Leptogenesis and Low Energy CP Violation in Neutrino Physics

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Abstract

Taking into account the recent progress in the understanding of the lepton flavour effects in leptogenesis, we investigate in detail the possibility that the CP-violation necessary for the generation of the baryon asymmetry of the Universe is due exclusively to the Dirac and/or Majorana CP-violating phases in the PMNS neutrino mixing matrix U, and thus is directly related to the low energy CP-violation in the lepton sector (e.g., in neutrino oscillations, etc.). We first derive the conditions of CP-invariance of the neutrino Yukawa couplings λ in the see-saw Lagrangian, and of the complex orthogonal matrix R in the "orthogonal" parametrisation of λ. We show, e.g. that under certain conditions i) real R and specific CP-conserving values of the Majorana and Dirac phases can imply CP-violation, and ii) purely imaginary R does not necessarily imply breaking of CP-symmetry. We study in detail the case of hierarchical heavy Majorana neutrino mass spectrum, presenting results for three possible types of light neutrino mass spectrum: i) normal hierarchical, ii) inverted hierarchical, and iii) quasi-degenerate. Results in the alternative case of quasi-degenerate in mass heavy Majorana neutrinos, are also derived. The minimal supersymmetric extension of the Standard Theory with right-handed Majorana neutrinos and see-saw mechanism of neutrino mass generation is discussed as well. We illustrate the possible correlations between the baryon asymmetry of the Universe and i) the rephasing invariant $J_{\rm CP}$ controlling the magnitude of CP-violation in neutrino oscillations, or ii) the effective Majorana mass in neutrinoless double beta decay, in the cases when the only source of CP-violation is respectively the Dirac or the Majorana phases in the neutrino mixing matrix.

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1 Introduction

Baryogenesis through Leptogenesis [1] is a simple mechanism to explain the observed baryon asymmetry of the Universe. A lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to $(B + L)$-violating sphaleron interactions [2] which exist within the Standard Model (SM). A simple scheme in which this mechanism can be implemented is the "seesaw"(type I) model of neutrino mass generation [3]. In its minimal version it includes the Standard Model (SM) plus two or three right-handed (RH) heavy Majorana neutrinos. Thermal leptogenesis [4, 5, 6] can take place, for instance, in the case of hierarchical spectrum of the heavy RH Majorana neutrinos. The lightest of the RH Majorana neutrinos is produced by thermal scattering after inflation. It subsequently decays out-of-equilibrium in a lepton number and CP-violating way, thus satisfying Sakharov's conditions [7]. In grand unified theories (GUT) the masses of the heavy RH Majorana neutrinos are typically by a few to several orders of magnitude smaller than the scale of unification of the electroweak and strong interactions, $M_{\mathrm{GUT}} \cong 2 \times 10^{16}$ GeV. This range coincides with the range of values of the heavy Majorana neutrino masses, required for a successful thermal leptogenesis.

Compelling evidence for existence non-zero neutrino masses and non-trivial neutrino mixing have been obtained during the last several years in the experiments studying oscillations of solar, atmospheric, reactor and accelerator neutrinos [8, 9, 10, 11, 12]. The currently existing data imply the presence of 3-neutrino mixing in the weak charged-lepton current (see, e.g., [13]):

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj} \, \nu_{j\mathrm{L}}, \quad l = e, \mu, \tau, \tag{1}$$

where ν_{lL} are the flavour neutrino fields, $\nu_{j\mathrm{L}}$ is the field of neutrino ν_j having a mass m_j and U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [14]. The existing data, including the data from the ^{3}H β-decay experiments [15], show that the massive neutrinos ν_j are significantly lighter than the charged leptons and quarks [2]. The see-saw mechanism of neutrino mass generation [3] provides a natural explanation of the smallness of neutrino masses: integrating out the heavy RH Majorana neutrinos generates a mass term of Majorana type for the left-handed flavour neutrinos, which is inversely proportional to the large mass of the RH ones.

Establishing a connection between the low energy neutrino mixing parameters and high energy leptogenesis parameters has received much attention in recent years [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. These studies showed, in particular, that the number of phenomenological parameters of the seesaw mechanism is significantly larger than the number of quantities measurable in the "low energy" neutrino experiments.

In the present article we investigate the link between the high energy CP-violation responsible for the generation of baryon asymmetry through leptogenesis and the leptonic

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[2]The data from the ^{3}H β-decay experiments [15] imply $m_j < 2.3$ eV (95%C.L.). More stringent upper limit on m_j follows from the constraints on the sum of neutrino masses obtained from cosmological/astrophysical observations, namely, from the CMB data of the WMAP experiment [16] combined with data from large scale structure surveys (2dFGRS, SDSS) (see, e.g., [17]).

CP violation at low energy, which can manifest itself in non-zero CP-violating asymmetry in neutrino oscillations and, in an indirect way, in the effective Majorana mass in neutrinoless double beta $((\beta\beta)_{0\nu}\text{-})$ decay, $|\langle m \rangle|$ (see, e.g., [30, 31]).

It was concluded in a large number of studies of leptogenesis performed in the so-called "one-flavor approximation", that no direct link exists between the leptogenesis CP-violating parameters and the CP-violating Dirac and Majorana phases in the PMNS neutrino mixing matrix, measurable at low energies. In particular, an observation of leptonic low energy CP-violating phases would not automatically imply a non-vanishing baryon asymmetry through leptogenesis in the one-flavour case. This conclusion, however, does not universally hold [32, 33, 34]. Moreover, as was shown in [35] (see also [36]) and will be discussed in detail in the present article, the low-energy Dirac and/or Majorana CP-violating phases in U, which enter into the expressions respectively of the leptonic CP-violating rephasing invariant $J_{\rm CP}$ [37], controlling the magnitude of the CP-violation effects in neutrino oscillations, and of the effective Majorana mass $|\langle m \rangle|$ [38, 31, 39] can be the CP-violating parameters responsible for the generation of the baryon asymmetry of the Universe. Consequently, the leptogenesis mechanism can be maximally connected to the low energy CP-violating phases in U: within the scenario under discussion, the observation of CP-violation in the lepton (neutrino) sector would generically ensure the existence of a baryon asymmetry.

The possibility of a direct connection between the high-energy leptogenesis and the low-energy leptonic CP-violation is based on the fact that a new ingredient has been recently accounted for in the leptogenesis mechanism, namely, the lepton flavour effects [32, 33, 34]. As we have indicated above, the dynamics of leptogenesis was usually addressed within the 'one-flavour' approximation. In the latter, the Boltzmann equations are written for the abundance of the lightest RH Majorana neutrino, N_1, responsible for the out of equilibrium and CP-asymmetric decays, and for the *total lepton charge* asymmetry. However, this approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. Supposing that leptogenesis takes place at temperatures $T \sim M_1$, where M_1 is the mass of N_1, the 'one-flavour' approximation holds only for $T \sim M_1 \gtrsim 10^{12}$ GeV. For $M_1 \gtrsim 10^{12}$ GeV, i.e., at temperatures higher than 10^{12} GeV, all lepton flavours are indistinguishable - there is no notion of flavour. The lepton asymmetry generated in N_1 decays is effectively "stored" in one lepton flavour. However, for $T \sim M_1 \sim 10^{12}$ GeV, the interactions mediated by the τ-lepton Yukawa couplings come into equilibrium, followed by those mediated by the muon Yukawa couplings at $T \sim M_1 \sim 10^9$ GeV, and the notion of lepton flavour becomes physical.

The impact of flavour in thermal leptogenesis has been recently investigated in detail in [40, 32, 33, 34] and in [29, 41] including the quantum oscillations/correlations of the asymmetries in lepton flavour space [32]. The Boltzmann equations describing the asymmetries in flavour space have additional terms which can significantly affect the result for the final baryon asymmetry. The ultimate reason is that realistic leptogenesis is a dynamical process, involving the production and destruction of the heavy RH Majorana neutrinos, and of a lepton asymmetry that is distributed among *distinguishable* lepton flavours. Contrary to what is generically assumed in the one-single flavour approximation, the $\Delta L = 1$ inverse decay processes which wash-out the net lepton number are flavour dependent, that

is the lepton asymmetry carried by, say, electrons can be washed out only by the inverse decays involving the electron flavour. The asymmetries in each lepton flavour, are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of wash-out in the one-flavour approximation, where the flavours are taken indistinguishable, thus obtaining the unphysical result that, e.g., an asymmetry stored in the electron lepton charge may be washed out by inverse decays involving the muon or the tau charges.

When flavour effects are accounted for, the final value of the baryon asymmetry is the sum of three contributions. Each term is given by the CP asymmetry in a given lepton flavour l, properly weighted by a wash-out factor induced by the same lepton number violating processes. The wash-out factors are also flavour dependent. In the present article we perform a detailed analysis of the indicated flavour effects in leptogenesis. We show that the low energy Dirac and/or MajoranaCP-violating phases in U can be responsible for the generation of the baryon asymmetry of the Universe. We study also in detail the possible correlations between the physical low energy observables which depend on the CP-violating phases in U_{PMNS} - the rephasing invariant J_{CP} and the effective Majorana mass in neutrinoless double beta decay $|\langle m \rangle|$, and the baryon asymmetry.

The paper is organized as follows. In Section 2 we briefly review the existing data on the neutrino mixing parameters and the phenomenology of the low energy Dirac and Majorana CP-violation in the lepton sector. We note, in particular, that searching for CP-violation effects in ν- oscillations is the only practical way to get information about the CP-violation due to the Dirac phase in the neutrino mixing matrix U (Dirac CP-violation), and that the only feasible experiments which at present have the potential of establishing the Majorana nature of light neutrinos ν_j and of providing information on the Majorana CPV phases in U are the the neutrinoless double beta decay experiments searching for the process $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$. In Sections 3-7 we present a detailed discussion of the possible connection between the CP-violation at high energy in leptogenesis and the CP-violation in the lepton sector at low energies. We first derive the conditions of CP-invariance of the see-saw Lagrangian, i.e. of the neutrino Yukawa couplings, taking into account that the light and heavy neutrinos with definite mass are Majorana particles, and thus have definite CP-parities in the case of exact CP-symmetry (Section 3). We review briefly the arguments, based on the "one-flavour" approximation, leading to the conclusion that the connection between leptogenesis CP-violating parameters and the CP-violating phases in the PMNS matrix generically does not hold. We next discuss the conditions under which the lepton flavour effects in leptogenesis become important and present a brief summary of the results obtained recently on these effects, which are used in our analysis (Section 4). In Sections 5, 6 and 7 we investigate the possibility that the CP-violation necessary for the generation of the baryon asymmetry of the Universe is due exclusively to the Dirac and/or Majorana CP-violating phases in the PMNS matrix, and thus is directly related to the low energy CP-violation in the lepton sector (e.g., in neutrino oscillations, etc.). The case of hierarchical heavy Majorana neutrino mass spectrum is studied in detail in Section 5, where we present results for three possible types of light neutrino mass spectrum: i) normal hierarchical, ii) inverted hierarchical, and iii) quasi-degenerate. In Section 6 results

in the alternative case of quasi-degenerate in mass heavy Majorana neutrinos are derived, while in Section 7 we discuss how the results on leptogenesis in Section 4 - 6 will be modified in the minimal supersymmetric extension of the Standard Theory with right-handed Majorana neutrinos and see-saw mechanism of neutrino mass generation. Section 8 represents Conclusions.

2 Neutrino Mixing Parameters and Low Energy Dirac and Majorana CP-Violation

We will use the standard parametrisation of the PMNS matrix:

$$
U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) \quad (2)
$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\theta_{ij} = [0, \pi/2]$, $\delta = [0, 2\pi]$ is the Dirac CP-violating (CPV) phase and and α_{21} and α_{31} two Majorana CPV phases [42, 43]. We standardly identify $\Delta m_\odot^2 = \Delta m_{21}^2 \equiv m_2^2 - m_1^2 > 0$, where $\Delta m_\odot^2$ drives the solar neutrino oscillations. In this case $|\Delta m_A^2| = |\Delta m_{31}^2| \cong |\Delta m_{32}^2|$, $\theta_{23} = \theta_A$ and $\theta_{12} = \theta_\odot$, $|\Delta m_A^2|$, θ_A and $\theta_\odot$ being the ν-mass squared difference and mixing angles responsible respectively for atmospheric and solar neutrino oscillations, while θ_{13} is the CHOOZ angle [44]. The existing neutrino oscillation data, including the recent results of the MINOS experiment [45], allow us to determine $\Delta m_\odot^2$, $|\Delta m_A^2|$, $\sin^2\theta_{12}$ and $\sin^2 2\theta_{23}$ with a relatively good precision and to obtain rather stringent limits on $\sin^2\theta_{13}$ (see, e.g., [46, 47, 48]). The best fit values and the 95% C.L. allowed ranges of $\Delta m_\odot^2$, $\sin^2\theta_{12}$ $|\Delta m_A^2|$ and $\sin^2 2\theta_{23}$ read:

$$(|\Delta m_A^2|)_{\mathrm{BF}} = 2.5 \times 10^{-3} \ \mathrm{eV}^2, \quad 2.1 \times 10^{-3} \ \mathrm{eV}^2 \le |\Delta m_A^2| \le 2.9 \times 10^{-3} \ \mathrm{eV}^2, \quad (3)$$

$$(\Delta m_\odot^2)_{\mathrm{BF}} = 8.0 \times 10^{-5} \ \mathrm{eV}^2, \quad 7.3 \times 10^{-5} \ \mathrm{eV}^2 \le \Delta m_\odot^2 \le 8.5 \times 10^{-5} \ \mathrm{eV}^2, \quad (4)$$

$$(\sin^2\theta_{12})_{\mathrm{BF}} = 0.31, \quad 0.26 \le \sin^2\theta_{12} \le 0.36 \quad (5)$$

$$(\sin^2 2\theta_{23})_{\mathrm{BF}} = 1, \quad \sin^2 2\theta_{23} \ge 0.90. \quad (6)$$

A combined 3-ν oscillation analysis of the global neutrino oscillation data gives [46, 47]

$$\sin^2\theta_{13} < 0.027 \ (0.041) \quad \mathrm{at} \quad 95\% \ (99.73\%) \ \mathrm{C.L.} \quad (7)$$

The neutrino oscillation parameters discussed above can (and very likely will) be measured with much higher accuracy in the future (see, e.g., [13]).

Depending on the sign of $\Delta m_A^2 \equiv \Delta m_{31}^2 \cong \Delta m_{32}^2$ which cannot be determined from presently existing data, the ν-mass spectrum can be of two types:
– *with normal ordering* $m_1 < m_2 < m_3$, $\Delta m_A^2 = \Delta m_{31}^2 > 0$, and
– *with inverted ordering* $m_3 < m_1 < m_2$, $\Delta m_A^2 = \Delta m_{32}^2 < 0$, $m_j > 0$,
where we have employed the standardly used convention to define the two spectra. Depending on the sign of Δm_A^2, $\mathrm{sgn}(\Delta m_A^2)$, and on the value of the lightest neutrino mass

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(i.e., absolute neutrino mass scale), $\min(m_j)$, the ν-mass spectrum can be

– *Normal Hierarchical*: $m_1 \ll m_2 \ll m_3$, $m_2 \cong (\Delta m_\odot^2)^{\frac{1}{2}} \sim 0.009$ eV, $m_3 \cong |\Delta m_A^2|^{\frac{1}{2}} \sim 0.05$ eV;

– *Inverted Hierarchical*: $m_3 \ll m_1 < m_2$, with $m_{1,2} \cong |\Delta m_A^2|^{\frac{1}{2}} \sim 0.05$ eV;

– *Quasi-Degenerate*: $m_1 \cong m_2 \cong m_3 \cong m$, $m_j^2 \gg |\Delta m_A^2|$, $m \gtrsim 0.10$ eV.

Determining the nature of massive neutrinos, obtaining information on the type of ν-mass spectrum, absolute ν-mass scale and on the status of CP-symmetry in the lepton sector are among the fundamental problems in the studies of neutrino mixing [13].

2.1 CP and T Violation in Neutrino Oscillations

Searching for CP-violation effects in ν- oscillations is the only practical way to get information about Dirac CP-violation in the lepton sector, associated with the phase δ in U. A measure of CP- and T- violation is provided by the asymmetries [49, 42, 50, 37]:

$$A_{\mathrm{CP}}^{(l,l')} = P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'}), \quad A_{\mathrm{T}}^{(l,l')} = P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l), \quad l \neq l' = e, \mu, \tau. \tag{8}$$

For 3-ν oscillations in vacuum, which respect the CPT-symmetry, one has [37]:

$$A_{\mathrm{T}}^{(e,\mu)} = A_{\mathrm{T}}^{(\mu,\tau)} = -A_{\mathrm{T}}^{(e,\tau)} = J_{\mathrm{CP}} \, F_{\mathrm{osc}}^{\mathrm{vac}}, \quad A_{\mathrm{CP}}^{(l,l')} = A_{\mathrm{T}}^{(l,l')}, \tag{9}$$

$$J_{\mathrm{CP}} = \mathrm{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{4} \sin 2\theta_{12} \sin 2\theta_{23} \cos^2 \theta_{13} \sin \theta_{13} \sin \delta, \tag{10}$$

$$F_{\mathrm{osc}}^{\mathrm{vac}} = \sin \left(\frac{\Delta m_{21}^2}{2E} L \right) + \sin \left(\frac{\Delta m_{32}^2}{2E} L \right) + \sin \left(\frac{\Delta m_{13}^2}{2E} L \right). \tag{11}$$

Thus, the magnitude of CP-violation effects in neutrino oscillations is controlled by the rephasing invariant associated with the Dirac phase δ, J_{CP} [37]. The existence of Dirac CP-violation in the lepton sector would be established if, e.g., some of the vacuum oscillation asymmetries $A_{\mathrm{CP(T)}}^{(e,\mu)}$, etc. are proven experimentally to be nonzero. This would imply that $J_{\mathrm{CP}} \neq 0$, and, consequently, that $\sin \theta_{13} \sin \delta \neq 0$ [3]. One of the major goals of the future experimental studies of neutrino oscillations is the searches for CP-violation effects due to the Dirac phase in U (see, e.g., [13, 52]).

2.2 Majorana CP-Violating Phases and $(\beta\beta)_{0\nu}$-Decay

As is well-known, the theories with see-saw mechanism of neutrino mass generation [3] of interest for our discussion, predict the massive neutrinos ν_j to be Majorana particles. We will assume in what follows that the fields $\nu_j(x)$ satisfy the Majorana condition:

$$C(\bar{\nu}_j)^T = \nu_j, \; j = 1, 2, 3, \tag{12}$$

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[3] Let us note that the oscillations in matter, e.g., in the Earth, are neither CP- nor CPT- invariant [51] as a consequence of the fact that the Earth matter is not charge-symmetric (it contains e^-, p and n, but does not contain their antiparticles). This complicates the studies of CP-violation due to the Dirac phase δ in ν-oscillations in matter (Earth) (see, e.g., [52]). The matter effects in ν-oscillations in the Earth to a good precision are not T-violating [37], however. In matter with constant density, e.g., Earth mantle, one has [37]: $A_{\mathrm{T}}^{(e,\mu)} = J_{\mathrm{CP}}^{\mathrm{m}} F_{\mathrm{osc}}^{\mathrm{m}}$, $J_{\mathrm{CP}}^{\mathrm{m}} = J_{\mathrm{CP}} \, R_{\mathrm{CP}}$, where the function R_{CP} does not depend on θ_{23} and δ.

where C is the charge conjugation matrix: $C^{-1}\gamma_\mu C = -\gamma_\mu^T$, $C^T = -C$, $C^\dagger = C^{-1}$.

Determining the nature of massive neutrinos is one of the most formidable and pressing problems in today's neutrino physics (see, e.g., [13, 53]). If ν_j are proven to be Majorana fermions, getting experimental information about the Majorana CPV phases in U, α_{21} and α_{31}, would be a remarkably difficult problem. The oscillations of flavour neutrinos, $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$, $l, l' = e, \mu, \tau$, are insensitive to the phases $\alpha_{21,31}$ [42, 51]. The Majorana phases of interest $\alpha_{21,31}$ can affect significantly the predictions for the rates of (LFV) decays $\mu \to e + \gamma$, $\tau \to \mu + \gamma$, etc. in a large class of supersymmetric theories with see-saw mechanism of ν-mass generation (see, e.g., [54]).

In the case of 3-ν mixing under discussion there are, in principle, three independent CP violation rephasing invariants. The first is $J_{\rm CP}$ - the Dirac one, associated with the Dirac phase δ, we have discussed in the preceding subsection. The existence of two additional invariants, S_1 and S_2 is related to the two Majorana CP violation phases in U. The invariants S_1 and S_2 can be chosen as [4] [55, 56, 31]:

$$S_1 = {\rm Im}\left\{U_{\tau 1}^* U_{\tau 2}\right\}\ ,\qquad S_2 = {\rm Im}\left\{U_{\tau 2}^* U_{\tau 3}\right\}\ . \tag{13}$$

The rephasing invariants associated with the Majorana phases are not uniquely determined. Instead of S_1 defined above we could have chosen, e.g., $S_1' = {\rm Im}\left\{U_{e1}U_{e2}^*\right\}$ or $S_1'' = {\rm Im}\left\{U_{\mu 1}U_{\mu 2}^*\right\}$, while instead of S_2 we could have used $S_2' = {\rm Im}\left\{U_{e2} U_{e3}^*\right\}$, or $S_2'' = {\rm Im}\left\{U_{\mu 2} U_{\mu 3}^*\right\}$. The Majorana phases α_{21} and α_{31}, or α_{21} and $\alpha_{32} \equiv (\alpha_{31} - \alpha_{21})$, can be expressed in terms of the rephasing invariants thus introduced [31]: $\cos\alpha_{21} = 1 - (S_1')^2/(|U_{e1}U_{e2}|^2)$, etc. The expression for $\cos\alpha_{21}$ in terms of S_1 is somewhat more cumbersome (it involves also $J_{\rm CP}$) and we will not give it here. Note that CP-violation due to the Majorana phase α_{21} requires that both $S_1 = {\rm Im}\left\{U_{\tau 1}U_{\tau 2}^*\right\} \neq 0$ and ${\rm Re}\left\{U_{\tau 1}U_{\tau 2}^*\right\} \neq 0$. Similarly, $S_2 = {\rm Im}\left\{U_{\tau 2}^* U_{\tau 3}\right\} \neq 0$ would imply violation of the CP-symmetry only if in addition ${\rm Re}\left\{U_{\tau 2}^* U_{\tau 3}\right\} \neq 0$.

The only feasible experiments which at present have the potential of establishing the Majorana nature of light neutrinos ν_j and of providing information on the Majorana CPV phases in U are the experiments searching for the neutrinoless double beta $((\beta\beta)_{0\nu}\text{-})$ decay, $(A, Z) \to (A, Z+2) + e^- + e^-$ (see, e.g., [30, 53, 39]). The $(\beta\beta)_{0\nu}$-decay effective Majorana mass, $|\langle m \rangle|$ (see, e.g., [30]), which contains all the dependence of the $(\beta\beta)_{0\nu}$-decay amplitude on the neutrino mixing parameters, is given by the following expressions for the normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD) neutrino mass spectra (see, e.g., [39]):

$$|\langle m \rangle| \;\cong\; \left|\sqrt{\Delta m_\odot^2}\,\sin^2\theta_{12}e^{i\alpha_{21}} + \sqrt{\Delta m_{\rm A}^2}\,\sin^2\theta_{13}e^{i(\alpha_{31}-2\delta)}\right|\ ,\ m_1 \ll m_2 \ll m_3\ {\rm (NH)}, \tag{14}$$

$$|\langle m \rangle| \;\cong\; \sqrt{|\Delta m_{\rm A}^2|}\,\left|\cos^2\theta_{12} + e^{i\alpha_{21}}\,\sin^2\theta_{12}\right|\ ,\quad m_3 \ll m_1 < m_2\ {\rm (IH)}, \tag{15}$$

$$|\langle m \rangle| \;\cong\; m\,\left|\cos^2\theta_{12} + e^{i\alpha_{21}}\,\sin^2\theta_{12}\right|\ ,\quad m_{1,2,3} \cong m \gtrsim 0.10\ {\rm eV}\ {\rm (QD)}. \tag{16}$$

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[4]The expressions for the invariants $S_{1,2}$ we give and will use further correspond to the Majorana conditions (12) for the fields of neutrinos ν_j, which do not contain phase factors, see, e.g., [31].

Obviously, $|\langle m \rangle|$ depends strongly on the Majorana CPV phase(s): the CP-conserving values of $\alpha_{21} = 0, \pm\pi$ [57], for instance, determine the range of possible values of $|\langle m \rangle|$ in the cases of IH and QD spectrum, while the CP-conserving values of $(\alpha_{31} - \alpha_{21}) \equiv \alpha_{32} = 0, \pm\pi$, can be important in the case of NH spectrum. As is well-known, in the case of CP-invariance the phase factors

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1 \,, \ \ \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1 \,, \ \ \eta_{32} \equiv e^{i\alpha_{32}} = \pm 1 \,, \tag{17}$$

have a very simple physical interpretation [57, 30]: η_{ik} is the relative CP-parity of Majorana neutrinos ν_i and ν_k, $\eta_{ik} = (\eta_i^{\nu\mathrm{CP}})^* \eta_k^{\nu\mathrm{CP}}$, $\eta_{i(k)}^{\nu\mathrm{CP}} \pm i$ being the CP-parity of $\nu_{i(k)}$.

As can be shown, in the general case of arbitrary $\min(m_j)$, $|\langle m \rangle|$ depends on the two invariants S_1 and S_2 [31]. In the chosen parametrisation of the PMNS matrix, Eq. (2), $|\langle m \rangle|$ depends also on J_{CP}.

The $(\beta\beta)_{0\nu}$-decay experiments of the next generation, which are under preparation at present (see, e.g., [53]), are aiming to probe the QD and IH ranges of $|\langle m \rangle|$. If the $(\beta\beta)_{0\nu}$-decay will be observed in these experiments, the measurement of the $(\beta\beta)_{0\nu}$-decay half-life might allow to obtain constraints on the Majorana phase α_{21} [31, 38, 58].

3 The CP-Invariance Constraints

In the next Section, we will summarize the arguments leading to the conclusions that the leptogenesis CPV phases, responsible for the generation of the baryon asymmetry, can indeed be directly connected to the low energy CPV phases in U and, correspondingly, to CP violating phenomena, e.g., in neutrino physics. The starting point of our discussion is the Lagrangian of the Standard Model (SM) with the addition of three heavy right-handed Majorana neutrinos N_i $(i = 1, 2, 3)$ with masses $M_3 > M_2 > M_1 > 0$ and Yukawa couplings λ_{il}. It will be assumed (without loss of generality) that the fields N_j satisfy the Majorana condition:

$$C(\overline{N}_j)^T = N_j, \ \ j = 1, 2, 3. \tag{18}$$

We will work in the basis in which the Yukawa couplings for the charged leptons are flavour-diagonal. In this basis the leptonic part of the Lagrangian of interest reads:

$$\mathcal{L}^{\mathrm{lep}}(x) = \mathcal{L}_{\mathrm{CC}}(x) + \mathcal{L}_{\mathrm{Y}}(x) + \mathcal{L}_{\mathrm{M}}^{\mathrm{N}}(x) \,, \tag{19}$$

where $\mathcal{L}_{\mathrm{CC}}(x)$ and $\mathcal{L}_{\mathrm{Y}}(x)$ are the charged current (CC) weak interaction and Yukawa coupling Lagrangians, while $\mathcal{L}_{\mathrm{M}}^{\mathrm{N}}(x)$ is the mass term of the heavy Majorana neutrinos N_i:

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \overline{l}_L(x) \gamma_\alpha \nu_{lL}(x) W^{\alpha\dagger}(x) + \mathrm{h.c.} \,, \tag{20}$$

$$\mathcal{L}_{\mathrm{Y}}(x) = \lambda_{il} \overline{N_{iR}}(x) H^\dagger(x) \psi_{lL}(x) + h_l H^c(x) \overline{l_R}(x) \psi_{lL}(x) + \mathrm{h.c.} \,, \tag{21}$$

$$\mathcal{L}_{\mathrm{M}}^{\mathrm{N}}(x) = -\frac{1}{2} M_i \overline{N_i}(x) N_i(x) \,. \tag{22}$$

Here ψ_{lL} and l_R denote respectively the left-handed (LH) lepton doublet and right-handed lepton singlet fields of flavour $l = e, \mu, \tau$, $\psi_{lL}^{\mathrm{T}} = (\nu_{lL} \ l_L)$, $W^\alpha(x)$ is the $W^\pm$-boson field,

and H is the Higgs doublet field whose neutral component has a vacuum expectation value equal to $v = 174$ GeV. Obviously, $\mathcal{L}_Y(x) + \mathcal{L}_M^N(x)$ includes all the necessary ingredients of the see-saw mechanism. At energies below the heavy Majorana neutrino mass scale M_1, the heavy Majorana neutrino fields are integrated out and after the breaking of the electroweak symmetry, a Majorana mass term for the LH flavour neutrinos is generated:

$$m_\nu = v^2 \lambda^T \, M^{-1} \, \lambda = U^* \, m \, U^\dagger \,, \tag{23}$$

where M and m are diagonal matrices formed by the masses of N_j and ν_j, $M \equiv \text{Diag}(M_1, M_2, M_3)$, $m \equiv \text{Diag}(m_1, m_2, m_3)$, $M_j > 0$, $m_k \geq 0$, and U is the PMNS matrix. The diagonalisation of the Majorana mass matrix m_ν, Eq. (23), leads to the relation (1) between the LH flavour neutrino fields ν_{lL} and the fields ν_{jL} of neutrinos with definite mass and, correspondingly, to the appearance of the PMNS neutrino mixing matrix in the charged current weak interaction Lagrangian $\mathcal{L}_{CC}(x)$.

