
Y_{22} &\simeq \frac{0.4}{g_*}\epsilon_{22}K_{22} - \frac{1}{K}\left[K_{12}\frac{\epsilon_{21}}{2g_* z_\Lambda}I(\Lambda) + K_{21}\frac{\epsilon_{12}}{2g_* z_\Lambda}I(\Lambda)^* - K_{12}K_{21}\mathrm{Tr}[Y(z_\Lambda)]\right] && (43)
\end{aligned}
$$

where

$$
I(\Lambda) = \int_{z_\Lambda}^{\infty} \mathrm{d}z z^3 \mathrm{K}_1(z) e^{-i(z-z_\Lambda)\mathrm{Re}(\Lambda)} e^{-(z-z_\Lambda)|\mathrm{Im}(\Lambda)|} \tag{44}
$$

satisfies the property $I(\Lambda \to \infty) = 0$. As in the previous case, one first finds an expression for the trace of Y and then uses it to write the diagonal entries in an explicit form

$$
\begin{aligned}
Y_{11} &\simeq \frac{1}{2g_*\bar{z}_1}\frac{\epsilon_{11}}{K_{11}} + \\
&+ \frac{e^{-(\bar{z}_1 - z_\Lambda)|\mathrm{Im}(\Lambda)|}}{g_* K_{11}^2\left[1 - \frac{K_{12}K_{21}}{K_{11}}\mathrm{Re}(I(\Lambda))\right]}\left[0.4\epsilon_{22}K_{22}K_{12}K_{21}\cos\left[(\bar{z}_1 - z_\Lambda)\mathrm{Re}(\Lambda)\right]\right. \\
&\left. - \frac{1}{2z_\Lambda}\left(K_{21}\epsilon_{12}e^{i(\bar{z}_1 - z_\Lambda)\mathrm{Re}(\Lambda)} + \text{c.c.}\right)\right], && (45)\\
Y_{22} &\simeq \frac{0.4}{g_*}\frac{\epsilon_{22}K_{22}}{\left[1 - \frac{K_{12}K_{21}}{K_{11}}\mathrm{Re}(I(\Lambda))\right]} + \\
&+ \frac{1}{g_*\left[1 - \frac{K_{12}K_{21}}{K_{11}}\mathrm{Re}(I(\Lambda))\right]}\left[\frac{1}{\bar{z}_1}\frac{K_{12}K_{21}}{K_{11}^2}\epsilon_{11}\mathrm{Re}(I(\Lambda))\right. \\
&\left. - \frac{1}{2z_\Lambda K_{11}}\left(K_{21}\epsilon_{12}I(\Lambda) + \text{c.c.}\right)\right]. && (46)
\end{aligned}
$$