In what follows we will often use the well-known "orthogonal parametrisation" of the matrix of neutrino Yukawa couplings [59]:

$$\lambda = \frac{1}{v} \, \sqrt{M} \, R \, \sqrt{m} \, U^\dagger \,, \tag{24}$$

where R is, in general, a complex orthogonal matrix, $R \, R^T = R^T \, R = \mathbf{1}$.

Before discussing leptogenesis and the violation of CP-symmetry associated with it, it proves useful to analyze the constraints which the requirement CP-invariance imposes on the Yukawa couplings λ_{jl}, on the PMNS matrix U and on the matrix R. If the CP-symmetry is unbroken, the Majorana fields N_j and ν_k have definite CP-parities (see, e.g., [30, 57]) $\eta_j^{NCP} = \pm i$, $\eta_k^{\nu CP} = \pm i$, and transform in the following way under the CP-symmetry transformation:

$$U_{CP} \, N_j(x) \, U_{CP}^\dagger \;=\; \eta_j^{NCP} \, \gamma_0 \, N_j(x') \,, \quad \eta_j^{NCP} = i\rho_j^N = \pm i \,, \tag{25}$$

$$U_{CP} \, \nu_k(x) \, U_{CP}^\dagger \;=\; \eta_k^{\nu CP} \, \gamma_0 \, \nu_k(x') \,, \quad \eta_k^{\nu CP} = i\rho_k^\nu = \pm i \,. \tag{26}$$

The sign factors ρ_j^N and ρ_k^ν, and correspondingly, the CP-parities of the heavy and light Majorana neutrinos N_j and ν_k, are determined by the properties of the corresponding RH and LH flavour neutrino Majorana mass matrices [30]. They will be treated as free discrete parameters in what follows.

Obviously, the mass term $\mathcal{L}_M^N(x)$, Eq. (22), is CP-invariant. The requirement of CP-invariance of the Lagrangian $\mathcal{L}_Y(x)$, Eq. (21), leads, as can be shown, to the following constraint on the neutrino Yukawa couplings:

$$\lambda_{jl}^* = \lambda_{jl} \, (\eta_j^{NCP})^* \, \eta^l \, \eta^{H*} \,, \quad j = 1, 2, 3, \ l = e, \mu, \tau, \tag{27}$$

where η^l and η^H are unphysical phase factors which appear in the CP-transformations of the LH lepton doublet and Higgs doublet fields, $\psi_{lL}(x)$ and $H(x)$, respectively. These phase factors do not affect any of the physical observables and, for convenience, we will set them to i and 1: $\eta^l = i$, $\eta^H = 1$. Thus, in what follows we will work with the constraint (27) in the form

$$\lambda_{jl}^* = \lambda_{jl} \, \rho_j^N \,, \quad \rho_j^N = \pm 1 \,, \quad j = 1, 2, 3, \ l = e, \mu, \tau, \tag{28}$$

9

where we have used Eq. (25). Note that if the CP-parity of a given heavy Majorana neutrino N_j is equal to $(-i)$, i.e., if $\rho_j^N = -1$, the elements λ_{jl}, $l = e, \mu, \tau$, of the matrix of neutrino Yukawa couplings λ will be *purely imaginary*.

It follows from Eqs. (23) and (28) that if CP-invariance holds, the Majorana mass matrix of the LH flavour neutrinos generated by the see-saw mechanism is real (in the convention for the various unphysical phase factors employed by us): $m_{\nu*} = m_\nu$. This leads to the following CP-invariance constraint on the PMNS matrix U [30]:

$$U_{lj}^* = U_{lj}\, \rho_j^\nu,, \quad j = 1, 2, 3, \ l = e, \mu, \tau\,. \tag{29}$$

In the parametrisation (2) we are using these conditions imply that the Dirac phase $\delta = \pi q$, $q = 0, 1, 2, ...$, and that the Majorana phases should satisfy: $\alpha_{21} = \pi q'$, $\alpha_{31} = \pi q''$, $q', q'' = 0, 1, 2, ...$

Using Eqs. (23), (28) and (29) it is easy to derive the constraints on the matrix R following (in the convention we are using) from the requirement of CP-invariance of the "high-energy" Lagrangian of interest (19):

$$R_{jk}^* = R_{jk}\, \rho_j^N\, \rho_k^\nu, \quad j, k = 1, 2, 3\,. \tag{30}$$

Obviously, this would be a condition of reality of the matrix R only if $\rho_j^N \rho_k^\nu = 1$ for any $j, k = 1, 2, 3$. However, we can also have $\rho_j^N \rho_k^\nu = -1$ for some j and k and in that case the corresponding elements of R will be *purely imaginary*.

The preceding results lead to the following (perhaps obvious) conclusions.
i) If CP-invariance holds at "high" energy, i.e., if the neutrino Yukawa couplings satisfy (in the convention used) the constraints (28), it will also hold at "low" energy in the lepton sector, i.e., the elements of the PMNS matrix will satisfy (in the same convention) Eq. (29).
ii) If the CP-symmetry is violated at "low" energy, i.e., if the PMNS matrix does not satisfy conditions (29) and the CC weak interaction is not CP-invariant, it will also be violated at "high" energy, i.e., the neutrino Yukawa couplings will not satisfy Eq. (28) and the Lagrangian $\mathcal{L}_Y(x)$ will not be CP-invariant. Obviously, it is not possible to use the matrix R in order to render the Yukawa couplings CP-conserving.
iii) If CP-invariance holds at "low" energy, i.e., if the CC weak interaction (20) is CP-invariant and the PMNS matrix satisfy conditions (29), it can still be violated at "high" energy, i.e., the neutrino Yukawa couplings will not necessarily satisfy Eq. (28) and the Lagrangian $\mathcal{L}_Y(x)$ will not necessarily be CP-invariant. The CP-violation in this case can be due to the matrix R.

As we will see, of interest for our further analysis is, in particular, the product

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U_{lk}^*\, U_{lm}\,, \quad k \neq m. \tag{31}$$

If CP-invariance holds, we find from Eqs. (29) and (30) that P_{jkml} is real:

$$P_{jkml}^* = P_{jkml}\, (\rho_j^N)^2\, (\rho_k^\nu)^2\, (\rho_m^\nu)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,. \tag{32}$$

Let us consider next the case when conditions (29) are satisfied and $\mathrm{Re}(U_{\tau k}^* U_{\tau m}) = 0$, $k < m$, i.e., $U_{\tau k}^* U_{\tau m}$ is *purely imaginary*. This can be realised, as Eqs. (2) and (29) show,

for $\delta = \pi q$, $q = 0, 1, 2, ...$, and $\rho_k^\nu \rho_m^\nu = -1$, i.e., if the relative CP-parity of the light Majorana neutrinos ν_k and ν_m is equal to (-1), or, correspondingly, the Majorana phase $\alpha_{mk} = \pi(2q' + 1)$, $q' = 0, 1, 2, ...$ If $J_{\rm CP} = 0$ and both $\mathrm{Re}(U_{\tau 1}^* U_{\tau 2}) = 0$ and $\mathrm{Re}(U_{\tau 2}^* U_{\tau 3}) = 0$, CP-invariance holds in the lepton sector at low energies. In order for CP-invariance to hold at "high" energy, i.e., for P_{jkml} to be real, the product $R_{jk}R_{jm}$, in the case under consideration, has also to be *purely imaginary*, $\mathrm{Re}(R_{jk}R_{jm}) = 0$. Thus, *purely imaginary* $U_{\tau k}^* U_{\tau m} \neq 0$ and *purely real* $R_{jk}R_{jm} \neq 0$, i.e., $\mathrm{Re}(U_{\tau k}^* U_{\tau m}) = 0$, $\mathrm{Im}(R_{jk}R_{jm}) = 0$, in particular, *imply violation of the CP-symmetry at "high" energy by the matrix R.*

4 Leptogenesis and the Baryon Asymmetry

In this Section we will repeat briefly the arguments leading to the conclusion that the connection between leptogenesis CP-violating parameters and the CP-violating phases in the PMNS matrix generically does not hold. We will therefore first restrict our discussion to the "one-flavour" approximation. Then, we will review the recent results on leptogenesis which take into account the lepton flavour effects [32, 33, 34]. In this case, the baryon asymmetry depends explicitly on the parameters of the lepton mixing matrix U.

4.1 The One Flavour Case

We assume that the heavy Majorana neutrinos N_i have a hierarchical mass spectrum, $M_{2,3} \gg M_1$, so that studying the evolution of the number density of N_1 suffices. The final amount of $(B - L)$ asymmetry can be parametrized as $Y_{B-L} = n_{B-L}/s$, where $s = 2\pi^2 g_* T^3/45$ is the entropy density and g_* counts the effective number of spin-degrees of freedom in thermal equilibrium ($g_* = 217/2$ in the SM with a single generation of right-handed neutrinos). After reprocessing by sphaleron transitions, the baryon asymmetry is related to the $(B - L)$ asymmetry by [60] $Y_B = (12/37)\,(Y_{B-L})$.

One defines the CP asymmetry generated by N_1 decays as

$$\epsilon_1 \equiv \frac{\sum_l [\Gamma(N_1 \to H\ell_l) - \Gamma(N_1 \to \overline{H\ell_l})]}{\sum_l [\Gamma(N_1 \to H\ell_l) + \Gamma(N_1 \to \overline{H\ell_l})]} = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\mathrm{Im}\left[(\lambda\lambda^\dagger)_{j1}^2\right]}{[\lambda\lambda^\dagger]_{11}} g\left(\frac{M_j^2}{M_1^2}\right) . \tag{33}$$

The wavefunction plus vertex contributions are included giving $g(x) \simeq -\frac{3}{2\sqrt{x}}$ for $x \gg 1$. Notice, in particular, that ϵ_1 denotes the CP asymmetry in the total lepton charge (i.e., trace over the lepton flavour index).

Besides the CP parameter ϵ_1, the final baryon asymmetry depends on a single parameter,

$$\left(\frac{\widetilde{m_1}}{\widetilde{m_*}}\right) \equiv \frac{\sum_l \Gamma(N_1 \to H\ell_l)}{H(M_1)}, \tag{34}$$

where $H(M_1)$ denotes the value of the Hubble rate evaluated at a temperature $T = M_1$, $\widetilde{m_*} \sim 3 \times 10^{-3}$ eV and

$$\widetilde{m_1} \equiv \frac{(\lambda\lambda^\dagger)_{11}v^2}{M_1} \tag{35}$$

is proportional to the total decay rate of the heavy RH Majorana neutrino N_1. Notice again that $\widetilde{m_1}$ is obtained by computing the total decay rate of the N_1, that is by summing over all the flavours.

The asymmetry in the total lepton charge is then given by

$$Y_L \simeq \frac{\epsilon_1}{g_*} \eta\left(\widetilde{m_1}\right) , \tag{36}$$

where $\eta\left(\widetilde{m_1}\right)$ accounts for the washing out of the total lepton asymmetry due to inverse decays.

The final baryon asymmetry in the "one flavour approximation" depends always upon the trace of the CP asymmetries over flavours, ϵ_1, times a function of the trace over flavours of the decay rate of the RH neutrino N_1. This is an unexpected result because it suggests, in particular, that the lepton asymmetry, say, in the electron lepton number can be washed out by inverse decays involving the second and/or the third family (which erase only the muon and/or tau lepton charges).

Let us turn now to the issue of the connection between the CP violating parameters in leptogenesis and the low energy CP-violating phases in the lepton sector, i.e., in the PMNS matrix U. The CP asymmetry ϵ_1 can be written in terms of the diagonal light and heavy Majorana neutrino mass matrices introduced earlier, $m = \mathrm{Diag}(m_1, m_2, m_3)$ and $M = \mathrm{Diag}(M_1, M_2, M_3)$, and the orthogonal complex matrix R:

$$
\begin{aligned}
\epsilon_1 &= -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{l\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U_{l\beta}^* U_{l\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_\rho m_\rho^2 R_{1\rho}^2\right)}{\sum_\beta m_\beta |R_{1\beta}|^2} ,
\end{aligned}
\tag{37}
$$

where we have summed over all lepton flavours $l = e, \mu, \tau$. In particular, if the elements of the matrix R satisfy CP-invariance condition (30), i.e., if $R_{1\rho}$, $\rho = 1, 2, 3$, is *real or purely imaginary*, the leptogenesis CP-asymmetry $\epsilon_1 = 0$. In the top-down approach, working in the basis in which the matrix of charged lepton Yukawa couplings and the RH neutrino Majorana mass matrix are diagonal, the matrix of neutrino Yukawa couplings can be written as $\lambda = V_R^\dagger \mathrm{Diag}(\lambda_1, \lambda_2, \lambda_3) V_L$ and the low energy leptonic phases can arise from the phases in V_L, i.e. in the left-handed (LH) sector, in V_R, i.e. in the RH sector, or from both V_L and V_R. However, from $\lambda\lambda^\dagger = V_R^\dagger \mathrm{Diag}(\lambda_1^2, \lambda_2^2, \lambda_3^2) V_R = M^{1/2} R\, m\, R^\dagger M^{1/2}/v^2$, one sees that the phases of R are related to those of V_R. This means that in the "one-flavour" approximation a future possible observation of CP-violating phases at low energies in the neutrino sector does not imply the existence of a baryon asymmetry. Indeed, low energy CP-violating phases might stem entirely from the LH sector, i.e. V_L, and hence be irrelevant for leptogenesis which would be driven by the phases in R, i.e. of the RH sector.

4.2 Taking the Flavour Effects into Account

As we mentioned in the Introduction, the 'one-flavour' approximation rigorously holds only when the interactions mediated by the charged lepton Yukawas are out of equilibrium, that is at $T \sim M_1 \gtrsim 10^{12}$ GeV. In this regime, flavours are indistinguishable and one may indeed perform a rotation is flavour space to store all the asymmetry in a single flavour. At smaller temperatures, though, this operation is not possible. The τ (μ) lepton doublet is a distinguishable mass eigenstate for $T \sim M_1 \lesssim 10^{12} (10^9)$ GeV. Flavoured Boltzmann equations may be written down for the asymmetry Y_{Δ_l} in each flavour $\Delta_l = B/3 - L_l$ [32, 33, 34]. For the case of hierarchical RH neutrinos, they read [5]

$$\frac{\mathrm{d}Y_{N_1}}{\mathrm{d}z} = \frac{-z}{sH(M_1)}(\gamma_D + \gamma_{\mathrm{S},\Delta L=1})\left(\frac{Y_{N_1}}{Y_{N_1}^{\mathrm{eq}}} - 1\right), \tag{38}$$

$$\frac{\mathrm{d}Y_{\Delta_l}}{\mathrm{d}z} = \frac{-z}{sH(M_1)}\left[\epsilon_l(\gamma_D + \gamma_{\mathrm{S},\Delta L=1})\left(\frac{Y_{N_1}}{Y_{N_1}^{\mathrm{eq}}} - 1\right) - \left(\frac{\gamma_D^l}{2} + \gamma_{\mathrm{W},\Delta L=1}^l\right)\frac{\sum_\beta A_{lm}Y_{\Delta_m}}{Y_\ell^{\mathrm{eq}}}\right], \tag{39}$$

where $z = M_1/T$, T being the temperature, and there is no sum over l in the last term in the right-side of Eq. (39). In the above equation Y_{N_1} is the density of the lightest right-handed neutrino N_1 with mass M_1. Y_{Δ_l} are defined as $Y_{\Delta_l} \equiv Y_B/3 - Y_{L_l}$, where Y_{L_l} are the total lepton number densities for the flavours $l = e, \mu, \tau$ and Y_B is the total baryon density. One has to solve the Boltzmann equations for Y_{Δ_l} instead of for the number densities Y_l of the lepton doublets ℓ_l, since ($\Delta_l \equiv B/3 - L_l$) is conserved by sphalerons and by the other SM interactions. Furthermore, all number densities Y, normalization to the entropy density s is understood and $Y_{N_1}^{\mathrm{eq}}$ and Y_ℓ^{eq} stand for the corresponding equilibrium number densities; γ_D is the thermally averaged total decay rate of N_1 and $\gamma_{\mathrm{S},\Delta L=1}$ represents the rates for the $\Delta L = 1$ scattering processes in the thermal bath. Notice, in particular, that $\gamma_{\mathrm{S},\Delta L=1}$ contributes to the asymmetry, as was recently pointed out in [34]. The corresponding flavour-dependent rates for wash-out processes involving the lepton flavour l are γ_D^l (from inverse decays involving leptons ℓ_l) and $\gamma_{\mathrm{W},\Delta L=1}^l$, while $\Delta L = 2$ scatterings may be safely neglected. The matrix A, which appears in the wash-out term connects the asymmetries in the lepton doublets to the asymmetries in the charges Δ_l by $Y_l = \sum_m A_{lm} Y_{\Delta_m}$. The values of its elements depend on which interactions, in addition to the weak and strong sphalerons, are in thermal equilibrium at the temperatures where leptogenesis takes place. Below 10^9 GeV in the SM, A is given by [34]

$$A = \begin{pmatrix} -151/179 & 20/179 & 20/179 \\ 25/358 & -344/537 & 14/537 \\ 25/358 & 14/537 & -344/537 \end{pmatrix}. \tag{40}$$

In what regards the charged leptons, in the SM only the interaction mediated by the τ Yukawa coupling is in equilibrium between 10^9 and 10^{12} GeV, and the lepton asymmetries

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[5]The equations of motion should include the terms accounting for the quantum oscillations among the different flavours [32]. They become relevant only in the proximity of the transition between the one-single flavour and the two-flavour state when the τ flavour becomes distinguishable.

and $B/3 - L_l$ asymmetries in the e and μ flavour can be combined to $Y_2 \equiv Y_{e+\mu}$ and $Y_{\Delta_2} \equiv Y_{\Delta_e + \Delta_\mu}$. In this temperature range, A is given by [34]

$$A = \begin{pmatrix} -920/589 & 120/589 \\ 30/589 & -390/589 \end{pmatrix}. \tag{41}$$

Above 10^{12} GeV, all asymmetries can be combined to Y_{B-L}, and A is given by $A = -1$.

The asymmetry in each flavour is given by

$$\epsilon_l = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U_{l\beta}^* U_{l\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}, \quad l = e, \mu, \tau. \tag{42}$$

Obviously, the trace over the flavours of ϵ_l coincides with ϵ_1. It should be clear from Eq. (42) that we can have $\epsilon_l \neq 0$ even if, e.g., R is a real matrix and $R \neq \mathbf{1}$. If, however, R is a diagonal matrix, e.g., if $R = \mathbf{1}$, all three lepton number asymmetries vanish: $\epsilon_l = 0$, $l = e, \mu, \tau$.

Similarly, one has to define a "wash-out mass parameter" for each flavour l [32, 34]:

$$\left(\frac{\widetilde{m_l}}{3 \times 10^{-3}\,\text{eV}}\right) \equiv \frac{\Gamma(N_1 \to H\,l)}{H(M_1)},$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau. \tag{43}$$

The quantity $\widetilde{m_l}$ parametrizes the decay rate of N_1 to the leptons of flavour l. The trace $\sum_l \widetilde{m_l}$ coincides with the $\widetilde{m_1}$ parameter defined in the previous section.

What is more relevant is that the total baryon asymmetry is the sum of each individual lepton asymmetry. In the rest of the paper we will be concerned with temperatures $(10^9 \lesssim T \sim M_1 \lesssim 10^{12})$ GeV. In this range only the interactions mediated by the τ Yukawa coupling are in equilibrium and the final baryon asymmetry is well approximated by [34]

$$Y_B \simeq -\frac{12}{37g_*}\left(\epsilon_2\,\eta\left(\frac{417}{589}\,\widetilde{m_2}\right) + \epsilon_\tau\,\eta\left(\frac{390}{589}\,\widetilde{m_\tau}\right)\right), \tag{44}$$

where $\epsilon_2 = \epsilon_e + \epsilon_\mu$, $\widetilde{m_2} = \widetilde{m_e} + \widetilde{m_\mu}$ and

$$\eta\left(\widetilde{m_l}\right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25 \times 10^{-3}\,\text{eV}}\right)^{-1} + \left(\frac{0.2 \times 10^{-3}\,\text{eV}}{\widetilde{m_l}}\right)^{-1.16}\right)^{-1}. \tag{45}$$

Notice that the wash-out masses $\widetilde{m_2}$ and $\widetilde{m_\tau}$ in Eq. (44) are multiplied by some numerical coefficients which account for the dynamics involving the lepton doublet asymmetries and the asymmetries stored in the charges $\Delta_l = (1/3)B - L_l$ [34].

From the generic expression for the baryon asymmetry, we deduce that the CP asymmetry in each flavour is weighted by the corresponding wash-out parameter. Therefore, the total baryon number is generically not proportional to ϵ_1.

The dependence of ϵ_l on the PMNS matrix elements is such that nonvanishing low energy leptonic CP-violating phases imply, in the context of leptogenesis and barring accidental cancellations, a nonvanishing baryon asymmetry. In the general case of complex matrix R, the low energy phases in the neutrino mixing matrix U could stem from the phases in the left-handed sector, in the RH sector, or in both sectors. This result only follows when the lepton flavour effects are correctly taken into account in the Boltzmann equations. In previous analyses of leptogenesis ignoring the flavour effects, the observation of low energy CP violation did not automatically imply the existence of a baryon asymmetry, since the possibility existed that the low energy phases could stem exclusively from the left-handed sector and hence be irrelevant for leptogenesis.

This conclusion is by itself already quite interesting. However, we can go even further. Let us consider, for instance, the case in which the CP parities of the heavy and light Majorana neutrinos are such that $\rho_i^N = \rho_j^\nu = 1$ for all $i, j = 1, 2, 3$. In such a case, CP invariance corresponds to having all the elements of the matrix R real, see Eq. (30) and $\delta = \alpha_{21} = \alpha_{31} = 0 \pmod{2\pi}$[6]. In the top-down approach, a real matrix R would correspond to the class of models where CP is an exact symmetry in the RH neutrino sector. The reason for this can be more easily understood working in the basis where the charged lepton Yukawa coupling and the RH mass matrix are diagonal, so that the neutrino Yukawa matrix is the only coupling in the leptonic Lagrangian that violates CP. More specifically, since the neutrino Yukawa coupling can be written in its singular value decomposition, $\lambda = V_R^\dagger \text{Diag}(\lambda_1, \lambda_2, \lambda_3) V_L$, CP violation in the RH neutrino sector is encoded in the phases in V_R, that can be extracted from diagonalizing the combination $\lambda\lambda^\dagger = V_R^\dagger \text{Diag}(\lambda_1^2, \lambda_2^2, \lambda_3^2) V_R$. On the other hand, as we have seen, using the parametrization of the Yukawa coupling (24), this same combination of matrices can be written as $\lambda\lambda^\dagger = M^{1/2} R m R^\dagger M^{1/2}/v^2$. Comparing the two expressions it is apparent that R is real if and only if V_R is real, i.e. when there is no CP violation in the RH sector. It has been recently pointed out that the case of a real R matrix is naturally realized within the class of models based on sequential dominance [61]. In the case of R real, the flavour CP asymmetries and the baryon asymmetry depend exclusively on the phases of the left-handed sector, that are in turn uniquely determined by the low energy phases. Consequently, for real matrix $R \neq \mathbf{1}$, the leptogenesis mechanism is directly connected to the low energy CP-violating phases in U_{PMNS}. This connection is more apparent from the expression of the flavour CP asymmetries in the parametrization (24):

$$
\begin{aligned}
\epsilon_l &= -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_\beta \sqrt{m_\beta} R_{1\beta} U_{l\beta}^* \sum_\rho \sqrt{m_\rho^3} R_{1\rho} U_{l\rho}\right)}{\sum_\gamma m_\gamma |R_{1\gamma}|^2} \\
&= -\frac{3M_1}{16\pi v^2} \frac{\sum_\beta \sum_{\rho>\beta} \sqrt{m_\beta m_\rho}(m_\rho - m_\beta) R_{1\beta} R_{1\rho} \text{Im}\left(U_{l\beta}^* U_{l\rho}\right)}{\sum_\gamma m_\gamma |R_{1\gamma}|^2}.
\end{aligned}
\tag{46}
$$

If $R_{1\beta} R_{1\rho}$ is purely imaginary, $R_{1\beta} R_{1\rho} = \pm i |R_{1\beta} R_{1\rho}|$, we would get $\pm(m_\rho +$

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[6]Alternatively, one may consider the case in which the CP parities of the heavy and light neutrinos are such that CP invariance is realized for the elements of the matrix R purely imaginary and $\delta = 0$, $\alpha_{21} = \alpha_{31} = \pi \pmod{2\pi}$.

$m_\beta)|R_{1\beta}R_{1\rho}|\mathrm{Re}(U_{l\beta}^* U_{l\rho})$ instead of $(m_\rho - m_\beta)R_{1\beta}R_{1\rho}\mathrm{Im}(U_{l\beta}^* U_{l\rho})$ in Eq. (46). Purely imaginary $R_{1\beta}R_{1\rho}$ and real $(U_{l\beta}^* U_{l\rho})$ implies violation of CP-invariance. In order for the CP-symmetry to be broken at low energies, we should have both $\mathrm{Re}(U_{l\beta}^* U_{l\rho}) \neq 0$ and $\mathrm{Im}(U_{l\beta}^* U_{l\rho}) \neq 0$.

In the next Section we will study in greater detail the possibility that the baryon asymmetry stems only from the low energy measurable CP violating phases in the neutrino mixing matrix U.

5 Baryon Asymmetry from Low Energy CP-Violating Dirac and Majorana Phases in U_{PMNS}: RH Neutrinos with Hierarchical Mass Spectrum

In what follows we shall assume that the matrix R has real and/or purely imaginary elements and that the heavy Majorana neutrinos possess a hierarchical mass spectrum, $M_1 \ll M_2 \ll M_3$, with M_1 having a value in the interval of interest, 10^9 GeV $\lesssim M_1 \lesssim 10^{12}$ GeV. We will investigate the case when the RG running of m_j and of the parameters in U from M_Z to M_1 is relatively small and can be neglected. This possibility is realised (in the class of theories under discussion) for sufficiently small values of the lightest neutrino mass $\min(m_j)$ [62], e.g., for $\min(m_j) \lesssim 0.05$ eV. The latter condition is fulfilled for the normal hierarchical (NH) and inverted hierarchical (IH) light neutrino mass spectrum. Under the indicated condition m_j, and correspondingly Δm_{A}^2 and $\Delta m_{\odot}^2$, and U in Eqs. (46) and Eqs. (43) can be taken at the scale $\sim M_Z$, at which the neutrino mixing parameters are measured.