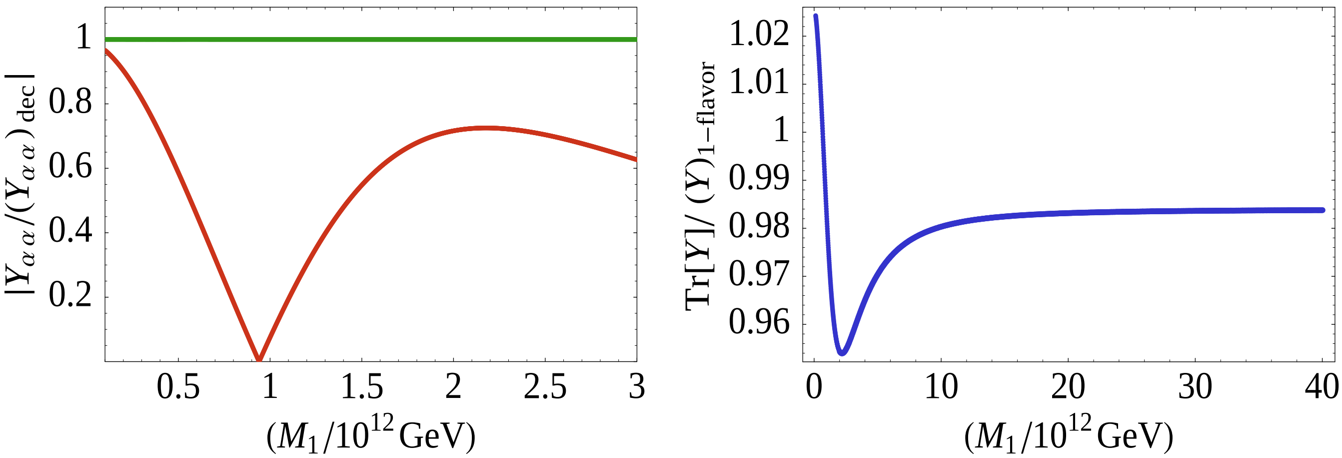

Figure 5: The ratio between the lepton asymmetries Y_{11} (green) and Y_{22} (red) computed including the off-diagonal terms of Eqs. (28) and (29) and the ones neglecting them (see text) as a function of M_1 *(left)*. The trace of the lepton asymmetry divided by the same trace computed in the single-flavour approximation (see text) as a function of M_1 *(right)*. The parameters are $K_{11} = 30$, $K_{22} = 10^{-2}$, $K_{12} = K_{21} = 0.6$, $\epsilon_{11} = 0.3$, $\epsilon_{22} = 5 \times 10^{-3}$, $\epsilon_{12} = \epsilon_{21} = 0.006$.

If $\Lambda \to \infty$ the previous expressions reduce to those usually found in the literature [11], where the off-diagonal correlations are neglected and the two flavours are completely decoupled

$$Y_{11} \simeq \frac{1}{2g_* \bar{z}_1} \frac{\epsilon_{11}}{K_{11}}\,, \qquad Y_{22} \simeq \frac{0.4}{g_*} \epsilon_{22} K_{22}\,. \tag{47}$$

Figure 5 on the right shows $\mathrm{Tr}[Y]/Y_{1-\mathrm{flavour}}$ as a function of M_1. We see that the ratio tends to unity for large values of M_1, as expected and it does it very fast, in agreement with our previous findings that, as soon $M_1 \gtrsim 10^{12}$ GeV, then the two-flavour regime is reached. Figure 5 on the left shows $Y_{\alpha\alpha}/(Y_{\alpha\alpha})_{\rm dec}$ as functions of M_1. The analytical results reproduce the numerical ones within 10%. From this figure we deduce that neglecting the off-diagonal terms in evaluating the diagonal terms of the matrix Y is a good approximation for the strongly washed-out flavour for values of $M_1 \sim 10^{12}$ GeV. For the weakly coupled flavour the transition occurs at $M_1 \gtrsim (10/K)10^{12}$ GeV $\sim 3 \times 10^{11}$ GeV, as derived in Section 4. This time the transition does not occur for values of $M_1 \sim 10^{12}$ GeV because, for the set of parameters chosen, the asymmetry stored in the weakly coupled flavour is comparable with the one stored in the off-diagonal terms. This illustrates the fact that the contribution from the off-diagonal terms may influence the final asymmetry in the weakly coupled flavour if the choice of the parameters is such that the off-diagonal CP asymmetries and wash out factors are not too