Taking into account that in the case under discussion $\epsilon_2 = \epsilon_e + \epsilon_\mu = -\epsilon_\tau$, we can cast Eq. (44) in a somewhat more convenient form:

$$Y_B = -\frac{12}{37}\frac{\epsilon_\tau}{g_*}\left(\eta\left(\frac{390}{589}\widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right)\right),\tag{47}$$

where $\widetilde{m_2} = \widetilde{m_e} + \widetilde{m_\mu}$ and $\widetilde{m_l}$ and $\eta(\widetilde{m_l})$ are given in Eqs. (43) and (45).

5.1 Normal Hierarchical Light Neutrino Mass Spectrum

Given the inequalities $m_1 \ll m_2 \ll m_3$, we will assume that the terms $\propto \sqrt{m_1}$ give subleading contributions to the lepton flavour asymmetries ϵ_l, Eq. (46), and to the mass parameters $\widetilde{m_l}$ related to the wash-out effects, and we neglect them with respect to those $\propto \sqrt{m_{2,3}}$ giving the dominant contribution. This requires that $\sqrt{m_1}|R_{11}| \ll \sqrt{m_2}|R_{12}|, \sqrt{m_3}|R_{13}|$. In the indicated approximation, we find using Eq. (46) and the fact that $m_2 \cong \sqrt{\Delta m_{\odot}^2}$,

$m_3 \cong \sqrt{\Delta m_{\rm A}^2}$:

$$\epsilon_l \simeq -\frac{3M_1\sqrt{\Delta m_{\rm A}^2}}{16\pi v^2}\left(1-\frac{\sqrt{\Delta m_\odot^2}}{\sqrt{\Delta m_{\rm A}^2}}\right)\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2}\right)^{\frac{1}{4}}\frac{|R_{12}R_{13}|}{\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2}\right)^{\frac{1}{2}}|R_{12}|^2+|R_{13}|^2}$$

$$\times\left[{\rm Im}\left(e^{i\beta_{23}}U_{l2}^*U_{l3}\right)+\frac{2\sqrt{\Delta m_\odot^2}}{\sqrt{\Delta m_{\rm A}^2}-\sqrt{\Delta m_\odot^2}}{\rm Im}\left(e^{i\beta_{23}}{\rm Re}\left(U_{l2}^*U_{l3}\right)\right)\right] \qquad (48)$$

$$= -\frac{3M_1\sqrt{\Delta m_{\rm A}^2}}{16\pi v^2}\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2}\right)^{\frac{1}{4}}\frac{|R_{12}R_{13}|}{\left(\frac{\Delta m_\odot^2}{\Delta m_{\rm A}^2}\right)^{\frac{1}{2}}|R_{12}|^2+|R_{13}|^2}$$

$$\times{\rm Im}\left[\left(1-\frac{\sqrt{\Delta m_\odot^2}}{\sqrt{\Delta m_{\rm A}^2}}\right)e^{i(\beta_{23}+\frac{\pi}{2})}{\rm Im}\left(U_{l2}^*U_{l3}\right)+\left(1+\frac{\sqrt{\Delta m_\odot^2}}{\sqrt{\Delta m_{\rm A}^2}}\right)e^{i\beta_{23}}{\rm Re}\left(U_{l2}^*U_{l3}\right)\right], (49)$$

where $\beta_{23}\equiv\widetilde{\beta_{12}}+\widetilde{\beta_{13}}\equiv\arg(R_{12}R_{13})$, $\widetilde{\beta_{1j}}\equiv\arg(R_{1j})$. The phase β_{23} parametrises the effect of CP-violation due to the matrix R in the asymmetry ϵ_l. In the case of CP-invariance, $\beta_{23}=0$ or $\pi/2$ [7] depending on whether ${\rm Im}(U_{l2}^*U_{l3})=0$ or ${\rm Re}(U_{l2}^*U_{l3})=0$, respectively, and $\epsilon_l=0$, $l=e,\mu,\tau$.

From Eq. (2), it is straightforward to obtain ${\rm Im}(e^{i\beta_{23}}U_{l2}^*U_{l3})$ and ${\rm Im}(e^{i\beta_{23}}{\rm Re}(U_{l2}^*U_{l3}))$. The expressions, e.g., for ${\rm Im}(e^{i\beta_{23}}U_{l2}^*U_{l3})$ read:

$${\rm Im}\left(e^{i\beta_{23}}U_{e2}^*U_{e3}\right) = -s_{12}c_{13}s_{13}\sin\left(\delta-\left(\frac{\alpha_{32}}{2}+\beta_{23}\right)\right), \qquad (50)$$

$${\rm Im}\left(e^{i\beta_{23}}U_{\mu2}^*U_{\mu3}\right) = -c_{13}\left[-c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}+\beta_{23}\right)-s_{23}^2s_{12}s_{13}\sin\left(\delta-\left(\frac{\alpha_{32}}{2}+\beta_{23}\right)\right)\right], \qquad (51)$$

$${\rm Im}\left(e^{i\beta_{23}}U_{\tau2}^*U_{\tau3}\right) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}+\beta_{23}\right)-c_{23}^2s_{12}s_{13}\sin\left(\delta-\left(\frac{\alpha_{32}}{2}+\beta_{23}\right)\right)\right], \qquad (52)$$

where $\alpha_{32}\equiv\alpha_{31}-\alpha_{21}$. Thus, ϵ_l depend on the same Majorana phase difference (or phase) the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay, $|\langle m\rangle|$, depends (see Eq. (15)) on. It follows from Eq. (49) that, as could be expected, for real or purely imaginary R_{12} and R_{13} we have $\epsilon_e+\epsilon_\mu+\epsilon_\tau=0$.

We are interested in the case in which the CP-violating phases in U play the role of leptogenesis CP-violating parameters. It follows from the preceding discussions that one has to consider two cases: i) $\beta_{23}=0$ (or more generally, $\beta_{23}=\pi q$, $q=0,1,2,...$), and ii) $\beta_{23}=\pi/2$ (or more generally, $\beta_{23}=(2q+1)\pi/2$, $q=0,1,2,...$). We have to remember that if ${\rm Im}(U_{l2}^*U_{l3})\neq0$ (${\rm Re}(U_{l2}^*U_{l3})\neq0$) but ${\rm Re}(U_{l2}^*U_{l3})=0$ (${\rm Im}(U_{l2}^*U_{l3})=0$), the case of $\beta_{23}=0,\pi$ ($\beta_{23}=\pi/2,3\pi/2$) *corresponds to violation of the CP-symmetry by the matrix R.*

Consider first the possibility of $\beta_{23}=0$ (π). It is quite remarkable that, as it follows from Eqs. (49) and (52), for $\beta_{23}=0$ (π) the asymmetry ϵ_τ depends on the rephasing invariant S_2,

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[7] More precisely, in the case of CP-invariance we have $\beta_{23}=\pi q$ or $(2q+1)\pi/2$, $q=0,1,2,....$

Eq. (13): we have $\epsilon_\tau \propto \mathrm{Im}\,(U_{\tau 2}^* U_{\tau 3}) \equiv S_2$. The requirement of a nonzero asymmetry, $\epsilon_\tau \neq 0$, implies $S_2 \neq 0$, and correspondingly $\alpha_{32} \neq 2\pi k$ and/or $(\delta - \alpha_{32}/2) \neq \pi k'$, $k, k' = 0, 1, ...$ In order to reproduce the observed value of the baryon asymmetry, $|\sin(\alpha_{32}/2)|$ and/or $|\sin\theta_{13}\sin(\delta - \alpha_{32}/2)|$ have to be sufficiently large (see further). Since in the case under study $\beta_{23} = 0$ or π, the sign of ϵ_τ is not uniquely determined by the sign of $\mathrm{Im}\,(U_{\tau 2}^* U_{\tau 3})$.

In the alternative possibility of $\beta_{23} = \pi/2$ $(3\pi/2)$ we have $|\epsilon_\tau| \propto |\mathrm{Re}(U_{\tau 2}^* U_{\tau 3})|$. Now $\epsilon_\tau \neq 0$ provided $\alpha_{32} \neq \pi(2k + 1)$ and/or $(\delta - \alpha_{32}/2) \neq (\pi/2)(2k' + 1)$, $k, k' = 0, 1, ...$ In this case $|\cos(\alpha_{32}/2)|$ and/or $|\cos\theta_{13}\cos(\delta - \alpha_{32}/2)|$ have to be sufficiently large in order for leptogenesis to be successful. The sign of ϵ_τ is again not uniquely determined by the sign of $\mathrm{Re}\,(U_{\tau 2}^* U_{\tau 3})$.

The maximal value of $|\mathrm{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})|$ and of $|\epsilon_\tau|$, is reached for $\beta_{23} = 0$; π $(\beta_{23} = \pi/2$; $3\pi/2)$ at [8] $\alpha_{32} = \pi(2k + 1)$ and $\delta = 2\pi k$ $(\alpha_{32} = 2\pi k$ and $\delta = \pi(2k + 1))$, $k = 0, 1, ...$:

$$\max \left|\mathrm{Im}\,\left(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3}\right)\right| = c_{23}c_{13}\,\left(s_{23}c_{12} + c_{23}s_{12}s_{13}\right) \lesssim 0.47\,, \quad \beta_{23} = \frac{\pi}{2}\,q\,, q = 0, 1, 2, ...,$$
(53)

where we have used the best fit values of $\sin^2 2\theta_{23}$ and $\sin^2 \theta_{12}$ given in Eq. (5) and the upper limit $s_{13} \leq 0.2$ (see Eq. (7)). For $s_{13} = 0$ we get $|\mathrm{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})| \lesssim 0.42\,|\sin(\alpha_{32}/2)| \leq 0.42$ while if $\alpha_{32} = 0$, one obtains $|\mathrm{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})| \cong 0.27\,|s_{13}\sin\delta| \lesssim 0.054$. Thus, the effect of the term $\propto |s_{13}\sin\delta|$ in $|S_2|$, and correspondingly in $|\epsilon_\tau|$, can be significant only if $|\sin(\alpha_{32}/2)| \lesssim 0.20$. Note that for $\beta_{23} = 0$; π $(\beta_{23} = \pi/2$; $3\pi/2)$ $\max\,(|\mathrm{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})|)$ corresponds to $\mathrm{Re}(U_{l2}^* U_{l3}) = 0$ $(\mathrm{Im}(U_{l2}^* U_{l3}) = 0)$, $l = e, \mu, \tau$, i.e., to CP-conserving values of α_{32} and δ. Nevertheless, in this case we still have $|\epsilon_\tau| \neq 0$ as a consequence of the violation of the CP-symmetry by the matrix R.

We turn next to the dependence on the parameters in R. Taking into account that R is, in general, a complex orthogonal matrix, we get for the elements of R of interest, R_{1j}, $j = 1, 2, 3$, obeying the CP-invariance constraints:

$$\left(|R_{11}|^2\,\rho_1^\nu + |R_{12}|^2\,\rho_2^\nu + |R_{13}|^2\,\rho_3^\nu\right)\,\rho_1^N = 1\,.$$
(54)

The CP-asymmetry $|\epsilon_\tau|$ is proportional to the factor r:

$$r \equiv \frac{|R_{12}R_{13}|}{\left(\dfrac{\Delta m_\odot^2}{\Delta m_{\mathrm{A}}^2}\right)^{\frac{1}{2}}\,|R_{12}|^2 + |R_{13}|^2}\,.$$
(55)

It is clear that as long as $|R_{12}| \sim |R_{13}|$, r will not act as a suppression factor for the asymmetry $|\epsilon_\tau|$; the latter can be suppressed if, for instance, $|R_{12}| \ll |R_{13}|$ or $|R_{13}| \ll (\Delta m_\odot^2/\Delta m_{\mathrm{A}}^2)^{\frac{1}{4}}|R_{12}|$.

We now focus on the wash-out effects. The mass parameters $\widetilde{m_l}$ related to the wash-out effects for the three lepton number asymmetries read:

$$\widetilde{m_l} \cong \sqrt{\Delta m_{\mathrm{A}}^2}\left|\left(\frac{\Delta m_\odot^2}{\Delta m_{\mathrm{A}}^2}\right)^{\frac{1}{4}} R_{12}\,U_{l2}^* + R_{13}\,U_{l3}^*\right|^2\,, \quad l = e, \mu, \tau\,.$$
(56)

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[8]Note that for $\beta_{23} = \pi/2$; $3\pi/2$ we have $|\mathrm{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})| = |\mathrm{Re}(U_{\tau 2}^* U_{\tau 3})|$, and the maximum of $|\mathrm{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})|$ coincides with the maximum of $|\mathrm{Re}(U_{\tau 2}^* U_{\tau 3})|$.

Using the unitarity of the PMNS matrix U we obtain:

$$\widetilde{m_e} + \widetilde{m_\mu} + \widetilde{m_\tau} = \sqrt{\Delta m_A^2} \left[\left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2 \right]. \tag{57}$$

Since $\widetilde{m_l} \geq 0$, $l = e, \mu, \tau$, each of the individual wash-out mass parameters is limited from above by the expression in the right-hand side of Eq. (57).

Except for strong fine tuning, the contribution due to $\sin \theta_{13}$ is subdominant and can be neglected at first order. Therefore, for simplicity, we take $\sin \theta_{13} = 0$ in the following analysis and we get:

$$\widetilde{m_2} \simeq \sqrt{\Delta m_A^2} \left(\left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{2}} |R_{12}|^2 (1 - c_{12}^2 s_{23}^2) + |R_{13}|^2 s_{23}^2 \right. \tag{58}$$

$$+ 2 \left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{4}} |R_{12} R_{13}| \ s_{23} c_{23} c_{12} \cos((\widetilde{\beta_{13}} - \widetilde{\beta_{12}} - \frac{\alpha_{32}}{2})) \bigg),$$

$$\widetilde{m_\tau} \simeq \sqrt{\Delta m_A^2} \left(\left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{2}} |R_{12}|^2 c_{12}^2 s_{23}^2 + |R_{13}|^2 c_{23}^2 \right. \tag{59}$$

$$- 2 \left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{4}} |R_{12} R_{13}| \ s_{23} c_{23} c_{12} \cos((\widetilde{\beta_{13}} - \widetilde{\beta_{12}} - \frac{\alpha_{32}}{2})) \bigg).$$

It follows from Eq. (44) that the baryon asymmetry $|Y_B|$ can be zero if $|\eta(0.66\widetilde{m_\tau}) - \eta(0.71\widetilde{m_2})| = 0$, although $|\epsilon_\tau| \neq 0$ and $|\epsilon_e + \epsilon_\mu| \neq 0$. This corresponds to the physical case when the asymmetries generated in the lepton doublet charges τ and $(e + \mu)$ are equal in magnitude, but have opposite signs. Such a possibility can occur if the following relation holds:

$$\eta \left(\frac{390}{589} \widetilde{m_\tau} \right) = \eta \left(\frac{417}{589} \widetilde{m_2} \right). \tag{60}$$

One solution is given by:

$$\widetilde{m_2} = \frac{390}{417} \widetilde{m_\tau} \cong 0.935 \, \widetilde{m_\tau}. \tag{61}$$

For fixed $|R_{12}|^2$ and $|R_{13}|^2$, Eqs. (57) and (61) determine the value of $\widetilde{m_\tau}$ for which we can have $|Y_B| = 0$:

$$\widetilde{m_\tau} = \frac{\sqrt{\Delta m_A^2}}{1 + 0.935} \left[\left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2 \right]. \tag{62}$$

Since $\widetilde{m_\tau}$ is calculated from Eq. (56), given the neutrino oscillation parameters and $|R_{12}|$ and $|R_{13}|$, the requirement that the value of $\widetilde{m_\tau}$ obtained from Eq. (56) coincides with that determined by Eq. (62) leads to a combined constraint on the quantities $|U_{\tau 2}|^2$ and $2 \operatorname{Re}(R_{12} U_{\tau 2}^* R_{13}^* U_{\tau 3})$ which depend on the phases δ and α_{32}. The phase factors associated with the latter vary only between (-1) and 1. Therefore it is not guaranteed that condition (61) can be satisfied and we can have $|Y_B| = 0$ with $|\epsilon_\tau| \neq 0$ and $|\epsilon_e + \epsilon_\mu| \neq 0$ for any possible

values of R_{12} and R_{13}. Taking for simplicity $(\widetilde{\beta}_{13} - \widetilde{\beta}_{12} - \alpha_{32}/2) = \pi/2$, a solution is given by:

$$|R_{12}|^2 = 0.94 |R_{13}|^2 \frac{\cos 2\theta_{23} - 0.069 s_{23}^2}{1 - 2.069 c_{12}^2 s_{23}^2}, \tag{63}$$

which holds for $2.069\, c_{12}^2 s_{23}^2 \neq 1$ and $(\cos 2\theta_{23} - 0.069 s_{23}^2)/(1 - 2.069 c_{12}^2 s_{23}^2) \geq 0$. For example, if $c_{12}^2 = 0.69$, it can be satisfied for $s_{23}^2 \lesssim 0.48$.

A general solution of Eq. (60) can also be found and is given by:

$$\widetilde{m}_2 \,\widetilde{m}_\tau \simeq \frac{589^2}{390\ 417} 10^{-6} \ \mathrm{eV}^2 \left(\frac{y-1}{y^{1.16}-1} y^{0.08} \right)^{\frac{2}{2.16}} \tag{64}$$

where $y \equiv (417/390)\widetilde{m}_2/\widetilde{m}_\tau$. If, for instance, $\widetilde{\beta}_{13} - \widetilde{\beta}_{12} - \alpha_{32}/2 = \pi/2$ and $|R_{13}|^2$ dominates in $\widetilde{m}_\tau$ and $\widetilde{m}_2$, y does not depend on the parameters in R and we can solve Eq. (64):

$$|R_{13}| \simeq \left(\frac{10^{-6}\ \mathrm{eV}^2}{\Delta m_{\mathrm{A}}^2} \frac{1}{c_{23}^2 s_{23}^2} \frac{589}{390} \frac{589}{417} \left(\frac{y-1}{y^{1.16}-1} y^{0.08} \right)^{\frac{2}{2.26}} \right)^{\frac{1}{4}} \simeq 0.25 \ . \tag{65}$$

We will analyse next the dependence of the baryon asymmetry on the parameters in R. For simplicity, we take the values of α_{32} and of $\widetilde{\beta}_{12}, \widetilde{\beta}_{13} = 0, \pi/2$, which maximize the CP-asymmetry in Eq. (49). The wash-out parameters read:

$$\widetilde{m}_2 \simeq \sqrt{\Delta m_{\mathrm{A}}^2} \left(\sqrt{\frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2}} |R_{12}|^2 (1 - c_{12}^2 s_{23}^2) + |R_{13}|^2 s_{23}^2 \right) , \tag{66}$$

$$\widetilde{m}_\tau \simeq \sqrt{\Delta m_{\mathrm{A}}^2} \left(\sqrt{\frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2}} |R_{12}|^2 c_{12}^2 s_{23}^2 + |R_{13}|^2 c_{23}^2 \right) . \tag{67}$$

Case A: Strong wash-out. This case is realized if $\widetilde{m}_{2,\tau} \gg 2 \times 10^{-3}$ eV. The latter condition is satisfied for $|R_{12}|^2 \gg 2 \times 10^{-3} \mathrm{eV}/(\sqrt{\Delta m_{\odot}^2} c_{12}^2 s_{23}^2) \cong 0.64$, and/or for $|R_{13}|^2 \gg 2 \times 10^{-3}$ eV$/(\sqrt{\Delta m_{\mathrm{A}}^2} c_{23}^2) \cong 0.08$. The baryon asymmetry can be approximated as:

$$|Y_B| \sim \mathcal{C} \frac{|R_{12}||R_{13}|}{\left(\frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2} \right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \left| \left(\frac{589}{390} \frac{2 \times 10^{-4}\ \mathrm{eV}}{\widetilde{m}_\tau} \right)^{1.16} - \left(\frac{589}{417} \frac{2 \times 10^{-4}\ \mathrm{eV}}{\widetilde{m}_2} \right)^{1.16} \right| . \tag{68}$$

The constant $\mathcal{C}$ is defined as

$$\mathcal{C} \equiv \frac{9 M_1}{74 \pi g_* v^2} \sqrt{\Delta m_{\mathrm{A}}^2} \left(\frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2} \right)^{\frac{1}{4}} c_{23}\, s_{23}\, c_{12} \simeq 1.1 \times 10^{-8} \left(\frac{M_1}{10^{11}\ \mathrm{GeV}} \right) , \tag{69}$$

where we have used the present best fit values of the neutrino oscillation parameters. If the $|R_{13}|^2$ term in the wash-out factors dominates, i.e. if $|R_{12}|^2/|R_{13}|^2 \ll \left(\Delta m_{\mathrm{A}}^2/\Delta m_{\odot}^2 \right)^{1/2} s_{23}^2/(1 - c_{12}^2 s_{23}^2) \cong 4.3$, the dependence on the R parameters becomes:

$$|Y_B| \sim 2.7 \times 10^{-3}\ \mathcal{C}\ \frac{|R_{12}|}{|R_{13}|^{3.32} c_{23}^{2.32}} \left| 1 - \left(\frac{390}{417} \frac{c_{23}^2}{s_{23}^2} \right)^{1.16} \right| . \tag{70}$$

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For maximal atmospheric neutrino mixing there is a strong cancellation and the resulting baryon asymmetry has an additional suppression factor ~ 0.075:

$$|Y_B| \sim 4.5 \times 10^{-4} \, \mathcal{C} \, \frac{|R_{12}|}{|R_{13}|^{3.32}} . \tag{71}$$

Taking into account the dependence of ϵ_τ on $\sin 2\theta_{23}$, we find that for $\sin^2 \theta_{23} = 0.34$ (0.66) $|Y_B|$ is larger by a factor of 9 (11). Notice that the asymmetry changes sign when $\sin^2 \theta_{23}$ increases from 0.34 to 0.66. Thus, there is also a value of $\sin^2 \theta_{23}$ in the interval [0.34,0.66], for which $Y_B = 0$ (up to corrections $\sim s_{13}$). The asymmetry decreases very rapidly with $|R_{13}|$ and therefore the maximal asymmetry should correspond to relatively small $|R_{13}|$.

In the alternative case when $|R_{12}|^2$ dominates in $\widetilde{m_{\tau,2}}$, which is realised for $|R_{12}|^2 \gg 7.9 \, |R_{13}|^2$, the asymmetry $|Y_B|$ is proportional to:

$$|Y_B| \sim 2.7 \times 10^{-3} \, \mathcal{C} \, \left(\frac{\Delta m_A^2}{\Delta m_\odot^2}\right)^{1.08} \frac{|R_{13}|}{|R_{12}|^{3.32}} \left| \left(c_{12}^2 s_{23}^2\right)^{-1.16} - \left(\frac{417}{390}(1 - c_{12}^2 s_{23}^2)\right)^{-1.16} \right| . \tag{72}$$

In this case, $|Y_B|$ is not suppressed for maximal atmospheric neutrino mixing. Varying the solar and atmospheric angles within their 95% C.L. ranges induces a factor of a few in the predicted baryon asymmetry. Also in this case the baryon asymmetry is a decreasing function of $|R_{12}|$.

Case B: weak wash-out. For sufficiently small values of $|R_{12}|$ and $|R_{13}|$, one enters the weak wash-out regime. The asymmetry is given approximately by:

$$|Y_B| \sim \mathcal{C} \, \frac{|R_{12}||R_{13}|}{\left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2} \left| \frac{390}{589} \frac{\widetilde{m_\tau}}{8.25 \times 10^{-3} \text{ eV}} - \frac{417}{589} \frac{\widetilde{m_2}}{8.25 \times 10^{-3} \text{ eV}} \right| . \tag{73}$$

If the term $\propto |R_{13}|^2$ dominates in $\widetilde{m_{\tau,2}}$, we can have again a partial cancellation between the two leading terms in $|Y_B|$ for maximal atmospheric neutrino mixing, $\theta_{23} = \pi/4$:

$$|Y_B| \simeq 4.0 \, \mathcal{C} \, |R_{12}||R_{13}| \left| \cos 2\theta_{23} - \frac{27}{390} s_{23}^2 \right| \simeq 0.12 \, \mathcal{C} \, |R_{12}||R_{13}| . \tag{74}$$

The asymmetry $|Y_B|$ is bigger by approximately a factor of 10 if the atmospheric mixing angle assumes the values $\sin^2 \theta_{23} = 0.34$, 0.66. The asymmetry has opposite sign for these two values, indicating that $|Y_B|$ goes through zero (up to corrections $\sim s_{13}$) when $\sin^2 \theta_{23}$ is varied from 0.34 to 0.66.

If the term $\propto |R_{12}|^2$ is the dominant one in $\widetilde{m_{\tau,2}}$, one finds:

$$|Y_B| \sim 4.0 \, \mathcal{C} \, \frac{\Delta m_\odot^2}{\Delta m_A^2} |R_{12}||R_{13}| \left(\frac{417}{390} + \frac{27}{390} c_{12}^2 s_{23}^2\right) \simeq 0.14 \, \mathcal{C} \, |R_{12}||R_{13}| . \tag{75}$$

We notice that in the case of weak wash-out one finds that the asymmetry increases with the values of $|R_{12}|$ or $|R_{13}|$.

On the basis of the above discussion, we can conclude that more than one local maximum of $|Y_B|$ is present and that the absolute one is found, in general, in a regime which interpolates between the weak and strong wash-out regimes. However, it is possible to show that in this region Y_B can be even zero if Eq. (60) is satisfied.

5.2 The case of N_3 decoupling

We will assume further for simplicity that $|R_{11}|^2 \ll ||R_{12}|^2 \pm |R_{13}|^2|$. This corresponds to the case of "decoupling" of the heaviest Majorana neutrino N_3 [63, 64, 28]. For $\beta_{23} = 0$; π ($\beta_{23} = \pi/2$; $3\pi/2$), which implies $\rho_2^\nu \rho_3^\nu = 1$ (-1), we get:

$$|R_{12}|^2 \underset{(-)}{\overset{+}{}} |R_{13}|^2 = \rho_2^\nu \, \rho_1^N \,. \tag{76}$$

Obviously, if $\beta_{23} = 0$; π we have $|R_{12}|^2 + |R_{13}|^2 = 1$, while for $\beta_{23} = \pi/2$; $3\pi/2$, one finds $|R_{12}|^2 - |R_{13}|^2 = \pm 1$. It is not difficult to find the maximal value of r for $\rho_2^\nu \rho_1^N = 1$:

$$\max r = \frac{1}{2} \left(\frac{\Delta m_{\rm A}^2}{\Delta m_\odot^2} \right)^{\frac{1}{4}} \cong 1.2 \,, \quad \rho_2^\nu \rho_1^N = 1 \,, \quad \beta_{23} = \frac{\pi}{2} k \,, k = 0, 1, 2, ... \tag{77}$$

The maximum is reached when $\beta_{23} = 0$; π at $|R_{12}|^2 = (1 + \sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_{\rm A}^2})^{-1} \cong 0.85$, while for $\beta_{23} = \pi/2$; $3\pi/2$ it corresponds to $|R_{12}|^2 = (1 - \sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_{\rm A}^2})^{-1} \cong 1.22$. In the latter case $|R_{12}|^2 - |R_{13}|^2 = 1$ and $|R_{12}|$, $|R_{13}|$, in general, are not limited from above [9]. However, it is not difficult to convince oneself that we can have successful leptogenesis only if $|R_{12}|$ and $|R_{13}|$ are not exceedingly large, namely for $|R_{12}|, |R_{13}| \lesssim 10$. Indeed, barring accidental cancellations, the wash-out mass parameters $\widetilde{m}_l$, $l = e, \mu, \tau$, increase with the increasing of $|R_{12}|$ ($|R_{13}|^2 = |R_{12}|^2 - 1$), and for $|R_{12}| \sim |R_{13}| \sim 10$, one would typically have $\widetilde{m}_l \sim 1$ eV. The corresponding efficiency factors would be exceedingly small and $|\eta \left(\frac{390}{589} \widetilde{m}_\tau \right) - \eta \left(\frac{417}{589} \widetilde{m}_2 \right)| \sim 10^{-4}$ (extremely strong wash-out regime), which makes it impossible to reproduce the observed value of the baryon asymmetry for $M_1 \lesssim 10^{12}$ GeV. The preceding rather qualitative analysis can obviously be refined to obtain more precise upper limits on $|R_{12}|$ and $|R_{13}|$ satisfying $|R_{12}|^2 - |R_{13}|^2 = 1$.

If $\beta_{23} = \pi/2$; $3\pi/2$ and $\rho_2^\nu \rho_1^N = -1$, one always has $r < 1$. In this case $|R_{12}|^2 < |R_{13}|^2$. For $|R_{12}| \ll |R_{13}|$ we have $r \ll 1$ and the asymmetry $|\epsilon_\tau|$ will be suppressed by the r-factor, Eq. (55). We will not analise this case further.

Given the fact that $\sqrt{\Delta m_{\rm A}^2} \cong 0.05$ eV, $\sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_{\rm A}^2} \cong 0.18$, $r \lesssim 1.2$, Eq. (77), and $|{\rm Im}(U_{\tau 2}^* U_{\tau 3})| \lesssim 0.47$, we obtain for the maximal value of the asymmetry $|\epsilon_\tau|$ in the cases of interest ($|R_{12}|^2 \underset{(-)}{\overset{+}{}} |R_{13}|^2 = 1$, $\beta_{23} = (\pi/2)k$, $k = 0, 1, 2, ...$):

$$|\epsilon_\tau| \leq \frac{3 M_1 \sqrt{\Delta m_{\rm A}^2}}{32 \pi v^2} \left(1 \underset{(+)}{\overset{-}{}} \frac{\sqrt{\Delta m_\odot^2}}{\sqrt{\Delta m_{\rm A}^2}} \right) \left| {\rm Im} \left(e^{i\beta_{23}} U_{l2}^* U_{l3} \right) \right| \tag{78}$$

$$\lesssim 0.19 \, (0.27) \frac{3 M_1 \sqrt{\Delta m_{\rm A}^2}}{16 \pi v^2} \simeq 1.9 \, (2.7) \times 10^{-8} \left(\frac{\sqrt{\Delta m_{\rm A}^2}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) , \tag{79}$$

where the minus (plus) sign and the value 0.19 (0.27) correspond to $\beta_{23} = \pi k$, $k = 0, 1, 2, ...$ ($\beta_{23} = (\pi/2)(2k + 1)$, $k = 0, 1, 2, ...$).

---

[9]Note that if $|R_{12}|^2 + |R_{13}|^2 = 1$, one can obviously use the parametrisation $R_{12} = \cos \omega$, $R_{13} = \sin \omega$, while in the case of $|R_{12}|^2 - |R_{13}|^2 = 1$, we have $R_{12} = \cos(i\omega) = \cosh \omega$, $R_{13} = \sin(i\omega) = i \sinh \omega$, ω being a real parameter.

Let us note that, although for the values of the parameters we have used the asymmetry $|\epsilon_\tau|$ has a maximal value, the baryon asymmetry $|Y_B|$ does not necessarily has a maximum since the maximum of $|\epsilon_\tau|$ does not correspond, in general, to a maximum of the effective wash-out factor $|\eta(0.66\widetilde{m_\tau}) - \eta(0.71\widetilde{m_2})|$ in the expression for $|Y_B|$ (see Eq. (44)). As can be shown and Figs. 1 and 2 illustrate, however, for real R_{12} and R_{13} ($\beta_{23} = 0;\ \pi$) satisfying $|R_{12}|^2 + |R_{13}|^2 = 1$, the maximum of $|\epsilon_\tau|$ with respect to the parameter R_{12} practically coincides for $\delta = 0$ ($\delta = \pi k$, $k = 0, 1, ...$) and CP-violation due only to the Majorana phase α_{32}, with the maximum of $|Y_B|$. In the case of CP-violation generated only by the Dirac phase δ ($\alpha_{32} = 0$), the maximum of $|Y_B|$ occurs for $\beta_{23} = 0$ at slightly smaller value of $|R_{12}|$, namely at $|R_{12}| \cong 0.86$, compared to the value of $|R_{12}| \cong 0.92$ at which the maximum of $|\epsilon_\tau|$ takes place. We will work for convenience with R_{12} and R_{13} maximising $|\epsilon_\tau|$, for which we have simple analytic expressions in terms of the ratio $\Delta m_\odot^2/\Delta m_A^2$. In most of the cases we will discuss these values of R_{12} and R_{13} maximise also $|Y_B|$.

Consider next the wash-out parameters $\widetilde{m_l}$, $l = e, \mu, \tau$. In the case of real R_{12} and R_{13} we have $\beta_{23} = 0;\ \pi$ and $|R_{12}|^2 + |R_{13}|^2 = 1$. Taking into account this constraint we get from Eq. (57) [28, 63]:

$$\sqrt{\Delta m_\odot^2} \leq \widetilde{m_e} + \widetilde{m_\mu} + \widetilde{m_\tau} \leq \sqrt{\Delta m_A^2}. \tag{80}$$

The maximum and the minimum are reached respectively for $|R_{12}|^2 = 0$ and $|R_{13}|^2 = 0$, and in both cases $|\epsilon_\tau| = 0$. If $\beta_{23} = \pi/2;\ 3\pi/2$, we have $|R_{12}|^2 - |R_{13}|^2 = 1$ and for $\widetilde{m_e} + \widetilde{m_\mu} + \widetilde{m_\tau}$ we get the same lower bound as in Eq. (80), but no upper bound, in general, since $|R_{12}|^2$ is not limited from above. The requirement of successful leptogenesis leads to an upper bound roughly of the order of 1 eV. For $|R_{12}|^2 = (1 \overset{+}{_{(-)}} \sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_A^2})^{-1}$, corresponding to the maximum of r and of $|\epsilon_\tau|$, we find

$$\widetilde{m_e} + \widetilde{m_\mu} + \widetilde{m_\tau} = 2\sqrt{\Delta m_\odot^2}\left[1 \overset{+}{_{(-)}} \left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^{\frac{1}{2}}\right]^{-1} \cong 0.30\,(0.44)\sqrt{\Delta m_A^2}. \tag{81}$$

In what follows we will analyse the case of real R_{12} and R_{13}. For real R_{12} and R_{13} we have $\beta_{23} = 0;\ \pi$ corresponding to $\mathrm{sgn}(R_{12}R_{13}) = +1;\ (-1)$.

5.2.1 Leptogenesis due to Majorana CP-Violation in U_{PMNS}

We shall consider first the interesting possibility of CP-symmetry being violated by the Majorana phase α_{32} in U, and not by the Dirac phase δ. To be concrete, we choose $\delta = 0$ and *real* R_{12} and R_{13} ($\beta_{23} = 0;\ \pi$) which maximise r, $|\epsilon_\tau|$ and $|Y_B|$, i.e., $|R_{12}|^2 = (1 + \sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_A^2})^{-1} \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$. For $\alpha_{32} = 0;\ 2\pi$, the CP-symmetry is not violated. If α_{32} takes the CP-conserving value of $\alpha_{32} = \pi$, CP-symmetry is violated by R and $|\epsilon_\tau| \neq 0$. It should also be noted that the terms $\propto \sin\theta_{13}$ in the expressions for ϵ_τ and the wash-out mass parameters $\widetilde{m_l}$ are sub-dominant in the case being studied.

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With the choices for δ, R_{12} and R_{13} made we get using Eqs. (2), (52) and (56):

$$\left| \mathrm{Im}\, \left(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3}\right)\right| = c_{23}\, c_{13}\, (s_{23}c_{12} + c_{23}s_{12}s_{13}) \left|\sin \frac{\alpha_{32}}{2}\right| \cong \frac{1}{2}\, (c_{12} + s_{12}s_{13}) \left|\sin \frac{\alpha_{32}}{2}\right|, \tag{82}$$

$$\widetilde{m_\tau} = \sqrt{\Delta m_{\mathrm{A}}^2} \left| \left(\frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2}\right)^{\frac{1}{4}} |R_{12}|\, (s_{23}c_{12} + c_{23}s_{12}s_{13})\, e^{i\frac{\alpha_{32}}{2}} - \kappa\, |R_{13}|\, c_{23} \right|^2 \tag{83}$$

$$\cong \frac{1}{2} \frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{1 + \left(\frac{\Delta m_{\mathrm{A}}^2}{\Delta m_{\odot}^2}\right)^{\frac{1}{2}}} \left[1 + (c_{12} + s_{12}s_{13})(c_{12} + s_{12}s_{13} - 2\,\kappa\, \cos \frac{\alpha_{32}}{2})\right], \tag{84}$$

$$\widetilde{m_2} \equiv \widetilde{m_e} + \widetilde{m_\mu} = \frac{2\sqrt{\Delta m_{\mathrm{A}}^2}}{1 + \left(\frac{\Delta m_{\mathrm{A}}^2}{\Delta m_{\odot}^2}\right)^{\frac{1}{2}}} - \widetilde{m_\tau}, \tag{85}$$

where $\kappa \equiv e^{i\beta_{23}} = \mathrm{sgn}(R_{12}R_{13}) = \pm 1$ and we have set $s_{23} = c_{23} = 1/\sqrt{2}$, used $|R_{12}|$ and $|R_{13}|$ specified above and neglected the terms $\propto s_{13}^2$ in Eqs. (82) and (84).

It follows from Eqs. (49), (82) and (83) that in the case under discussion the following relations hold: $|\epsilon_\tau(\alpha_{32})| = |\epsilon_\tau(2\pi - \alpha_{32})|$, and $\widetilde{m_\tau}(\alpha_{32}, \beta_{23} = 0) = \widetilde{m_\tau}(2\pi - \alpha_{32}, \beta_{23} = \pi)$. As a consequence we have from Eq. (44): $|Y_B(\alpha_{32}, \beta_{23} = 0)| = |Y_B(2\pi - \alpha_{32}, \beta_{23} = \pi)|$. Thus, we will consider values of α_{32} in the interval $[0, 2\pi]$ and limit our discussion to $\beta_{23} = 0$ (i.e., $\kappa \equiv \mathrm{sgn}(R_{12}R_{13}) = 1$). After fixing the value of β_{23} the only free parameter left in the problem being studied is the Majorana phase α_{32}.

For $\alpha_{32} = 0$; 2π, obviously $|\epsilon_\tau| = 0$ and therefore $|Y_B| = 0$. We can have $|Y_B| = 0$ also even if $|\epsilon_\tau| \neq 0$, provided the efficiency factor in the expression for $|Y_B|$, Eq. (44), is zero, i.e., if the condition given in Eq. (61) is fulfilled. For the values of $|R_{12}|$ and $|R_{13}|$ considered and α_{32} having a value in the interval $[0, 2\pi]$, this condition is satisfied for $\alpha_{32} \cong 1.2\pi$. Thus, in this case we have $|\epsilon_\tau| \neq 0$ but $|Y_B| = 0$ for $\alpha_{32} \cong 1.2\pi$.

The absolute maximum of the baryon asymmetry $|Y_B|$ as a function of α_{32} is reached close to $\alpha_{32} \cong \pi/2$. We have at the absolute maximum for $s_{13} = 0\ (0.2)$:

$$|\epsilon_\tau| \cong 1.2 \times 10^{-8} \left(\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{0.05\ \mathrm{eV}}\right) \left(\frac{M_1}{10^9\ \mathrm{GeV}}\right), \tag{86}$$

$$|Y_B| \cong 2.0\,(2.2) \times 10^{-12} \left(\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{0.05\ \mathrm{eV}}\right) \left(\frac{M_1}{10^9\ \mathrm{GeV}}\right). \tag{87}$$

As we have already indicated, the values of $|R_{12}|$ and $|R_{13}|$ we have chosen in this analysis maximise not only ϵ_τ, but also $|Y_B|$. Thus, the observed baryon asymmetry having a value in the interval $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, can be reproduced for $M_1 \gtrsim 3.6 \times 10^{10}$ GeV. A second (local) maximum of $|Y_B|$ occurs close to $\alpha_{32} = 3\pi/2$ (at $\alpha_{32} \cong 1.7\pi$). At this maximum the value of $|Y_B|$ for $s_{13} = 0\ (0.2)$ is approximately by a factor of 3 (1.8) smaller than that at the absolute maximum in Eq. (87). Given the fact that the flavour

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effects of interest can be substantial for values of M_1 up to $M_1 \sim 10^{12}$ GeV, leptogenesis can be successful even for rather small values of the Majorana phase α_{32}, or more precisely, of $|\sin(\alpha_{32}/2)|$. If, for instance, $M_1 \sim 5 \times 10^{11}$ GeV, the observed baryon asymmetry can be generated for $|\sin(\alpha_{32}/2)| \cong 0.15$.

The results obtained in this subsection are illustrated in Figs. 1 and 2 we have already discussed, and in Fig. 3 where the dependence of the baryon asymmetry $|Y_B|$ on the Majorana phase α_{32} is shown for $M_1 \sim 5 \times 10^{10}$ GeV in the case analised above: $\delta = 0$, real R_{12} and R_{13} which maximise $|Y_B|$, i.e., $|R_{12}| \cong 0.92$ and $|R_{13}| \cong 0.39$, and $\beta_{23} = 0$, i.e., $\kappa \equiv \text{sgn}(R_{12}R_{13}) = 1$ (Fig. 3), and $\beta_{23} = \pi$, i.e., $\kappa = -1$ (Fig. 4). The figures have been obtained using the best fit values of the oscillation parameters Δm_A^2, $\Delta m_\odot^2$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$. Results for two values of $\sin \theta_{13}$ are presented: $\sin \theta_{13} = 0$; 0.20. Figure 3 shows, in particular, that in the case being studied, the predicted baryon asymmetry $|Y_B|$ exhibits very weak dependence on $\sin \theta_{13}$ for $\alpha_{32} \lesssim \pi$. If $\sin \theta_{13}$ has a value close to the existing upper limit, the effect of $\sin \theta_{13} \sim 0.2$ can be noticeable in the region of the local maximum of $|Y_B|$ at $\alpha_{32} \cong 1.7\pi$: it can lead to an increase of $|Y_B|$ by a factor of 1.7. For $M_1 = 10^{11}$ GeV, for instance, we can have successful leptogenesis for $\pi/4 \lesssim \alpha_{32} \lesssim 0.9\pi$, and, if $s_{13} \sim 0.2$, also for $1.4\pi \lesssim \alpha_{32} \lesssim 1.9\pi$. We note that, as Figs. 1 and 2 show, that the predicted value of $|Y_B|$ exhibits a relatively strong dependence on the elements of the matrix R.

It follows from the results obtained in the present subsection that as long as $|\sin(\alpha_{32}/2)|$ is not exceedingly small [10] and $M_1 \gtrsim 3.5 \times 10^{10}$ GeV, we can have successful leptogenesis even if $|s_{13} \sin \delta| = 0$ ($J_{\text{CP}} = 0$) and the only CP-violating parameter is the low energy Majorana phase α_{32}.

5.2.2 Dirac CP-Violation in U_{PMNS} and Leptogenesis

The next question we would like to address is under what conditions we could have a successful leptogenesis if the only CP-violating parameter is the Dirac phase in U, i.e., if, e.g., $\beta_{23} = 0, \pi$, the Majorana phase α_{32} takes a CP-conserving value and $\sin(\alpha_{32}/2) = 0$ [11], so that $\alpha_{32} = 2\pi k$, $k = 0, 1, ...$ We choose for concreteness again *real R_{12} and R_{13}* which for $\alpha_{32} = 0$ (2π) and $\beta_{23} = \pi$ (0) maximise $|\epsilon_\tau|$ and $|Y_B|$, i.e., $|R_{12}|^2 = (1 + \sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_A^2})^{-1} \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$. We will present results for the baryon asymmetry $|Y_B|$ also for the values of R_{12} and R_{13} which maximise $|Y_B|$ in the case of $\alpha_{32} = 0$ (2π) and $\beta_{23} = 0$ (π), $|R_{12}|^2 = 0.75$ and $|R_{13}|^2 = 0.25$. For $\delta = \pi q$, $q = 0, 1, 2, ...$, the CP-symmetry is not violated and $|\epsilon_\tau| = 0$.

For the chosen CP-conserving values of α_{23} and β_{23} we get from Eqs. (2), (52) and (56):

$$\left|\text{Im}\left(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3}\right)\right| = c_{23}^2\, s_{12}\, s_{13}\, |\sin \delta| \,, \tag{88}$$

---

[10] Obviously, α_{32} should have a value sufficiently different from the "special" one for which $|\eta(0.66\widetilde{m_\tau}) - \eta(0.71\widetilde{m_2})| = 0$.

[11] By imposing the condition $\sin(\alpha_{32}/2) = 0$ we exclude the possibility of α_{32} taking the CP-conserving values $\alpha_{32} = \pi, 3\pi$, in which case the term associated with the violation of CP-symmetry due to the matrix R will dominate in $|\text{Im}(U_{\tau 2}^* U_{\tau 3})|$, and correspondingly in $|\epsilon_\tau|$. If $\beta_{23} = \pi/2, 3\pi/2$, the indicated possibility would be avoided if α_{32} takes a CP-conserving value and $|\sin(\alpha_{32}/2)| = 1$.

$$
\begin{aligned}
\widetilde{m_\tau} &= \sqrt{\Delta m_{\text{A}}^2} \left| \left(\frac{\Delta m_\odot^2}{\Delta m_{\text{A}}^2} \right)^{\frac{1}{4}} |R_{12}| \left(s_{23}c_{12} + c_{23}s_{12}s_{13}\, e^{i\delta} \right) - \kappa' |R_{13}| c_{23}\, c_{13} \right|^2 \\
&\cong \frac{\sqrt{\Delta m_{\text{A}}^2}}{1 + \left(\frac{\Delta m_{\text{A}}^2}{\Delta m_\odot^2} \right)^{\frac{1}{2}}} \left| (c_{23}c_{13} - \kappa'\, s_{23}c_{12}) + c_{23}s_{12}s_{13}\, e^{i\delta} \right|^2, \quad \kappa' \equiv e^{i(\beta_{23} + \frac{\alpha_{32}}{2})} = \pm 1 \, ,
\end{aligned}
\tag{89}
$$

where in Eq. (89) we have used the values of $|R_{12}|$ and $|R_{13}|$ specified earlier. Taking into account that $|\text{Im}(e^{i\beta_{23}} U_{\tau 2}^* U_{\tau 3})| \cong 0.027 |s_{13} \sin\delta|$, we find from Eq. (79):

$$
|\epsilon_\tau| \cong 2.2 \times 10^{-9} \left(\frac{|s_{13} \sin\delta|}{0.20} \right) \left(\frac{\sqrt{\Delta m_{\text{A}}^2}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) .
\tag{90}
$$

Thus, the maximal asymmetry $|\epsilon_\tau|$ in the case of CP-violation due only to the Dirac phase δ in U_{PMNS} is approximately by a factor of 6 smaller than the maximal asymmetry due to violation of the CP-symmetry by the Majorana phase α_{32} of U_{PMNS}.

Given $\widetilde{m_\tau}$, one can determine $\widetilde{m_2}$ from Eq. (85). It follows from Eqs. (89) and (90) that $|Y_B(\delta)| = |Y_B(2\pi - \delta)|$. One has to analyze the cases of $\kappa' \equiv e^{i(\alpha_{32}/2 + \beta_{23})} = +1$ ($\alpha_{32}/2 + \beta_{23} = 2\pi k$, $k = 0, 1, ...$) and $\kappa' = -1$ ($\alpha_{32}/2 + \beta_{23} = \pi(2k + 1)$, $k = 0, 1, 2, ...$) separately.

For $\kappa' = -1$ we find $\widetilde{m_\tau} \cong 0.25 \sqrt{\Delta m_{\text{A}}^2} > \widetilde{m_2} \cong 0.05 \sqrt{\Delta m_{\text{A}}^2}$. For the values of the parameters employed in this analysis (chosen, in particular, to maximise $|\epsilon_\tau|$ and $|Y_B|$), $\widetilde{m_\tau}$ and $\widetilde{m_2}$ exhibit weak dependence on $\sin\theta_{13}$ (and therefore on δ), which can be neglected. This implies that the maximum of the baryon asymmetry $|Y_B|$ as a function of the Dirac phase δ, will take place at values of $\delta = (\pi/2)(2k + 1)$, $k = 0, 1, ...$, for which $|\epsilon_\tau|$ also has a maximum. Using Eq. (45) to calculate the relevant efficiency factors $\eta(0.66\widetilde{m_\tau})$ and $\eta(0.71\widetilde{m_2})$, we get from Eq. (44):

$$
|Y_B| \cong 2.8 \times 10^{-13} |\sin\delta| \left(\frac{s_{13}}{0.2} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) .
\tag{91}
$$

The asymmetry of interest is predominantly in the lepton number $L_e + L_\mu$. Thus, in order to reproduce the observed baryon asymmetry, taken to lie in the interval $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, $s_{13} |\sin\delta|$ and M_1, in the case analised, should satisfy

$$
2.9 \lesssim |\sin\delta| \left(\frac{s_{13}}{0.2} \right) \left(\frac{M_1}{10^{11} \text{ GeV}} \right) \lesssim 3.3 .
\tag{92}
$$

Given that $s_{13} |\sin\delta| \lesssim 0.2$, the lower bound in this inequality can be satisfied only for $M_1 \gtrsim 2.9 \times 10^{11}$ GeV. Recalling that the flavour effects in leptogenesis of interest are fully developed for $M_1 \lesssim 5 \times 10^{11}$ GeV, we obtain a *lower bound on the values of $|s_{13} \sin\delta|$ and s_{13} for which we can have successful leptogenesis in the case considered*:

$$
|\sin\theta_{13} \sin\delta| \gtrsim 0.11 , \quad \sin\theta_{13} \gtrsim 0.11 .
\tag{93}
$$

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The lower limit (93) corresponds to

$$|J_{\rm CP}| \gtrsim 2.4 \times 10^{-2}\,, \tag{94}$$

where we have used the best fit values of $\sin 2\theta_{12}$ and $\sin 2\theta_{23}$. Values of s_{13} in the range given in Eq. (93) can be probed in the forthcoming Double CHOOZ [65] and future reactor neutrino experiments [66]. CP-violation effects with magnitude determined by $|J_{\rm CP}|$ satisfying (94) are within the sensitivity of the next generation of neutrino oscillation experiments, designed to search for CP- or T- symmetry violations in the oscillations [67]. Actually, since in the case under discussion the wash-out factor $|\eta_B| \equiv |\eta(0.66\widetilde{m_\tau}) - \eta(0.71\widetilde{m_2})|$ in the expression for $|Y_B|$ practically does not depend on s_{13} and δ, while both $|Y_B| \propto |s_{13}\sin\delta|$ and $|J_{\rm CP}| \propto |s_{13}\sin\delta|$, there is a direct relation between $|Y_B|$ and $|J_{\rm CP}|$ for given neutrino oscillation parameters, R_{12}, R_{13} and M_1:

$$\frac{|Y_B|}{M_1/(10^{11}\ {\rm GeV})} \cong 3.0 \times 10^{-8}\,|\eta_B|\,|J_{\rm CP}| \cong 1.3 \times 10^{-9}\,|J_{\rm CP}|\,, \tag{95}$$

where we have used the best fit values of the neutrino oscillation parameters, $|R_{12}| = 0.92$, $|R_{13}| = 0.39$ and $\kappa' = -1$.

For $\kappa' = +1$, the maximum of $|Y_B|$ as a function of $|R_{12}|$ ($|R_{13}|$) takes place in the case being investigated at $|R_{12}| = 0.86$ ($|R_{13}| = 0.51$) and we will employ this value of $|R_{12}|$ ($|R_{13}|$) in the remaining part of the current analysis. If $\kappa' = +1$, a strong cancellation between the terms in the round bracket in Eq. (89) takes place and we have typically $\widetilde{m_\tau} \ll \widetilde{m_2}$. Indeed, for $s_{23} = c_{23} = 1/\sqrt{2}$, $s_{12} = 0.2$ and $\delta = \pi/2$ one finds $\widetilde{m_\tau} \cong 2.5 \times 10^{-2}\sqrt{\Delta m_{\rm A}^2}$ and $\widetilde{m_2} \cong 0.28\sqrt{\Delta m_{\rm A}^2}$. Using Eq. (45) to calculate the corresponding efficiency factors, we get from Eq. (47) for the baryon asymmetry:

$$|Y_B| \cong 3.6\ (0.38) \times 10^{-13}\,|\sin\delta|\ \left(\frac{M_1}{10^9\ {\rm GeV}}\right)\,. \tag{96}$$

The numerical factor in the round brackets corresponds to the value of $|R_{12}| = 0.92$ ($|R_{13}| = 0.39$) which maximises $|\epsilon_\tau|$, but does not maximise $|Y_B|$.

The result we get for the maximal value of $|Y_B|$, i.e., for $|R_{12}| = 0.86$ ($|R_{13}| = 0.51$), are quite similar to those we have obtained in the case of $\kappa' = -1$, the only difference being that now the generated lepton asymmetry is predominantly in the tau lepton charge. The observed baryon asymmetry lying in the interval $8.0 \times 10^{-11} \lesssim |Y_B| \lesssim 9.2 \times 10^{-11}$, can be reproduced if $M_1 \gtrsim 2.2 \times 10^{11}$ GeV. For the flavour effects to fully develop one must have $M_1 \lesssim 5 \times 10^{11}$ GeV, which, together with the requirement of successful leptogenesis, implies

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \quad \sin\theta_{13} \gtrsim 0.09\,. \tag{97}$$

This lower limit corresponds to

$$|J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}\,, \tag{98}$$

where we have used again the best fit values of $\sin 2\theta_{12}$ and $\sin 2\theta_{23}$. The ranges of values of $\sin\theta_{13}$ and of $|J_{\rm CP}|$ we find in the case being studied are also within the sensitivity

respectively of the planned θ_{13} reactor neutrino experiments [65, 66] and of the neutrino oscillation experiments on CP- (T-) violation [67].

It is interesting to note that, as it follows from Eq. (96), for $|R_{12}| = 0.92$ and $|R_{13}| = 0.39$, which maximise $|\epsilon_\tau|$ but do not maximise $|Y_B|$, the predicted baryon asymmetry $|Y_B|$ is smaller by approximately one order of magnitude in spite of the fact that the indicated values of $|R_{12}|$ and $|R_{13}|$ are rather close to the values $|R_{12}| = 0.86$, $|R_{13}| = 0.51$ which maximise $|Y_B|$. As $|R_{12}|$ increases beyond 0.86, $|Y_B|$ rapidly decreases, as is also clearly seen in Fig. 1. Actually, for $|R_{12}| = 0.92$ and $|R_{13}| = 0.39$, the observed baryon asymmetry cannot be reproduced for $M_1 \lesssim 10^{12}$ GeV and $\sin\theta_{13} \leq 0.20$: we get $|Y_B| \lesssim 4 \times 10^{-11}$. Figures 1 and 2 show also that for real R_{12} and R_{13}, satisfying $|R_{12}|^2 + |R_{13}|^2 = 1$, and CP-violation generated only by the Dirac phase in U_{PMNS}, we can have successful leptogenesis if both $\sin\theta_{13}$ and $|R_{12}|$ have relatively large values: the observed baryon asymmetry cannot be reproduced if $|R_{12}| \lesssim 0.6$ and/or $\sin\theta_{13} \ll 0.1$.

The results obtained in the present subsection are illustrated in Figs. 1 and 2, as well in Fig. 5. In Fig. 5 we show the dependence of $|Y_B|$ on the Dirac phase δ which was varied in the interval $[0, 2\pi]$, for $s_{13} = 0.1$; 0.2, $M_1 = 5 \times 10^{11}$ GeV and for $\kappa' = +1$ and $\kappa' = -1$. The correlation between the rephasing invariant J_{CP} which controls the magnitude of the CP-violation effects in neutrino oscillations and the baryon asymmetry Y_B is illustrated in Fig. 6 for $s_{13} = 0.2$, $M_1 = 5 \times 10^{11}$ GeV and $\kappa' = +1$. Both figures are obtained for real R_{12} and R_{13} which maximise $|Y_B|$.