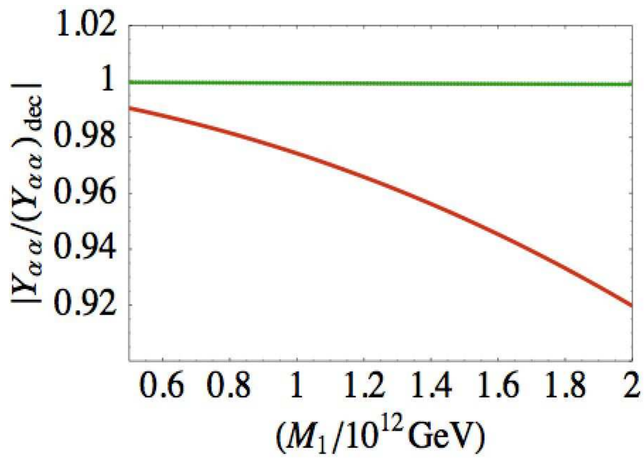

Figure 6: The ratio between the lepton asymmetries Y_{11} (green) and Y_{22} (red) computed including the off-diagonal terms and the ones neglecting them (see text) as a function of M_1 for $K_{11} = 2.4$, $K_{22} = 0.6$, $K_{12} = K_{21} = 1.2$, $\epsilon_{11} = 0.25$, $\epsilon_{22} = 0.06$, $\epsilon_{12} = \epsilon_{21} = 0.12$.

small. This might be relevant if the weakly coupled flavour gives the largest contribution to the final baryon asymmetry. On the other hand, one would expect that, when the asymmetry stored in the weakly coupled flavour is large enough, then the values of Y_{22} computed with and without taking into account the off-diagonal terms should be very close. This expectation is shown to be correct in Figure 6. It illustrates also our previous estimates that, if $K_{\alpha\alpha} \sim 3$, then the full two flavour regime should be recovered for $M_1 \lesssim (3/K)10^{12}$ GeV $\sim 10^{12}$ GeV. In Figs. 7, 8 and 9 we present the evolution of the asymmetries for a given choice of the parameters. Again, for large values of M_1 the off-diagonal terms die out for larger values of z. However, by the time the asymmetries stored in the diagonal terms are frozen out, the flavour oscillations have already been wiped out.

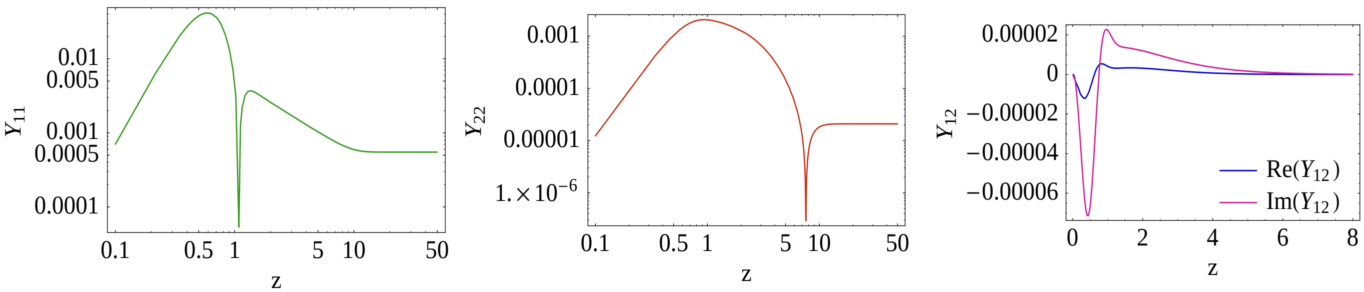

Figure 7: The time evolution of the asymmetries for $M_1 = 2 \times 10^{11}$ GeV, $K \simeq 30$, $K_{11} = 30$, $K_{22} = 10^{-2}$, $K_{12} = K_{21} = 0.6$, $\epsilon_{11} = 0.3$, $\epsilon_{22} = 5 \times 10^{-3}$, $\epsilon_{12} = \epsilon_{21} = 0.006$.