In conclusion of the present subsection we note that one can treat in a similar way the alternative possibility of $\beta_{23} = \pi/2$ $(3\pi/2)$. In this case we have $\epsilon_\tau \propto \mathrm{Re}(U_{\tau 2}^* U_{\tau 3})$. The CP symmetry will be violated at low energies if both $\mathrm{Re}(U_{\tau 2}^* U_{\tau 3}) \neq 0$ and $\mathrm{Im}(U_{\tau 2}^* U_{\tau 3}) \neq 0$. Maximal asymmetry $|\epsilon_\tau|$ is obtained for $\cos(\alpha_{32}/2) = \pm 1$, and, if s_{13} is non-negligible, for $\cos(\delta - \alpha_{32}/2) = \pm 1$. These conditions are satisfied for the CP-conserving values of α_{32} and δ: $\alpha_{32} = 0$; 2π, $\delta = 0$; π. The CP-symmetry is broken by the matrix R $(\beta_{23} = \pi/2$ $(3\pi/2))$. It is easy to convince oneself that the expression for the asymmetry $|\epsilon_l|$, $l = e, \mu, \tau$, for $\beta_{23} = \pi/2$ or $3\pi/2$, $\alpha_{32} = 0$ (2π) and $\delta = 0(\ \pi)$, coincides with the expression for the same asymmetry in the case respectively of $\beta_{23} = 0$ or π, $\alpha_{32} = \pi$ $(3\pi/2)$ and $\delta = 0$ (2π). Although, for $\beta_{23} = \pi/2$; $3\pi/2$ we also have $\max(r) \cong 1.2$, the maximum of r corresponds to $|R_{12}|^2 = (1 - \sqrt{\Delta m_\odot^2}/\sqrt{\Delta m_A^2})^{-1} \cong 1.22$, $|R_{13}|^2 = |R_{12}|^2 - 1 \cong 0.22$. Therefore the wash-out factors $\widetilde{m_l}$ will differ from those in the case of $\beta_{23} = 0$; π and maximal asymmetry $|\epsilon_\tau|$. The values of $|R_{12}|$ and $|R_{13}|$ which maximise $|\epsilon_\tau|$ are not guaranteed to maximise also the baryon asymmetry $|Y_B|$. Further investigation of this case is, however, outside the scope of the present study.

5.3   Inverted Hierarchical Light Neutrino Mass Spectrum

Given the inequalities $m_3 \ll m_1 < m_2$, it seems natural to assume that the terms $\propto \sqrt{m_3}$ are sufficiently small and give negligible contributions to the lepton flavour asymmetries ϵ_l and to the wash-out mass parameters $\widetilde{m_l}$. In the "one-flavour" approximation this case was studied in [28]. It was found that the lepton asymmetry is strongly suppressed by the factor

$\Delta m_\odot^2 / \Delta m_A^2$ and leptogenesis can produce the observed value of the baryon asymmetry only if the lightest heavy Majorana neutrino N_1 has a relatively large mass: $M_1 \gtrsim 7 \times 10^{12}$ GeV. Using Eq. (46) and the fact that in the case of interest we have $m_{1,2} \cong \sqrt{\Delta m_A^2}$ and $(m_2 - m_1) \cong \Delta m_\odot^2 / (2\sqrt{\Delta m_A^2})$, it is not difficult to obtain the conditions under which the indicated terms $\propto \sqrt{m_3}$ in ϵ_l can be neglected. The latter depend on whether the $R_{11} R_{12} \equiv e^{i\beta_{12}} |R_{11} R_{12}|$ is real or purely imaginary. In the case of $\beta_{12} \equiv \widetilde{\beta_{11}} + \widetilde{\beta_{12}} = \pi q$, $q = 0, 1, 2, ...$, we get:

$$2 \left(\frac{m_3}{\sqrt{\Delta m_\odot^2}} \right)^{\frac{1}{2}} \left(\frac{\Delta m_A^2}{\Delta m_\odot^2} \right)^{\frac{3}{4}} \frac{|R_{13}|}{|R_{12(11)}|} \ll 1 \,, \quad \beta_{12} = \pi q, \ q = 0, 1, 2, ... \tag{99}$$

Since $2(\Delta m_A^2 / \Delta m_\odot^2)^{\frac{3}{4}} \cong 26.4 \gg 1$, this inequality suggests two possibilities.

i) Equation (99) holds and the terms $\propto \sqrt{m_3}$ in ϵ_l and $\widetilde{m_l}$ are indeed negligible. The simplest realisation of this possibility corresponds to [28] setting $R_{13} = 0$, which in turn implies the decoupling of the heaviest RH Majorana neutrino N_3.

ii) The alternative possibility is that terms $\propto \sqrt{m_3}$ in ϵ_l and $\widetilde{m_l}$ are dominant in spite of the fact that $m_3 \ll m_1, m_2$. This would require the ratio in the left-hand side in Eq. (99) to be much bigger than 1. A possible simple realisation corresponds to setting $R_{11} = 0$ or $R_{12} = 0$. Since in the latter case $|\epsilon_l| \propto \sqrt{m_3 / m_2}$, the asymmetry $|\epsilon_l|$ will not be suppressed only if m_3 is sufficiently large. The latter condition is satisfied for values of m_3 in the interval $10^{-2} \sqrt{\Delta m_\odot^2} \lesssim m_3 \lesssim 0.5 \sqrt{\Delta m_\odot^2}$, for which we still have $m_3 \ll m_{1,2}$.

If, however, $\beta_{12} = \pi/2(2q+1)$, $q = 0, 1, 2, ...$, i.e., if $R_{11} R_{12} = \pm i \, |R_{11} R_{12}|$, we obtain a very different condition:

$$\left(\frac{m_3}{\sqrt{\Delta m_A^2}} \right)^{\frac{1}{2}} \frac{|R_{13}|}{|R_{12(11)}|} \ll 1 \,, \quad \beta_{12} = (\pi/2)(2q+1), \ q = 0, 1, 2, ... \tag{100}$$

For $m_3 \ll m_{1,2,}$, this condition can be naturally satisfied if $|R_{13}|$ is sufficiently small and, in particular, if $|R_{13}| = 0$.

5.3.1 The case of Real $R_{11} R_{12}$ and N_3 Decoupling $(R_{13} = 0)$

The terms $\propto \sqrt{m_3}$ in ϵ_l and $\widetilde{m_l}$ are negligible and we get:

$$\epsilon_l \simeq \frac{3 M_1 \sqrt{\Delta m_A^2}}{32 \pi v^2} \left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right) \left(\frac{\Delta m_\odot^2}{\Delta m_A^2} \right)^{\frac{1}{4}} \frac{|R_{11} R_{12}|}{|R_{11}|^2 + |R_{12}|^2} \, \text{Im} \left(e^{i\beta_{12}} U_{l1}^* U_{l2} \right) \,, \quad l = e, \mu, \tau \,, \tag{101}$$

where $\beta_{12} \equiv \arg(R_{11} R_{12})$. The phase β_{12} parametrises the effect of CP-violation due to the matrix R in the asymmetries ϵ_l. In the case of CP-invariance, $\beta_{12} = 0$ or $\pi/2$ depending on whether $\text{Im}(U_{l1}^* U_{l2}) = 0$ or $\text{Re}(U_{l1}^* U_{l2}) = 0$, respectively, and $\epsilon_l = 0$, $l = e, \mu, \tau$. Note that the asymmetries ϵ_l are suppressed by the factor $\Delta m_\odot^2 / (2\Delta m_A^2) \cong 1.6 \times 10^{-2}$ with respect to the asymmetries ϵ_l we have obtained in the case of NH light neutrino mass spectrum.

From Eq. (2) we get for the quantity $\mathrm{Im}(e^{i\beta_{12}}U^*_{\tau 1}U_{\tau 2})$ of interest:

$$\mathrm{Im}\left(e^{i\beta_{12}}U^*_{\tau 1}U_{\tau 2}\right) = -\mathrm{Im}\left[e^{i\beta_{12}}e^{i\frac{\alpha_{21}}{2}}\left(c_{12}s_{23}+s_{12}c_{23}s_{13}e^{i\delta}\right)\left(s_{12}s_{23}-c_{12}c_{23}s_{13}e^{-i\delta}\right)\right],$$

(102)

where α_{21} is the Majorana phase which enters also into the expression for $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$-decay. The wash-out mass parameters $\widetilde{m_l}$ for the three lepton asymmetries read [28]:

$$\widetilde{m_l} \cong \sqrt{\Delta m_{\mathrm{A}}^2}\,|R_{11}U^*_{l1}+R_{12}U^*_{l2}|^2\,,\quad l=e,\mu,\tau\,.$$

(103)

Using the unitarity of the PMNS matrix U we obtain:

$$\widetilde{m_e}+\widetilde{m_\mu}+\widetilde{m_\tau} = \sqrt{\Delta m_{\mathrm{A}}^2}\left(|R_{11}|^2+|R_{12}|^2\right)\,.$$

(104)

In what follows we limit our discussion to the case of real R_{11} and R_{12}. In this case we have $|R_{11}|^2+|R_{12}|^2=1$, and $\beta_{12}=0;\ \pi$ which correspond to $\mathrm{sgn}(R_{11}R_{12})=+1;\ (-1)$. Obviously, the values of $|R_{11}|=|R_{12}|=1/\sqrt{2}$ maximise the asymmetries ϵ_l. As can be shown, for generic values of the phases α_{21} and δ, these values of $|R_{11}|$ and $|R_{12}|$ maximise also $|Y_B|$. Now we have

$$\widetilde{m_e}+\widetilde{m_\mu}+\widetilde{m_\tau} = \sqrt{\Delta m_{\mathrm{A}}^2}\,.$$

(105)

Using the best values of the neutrino oscillation parameters $\sin^2 2\theta_{23}$, $\sin^2\theta_{12}$, $\Delta m_\odot^2$ and Δm_{A}^2, given in Eq. (5) and the upper limit $s_{13}\le 0.2$, we get maximal $|\epsilon_\tau|$ and $|Y_B|$ as can be shown, for $\alpha_{21}=\pi/2$, $\delta=\pi$ and $\beta_{12}=0$:

$$|\epsilon_\tau| \cong 1.5\times 10^{-10}\left(\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{0.05\ \mathrm{eV}}\right)\left(\frac{M_1}{10^9\ \mathrm{GeV}}\right),$$

(106)

$$|Y_B| \cong 2.2\times 10^{-14}\left(\frac{\sqrt{\Delta m_{\mathrm{A}}^2}}{0.05\ \mathrm{eV}}\right)\left(\frac{M_1}{10^9\ \mathrm{GeV}}\right).$$

(107)

Clearly, for $M_1 \lesssim 10^{12}$ GeV for which the flavour effects in leptogenesis can be substantial, it is impossible to reproduce the observed baryon asymmetry in the case of IH light neutrino mass spectrum, $R_{13}=0$ and real R_{11} and R_{12}, $|R_{11}|^2+|R_{12}|^2=1$. The main reason for this result lies in the fact that the lepton asymmetries ϵ_l are suppressed by the factor $\Delta m_\odot^2/(2\Delta m_{\mathrm{A}}^2) \cong 1.6\times 10^{-2}$, while the wash-out effects are rather large owning to the constraint (105).

5.3.2 Generating Y_B Compatible with the Observations

Consider next the case of $\beta_{12}=(\pi/2)(2q+1)$, $q=0,1,2,....$ Now the product $R_{11}R_{12}$ is purely imaginary: $R_{11}R_{12}=i\kappa|R_{11}R_{12}|$, $i\kappa\equiv e^{i\beta_{12}}$, $\kappa=\pm 1$. We shall assume further that condition (100) holds and thus the terms $\propto\sqrt{m_3}$ in ϵ_l and $\widetilde{m_l}$ are negligible. A simple

realisation of this scenario corresponds to $R_{13} = 0$ (N_3 decoupling), and to $|R_{11}|^2 - |R_{12}|^2 = 1$. Under the indicated conditions we have:

$$|\epsilon_\tau| \simeq \frac{3 M_1 \sqrt{\Delta m_A^2}}{16 \pi v^2} \frac{2 |R_{11} R_{12}|}{|R_{11}|^2 + |R_{12}|^2} |\mathrm{Re}\,(U_{\tau 1}^* U_{\tau 2})| , \quad l = e, \mu, \tau , \tag{108}$$

where

$$|\mathrm{Re}\,(U_{\tau 1}^* U_{\tau 2})| \cong \left| c_{12} s_{12} s_{23}^2 \cos \frac{\alpha_{21}}{2} - c_{23} s_{23} s_{13} \left[c_{12}^2 \cos \left(\frac{\alpha_{21}}{2} - \delta \right) - s_{12}^2 \cos \left(\frac{\alpha_{21}}{2} + \delta \right) \right] \right| , \tag{109}$$

and we have neglected the term $\propto s_{13}^2 c_{12} s_{12} c_{23}^2$ in the expression for $|\mathrm{Re}(U_{\tau 1}^* U_{\tau 2})|$. The maximal value of the factor $r' \equiv 2|R_{11} R_{12}|/(|R_{11}|^2 + |R_{12}|^2)$ is $r' = 1$ and is reached for $|R_{11}|^2 \gg 1$. However, even for $|R_{11}|^2 = 1.5$ and $|R_{12}|^2 = 0.5$, r' is rather close to its maximal value: $r' = \sqrt{3}/2 \cong 0.87$. In the case of purely Majorana or Dirac CP-violation from the PMNS matrix we obtain:

$$|\mathrm{Re}\,(U_{\tau 1}^* U_{\tau 2})| \cong s_{23} (c_{12} s_{12} s_{23} \pm s_{13} c_{23} \cos 2\theta_{12}) \left| \cos \frac{\alpha_{21}}{2} \right| , \quad \delta = 0, \pi, 2\pi , \tag{110}$$

$$|\mathrm{Re}\,(U_{\tau 1}^* U_{\tau 2})| \cong s_{13} c_{23} s_{23} |\sin \delta| , \quad \cos \frac{\alpha_{21}}{2} = 0 . \tag{111}$$

A. Majorana CP-Violation from U_{PMNS}

In what follows we first set $s_{13} = 0$. The CP-symmetry is violated by the Majorana phase α_{21} only. Both $|\epsilon_\tau|$ and $|Y_B|$ vanish for $\alpha_{21} = \pi(2q + 1)$, $q = 0, 1, ...$ The wash-out parameters $\widetilde{m_\tau}$ and $\widetilde{m_2}$ are given by

$$\widetilde{m_\tau} \cong \sqrt{\Delta m_A^2} \left[s_{12}^2 s_{23}^2 |R_{11}|^2 + c_{12}^2 s_{23}^2 |R_{12}|^2 + 2 \kappa c_{12} s_{12} s_{23}^2 |R_{11} R_{12}| \sin \frac{\alpha_{21}}{2} \right] , \tag{112}$$

$$\widetilde{m_2} = \sqrt{\Delta m_A^2} \left(|R_{11}|^2 + |R_{12}|^2 \right) - \widetilde{m_\tau} . \tag{113}$$

Obviously, we have $|\epsilon_\tau(\alpha_{21})| = |\epsilon_\tau(2\pi - \alpha_{21})|$, $\widetilde{m_\tau}(\alpha_{21}) = \widetilde{m_\tau}(2\pi - \alpha_{21})$, and therefore $|Y_B(\alpha_{21})| = |Y_B(2\pi - \alpha_{21})|$. Numerical calculations we have performed show that the maximal value of the baryon asymmetry $|Y_B|$ in the case under discussion corresponds to $|R_{11}|^2 \cong 1.1 - 1.4$ (see Fig. 7) and we will use these values of $|R_{11}|^2$ and the corresponding values of $|R_{12}|^2 = 0.1 - 0.4$, in our further analysis. The results we obtain for $|Y_B|$ depend strongly on whether $\kappa = +1$ or $\kappa = -1$.

For $\kappa = +1$ and $|R_{11}|^2 \cong 1.1 - 1.4$, one has $\widetilde{m_\tau}, \widetilde{m_2} \gg 2 \times 10^{-3}$ eV, which corresponds to a strong wash-out regime. The baryon asymmetry is maximal for $\alpha_{21} = 2\pi q$, $q = 0, 1, ...,$ for which both $|\epsilon_\tau|$ and the efficiency factor $|\eta_B| \equiv |\eta(0.66 \widetilde{m_\tau}) - \eta(0.71 \widetilde{m_2})|$ are maximal [12] ($|\cos(\alpha_{21}/2)| = 1$). Numerical studies show that the absolute maximum of $|Y_B|$ for $\kappa = +1$ is reached at $|R_{11}|^2 \cong 1.1$ (Figs. 7 and 8).

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[12]Note that for $\alpha_{21} = 2\pi q$, $q = 0, 1, ...,$ and $s_{13} = 0$ we have $\mathrm{Im}(U_{\tau 1}^* U_{\tau 2}) = 0$ and $J_{\mathrm{CP}} = 0$. Correspondingly, the CP-invariance is not violated by α_{21} and δ; it is violated at high energies by the matrix R.

In the case of $\kappa = -1$ and values of $|R_{11}|^2$ indicated above, which maximise $|Y_B|$, there is a strong partial compensation between the three terms in $\widetilde{m_\tau}$. As a consequence, the efficiency factor $|\eta_B|$ and the asymmetry $|Y_B|$ are for, e.g., $\alpha_{21} = \pi/2$ and the values of $|R_{11}|^2$ which maximise $|Y_B|$, approximately by a factor of 5 bigger than the values they have in the case of $\kappa = +1$. Moreover, the maximum of $|Y_B|$ takes place at $|R_{11}|^2 \cong 1.4$ and $\alpha_{21} \cong 2\pi/3;\ 4\pi/3$ (Figs. 7 and 9), rather than at $|R_{11}|^2 \cong 1.1$ and $\alpha_{21} = 2\pi q,\ q = 0, 1, ...$ In both cases of $\kappa = +1$ and $\kappa = -1$ we have $|Y_B| = 0$ for $\alpha_{21} = \pi(2q + 1)$.

We get for the maximal baryon asymmetry (i.e., at $|R_{11}|^2 \cong 1.1$ and $\alpha_{21} = 0;\ 2\pi$ for $\kappa = +1$, and at $|R_{11}|^2 \cong 1.4$ and $\alpha_{21} \cong 2\pi/3;\ 4\pi/3$ for $\kappa = -1$, see Figs. 7 - 9):

$$|Y_B| \cong 1.5\ (0.5) \times 10^{-12} \left(\frac{\sqrt{\Delta m_A^2}}{0.05\ \mathrm{eV}} \right) \left(\frac{M_1}{10^9\ \mathrm{GeV}} \right), \quad \kappa = -1\ (\kappa = +1). \tag{114}$$

It follows from the preceding analysis that the observed value of the baryon asymmetry $|Y_B| \gtrsim 8 \times 10^{-11}$ can be reproduced in the case being studied for $M_1 \gtrsim 5.3 \times 10^{10}$ GeV if $\kappa = -1$, and for $M_1 \gtrsim 1.6 \times 10^{11}$ GeV when $\kappa = +1$. Since both the baryon asymmetry $|Y_B|$ and the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay, $|\langle m \rangle|$ depend on the Majorana phase α_{21}, for given values of the other parameters there exists a direct correlation between the values of $|Y_B|$ and $|\langle m \rangle|$. The latter is illustrated in Fig. 10.

B. Dirac CP-Violation from U_{PMNS} ($\alpha_{21} = \pi$)

One can treat in a similar manner the case of $s_{13} \neq 0$ and CP-violation generated only by the Dirac phase δ in U_{PMNS}, $\alpha_{21} = \pi(2k + 1)$, $k = 0, 1, ...$. We have $e^{i\alpha_{21}/2} = i\kappa'$, where $\kappa' = 1$ for $k = 0, 2, 4, ...$, and $\kappa' = -1$ if $k = 1, 3, ...$. It follows from eqs. (108) and (110) that in this case the asymmetry $|\epsilon_\tau| \propto |s_{13} \sin \delta|$ and one can expect the baryon asymmetry $|Y_B|$ to be suppressed by s_{13}. Obviously, we have $|\epsilon_\tau| = 0$ and $|Y_B| = 0$ for $\delta = 0;\ \pi;\ 2\pi$. For the wash-out mass parameters $\widetilde{m_\tau}$ and $\widetilde{m_2}$ we get:

$$\widetilde{m_\tau} \cong \sqrt{\Delta m_A^2}\ \left| s_{23}\ (s_{12}\ |R_{11}| - \kappa\ \kappa'\ c_{12}\ |R_{12}|) - c_{23}\ s_{13}\ e^{-i\delta}\ (c_{12}\ |R_{11}| + \kappa\ \kappa'\ s_{12}\ |R_{12}|) \right|^2,$$
$$\tag{115}$$

and $\widetilde{m_2} = \sqrt{\Delta m_A^2}(|R_{11}|^2 + |R_{12}|^2) - \widetilde{m_\tau}$. The two possibilities $\kappa\kappa' = -1$ and $\kappa\kappa' = +1$ lead to drastically different results.

For $\kappa\kappa' = -1$, the term with the factor s_{23} in the expression for $\widetilde{m_\tau}$ in Eq. (115) gives the dominant contribution and determines the magnitude of $\widetilde{m_\tau}$. For, e.g., $\delta = \pi/2$ (for which $|\epsilon_\tau|$ is maximal), the maximum of the baryon asymmetry $|Y_B|$ takes place at $|R_{11}| \cong 1.05$ ($|R_{12}| \cong 0.32$). For these values of δ and $|R_{11}|$, both wash-out mass parameters satisfy $\widetilde{m_{\tau,2}} \gtrsim 2 \times 10^{-2}$ eV. Thus, the baryon asymmetry is generated in the strong wash-out regime. Correspondingly, the efficiency factor $|\eta_B|$ is relatively small, $|\eta_B| \lesssim 6 \times 10^{-3}$. The observed value of the baryon asymmetry $|Y_B| \cong (8.0 - 9.2) \times 10^{-11}$ can be reproduced only if $s_{13} \cong 0.2$ and $M_1 \cong (4 - 5) \times 10^{11}$ GeV; for $s_{13} \cong 0.1$, this requires a value of $M_1 \cong 8 \times 10^{11}$ GeV.

We obtain completely different results in the case of $\kappa\kappa' = +1$. In this case there can be a deep mutual compensation between the two terms in the bracket multiplied by s_{23}

in the right-hand side of Eq. (115) and we can have $\widetilde{m_\tau} \cong (1.5 - 2.0) \times 10^{-3}$ eV for the values of $|R_{11}|$ ($|R_{12}|$) which maximise the baryon asymmetry $|Y_B|$. Correspondingly, $|Y_B|$ can be generated in the weak wash-out regime and can be much larger than in the case of $\kappa\kappa' = -1$.

More specifically, the maximum of $|Y_B|$ with respect to the Dirac phase δ and $|R_{11}|$ ($|R_{12}|$) takes place approximately at (or relatively close to) $\delta \cong \pi/2$; $3\pi/2$ for any $s_{13} \lesssim 0.2$ of interest, and at $|R_{11}| \cong 1.30$; 1.60 ($|R_{12}| \cong 0.83$; 1.25) for $s_{13} = 0.20$; 0.10, respectively (Figs. 11 and 12); for $s_{13} \lesssim 0.02$ it is located at $|R_{11}| \cong 1.07$ ($|R_{12}| \cong 0.38$). We have used again the best fit values of Δm_A^2, $\sin 2\theta_{12}$ and $\sin 2\theta_{23}$ to obtain the location of the maxima in $|R_{11}|$. In the case of $s_{13} = 0.10$ there exists a second local maximum of $|Y_B|$ at $|R_{11}| \cong 1.15$ ($|R_{12}| \cong 0.57$), at which $|Y_B|$ is approximately by a factor of 1.4 smaller than at the absolute maximum at $|R_{11}| \cong 1.60$ (see Fig. 11). The positions of the indicated maxima of $|Y_B|$ are determined essentially by the position of the absolute maximum of the efficiency factor $|\eta_B| \cong |\eta(0.66\widetilde{m_\tau}) - \eta(0.71\widetilde{m_2})|$. The latter corresponds approximately to $\widetilde{m_\tau} \cong 1.5 - 1.8 \times 10^{-3}$ eV and negligible [13] $|\eta(0.71\widetilde{m_2})|$. At the maximum, $|\eta_B| \cong 6.7 \times 10^{-2}$. In the case of $s_{13} \cong 0.20$, as can be shown, $\widetilde{m_\tau} \gtrsim 2.3 \times 10^{-3}$ eV and the minimal value of $\widetilde{m_\tau}$ corresponds to negligible $s_{23}^2(s_{12}|R_{11}| - c_{12}|R_{12}|)^2 \cong 0$. The latter condition is fulfilled approximately for $|R_{11}|^2 \cong c_{12}^2/\cos 2\theta_{12} \cong 1.75$ and $|R_{12}|^2 \cong s_{12}^2/\cos 2\theta_{12} \cong 0.75$, which is very close to the value $|R_{11}|^2 \cong 1.69$ obtained by numerical calculations. Thus, for $s_{13} \cong 0.20$ we have for $\widetilde{m_\tau}$ which maximises $|\eta_B|$ ($|Y_B|$): $\widetilde{m_\tau} \cong \sqrt{\Delta m_A^2}\, c_{23}^2 s_{13}^2(c_{12}|R_{11}| + s_{12}|R_{12}|)^2 \cong (c_{23}^2 s_{13}^2/\cos 2\theta_{12})\sqrt{\Delta m_A^2}$.

In the case of $s_{13} \cong 0.10$, both terms in Eq. (115) "conspire" to produce $\widetilde{m_\tau} \cong 1.75 \times 10^{-3}$ eV, and thus maximal $|\eta_B|$ (and $|Y_B|$), at $|R_{11}| \cong 1.60$. The second local maximum of $|Y_B|$ at $|R_{11}| \cong 1.15$ corresponds to $\widetilde{m_\tau} \cong 10^{-3}$ eV. Again both terms in Eq. (115) contribute, the term $\propto c_{23}^2 s_{13}^2$ being approximately by a factor 1.5 smaller than the term $\propto s_{23}^2$. In contrast, the local minimum of $|Y_B|$ at $|R_{11}| \cong 1.28$ (Fig. 11) is associated with $\widetilde{m_\tau} \cong 6 \times 10^{-4}$ eV. In this case the contribution of the term $\propto s_{23}^2$ in $\widetilde{m_\tau}$ is negligible.

Finally, for $s_{13} \lesssim 0.02$, the term with the factor $\propto c_{23}s_{13}$ in the expression for $\widetilde{m_\tau}$, Eq. (115), plays no role in the determination of the maxima of $|\eta_B|$ and $|Y_B|$. Thus, in this case, in particular, $|Y_B|$ depends on s_{13} and δ only through ϵ_τ and we have $|Y_B| \propto s_{13}|\sin \delta|$.

For the maximal value of the baryon asymmetry $|Y_B|$ for $s_{13} = 0.20$ (0.10) and $\delta = \pi/2$ ($|R_{11}| \cong 1.30$ (1.60)) we get (Fig. 11):

$$|Y_B| \cong 1.7\ (1.0) \times 10^{-12} \left(\frac{\sqrt{\Delta m_A^2}}{0.05\ \text{eV}}\right)\left(\frac{M_1}{10^9\ \text{GeV}}\right), \quad \delta = \frac{\pi}{2}, \quad s_{13} = 0.20\ (0.10). \quad (116)$$

It follows from the above result that if, e.g., $s_{13} = 0.10$, we can obtain baryon asymmetry $|Y_B| \gtrsim 8 \times 10^{-11}$ compatible with the observations for $M_1 \gtrsim 8 \times 10^{10}$ GeV.

What is the minimal value of $s_{13}|\sin \delta|$ for which we can have successful leptogenesis? For sufficiently small values of s_{13} we get maximal $|\eta_B| \cong 6.5 \times 10^{-2}$ ($\widetilde{m_\tau} \cong 1.75 \times 10^{-3}$ eV) at $|R_{11}| \cong 1.07$ independently of the value of s_{13} (see the discussion preceding Eq. (116)).

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[13]Indeed, since $|R_{11}|^2 + |R_{12}|^2 > 1$, we have $\widetilde{m_2} > 5.0 \times 10^{-2}$ eV and therefore $|\eta(0.71\widetilde{m_2})| < 2 \times 10^{-3}$.

The baryon asymmetry at the maximum is given by:

$$|Y_B| \cong 8.1 \times 10^{-12}\, s_{13}\, |\sin\delta| \left(\frac{\sqrt{\Delta m_A^2}}{0.05 \text{ eV}} \right) \left(\frac{M_1}{10^9 \text{ GeV}} \right) . \tag{117}$$

Thus, for $M_1 \lesssim 5 \times 10^{11}$ GeV, we can have successful leptogenesis and get $|Y_B| \cong (8.0 - 9.2) \times 10^{-11}$ provided

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02 , \quad \sin\theta_{13} \gtrsim 0.02 . \tag{118}$$

The preceding lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3} . \tag{119}$$

Values of $\sin\theta_{13}$ and of $|J_{\text{CP}}|$ as small as 0.02 and 4.6×10^{-3}, respectively, can be probed in neutrino oscillation experiments at neutrino factories [67].

The results in the case under discussion are illustrated in Figs. 11 and 12. In Fig. 11 we show the baryon asymmetry Y_B as a function of $|R_{11}|$ for $\delta = \pi/2$, while Fig. 12 exhibits the dependence of $|Y_B|$ on the Dirac phase δ for the values of $|R_{11}|$ from the interval $|R_{11}| \cong (1.05 - 1.7)$, which maximise $|Y_B|$. The results presented in both figures are for $s_{13} = 0.1;\ 0.2$, $\alpha_{21} = \pi$ ($\kappa' = +1$), $\kappa = +1$, $\kappa = -1$ and $M_1 = 2 \times 10^{11}$ GeV. In Fig. 13 we show the correlation between the rephasing invariant J_{CP} (in blue) and the baryon asymmetry Y_{B} in the case under discussion, (CP-violation due to the Dirac phase $\delta\ \alpha_{21} = \pi$) for $s_{13} = 0.2$, $M_1 = 2 \times 10^{11}$ GeV, $\kappa = +1$ and $|R_{11}| = 1.3$. The Dirac phase δ is varied in the interval $[0,2\pi]$.

C. CP-Violation due the Dirac Phase in U_{PMNS} and R ($\alpha_{21} = 0;\ 2\pi$)

We will consider next briefly the "mixed" case of CP-violation corresponding to $\alpha_{21} = 2\pi k$, $k = 0, 1, ...,$ δ taking values in the interval $[0,2\pi]$, and purely imaginary $R_{11}R_{12}$, $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa \pm 1$. For $\delta = 0;\ \pi$, this case provides an interesting example of breaking of CP-symmetry due to $R_{11}R_{12}$ being purely imaginary.

It proves convenient to write $e^{i\alpha_{21}/2} = \kappa'$, where $\kappa' = \pm 1$. We have in the case under discussion:

$$|\text{Re}\,(U_{\tau 1}^* U_{\tau 2})| \cong \left| c_{12}\, s_{12}\, s_{23}^2 - c_{23}\, s_{23}\, \cos 2\theta_{12}\, s_{13}\, \cos\delta \right| . \tag{120}$$

Obviously, the term $\propto s_{13}\cos\delta$ plays a sub-dominant role in the asymmetry $|\epsilon_\tau|$. The wash-out mass parameter $\widetilde{m_\tau}$ reads:

$$
\begin{aligned}
\widetilde{m_\tau} \;=\; & \sqrt{\Delta m_A^2}\, \Big[s_{23}^2 \left(s_{12}^2\, |R_{11}|^2 + c_{12}^2\, |R_{12}|^2 \right) + c_{23}^2\, s_{13}^2 \left(c_{12}^2\, |R_{11}|^2 + s_{12}^2\, |R_{12}|^2 \right) \\
& - 2\, c_{23}\, s_{23}\, s_{13} \left(c_{12}\, s_{12}\, \cos\delta + \kappa\, \kappa'\, |R_{11}R_{12}|\, \sin\delta \right) \Big] .
\end{aligned}
\tag{121}
$$

It follows from eqs. (108), (120) and (121) that $|\epsilon_\tau(\delta)| = |\epsilon_\tau(2\pi - \delta)|$, $\widetilde{m_\tau}(\delta, \kappa\kappa') = \widetilde{m_\tau}(2\pi - \delta, -\kappa\kappa')$, and therefore $|Y_B(\delta, \kappa\kappa')| = Y_B(2\pi - \delta, -\kappa\kappa')|$.

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For $s_{13} \lesssim 0.01$, the terms $\propto s_{13}$ and $\propto s_{13}^2$ in the expressions for $|\epsilon_\tau|$ and $\widetilde{m_\tau}$ can be neglected. The CP-violation effects are practically due to the purely imaginary $R_{11}R_{12}$: the baryon asymmetry Y_B does not depend on s_{13} and δ. However, even in this case it is possible to have successful leptogenesis. The baryon asymmetry Y_B can be generated in a regime which is intermediate between the weak and strong wash-out ones: $\widetilde{m_\tau} \gtrsim 8 \times 10^{-3}$ eV. The asymmetry $|Y_B|$ has two very similar (in magnitude) maxima at $|R_{11}| \cong 1.05; 1.12$. At these maxima we obtain:

$$|Y_B| \cong 5.2 \ (5.1) \times 10^{-13} \left(\frac{\sqrt{\Delta m_A^2}}{0.05 \ \text{eV}} \right) \left(\frac{M_1}{10^9 \ \text{GeV}} \right), \quad s_{13} = 0, \quad |R_{11}| \cong 1.05 \ (1.12) . \quad (122)$$

Thus, one can have $|Y_B| = (8.0 - 9.2) \times 10^{-11}$ for $M_1 \gtrsim 1.6 \times 10^{11}$ GeV.

If $s_{13} \gg 0.01$, the effects of the Dirac CP-violating phase δ are non-negligible; for $s_{13} \cong (0.1 - 0.2)$, they are substantial for any value of δ. The magnitude of the baryon asymmetry $|Y_B|$ depends on the value of s_{13} and, through the wash-out mass parameters $\widetilde{m_\tau}$ and $\widetilde{m_2}$, on the sign factor $\kappa\kappa'$. In the case of $\delta = \pi/2$, the maximum of $|Y_B|$ is located approximately at $|R_{11}| \cong (1.05 - 1.10)$ (Fig. 14). For $\kappa\kappa' = +1$, the baryon asymmetry at the maximum is by a factor ~ 2 larger than that for $\kappa\kappa' = -1$. We can have successful leptogenesis in the case of $s_{13} = 0.1 \ (0.2)$ for $M_1 \gtrsim 1.2 \ (1.0) \times 10^{11}$ GeV. These results are illustrated in Figs. 14 and 15. In Fig. 16 we show the correlation between the rephasing invariant J_{CP} and the baryon asymmetry Y_B for $\alpha_{21} = 0$, $s_{13} = 0.2$, $\kappa = +1$, $|R_{11}| = 1.05$ and $M_1 = 2 \times 10^{11}$ GeV. The Dirac phase δ is varied in the interval $[0, 2\pi]$.

As we have indicated at the beginning of this subsection, the observed value of the baryon asymmetry can be reproduced in the case of real R_{1j}, $j = 1, 2, 3$, if the terms $\propto \sqrt{m_3}$ in ϵ_l and $\widetilde{m_l}$ are dominant in spite of the fact that $m_3 \ll m_1, m_2$. A simple realisation of this possibility corresponds to having $R_{11} = 0$ or $R_{12} = 0$, and sufficiently large m_3 still obeying the inequality $m_3 \ll m_{1,2}$. The latter conditions can be satisfied for m_3 possessing a value in the interval $10^{-2}\sqrt{\Delta m_\odot^2} \lesssim m_3 \lesssim 0.5\sqrt{\Delta m_\odot^2}$. A more detailed discussion of this case will be presented elsewhere.

5.4 Quasi-Degenerate Light Neutrino Mass Spectrum

We turn now to the QD spectrum, for which $m_1 \simeq m_2 \simeq m_3 \gg \sqrt{\Delta m_\odot^2}, \sqrt{\Delta m_A^2}$ and we take as conventional lower limit on the masses $m_1 \gtrsim 0.1$ eV. In this case the contribution of all the masses to the CP-asymmetry ϵ_τ needs to be taken into account. We study the two cases in which the products of the parameters in R entering in the asymmetry, $R_{1i}R_{1j}$, are either real or purely imaginary. This corresponds to having $\beta_{ij} \equiv \widetilde{\beta_{1i}} + \widetilde{\beta_{1j}} = 2k\pi/2, (2k+1)\pi/2, k = 0, 1, ...$, respectively. We consider first the case of $R_{1i}R_{1j}$ real and equal to $\pm|R_{1i}R_{1j}|$. The CP-asymmetry can be written as:

$$|\epsilon_\tau| = \frac{3M_1}{16\pi v^2} \frac{1}{\sum_i |R_{1i}|^2} \left| (m_2 - m_1)|R_{11}R_{12}|e^{i\beta_{12}}\text{Im}\left(U_{\tau 1}^* U_{\tau 2}\right) \right.$$
$$\left. + (m_3 - m_1)|R_{11}R_{13}|e^{i\beta_{13}}\text{Im}\left(U_{\tau 1}^* U_{\tau 3}\right) + (m_3 - m_2)|R_{12}R_{13}|e^{i\beta_{23}}\text{Im}\left(U_{\tau 2}^* U_{\tau 3}\right) \right|, \quad (123)$$

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$$
\simeq \frac{3M_1}{16\pi v^2} \frac{1}{\sum_i |R_{1i}|^2} \left| \frac{\Delta m_\odot^2}{2m_1} |R_{11}R_{12}| e^{i\beta_{12}} \mathrm{Im}\left(U_{\tau 1}^* U_{\tau 2} \right) \right.
$$
$$
\left. \pm \frac{\Delta m_A^2}{2m_1} |R_{11}R_{13}| e^{i\beta_{13}} \mathrm{Im}\left(U_{\tau 1}^* U_{\tau 3} \right) \pm \frac{\Delta m_A^2}{2m_1} |R_{12}R_{13}| e^{i\beta_{23}} \mathrm{Im}\left(U_{\tau 2}^* U_{\tau 3} \right) \right| \,, \quad (124)
$$

where we have neglected terms of order $\Delta m_\odot^2/\Delta m_A^2$. The signs $\pm$ in Eq. (124) refer to the quasi-degenerate spectrum with a normal or inverted hierarchy. This information could not be obtained in experiments which are sensitive to the overall neutrino mass scale, as neutrinoless double beta decay or direct neutrino mass searches. However, by exploiting matter effects, long baseline neutrino oscillation and atmospheric neutrino experiments might be able to establish if the spectrum is with normal or inverted hierarchy. Neglecting the terms proportional to $\Delta m_\odot^2$, we can further simplify the expression in Eq. (124):

$$
|\epsilon_\tau| \simeq \frac{3M_1 m_1}{16\pi v^2} \frac{1}{\sum_i |R_{1i}|^2} \frac{\Delta m_A^2}{2m_1^2} s_{23} c_{23} c_{13} |R_{13}|
$$
$$
\times \left| |R_{11}| \left(s_{12} \sin\frac{\alpha_{31}}{2} - c_{12}\frac{c_{23}}{s_{23}} s_{13} \sin\left(\frac{\alpha_{31}}{2} - \delta\right) \right) \pm |R_{12}| \left(-c_{12} \sin\frac{\alpha_{32}}{2} - s_{12}\frac{c_{23}}{s_{23}} s_{13} \sin\left(\frac{\alpha_{32}}{2} - \delta\right) \right) \right| (125)
$$

where $\pm$ refer to the case of $\widetilde{\beta_{11}} = \widetilde{\beta_{12}}$ and $\widetilde{\beta_{12}} = k\pi + \widetilde{\beta_{11}}$, $k = 0,1,...$, respectively. Notice that, in general, the asymmetry is suppressed by $\Delta m_A^2/m_1^2$. For comparison with the NH case studied in Section 5.1, we can have a mild suppression of order $(\Delta m_A^2)^{3/4}/(m_1(\Delta m_\odot^2)^{1/4}) \sim 0.2$ (1 eV/m_1) for large values of m_1. If R_{13} is negligible (N_3 decoupling), the dominant contribution is proportional to $\Delta m_\odot^2$ which amounts to an additional suppression factor of $\Delta m_\odot^2/\Delta m_A^2 \sim 0.03$ [28]. Typically, we get a CP-asymmetry of order

$$
|\epsilon_\tau| \sim 1.2 \times 10^{-6} \frac{M_1}{10^{11}\text{ GeV}} \frac{0.1\text{ eV}}{m_1} |R_{13}R_{11}|
$$
$$
\times \left| 0.55 \sin\frac{\alpha_{31}}{2} - 0.17\frac{s_{13}}{0.2} \sin\left(\frac{\alpha_{31}}{2} - \delta\right) - 0.84 \sin\frac{\alpha_{32}}{2} - 0.11\frac{s_{13}}{0.2} \sin\left(\frac{\alpha_{32}}{2} - \delta\right) \right| (126)
$$

where we have taken for definiteness $R_{12} = R_{11}$ and $\sum_i |R_{1i}|^2 = 1$. The asymmetry decreases linearly with m_1 and we have evaluated it for the minimal value of m_1 which is allowed for the QD spectrum. Notice that, as far as α_{31}, α_{32} is not too small, $\sin\frac{\alpha_{31}}{2} \gg 0.1$ ($s_{13}/0.2$), the contribution of the δ phase will be subdominant. The phases α_{21} and α_{31} enter in the effective Majorana mass parameter for neutrinoless double beta decay. For the QD light neutrino mass spectrum, we have

$$
|\langle m \rangle| = m_1 \left| c_{12}^2 c_{13}^2 + s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + s_{13}^2 e^{i(\alpha_{31}-2\delta)} \right| \,. \quad (127)
$$

The dependence on the phase difference $(\alpha_{31} - 2\delta)$ is suppressed by s_{13}^2 and can be neglected. In principle, the phase α_{21} can be measured in future neutrinoless double beta decay experiments if a sufficient precision will be achieved in the measurement of $|\langle m \rangle|$ and of the masses, and if the problem of the computation of nuclear matrix elements will be solved [58]. The prospects of measuring the other Majorana phase α_{31} are far beyond the sensitivities of the present and future planned experiments.

In the case of $R_{1i}R_{1j}$ purely imaginary, i.e. $\widetilde{\beta}_{1i}+\widetilde{\beta}_{1j} = (2k+1)\pi/2$, $k = 0, 1, ...$, in general, there is no suppression as the masses enter in the CP-asymmetry via the combination $m_i + m_j$. We get

$$|\epsilon_\tau| = \frac{3M_1 m_1}{8\pi v^2}\frac{1}{\sum_i |R_{1i}|^2}$$
$$\times \left| \pm |R_{11}R_{12}|\mathrm{Re}(U^*_{\tau 1}U_{\tau 2}) \pm |R_{11}R_{13}|\mathrm{Re}(U^*_{\tau 1}U_{\tau 3}) \pm |R_{12}R_{13}|\mathrm{Re}(U^*_{\tau 2}U_{\tau 3}) \right| , \quad (128)$$

where the $+(-)$ refer to $\beta_{ij} = \pi/2 \, (3\pi/2)$. The expression in Eq. (128) is rather lengthy but can be simplified if we neglect θ_{13}. In this case it reads:

$$|\epsilon_\tau| = \frac{3M_1 m_1}{8\pi v^2}\frac{1}{\sum_i |R_{1i}|^2}$$
$$\times \left| \pm|R_{11}R_{12}|(-s_{12}c_{12}s^2_{23}\cos\frac{\alpha_{21}}{2}) \pm |R_{11}R_{13}|(s_{12}s_{23}c_{23}\cos\frac{\alpha_{31}}{2}) \pm |R_{12}R_{13}|(-c_{12}s_{23}c_{23}\cos\frac{\alpha_{32}}{2}) \right| (129)$$

As we have neglected the terms $\propto s_{13}$, there is no dependence on the phase δ. However, both Majorana phases enter in the expression for ϵ_τ.

Let's now turn to the wash-out factors. Using the unitarity condition on U, we find:

$$\widetilde{m_2} + \widetilde{m_\tau} = m_1 \sum_k |R_{1k}|^2. \quad (130)$$

In the case of real R_{1k}, for instance, this implies [28] $\widetilde{m_2} + \widetilde{m_\tau} = m_1$. Therefore, we can expect that the wash-out mass parameters will typically be much larger than 3×10^{-3} eV, leading to a strong suppression of the baryon asymmetry. More specifically, $\widetilde{m_\tau}$ is given by:

$$\widetilde{m_\tau} = m_1 \left|R_{11}U^*_{\tau 1} + R_{12}U^*_{\tau 2} + R_{13}U^*_{\tau 3}\right|^2 . \quad (131)$$

We will study first the case of real R_{ij}. Taking, for example, $R_{11} = R_{12} = R_{13}$, we obtain $\widetilde{m_\tau} = 0.053\, m_1$, for $\alpha_{21} = \pi$ and $\alpha_{31} = \pi$. In this case the strong wash-out regime formulas apply and we have

$$\eta(390/589\,\widetilde{m_\tau}) = 3.3 \times 10^{-2}\left(\frac{0.1\text{ eV}}{m_1}\right)^{1.16} , \quad (132)$$

resulting in a baryon asymmetry which is substantially smaller than the observed one.

A larger efficiency factor can be achieved in the regime interpolating between the ones of strong and weak wash-out effects. However, it should be noticed that $\eta(\widetilde{m_\tau})$ and ϵ_τ depend on the same parameters and they cannot be maximized independently. In fact, for real R_{ij}, we can rewrite the CP-asymmetry as:

$$|\epsilon_\tau| \simeq \frac{3M_1 m_1}{32\pi v^2}\frac{\Delta m^2_A}{m^2_1 \sum_i |R_{1i}|^2}\left|\mathrm{Im}(R_{13}U_{\tau 3}\sum_\beta R_{1\beta}U^*_{\tau\beta})\right| ,$$

$$\leq \frac{3M_1 m_1}{32\pi v^2}\frac{\Delta m^2_A}{m^2_1 \sum_i |R_{1i}|^2}\sqrt{\frac{\widetilde{m_\tau}}{m_1}}\,|R_{13}U_{\tau 3}| . \quad (133)$$

Therefore, while the efficiency factor $\eta(\widetilde{m_\tau})$ increases when $\widetilde{m_\tau}$ decreases as the wash-out regime changes from strong to weak, $|\epsilon_\tau|$ decreases accordingly. In the strong wash-out regime in the τ flavour and $\widetilde{m_\tau} \ll m_1$, the baryon asymmetry has an upper bound:

$$|Y_B| \leq \frac{12}{37g_*} \frac{3M_1 m_1}{32\pi v^2} \frac{\Delta m_A^2}{m_1^2} |R_{13}| c_{23} \sqrt{\frac{\widetilde{m_\tau}}{m_1}} \left(\frac{0.2 \times 10^{-3} \text{ eV}}{\widetilde{m_\tau}} \frac{589}{390} \right)^{1.16} . \tag{134}$$

Conversely, for weak wash-out in the τ flavour, we have

$$|Y_B| \leq \frac{12}{37g_*} \frac{3M_1 m_1}{32\pi v^2} \frac{\Delta m_A^2}{m_1^2} |R_{13}| c_{23} \sqrt{\frac{\widetilde{m_\tau}}{m_1}} \left(\frac{\widetilde{m_\tau}}{0.2 \times 10^{-3} \text{ eV}} \frac{390}{589} \right) . \tag{135}$$

Thus, the maximal baryon asymmetry is obtained for intermediate wash-out effect. For definiteness we can estimate the upper bound on Y_B at $\widetilde{m_\tau} \simeq 3 \times 10^{-3}$ eV. For the smallest allowed value of m_1 for the QD spectrum, $m_1 = 0.1$ eV, we get:

$$|Y_B| \lesssim 5 \times 10^{-11} |R_{13}| \frac{M_1}{10^{11} \text{ GeV}} . \tag{136}$$

Requiring that $\widetilde{m_\tau} \ll m_1$ imposes a fine tuning on the values of the parameters R_{1i}. For simplicity, we search for the solution of the equation $\widetilde{m_\tau} \sim 3 \times 10^{-3}$ eV $\simeq 0$, which corresponds to:

$$R_{11}|U_{\tau 1}| - R_{12}|U_{\tau 2}| \cos\left(\frac{\alpha_{21}}{2}\right) + R_{13}|U_{\tau 3}| \cos\left(\frac{\alpha_{31}}{2}\right) \simeq 0 , \tag{137}$$

$$R_{12}|U_{\tau 2}| \sin\left(\frac{\alpha_{21}}{2}\right) - R_{13}|U_{\tau 3}| \sin\left(\frac{\alpha_{31}}{2}\right) \simeq 0 . \tag{138}$$

If $\alpha_{21}\,(\alpha_{31})\,[\alpha_{32}] = 0$, we have that $R_{13}\,(R_{12})\,[R_{11}] = 0$ as well. Otherwise, the solution of Eqs. (137) and (138) is given by:

$$R_{11} = R_{13} \frac{c_{23}}{s_{12}s_{23}} \frac{\sin\alpha_{32}/2}{\sin\alpha_{21}/2} \quad \text{and} \quad R_{12} = R_{13} \frac{c_{23}}{c_{12}s_{23}} \frac{\sin\alpha_{31}/2}{\sin\alpha_{21}/2} . \tag{139}$$

Another possibility consists in having $\widetilde{m_\tau} \simeq m_1$ and small $\widetilde{m_2}$. However, also in this case the baryon asymmetry is suppressed due to the values of the CP-violating phases required to have $\widetilde{m_2} \ll m_1$, namely, $\alpha_{21}, \alpha_{31} \sim 0, \pi$. In conclusion, for real R, it might be possible to reproduce the observed baryon asymmetry, but only for relatively large values of $M_1 \gg 10^{11}$ GeV and small values of m_1. A careful and detailed analysis should be performed on a case by case basis. A measurement of m_1 in the upper end of the range of allowed values $[0.1\,\text{eV}, 2.3\,\text{eV}]$ in the quasi-degenerate spectrum would strongly disfavour, if not rule out, the possibility of having leptogenesis due uniquely to the low energy CP-violating phases, for hierarchical RH neutrinos and real R.

For $R_{1i}R_{1j} = \pm i |R_{1i}R_{1j}|$, the CP-asymmetry is enhanced by a factor $4m_1^2/\Delta m_A^2$. Also in this case an upper bound on $|Y_B|$ can be derived, and it depends on $\widetilde{m_\tau}$. A sufficiently large baryon asymmetry might be obtained for relatively large values of M_1. A more detailed analysis of this case is beyond the scope of the present study.

6 Baryon Asymmetry from Low Energy CP-Violating Dirac and Majorana Phases in U: the Case of Quasi-Degenerate RH Neutrinos

In this Section we extend our previous findings to the case in which the RH neutrinos are quasi-degenerate in mass, $M_1 \simeq M_2 \simeq M_3$ and the matrix R is real. We therefore consider the case in which the CP parities of the heavy and light Majorana neutrinos are such that $\rho_i^N = \rho_j^\nu = 1$ for all $i, j = 1, 2, 3$. In such a case, indeed, CP invariance corresponds to having all the elements of the matrix R real, see Eq. (30), and $\delta = \alpha_{21} = \alpha_{31} = 0$. The degenerate pattern for the RH neutrino masses may arise if, for instance, there is a slightly broken SO(3) symmetry in the RH sector. The baryon asymmetry receives a contribution from the decay of all three RH neutrinos. The CP asymmetry in a given flavour l generated by the decay of the RH neutrino N_i ($i = 1, 2, 3$) is dominated by the one-loop self energy contribution [68] and reads

$$
\begin{aligned}
\epsilon_i^l &= -\sum_{j \neq i} \frac{M_i}{M_j} \frac{\Gamma_j}{M_j} S_{ij} I_{ij}^l \,, \quad \Gamma_j = \frac{\lambda \lambda_{jj}^\dagger M_j}{8\pi} \,, \quad (\lambda \lambda^\dagger)_{ii} = \frac{M_i}{v^2} \sum_\ell m_\ell R_{i\ell}^2 \,, \\
S_{ij} &= \frac{M_j^2 \Delta M_{ij}^2}{\left(\Delta M_{ij}^2\right)^2 + M_i^2 \Gamma_j^2} \,, \quad \Delta M_{ij}^2 = M_j^2 - M_i^2 \,, \\
I_{ij}^l &= \frac{1}{(\lambda \lambda^\dagger)_{ii}} \frac{1}{(\lambda \lambda^\dagger)_{jj}} \frac{M_i M_j}{v^4} \sum_\ell \left(R_{i\ell} R_{j\ell} m_\ell\right) \sum_{ts} \sqrt{m_t m_s} R_{it} R_{js} \mathrm{Im}\left(U_{ls} U_{lt}^*\right) \,. \quad (140)
\end{aligned}
$$

Notice, in particular, that $I_{ij}^l = -I_{ji}^l$ and $S_{ij} = -S_{ji}$. The CP asymmetry ϵ_i^l is resonantly enhanced when $\Gamma_j = \Delta M_{ij}^2 / M_i$. At the resonance

$$
S_{ij} \simeq \frac{M_i}{2\,\Gamma_j} \simeq \frac{M_j}{2\,\Gamma_j} \,, \quad \epsilon_{ij}^l \simeq -\frac{1}{2} \sum_{j \neq i} I_{ij}^l \,. \quad (141)
$$

The washing out of a given flavour l is now operated by the $\Delta L = 1$ scatterings involving all three RH neutrinos. Therefore, the parameter $\widetilde{m_l}$ is given by

$$
\widetilde{m_l} \simeq \sum_j \frac{|\lambda_{jl}|^2 v}{M_j} \simeq \frac{(\lambda^\dagger \lambda)_{ll} v}{M_1} = \sum_\ell m_\ell |U_{l\ell}|^2 \,, \quad (142)
$$

where we have set the nearly equal masses of the RH neutrinos approximately equal to M_1. If the resonance operates for all three RH neutrinos, the CP asymmetry in the flavour l is

$$
\epsilon_l = 2 \sum_{i<j} \epsilon_{ij}^l = -\sum_{i<j} I_{ij}^l = 2 \frac{\sum_{i<j} \sum_{\ell t s} R_{i\ell} R_{j\ell} R_{it} R_{js} m_\ell \sqrt{m_s m_t} \mathrm{Im}\left(U_{ls} U_{lt}^*\right)}{\left(\sum_p m_p R_{ip}^2\right) \left(\sum_q m_q R_{jq}^2\right)} \,. \quad (143)
$$

It does not depend upon the mass of the RH neutrinos (if the running of the parameters is neglected). The elements of the matrix R can be parametrized by introducing a real antisymmetric matrix A [54]

$$R \equiv e^A = \mathbf{1} + \frac{1 - \cos r}{r^2} A^2 + \frac{\sin r}{r} A \,, \quad r = \sqrt{A_{12}^2 + A_{23}^2 + A_{13}^2} \,, \qquad (144)$$

where $\mathbf{1}$ is the 3×3 unity matrix and $A_{ij} = -A_{ji}$ are the elements of the matrix A. In the limiting case where the RH neutrinos are exactly degenerate, one can perform an orthogonal rotation on the RH neutrino states which leaves the mass matrix of the RH neutrinos proportional to the unity matrix and defines a physically equivalent reparametrization of the RH neutrinos. This amounts to saying that for $M_1 = M_2 = M_3$, the real matrix R can be set equal to the unity matrix (or $A_{12} = A_{23} = A_{13} = 0$). The flavour asymmetries in Eq. (140) vanish if $R = \mathbf{1}$ which reflects the fact that no baryon asymmetry can be obtained in the exactly degenerate case. On the other hand, if the degeneracy is slightly broken, the elements of the matrix A are expected to be tiny, but not all vanishing and the baryon asymmetry is typically different from zero. We are now ready to study all three possible cases for the spectrum of the light neutrinos. We will restrict ourselves to the case in which the resonant condition is not satisfied for all couples of RH neutrinos and work under the condition that $M_1 \simeq M_2 \lesssim M_3$; the flavour asymmetries are generated resonantly only by the decays of N_1 and N_2 and we will set $I_{13} \simeq I_{23} \simeq 0$.