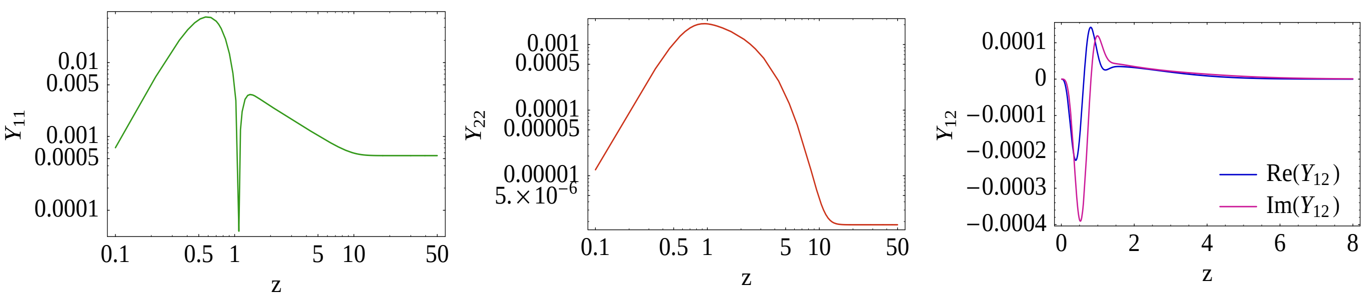

Figure 8: The time evolution of the asymmetries for $M_1 = 10^{12}$ GeV,, $K \simeq 30$, $K_{11} = 30$, $K_{22} = 10^{-2}$, $K_{12} = K_{21} = 0.6$, $\epsilon_{11} = 0.3$, $\epsilon_{22} = 5 \times 10^{-3}$, $\epsilon_{12} = \epsilon_{21} = 0.006$.

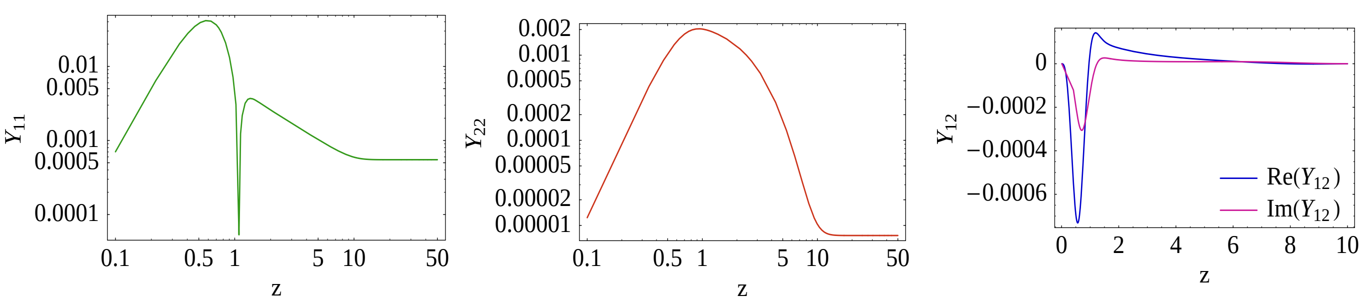

Figure 9: The time evolution of the asymmetries for$M_1 = 5 \times 10^{12}$ GeV, $K \simeq 30$, $K_{11} = 30$, $K_{22} = 10^{-2}$, $K_{12} = K_{21} = 0.6$, $\epsilon_{11} = 0.3$, $\epsilon_{22} = 5 \times 10^{-3}$, $\epsilon_{12} = \epsilon_{21} = 0.006$.

6 Comments and Conclusions

In this paper we have studied the impact of the oscillations among the lepton asymmetries in leptogenesis and investigated the transition from the one-flavour to the two-flavour states. We also accounted for the $\Delta L = 1$ scatterings both in the CP asymmetries and in the wash-out terms. The transition mimics the realistic one when the τ flavour becomes distinguishable from the other two flavours. We have first formally shown that for $M_1 \gtrsim 10^{12}$ GeV, the quantum correlators are relevant to reduce the system of Boltzmann equations to a single equation for the total lepton asymmetry. In this regime the one-flavour approximation holds. Subsequently, we have shown that in the regime $M_1 \ll 10^{12}$ GeV, the full two-flavour state is recovered thanks to the damping of the quantum correlators. We have subsequently solved both analytically and numerically the Boltzmann equations for the lepton asymmetries in flavour space. Particular attention has been devoted to the case $M_1 \sim 10^{12}$ GeV where we expected the role played by the quantum correlators to be maximal.