We find

$$
\begin{aligned}
\epsilon_l \simeq & -\sum_{I<j} \left[R_{i3}R_{j3} \left(\Delta m_{\rm A}^2 \right)^{1/2} - R_{i1}R_{j1} \left(\Delta m_{\odot}^2 \right)^{1/2} \right] \\
& \times \left[(R_{i3}R_{j2} - R_{i2}R_{j3}) \left(\Delta m_{\odot}^2 \right)^{1/4} \left(\Delta m_{\rm A}^2 \right)^{1/4} \text{Im} \left(U_{l2}U_{l3}^* \right) \right] \\
& \times \left[\left(\Delta m_{\odot}^2 \right)^{1/2} R_{i2}^2 + \left(\Delta m_{\rm A}^2 \right)^{1/2} R_{i3}^2 \right]^{-1} \\
& \times \left[\left(\Delta m_{\odot}^2 \right)^{1/2} R_{j2}^2 + \left(\Delta m_{\rm A}^2 \right)^{1/2} R_{j3}^2 \right]^{-1} \,, \\
\widetilde{m_l} \simeq & \left(\Delta m_{\odot}^2 \right)^{1/2} |U_{l2}|^2 + \left(\Delta m_{\rm A}^2 \right)^{1/2} |U_{l3}|^2
\end{aligned}
\qquad (145)
$$

in the normal hierarchical case;

$$
\begin{aligned}
\epsilon_l \simeq & -\sum_{i<j} \left[-R_{i3}R_{j3} \left(\Delta m_{\rm A}^2 \right)^{1/2} - R_{i1}R_{j1} \frac{\left(\Delta m_{\odot}^2 \right)}{2 \left(\Delta m_{\rm A}^2 \right)^{1/2}} \right] \\
& \times \left[(R_{i2}R_{j1} - R_{i1}R_{j2}) \left(\Delta m_{\rm A}^2 \right)^{1/2} \text{Im} \left(U_{l1}U_{l2}^* \right) \right] \\
& \times \left[\left(\Delta m_{\rm A}^2 \right)^{1/2} R_{i1}^2 + \left(\Delta m_{\rm A}^2 \right)^{1/2} R_{i2}^2 \right]^{-1} \\
& \times \left[\left(\Delta m_{\rm A}^2 \right)^{1/2} R_{j1}^2 + \left(\Delta m_{\rm A}^2 \right)^{1/2} R_{j2}^2 \right]^{-1} \,, \\
\widetilde{m_l} = & \left(\Delta m_{\rm A}^2 \right)^{1/2} |U_{l1}|^2 + \left(\Delta m_{\rm A}^2 \right)^{1/2} |U_{l2}|^2
\end{aligned}
\qquad (146)
$$

in the inverted hierarchical case and, finally,

$$
\begin{aligned}
\epsilon_l \;\simeq\; & -\frac{1}{2\,m^2}\sum_{i<j}\frac{1}{2\,m^2}\left[-R_{i3}R_{j3}\left(\Delta m_{\mathrm{A}}^2\right)-R_{i1}R_{j1}\left(\Delta m_{\odot}^2\right)\right] \\
\times\; & \left[\left(R_{i2}R_{j1}-R_{i1}R_{j2}\right)\mathrm{Im}\left(U_{l1}U_{l2}^{*}\right)\right. \\
+\; & \left(R_{i3}R_{j2}-R_{i2}R_{j3}\right)\mathrm{Im}\left(U_{l2}U_{l3}^{*}\right) \\
+\; & \left.\left(R_{i3}R_{j1}-R_{i1}R_{j3}\right)\mathrm{Im}\left(U_{l1}U_{l3}^{*}\right)\right], \\
\widetilde{m_l} \;\simeq\; & m
\end{aligned}
\tag{147}
$$

in the degenerate case. In this latter case, since the washing-out factors are approximately same, the expressions for the baryon asymmetries may be simplified if R is real. Indeed, the total asymmetry ϵ_1 vanishes and, if $(10^9 \lesssim M_1 \lesssim 10^{12})$ GeV, the flavour asymmetry $\epsilon_2 = -\epsilon_\tau$ while $\widetilde{m_2} \simeq 2m \simeq 2\widetilde{m_\tau}$. One finds

$$
Y_B \simeq -\frac{12}{37}\frac{222}{417}\,Y_\tau\,.
\tag{148}
$$

In Fig. 17, we show the correlation of the baryon asymmetry with the effective Majorana mass in neutrinoless double beta decay for the case of quasi-degenerate RH neutrinos and QD spectrum of light neutrinos. A number of projects aim to reach a sensitivity to $|\langle m_\nu\rangle| \sim (0.01-0.05)$ eV [69] and can certainly probe the region of values of $|\langle m_\nu\rangle|$ for successfull baryon asymmetry from the PMNS phases only. In particular, a direct information on the Majorana phase α_{21} may come from the measurement of $\langle m_\nu\rangle$, m, and $\sin^2 2\theta_{12}$,

$$
\sin^2\frac{\alpha_{21}}{2} \simeq \left(1-\frac{|\langle m_\nu\rangle|^2}{m^2}\right)\frac{1}{\sin^2 2\theta_{12}}\,,
\tag{149}
$$

and might tell us if enough baryon asymmetry may be generated uniquely from the PMNS Majorana phases.

7 Extension to the MSSM

The extension to our findings to the supersymmetric version of the SM, the so-called Minimal Supersymmetric Standard Model (MSSM) is rather straightforward [29]. One has to consider the presence of the supersymmetric partners of the RH heavy neutrinos, the so-called sneutrinos $\widetilde{N}_i$ $(i=1,2,3)$, which also give a contribution to the flavour asymmetries, and of the supersymmetric partners of the lepton doublets, the so-called slepton doublets. Since the effects of supersymmetry breaking may be safely neglected, the flavour CP asymmetries in the MSSM are twice those in the SM and double is also the possible channels by which a lepton flavour asymmetry is reproduced. However, the $\Delta L = 1$ scatterings washing out the asymmetries are also doubled and the number of relativistic degrees of freedom is almost twice the one for the SM case. As a result, introducing new degrees of freedom

and interactions does not appreciably change the flavour asymmetries with respect to the values obtained within the SM.

There are however two other and important differences with respect to the SM case. First, in the MSSM, the flavour-independent formulae can only be applied for temperatures larger than $(1 + \tan^2 \beta) \times 10^{12}$ GeV, where $\tan \beta$ indicates the ratio of the vacuum expectation values of the two Higgs fields of the MSSM. Indeed, the squared charged lepton Yukawa couplings in the MSSM are multiplied by $(1 + \tan^2 \beta)$. Consequently, charged μ and τ lepton Yukawa couplings are in thermal equilibrium for $(1 + \tan^2 \beta) \times 10^5$ GeV $\ll T \ll (1 + \tan^2 \beta) \times 10^9$ GeV and all flavours in the Boltzmann equations are to be treated separately. For $(1 + \tan^2 \beta) \times 10^9$ GeV $\ll T \ll (1 + \tan^2 \beta) \times 10^{12}$ GeV, only the τ Yukawa coupling is in equilibrium and only the τ flavour is treated separately in the Boltzmann equations, while the e and μ flavours are indistinguishable. This implies that the range of the RH (s)neutrino masses where flavour is relevant in leptogenesis is greater than the one in the SM by the factor $(1 + \tan^2 \beta)$ which is large even for moderate values of $\tan \beta$. As a consequence, the lower bounds given in Eqs. (93) - (94) change approximately to

$$| \sin \theta_{13} \, \sin \delta |, \sin \theta_{13} \gtrsim 0.11 \, (1 + \tan^2 \beta)^{-1} \,, \tag{150}$$

$$|J_{\mathrm{CP}}| \gtrsim 2.4 \times 10^{-2} \, (1 + \tan^2 \beta)^{-1} \,. \tag{151}$$

The lower bounds in Eqs. (97) - (98) and in Eqs. (118) - (119) change in a similar way. The shift of the range of the heavy Majorana neutrino masses, in which the lepton flavour effects are significant, to larger values has important implications also if the spectrum of the RH (s)neutrinos is hierarchical and the light neutrinos possess inverted hierachical (IH) spectrum. This is illustrated in Fig. 18, where we plot the baryon asymmetry versus the quantity J_{CP} for the IH spectrum of light neutrinos in the supersymmetric case for a given set of parameters. Let us recall that for real R matrix elements R_{11} and R_{12}, it was impossible to obtain baryon asymmetry compatible with the observations in the corresponding non-SUSY case. The reader should be also warned that, for values of $\tan \beta \gtrsim 30$, radiative corrections to the physical neutrino parameters should be accounted for [62].

Secondly, the relation between the baryon asymmetry and the lepton flavour asymmetries has to be modified to account for the presence of two Higgs fields. Between $(1 + \tan^2 \beta) \times 10^9$ and $(1 + \tan^2 \beta) \times 10^{12}$ GeV, the relation is

$$Y_B^{\mathrm{MSSM}} \simeq -\frac{10}{31 g_*} \left(\hat{\epsilon}_2 \eta \left(\frac{541}{761} \widetilde{m_2} \right) + \hat{\epsilon}_\tau \eta \left(\frac{494}{761} \widetilde{m_\tau} \right) \right), \tag{152}$$

where the hat superscripts indicates that the flavour lepton asymmetries are computed including leptons and sleptons. Notice that if the spectrum of RH (s)neutrinos is quasi-degenerate as well as that of the light neutrinos, the wash-out factor are also the approximately same and the expressions for the baryon asymmetries may be simpliflied if R is real. Indeed, the total asymmetry ϵ_1 vanishes and one of the flavour asymmetries may be expressed in terms of the others. Under these circumstances, one finds

$$Y_B^{\mathrm{MSSM}} \simeq -\frac{10}{31} \frac{447}{988} Y_\tau^{\mathrm{MSSM}} \,. \tag{153}$$

Finally, in the case of supersymmetric leptogenesis one should also face the problem arising from the so-called gravitino bound. The latter is posed by the possible overproduction of gravitinos during the reheating stage after inflation. Being only gravitationally coupled to the SM particles, gravitinos may decay very late jeopardising the successfull predictions of Big Bang nucleosynthesis. This does not happen, however, if gravitinos are not efficiently generated during reheating, that is if the reheating temperature T_{RH} is bounded from above, $T_{RH} \lesssim 10^{10}$ GeV [70]. The severe bound on the reheating temperature makes the generation of the RH neutrinos problematic (for a complete study in the one-flavour case see [4]), if the latter are a few times heavier than the reheating temperature, rendering the thermal leptogenesis scenario difficult. There are, though, two possible ways out to this problem. First, leptogenesis might occur in a non-thermal way, that is the RH neutrinos might be generated not through thermal scatterings, but by other mechanisms, e.g. at the preheating stage [71]. Alternatively, and maybe more interestingly, as we have previously seen if the two lightest RH neutrinos are quasi-degenerate in mass, the final baryon asymmetry does not depend upon their common mass. The latter, therefore, might be smaller than the largest possible reheating temperature and thermal leptogenesis might take place without any limitation from the gravitino bound.

8 Conclusions

In this paper we have systematically investigated the connection between the leptogenesis and the low energy CP-violation in the lepton (neutrino) sector. Our study was stimulated by the recent progress in the understanding of the importance of lepton flavour effects in leptogenesis. It lead to the realization that these effects can play a crucial role in the leptogenesis scenario, both from the quantitative and the qualitative point of view. When the lepton flavour effects are taken into account, the final baryon asymmetry is the sum of three different contributions given by the CP asymmetries generated in each flavour (lepton charge), properly weighted by the corresponding wash-out factor. In the one-flavour approximation, which holds only if leptogenesis is taking place at a temperature higher than about 10^{12} GeV, the final baryon asymmetry is proportional to the total baryon CP asymmetry (summed over the three flavours) and weighted by a single wash-out factor (obtained by summing the wash-out factor of the three lepton flavours).

There are many differences between the predictions for the baryon asymmetry $Y_{\rm B}$ obtained in the one-flavour approximation and in the case when the flavour effects are accounted for. The baryon asymmetry $Y_{\rm B}$, derived in the one-flavour approximation, for instance, vanishes if the light neutrinos are degenerate in mass. Correspondingly, $Y_{\rm B}$ has to be proportional to a difference of masses of the light neutrinos. In the "flavour" case this suppression can be absent even when the leptogenesis CP violation is due entirely to the low energy phases in the PMNS matrix. However, the most significant difference is that in the one flavour approximation there is no direct connection between the leptogenesis CP-violating parameters and the CP-violating parameters - Dirac and Majorana phases, present in the lepton (neutrino) sector. In particular, a possible future observation of CP violation in neutrino oscillations would not automatically imply, within the "one-flavour"

leptogenesis scenario, the existence of a baryon asymmetry. In the "flavoured" treatment of leptogenesis, however, this conclusion does not universally hold and the observation of CP violation in the lepton (neutrino) sector would generically imply a nonvanishing baryon asymmetry. Including the effects of lepton flavour, therefore, allows to build a new bridge between the CP violation in leptogenesis and the observables depending on the CP-violating Dirac and Majorana phases in the PMNS neutrino mixing matrix, such as the CP violating rephasing invariant J_{CP} which controls the magnitude of CP-violation effects in neutrino oscillations, the effective Majorana mass $|\langle m \rangle|$ in neutrinoless double beta decay, etc. The study of such a connection has been the main subject of our paper.

We have first derived the constraints the requirement of CP-invariance imposes on the neutrino Yukawa couplings λ and on the elements of the complex orthogonal matrix R appearing in the "orthogonal" parametrisation of λ. The CP-parities of the light and heavy Majorana neutrinos, which take the values $\pm i$, play a special role in these constraints. The CP-invariance constraints are useful for understanding the source of CP violation generating the CP asymmetries in the heavy Majorana neutrino decays. One example is the case of *real matrix R and specific CP-conserving values of the Majorana and Dirac phases in the PMNS neutrino mixing matrix*, which corresponds to violation of CP-symmetry at high energy in leptogenesis, leading to the generation of non-zero baryon asymmetry. The indicated constraints help to clarify also under which conditions the leptogenesis CP-asymmetries are due entirely to the low energy CP-violating phases of the PMNS matrix.

Taking into account the lepton flavour effects in leptogenesis, we have subsequently investigated in detail the possibility that the CP-violation necessary for the generation of the baryon asymmetry of the Universe is due exclusively to the Dirac and/or Majorana CP-violating phases in the PMNS matrix, and thus is directly related to the low energy CP-violation in the lepton sector (e.g., in neutrino oscillations, etc.). We have derived results for two types of spectrum of the heavy RH Majorana neutrinos: i) hierarchical, in which the lightest RH neutrino N_1 is much lighter than the other two RH neutrinos, and ii) quasi-degenerate, in which the two lightest RH neutrinos $N_{1,2}$ are almost degenerate in mass and have masses which are smaller than the mass of the third one. For each of the two cases, we have presented predictions for the baryon asymmetry for three types of spectra of the light Majorana neutrinos: normal hierarchical (NH), inverted hierarchical (IH) and quasi-degenerate (QD). In all numerical calculations we have used the best fit values of the solar and atmospheric neutrino oscillation parameters, Δm_A^2, $\Delta m_\odot^2$, $\sin^2\theta_{23}$ and $\sin^2\theta_{12}$, given in Section 2.

For hierarchical RH neutrino mass spectrum, the lepton flavour effects are relevant in leptogenesis for $M_1 \lesssim \times 10^{12}$ GeV. The predicted baryon asymmetry Y_B depends linearly on M_1. In order to reproduce the observed baryon asymmetry, generically values of $M_1 \gtrsim 3 \times 10^{10}$ GeV are required. We have shown that if the light neutrinos have a NH spectrum, the requisite baryon asymmetry can be produced if the only source of CP violation is either the Majorana phases or the Dirac phase in the PMNS matrix, U_{PMNS} (Figs. 1 - 6). When the only CP-violating parameter is the low energy Majorana phase $\alpha_{32} \equiv \alpha_{31} - \alpha_{21}$, we can have successful leptogenesis as long as $|\sin(\alpha_{32}/2)|$ is not exceedingly small and $M_1 \gtrsim 3.5 \times 10^{10}$ GeV. If the only source of CP violation is the Dirac phase δ in U_{PMNS}, the observed baryon

asymmetry can be reproduced provided $M_1 \gtrsim 2 \times 10^{11}$ GeV and $|\sin\theta_{13}\sin\delta| \gtrsim 0.1$, θ_{13} being the CHOOZ angle. This condition leads to the inequality $\sin\theta_{13} \gtrsim 0.1$ and to the following lower bound on the CP-violating rephasing invariant $J_{\rm CP}$, associated with the Dirac phase in the PMNS matrix: $|J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}$. Values of $\sin\theta_{13} \gtrsim 0.1$ can be probed in the forthcoming Double CHOOZ and Daya Bay reactor neutrino experiments. CP-violation effects with magnitude determined by $|J_{\rm CP}| \gtrsim 2.0 \times 10^{-2}$ are within the sensitivity of the next generation of neutrino oscillation experiments, designed to search for CP- or T- symmetry violation in the oscillations. Moreover, since in this case both $|Y_{\rm B}| \propto |\sin\theta_{13}\sin\delta|$ and $|J_{\rm CP}| \propto |\sin\theta_{13}\sin\delta|$, given the other parameters on which $|Y_{\rm B}|$ and $|J_{\rm CP}|$ depend, there exists a correlation between the rephasing invariant $J_{\rm CP}$, which controls the magnitude of the CP-violation effects in neutrino oscillations, and the baryon asymmetry Y_B (Fig. 6).

In the case of IH light neutrino mass spectrum and negligible lightest neutrino mass m_3, the observed baryon asymmetry cannot be reproduced if the product of the elements of the matrix R, $R_{11}R_{12}$, is purely real: the generated baryon asymmetry is generically small, being suppressed by the additional factor $\Delta m_\odot^2/\Delta m_{\rm A}^2$. However, if $R_{11}R_{12}$ is purely imaginary (and CP-conserving), a sufficiently large baryon asymmetry compatible with the observations can be obtained both when the only source of CP-violation is the Majorana phase α_{21}, or the Dirac phase δ, in $U_{\rm PMNS}$ (Figs. 7 - 13). In the case of Majorana CP-violation, depending on the sgn(Im($R_{11}R_{12}$)), values of $M_1 \gtrsim 5 \times 10^{10}$ GeV or somewhat larger (e.g., $M_1 \gtrsim 1.6 \times 10^{11}$ GeV) are required. Since both the baryon asymmetry $|Y_B|$ and the effective Majorana mass in $(\beta\beta)_{0\nu}$-decay, $|\langle m \rangle|$, depend on the Majorana phase α_{21}, for given values of the other parameters there exists a direct correlation between the values of $|Y_B|$ and $|\langle m \rangle|$ (Fig. 10). We have shown that one can have successful leptogenesis in the case under discussion also if $s_{13} \neq 0$ and the CP-violation is generated only by the Dirac phase δ in $U_{\rm PMNS}$ (Figs. 11 and 12). For $M_1 \lesssim 5 \times 10^{11}$ GeV, the observed baryon asymmetry can be reproduced if $|\sin\theta_{13}\sin\delta| \gtrsim 0.02$. This requirement implies that we should have also $\sin\theta_{13} \gtrsim 0.02$ and $|J_{\rm CP}| \gtrsim 4.6 \times 10^{-3}$. Values of $\sin\theta_{13}$ and of $|J_{\rm CP}|$ as small as 0.02 and 4.6×10^{-3}, respectively, can be probed in neutrino oscillation experiments at neutrino factories. There exists a correlation between the rephasing invariant $J_{\rm CP}$ and the baryon asymmetry Y_B in this case as well (Fig. 13).

The analysis we have performed showed that if the light neutrinos have QD spectrum, the baryon asymmetry is generically too small mainly due to the large wash-out suppression factor.

For heavy RH Majorana neutrinos with QD spectrum, leptogenesis takes place through a resonance effect. The main new feature is that the final baryon asymmetry does not depend on the mass of the RH neutrinos. This property is crucial, allowing the generation of a sufficiently large baryon asymmetry even in the case of QD light neutrino mass spectrum. In the latter case the predicted baryon asymmetry is correlated with the effective Majorana mass in the neutrinoless double beta decay (Fig. 17).

Finally, we have discussed how the results on leptogenesis we have obtained will be modified in the minimal supersymmetric extension of the Standard Theory (MSSM) with right-handed Majorana neutrinos and see-saw mechanism of neutrino mass generation. We

have noticed, in particular, that for hierarchical heavy Majorana neutrino mass spectrum, the range of the lightest RH neutrino mass M_1 for which the lepton flavour effects are relevant in leptogenesis is greater than the one in the non-supersymmetric case by the factor $(1+\tan^2\beta)$, $M_1 \lesssim (1+\tan^2\beta)\times 10^{12}$ GeV, $\tan\beta$ being the ratio of the vacuum expectation values of the two Higgs fields of the MSSM. This can have important implications especially in the cases when the generation of the baryon asymmetry in the non-supersymmetric case is strongly suppressed. We have also stressed that a quasi-degenerate spectrum of the heavy RH Majorana neutrinos is welcome in the case of supersymmetric leptogenesis since it renders the gravitino bound harmless.

The results obtained in the present article underline the importance of understanding the status of the CP-symmetry in the lepton sector and, correspondingly, of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.

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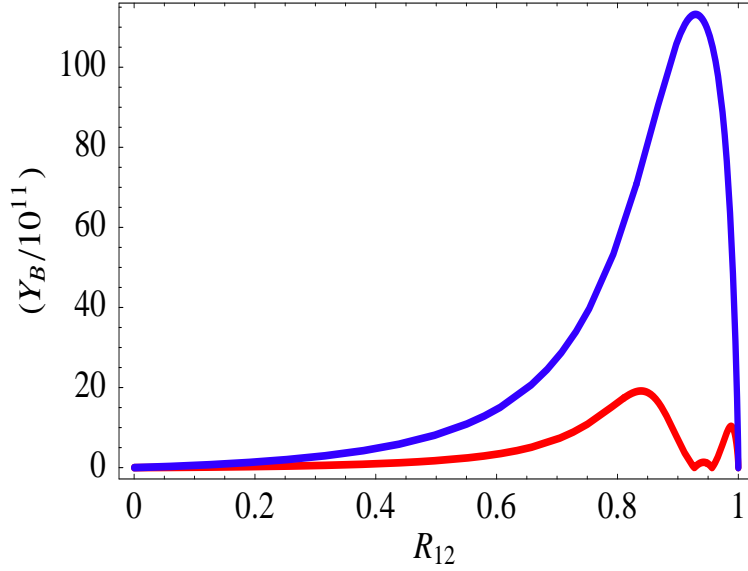
Figure 1: The baryon asymmetry Y_B as a function of R_{12} in the case of real R_{12} and R_{13}, sign $(R_{12}R_{13}) = +1$ $(\beta_{23} = 0)$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$, hierarchical RH neutrinos and NH light neutrino mass spectrum and a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ $(\kappa = +1)$, and b) Dirac CP-violation (red l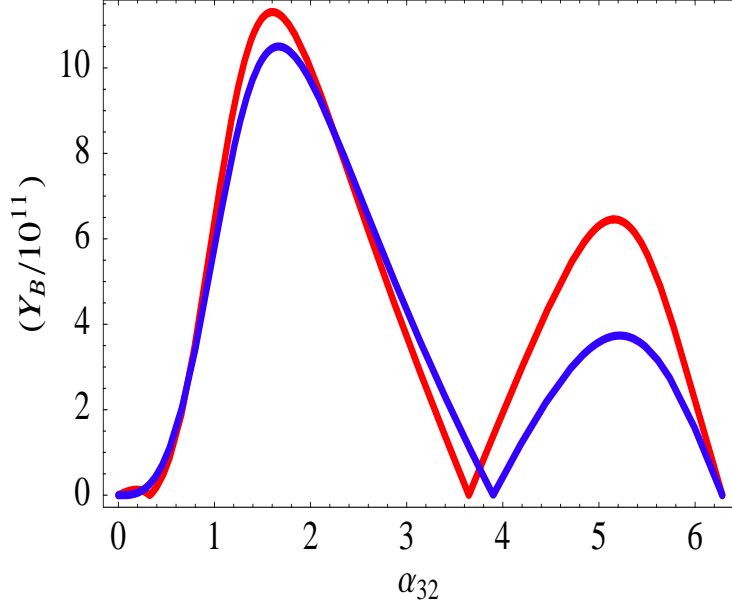ine), $\delta = \pi/2$ and $\alpha_{32} = 0$ $(\kappa' = +1)$, for $M_1 = 5 \times 10^{11}$ GeV. The neutrino oscillation parameters $\Delta m_\odot^2$, $\sin^2 \theta_{12}$, Δm_A^2 and $\sin^2 2\theta_{23}$ are fixed at their best fit values.
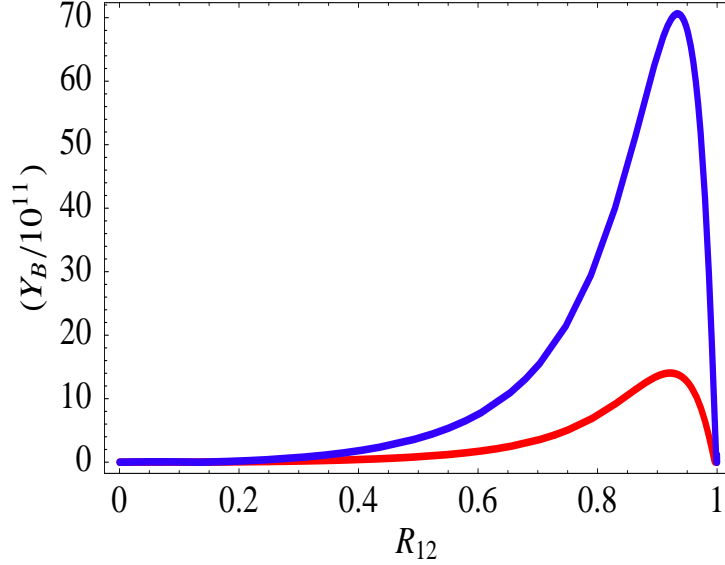
Figure 2: The same as in Fig. 1 but for sign $(R_{12}R_{13}) = -1$ $(\beta_{23} = \pi)$ and a) Majorana CP-violation (blue line), $\delta = 0$ and $\alpha_{32} = \pi/2$ $(\kappa = -1)$, and b) Dirac CP-violation (red line), $\delta = \pi/2$ and $\alpha_{32} = 0$ $(\kappa' = -1)$.