Let us summarize our results. If all flavours are in the weak wash-out regime, the two flavour state is reached and the flavour oscillations may be safely neglected if $M_1 \lesssim 10^{12}$ GeV. If all flavours are in the strong wash-out regime, we have estimated analytically that the two flavour state is reached and the flavour oscillations may be safely neglected if $M_1 \lesssim (K_{\alpha\alpha}/K)10^{12}$ GeV. We point out however that our numerical studies show that the real bound is weaker. The two flavour state is reached even for values of M_1 close to 10^{12} GeV. The flavour oscillations seem to efficiently project the lepton state on the flavour basis. To our understanding this is due to the short timescale of the flavour oscillations compared to the damping timescale. Flavour oscillations decay and have a rapid oscillatory behaviour, thus restricting the range of time integration. This suppresses the contribution from the flavour oscillations to all the dynamics, rendering the transition easier.

We conclude that for the strong wash out case it is a good approximation to solve the Boltzmann equations just for the asymmetries stored in the lepton doublets. This procedure is usually followed in the recent literature regarding the flavoured leptogenesis. Our results justify it.

The same conclusion is obtained if all the flavours are in the so-called mild regime. This occurs when the lepton asymmetry is generated only by the low energy CP violating phases in the PMNS matrix [13].

In the extreme case in which one of the flavour is very weakly coupled and the other is strongly coupled, the approximation of neglecting the flavour oscillations is a good one for the strongly coupled flavour even for $M_1 \sim 10^{12}$ GeV. For the weakly coupled flavour neglecting the off-diagonal terms may be too drastic for $M_1 \sim 10^{12}$ GeV, especially if the parameters of the off-diagonal terms are such that they induce large asymmetries. However, as soon as M_1 is smaller than the analytically estimated value $\sim (10/K)10^{12}$ GeV, neglecting the off-diagonal terms is safe.

Our findings therefore indicate that the flavour effects in leptogenesis become generically relevant at $M_1 \sim 10^{12}$ GeV. Let us conclude with some comments. In this paper we have dealt with classical Boltzmann equations. However, a full treatment based on the quantum Boltzmann equations would be welcome to study in detail the transition from one- to the two-flavour state. A full quantum treatment usually introduces memory effects [19] leading to relaxation times which are longer than the one dictated by the thermalization rates of the particles in the plasma. In the quantum approach, particle number densities are replaced by Green functions. The latter are subject both to exponential decays and to an oscillatory behaviour which restrict the range of time integration for the scattering terms, thus leading to larger relaxation times and to a decrease of the wash-out rates. This might further help the flavour oscillations to efficiently project the lepton state on the flavour basis.

If the RH spectrum is quasi-degenerate, leptogenesis takes place through a resonance effect. In such a case the final baryon asymmetry does not depend any longer on the mass of the RH neutrinos. Therefore, M_1 may be chosen to well reproduce the full flavour regime without causing any suppression in the final baryon asymmetry.

Finally, let us comment about the upper bound on the neutrino mass from leptogenesis. In the one-flavour approximation there is a bound on the largest light neutrino mass $\overline{m}$ because the total CP asymmetry is bounded from above. The upper limit scales like $M_1/\overline{m}$ [20]. Therefore, larger values of $\overline{m}$ needs larger values of M_1 to explain the observed baryon asymmetry. However, M_1 may not be increased indefinitely, because at $M_1 \sim (\mathrm{eV}/\overline{m})^2\, 10^{10}$ GeV, $\Delta L = 2$ scatterings enter in thermal equilibrium and wipe out the asymmetry. This leads to the upper bound $\overline{m} \lesssim$ 0.15 eV. In flavour leptogenesis the bound on the individual CP asymmetries (11) scales like $\overline{m}$ and therefore it was concluded that no bound stringent exists on the largest light neutrino mass [9]. From these considerations it is clear that the bound on $\overline{m}$ depends very much on which regime leptogenesis is occuring, i.e either the one-flavor or the two-flavour regime. For large values of $\overline{m}$, the strong wash-out regime applies and, as we have seen in Sec. 4, the full flavour regime roughly (because our numerical results indicate that the bound is weaker) holds only for $M_1 \lesssim (K_{\alpha\alpha}/K)10^{12}$ GeV. Therefore, one would expect that, again, $\overline{m}$ cannot be large at will since K scales as $\overline{m}$. Indeed, at $\overline{m} \sim 2$ eV the full flavour regime would seem not to apply [17]. To get this estimate it is assumed that both flavours are in the strong wash-out regime, have roughly the same CP asymmetries, but that one of the two has a wash-out coefficient much smaller than the other, $1 \ll K_{\alpha\alpha} \ll K_{\beta\beta}$. Under these circumstances the final baryon asymmetry Y_B is dominated by the flavour α