Figure 3: The baryon asymmetry as a function of the Majorana phase α_{32} varying in the interval $\alpha_{32} = [0, 2\pi]$ in the case of Majorana CP-violation, hierarchical RH neutrinos and NH light neutrino mass spectrum, for $\delta = 0$, real R_{12} and R_{13}, $|R_{12}| = 0.92$, $|R_{13}| = 0.39$, $\text{sgn}(R_{12}R_{13}) = +1$ ($\beta_{23} = 0$, $\kappa = +1$), $M_1 = 5 \times 10^{10}$ GeV, and two values of s_{13}: $s_{13} = 0$ (blue line) and 0.2 (red line).
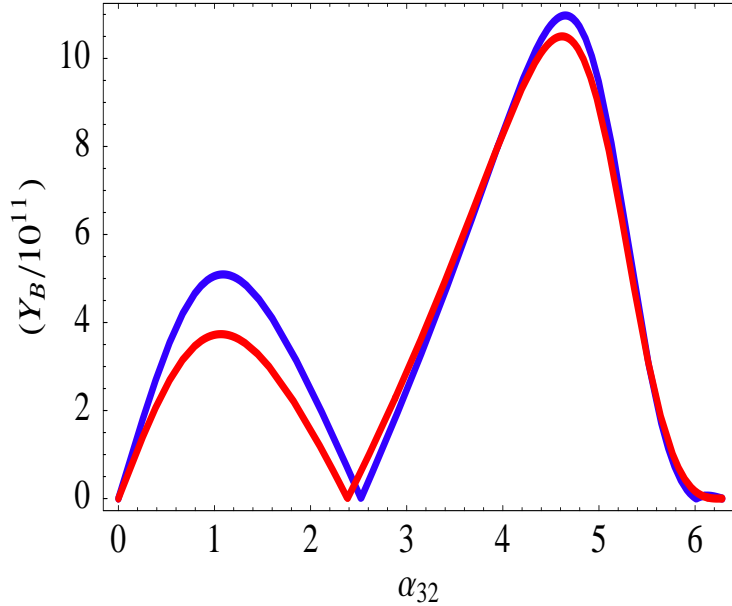

Figure 4: The same as in Fig. 3 but for real R_{12} and R_{13} having opposite signes, $\text{sgn}(R_{12}R_{13}) = -1$ ($\beta_{23} = \pi$, $\kappa = -1$), $|R_{12}| = 0.92$, $|R_{13}| = 0.39$, and two values of s_{13}: $s_{13} = 0$ (red line) and $s_{13} = 0.1$ (blue line).
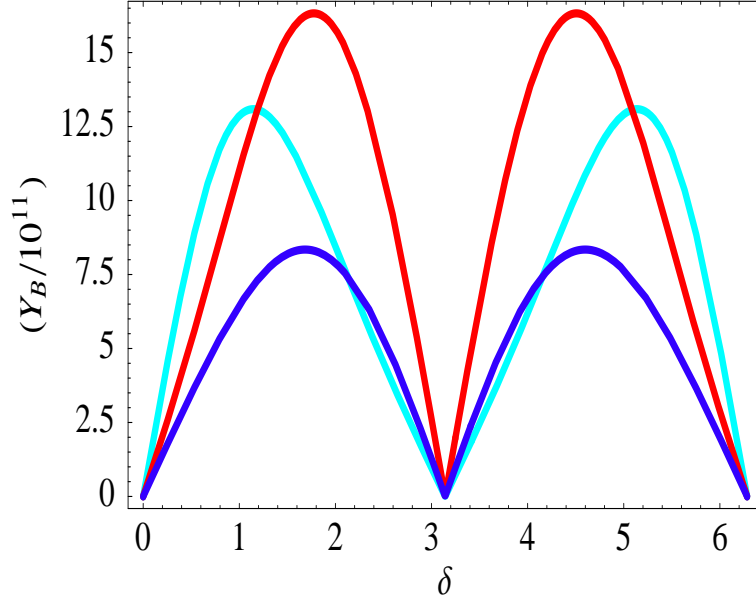
Figure 5: The baryon asymmetry $|Y_B|$ as a function of the Dirac phase δ varying in the interval $\delta = [0, 2\pi]$ in the case of Dirac CP-violation, $\alpha_{32} = 0;\ 2\pi$, hierarchical RH neutrinos and NH light neutrino mass spectrum, for $M_1 = 5 \times 10^{11}$ GeV, real R_{12} and R_{13} satisfying $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, sign $(R_{12}R_{13}) = +1$, and for i) $\alpha_{32} = 0$ ($\kappa' = +1$), $s_{13} = 0.2$ (red line) and $s_{13} = 0.1$ (dark blue line), ii) $\alpha_{32} = 2\pi$ ($\kappa' = -1$), $s_{13} = 0.2$ (light blue line).
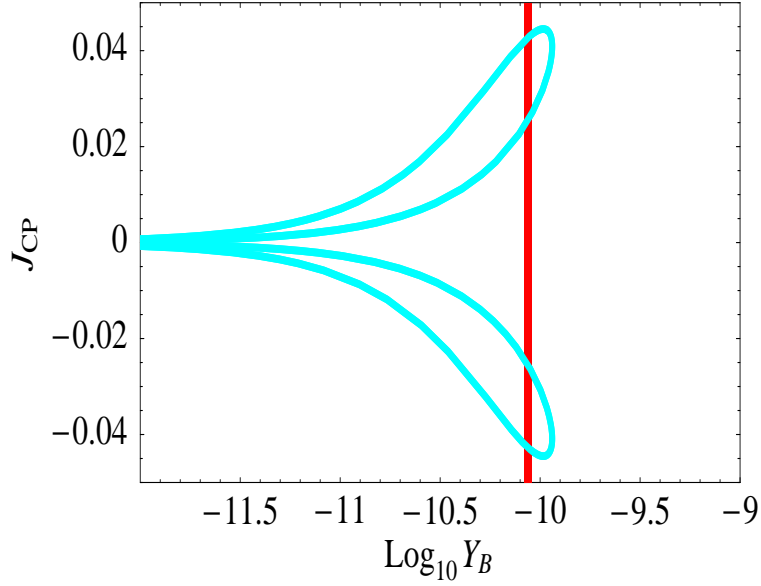
Figure 6: The correlation between the rephasing invariant $J_{\rm CP}$ (in blue) and the baryon asymmetry $Y_{\rm B}$ when varying the Dirac phase $\delta = [0, 2\pi]$, in the case of hierarchical RH neutrinos and NH light neutrino mass spectrum and for $s_{13} = 0.2$, $\alpha_{32} = 0\ (2\pi)$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\mathrm{sign}\,(R_{12}R_{13}) = +1\ (-1)\ (\beta_{23} = 0\ (\pi),\ \kappa' = +1)$, $M_1 = 5 \times 10^{11}$ GeV . The red region denotes the 2σ allowed range of $Y_{\rm B}$.
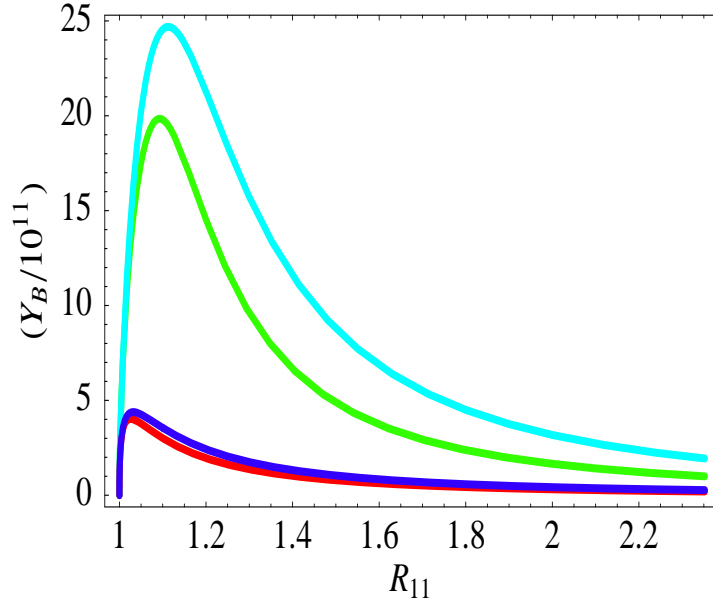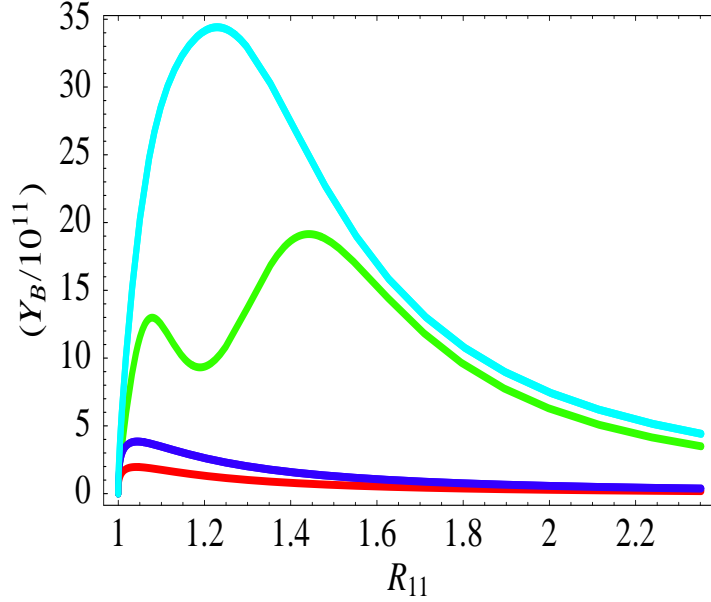

Figure 7: The baryon asymmetry Y_B as a function of $|R_{11}|$ in the case of hierarchical RH neutrinos and IH light neutrino mass spectrum, Majorana CP-violation, $\delta = 0$ and $\alpha_{32} = \pi/2$, $M_1 = 2 \times 10^{11}$ GeV, purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$ and $\kappa = +1$ (dark blue and red lines), $\kappa = -1$ (light blue and green lines), $|R_{12}|^2 - |R_{13}|^2 = 1$, and for $s_{13} = 0.2$ (green and red lines) and $s_{13} = 0$ (light and dark blue lines).
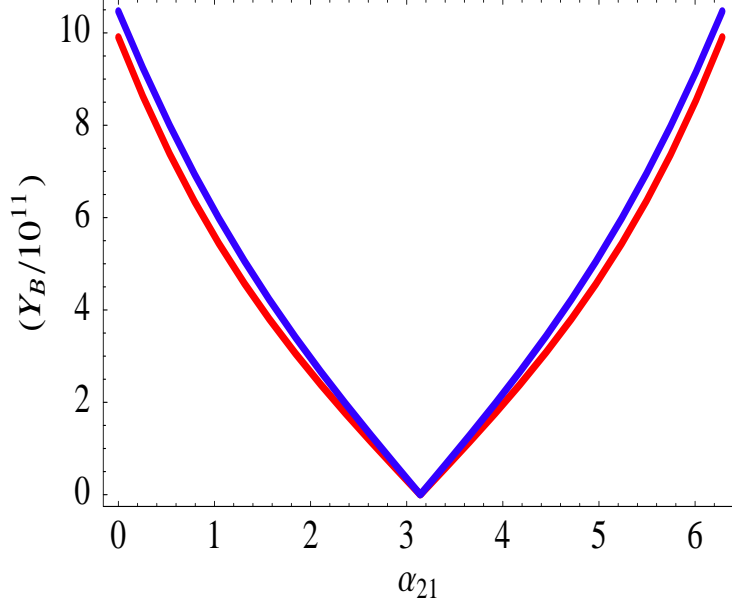
Figure 8: The baryon asymmetry as a function of the Majorana phase α_{32} varying in the interval $\alpha_{32} = [0, 2\pi]$ in the case of hierarchical RH neutrinos and IH light neutrino mass spectrum, Majorana CP-violation, $\delta = 0$, purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ ($\beta_{12} = \pi/2$), $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$, and for $M_1 = 2 \times 10^{11}$ GeV, and two values of s_{13}: $s_{13} = 0$ (blue line) and 0.2 (red line).
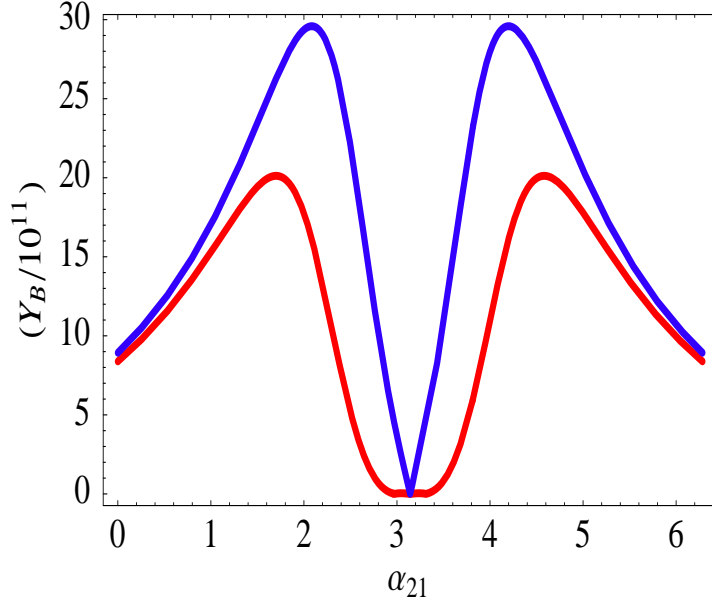
Figure 9: The same as in Fig. 8, but for $\kappa = -1$ ($\beta_{12} = 3\pi/2$) and $|R_{11}| = 1.2$.
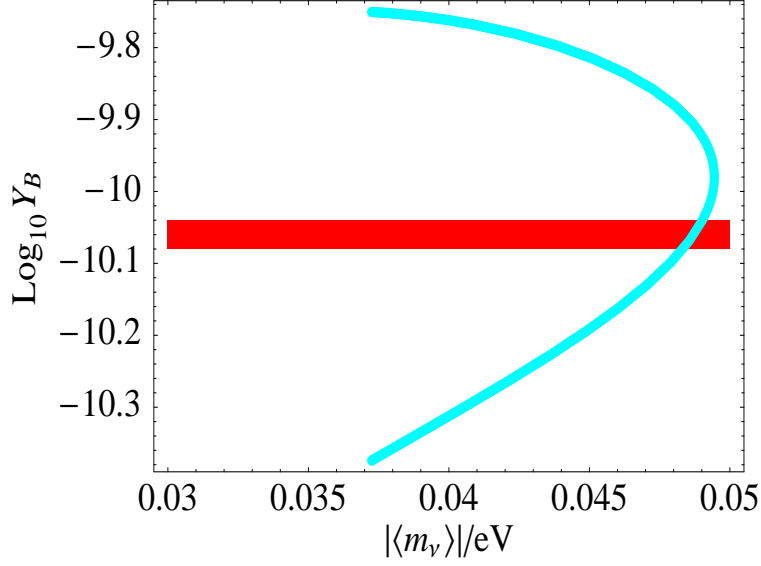
Figure 10: The baryon asymmetry $|Y_B|$ versus the effective Majorana mass in neutrinoless double beta decay, $|\langle m \rangle|$, in the case of Majorana CP-violation, hierarchical RH neutrinos and IH light neutrino mass spectrum, for $\delta = 0$, $s_{13} = 0$, purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ ($\beta_{12} = \pi/2$), $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$ and $M_1 = 2 \times 10^{11}$ GeV. The Majorana phase α_{21} is varied in the interval $[-\pi/2, \pi/2]$.

Figure 11: The baryon asymmetry $|Y_B|$ as a function of $|R_{11}|$ in the case of hierarchical RH neutrinos and IH light neutrino mass spectrum and Dirac CP-violation, $\delta = \pi/2$, $\alpha_{21} = \pi$ ($\kappa' = +1$), purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$ ($|R_{12}|^2 - |R_{13}|^2 = 1$), for $\kappa = -1$ (red and dark blue lines) and $\kappa = +1$ (light blue and green lines), $s_{13} = 0.2$ (light blue and dark blue lines) and $s_{13} = 0.1$ (green and red lines), and $M_1 = 2 \times 10^{11}$ GeV.
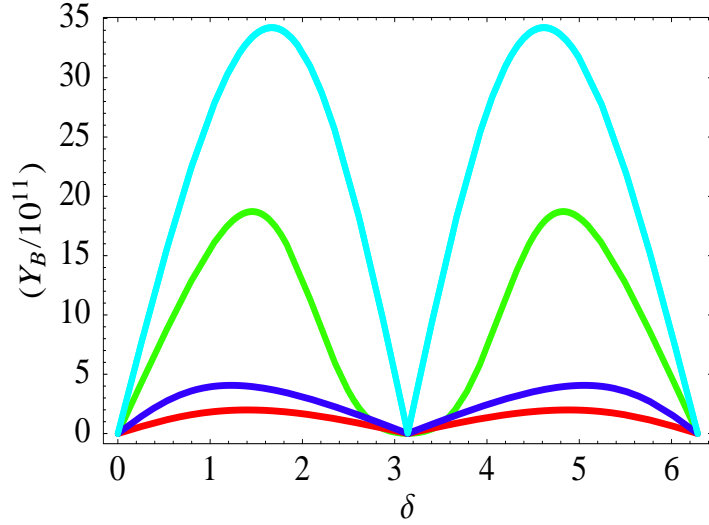
Figure 12: The asymmetry $|Y_B|$ as a function of the Dirac phase δ in the case of hierarchical RH neutrinos, IH light neutrino mass spectrum, Dirac CP-violation, $\alpha_{21} = \pi$ ($\kappa' = +1$), $R_{11}R_{12} = i \, \kappa \, |R_{11}R_{12}|$ ($|R_{11}|^2 - |R_{12}|^2 = 1$), $\kappa = -1$ (red and dark blue lines), $\kappa = +1$ (light blue and green lines), for $M_1 = 2 \times 10^{11}$ GeV, and $s_{13} = 0.1$ (red and green lines) and $s_{13} = 0.2$ (dark blue and light blue lines). Values of $|R_{11}|$, which maximise $|Y_B|$ have been used: $|R_{11}| = 1.05$ in the case of $\kappa = -1$, and $|R_{11}| = 1.3$ (1.6) for $\kappa = +1$ and $s_{13} = 0.2$ (0.1).
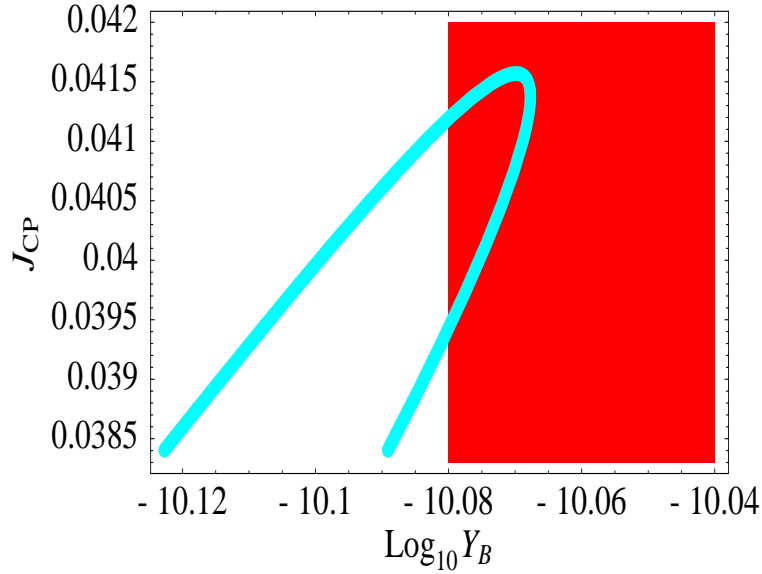
Figure 13: The correlation between the rephasing invariant J_{CP} (in blue) and the asymmetry Y_{B} in the case of hierarchical RH neutrinos, IH light neutrino mass spectrum, Dirac CP-violation, $\alpha_{21} = \pi$ ($\kappa' = +1$), $R_{11}R_{12} = i \, \kappa \, |R_{11}R_{12}|$ ($|R_{11}|^2 - |R_{12}|^2 = 1$), $\kappa = +1$, and for $s_{13} = 0.2$, $M_1 = 5 \times 10^{10}$ GeV and $|R_{11}| = 1.3$. The Dirac phase δ is varied in the interval $[0, 2\pi]$. The red region denotes the 2σ allowed range of Y_{B}.
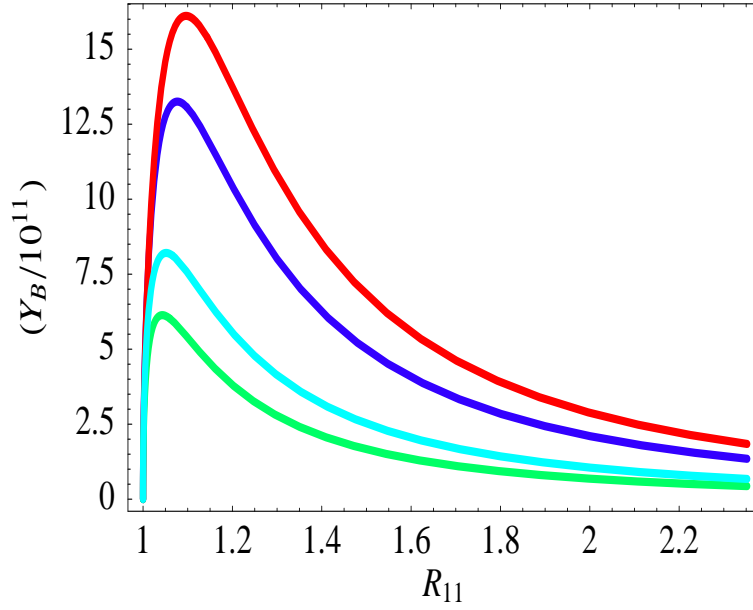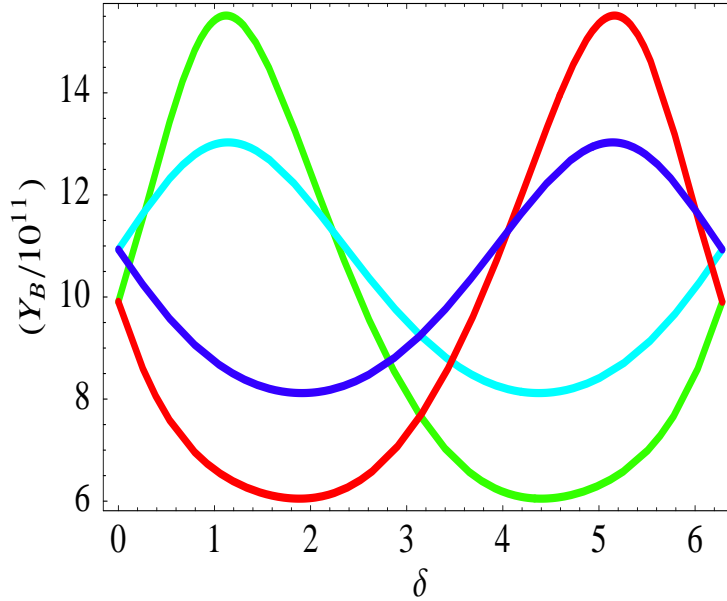
Figure 14: The baryon asymmetry Y_B as a function of $|R_{11}|$ in the case of hierarchical RH neutrinos and IH light neutrino mass spectrum, $\delta = \pi/2$, $\alpha_{21} = 0$, and purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$ satisfying $|R_{12}|^2 - |R_{13}|^2 = 1$, for $\kappa = +1$ (red and dark blue lines), $\kappa = -1$ (light blue and green lines), $M_1 = 2 \times 10^{11}$ GeV and two values of s_{13}: $s_{13} = 0.2$ (red and light blue lines) and $s_{13} = 0.1$ (dark blue and green lines).

Figure 15: The baryon asymmetry as a function of the Dirac phase δ in the case of hierarchical RH neutrinos and IH light neutrino mass spectrum, for $\alpha_{21} = 0$, $M_1 = 2 \times 10^{11}$ GeV, purely imaginary $R_{11}R_{12} = i \kappa |R_{11}R_{12}|$ $(|R_{11}|^2 - |R_{12}|^2 = 1)$, with $\kappa = +1$ (green and light blue lines) and $\kappa = -1$ (red and dark blue lines), and for $|R_{11}| = 1.05$ and two values of s_{13}: $s_{13} = 0.1$ (light and dark blue lines), $s_{13} = 0.2$ (red and green lines).
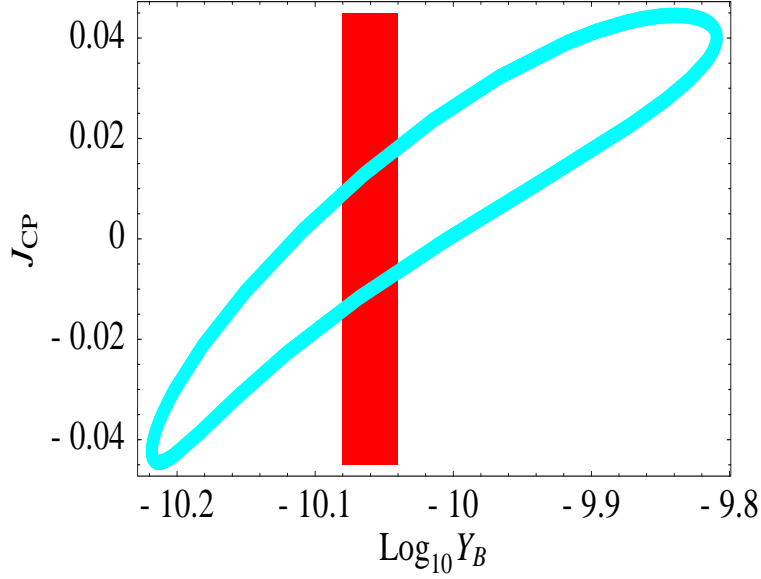
Figure 16: The correlation between the rephasing invariant $J_{\rm CP}$ (in blue) and the baryon asymmetry $Y_{\rm B}$ in the case of hierarchical RH neutrinos and IH light neutrino mass spectrum for $\alpha_{21} = 0$, $s_{13} = 0.2$, $M_1 = 2 \times 10^{11}$ GeV, $R_{11}R_{12} = i\,\kappa\,|R_{11}R_{12}|$ $(|R_{11}|^2 - |R_{12}|^2 = 1)$, and for $\kappa = +1$ and $|R_{11}| = 1.05$. The Dirac phase δ is varied in the interval $[0, 2\pi]$. The red region denotes the 2σ allowed range of $Y_{\rm B}$.
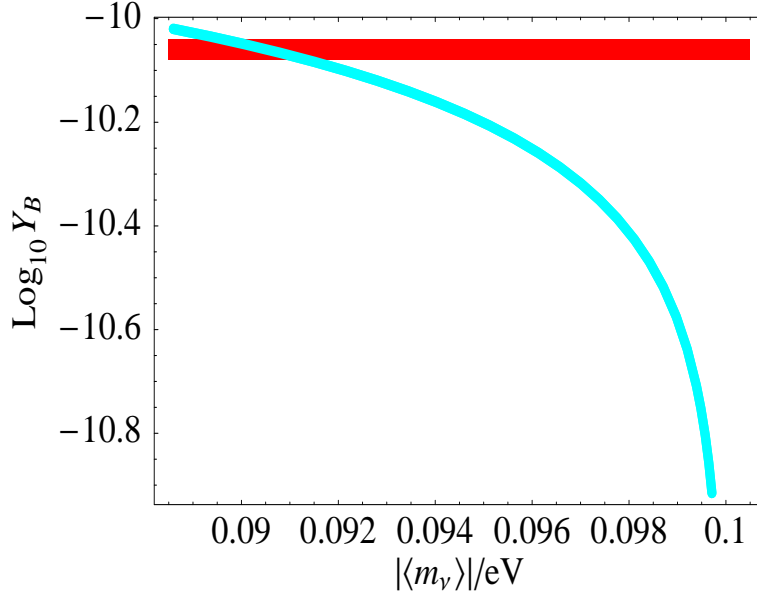

Figure 17: The baryon asymmetry versus the $(\beta\beta)_{0\nu}$-decay effective Majorana mass $|\langle m_\nu \rangle|$ in the case of QD heavy RH neutrinos and QD light neutrino mass spectrum, and for $\delta = \pi/3$, $s_{13} = 0.01$, $M_1 = 10^{10}$ GeV and $m = 0.1$ eV. The Majorana phase α_{32} is varied in the interval $[0, \pi/3]$.
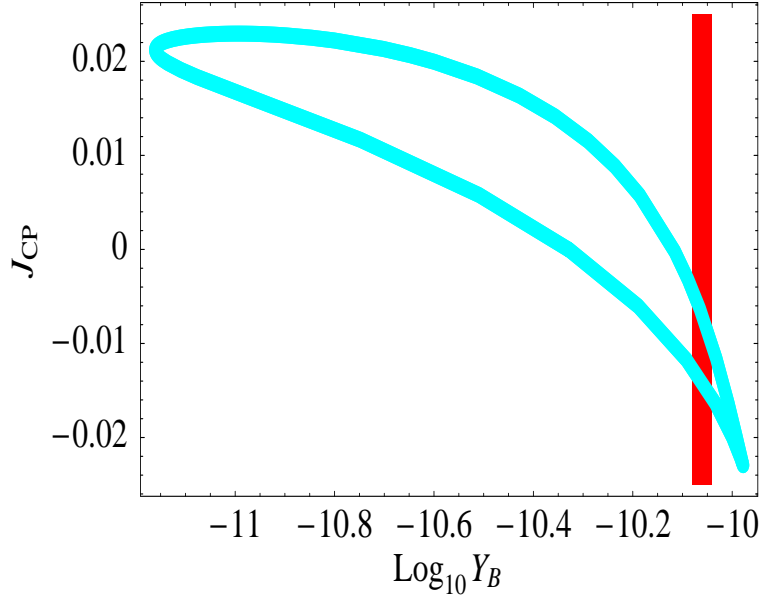
Figure 18: The rephasing invariant J_{CP} versus the baryon asymmetry in the case of supersymmetric hierarchical RH neutrinos and IH light neutrino mass spectrum and for $\alpha_{32} = \pi/4$, $s_{13} = 0.1$, $R_{12} = 0.86$, $R_{13} = 0.5$, $\text{sign}(R_{12}R_{13}) = +1$ and $M_1 = 6 \times 10^{12}$ GeV. The Dirac phase δ is varied in the interval $[0,2\pi]$.