$$Y_B \lesssim \frac{0.1}{g_* \, K_{\alpha\alpha}^{1.16}} \frac{3M_1 \overline{m}}{8\pi v^2} \sqrt{\frac{K_{\alpha\alpha}}{K}} \,, \quad M_1 \lesssim 10^{12} \, \frac{K_{\alpha\alpha}}{K} \, \mathrm{GeV} \tag{48}$$

where we have applied the upper bound (11) and remind the reader about the bound on M_1 for the full flavour regime to hold. Since the upper bound is inversely proportional to $K_{\alpha\alpha}$, the most favourable value for the wash-out factor of the flavour α in the strong wash-out regime is $K_{\alpha\alpha} \sim 3.3$. Therefore, the maximal baryon asymmetry would be

$$Y_B \simeq 0.1 \, \frac{(3.3)^{0.34}}{g_*} \frac{3 \, \overline{m}}{8\pi v^2} \, K^{-3/2} \, 10^{12} \, \mathrm{GeV} \,. \tag{49}$$

Setting $K \simeq (\overline{m}/0.5 \times 10^{-3} \, \mathrm{eV})$, we reproduce the statement that for $\overline{m} \gtrsim 2$ eV one is entering the one-flavour regime [17]. This conclusion would seem to indicate that a bound on the light neutrino mass $\overline{m}$ from leptogenesis might be present (even though not useful, given the conservative upper bound $\overline{m} \lesssim 2$ eV from cosmology [21]). We notice, however, that upper limit on M_1 to be in the two-flavour regime becomes weaker if all flavours have the same wash-out term. Assume that the total CP asymmetry ϵ_1 is very close to zero (for exactly degenerate light neutrino masses $\epsilon_1 = 0$ and $\epsilon_{\alpha\alpha} = -\epsilon_{\beta\beta}$). As before, all flavours are in the strong wash-out regime, but this time we suppose that $K_{\alpha\alpha} \simeq K_{\beta\beta}$ [9]. Under these circumstances the final baryon asymmetry reads

$$Y_B \simeq \frac{0.1}{g_*} \, \frac{222}{417} \, \frac{\epsilon_{\alpha\alpha}}{K_{\alpha\alpha}^{1.16}} \,, \quad M_1 \lesssim 10^{12} \, \frac{K_{\alpha\alpha}}{K} \, \mathrm{GeV} \,, \tag{50}$$

where the flavour α can be identified with the τ-flavour and we have applied the formulae in Ref. [11] which account for the connection among the asymmetries in the lepton doublets and the ones in the Δ_α charges. Taking $K_{\alpha\alpha}/K \simeq 1/2$, $K_{\alpha\alpha} \simeq (\overline{m}/0.5 \times 10^{-3} \, \mathrm{eV})$, and, for instance, $M_1 \sim 5 \times 10^{10}$ GeV (which is much larger than $10^{12}(3/K)$ GeV $\sim 10^9$ GeV), we are well in the full flavour regime. Using the condition (11), the following maximal value of the baryon asymmetry is achieved

$$Y_B \simeq 6 \left(\frac{\mathrm{eV}}{\overline{m}} \right)^{0.16} \times 10^{-11} \,, \tag{51}$$

It shows that, even for light neutrino masses in the few eV range, a large baryon asymmetry is generated. We therefore conclude that the bound on the largest of light neutrino mass is evaded in flavour leptogenesis.

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References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[2] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov. Phys. Lett. B **155** 36 (1985).

[3] P. Minkowski, Phys. Lett. B **67** (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop*, New York 1979, eds. P. van Nieuwenhuizen and D. Freedman; T. Yanagida, *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan 1979, eds. A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, Phys.Rev.Lett. **44** (1980) 912.

[4] See, e.g. G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B **685**, 89 (2004) [arXiv:hep-ph/0310123].

[5] W. Buchmuller, P. Di Bari and M. Plumacher, Annals Phys. **315** (2005) 305 [arXiv:hep-ph/0401240].

[6] A partial list: W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B **643** (2002) 367 [arXiv:hep-ph/0205349]; J. R. Ellis, M. Raidal and T. Yanagida, Phys. Lett. B **546** (2002) 228 [arXiv:hep-ph/0206300]; G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B **640** (2002) 202 [arXiv:hep-ph/0202030]; R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **615** (2005) 231 [arXiv:hep-ph/0502026]; A. Broncano, M. B. Gavela and E. Jenkins, Nucl. Phys. B **672** (2003) 163 [arXiv:hep-ph/0307058]; A. Pilaftsis, Phys. Rev. D **56** (1997) 5431 [arXiv:hep-ph/9707235]; E. Nezri and J. Orloff, JHEP **0304** (2003) 020 [arXiv:hep-ph/0004227]; S. Davidson and A. Ibarra, Nucl. Phys. B **648**, 345 (2003) [arXiv:hep-ph/0206304]; S. Davidson, JHEP **0303** (2003) 037 [arXiv:hep-ph/0302075]; S. T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, Nucl. Phys. B **739** (2006) 208 [arXiv:hep-ph/0510404].

[7] A. D. Sakharov. *JETP Lett.*, 5:24, 1967. For a review, see A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49**, 35 (1999) [arXiv:hep-ph/9901362].

[8] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B **575** (2000) 61 [arXiv:hep-ph/9911315]; T. Endoh, T. Morozumi and Z.h. Xiong, Progr. Theor. Phys. **111**, 123 (2004).

[9] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP **0604** (2006) 004 [arXiv:hep-ph/0601083].

[10] E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP **0601**, 164 (2006) [arXiv:hep-ph/0601084].

[11] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, JHEP **0609**, 010 (2006) [arXiv:hep-ph/0605281].

[12] S. Antusch, S. F. King and A. Riotto, arXiv:hep-ph/0609038.

[13] S. Pascoli, S. T. Petcov and A. Riotto, arXiv:hep-ph/0609125; G.C. Branco, R.G. Felipe and F.R. Joaquim, arXiv:hep-ph/0609297; S. Antusch and A. M. Teixeira, arXiv:hep-ph/0611232; S. Pascoli, S. T. Petcov and A. Riotto, arXiv:hep-ph/0611338.

[14] N. F. Bell, R. F. Sawyer and R. R. Volkas, Phys. Lett. B **500**, 16 (2001) [arXiv:hep-ph/0011068].

[15] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169 [arXiv:hep-ph/9605319].

[16] J. M. Cline, K. Kainulainen and K. A. Olive, Phys. Rev. D **49** (1994) 6394 [arXiv:hep-ph/9401208]; P. Elmfors, K. Enqvist, A. Riotto and I. Vilja, Phys. Lett. B **452**, 279 (1999) [arXiv:hep-ph/9909529];

[17] S. Blanchet, P. Di Bari, G. G. Raffelt, arXiv:hep-ph/0611337.

[18] A. Strumia, arXiv:hep-ph/0608347.

[19] See A. Riotto, Phys. Rev. D **53**, 5834 (1996) [arXiv:hep-ph/9510271]; A. Riotto, Nucl. Phys. B **518**, 339 (1998) [arXiv:hep-ph/9712221].

[20] S. Davidson and A. Ibarra, Phys. Lett. B **535**, 25 (2002) [arXiv:hep-ph/0202239].

[21] C. Zunckel and P. G. Ferreira, arXiv:astro-ph/0610597.