THE MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY

Sir Isaac Newton

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THE MATHEMATICAL PRINCIPLES OF Natural Philosophy.

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OF THE

MOTION OF BODIES.

BOOK II.

SECTION I.

Of the Motion of Bodies that are reſiſted in the ratio of the Velocity.

PROPOSITION I. THEOREM I.

If a body is reſiſted in the ratio of its velocity, the motion loſt by reſiſtance is as the ſpace gone over in its motion.

FOR ſince the motion loſt in each equal particle of time is as the velocity, that is, as the particle of ſpace gone over; then, by compoſition, the motion loſt in the whole time will be as the whole ſpace gone over. *Q. E. D.*

Cor. Therefore if the body, destitute of all gravity, move by its innate force only in free spaces, and there be given both its whole motion at the beginning, and also the motion remaining after some part of the way is gone over; there will be given also the whole space which the body can describe in an infinite time. For that space will be to the space now described, as the whole motion at the beginning is to the part lost of that motion.

Lemma I.

Quantities proportional to their differences are continually proportional.

Let *A* be to *A—B* as *B* to *B—C* and *C* to *C—D*, &c. and, by conversion, *A* will be to *B* as *B* to *C* and *C* to *D*, &c. *Q. E. D.*

Proposition II. Theorem II.

If a body is resisted in the ratio of its velocity, and moves, by its vis insita *only, through a similar medium, and the times be taken equal; the velocities in the beginning of each of the times are in a geometrical progression, and the spaces described in each of the times are as the velocities.*

Case 1. Let the time be divided into equal particles; and if at the very beginning of each particle we suppose the resistance to act with one single impulse which is as the velocity; the decrement of the velocity in each of the particles of time will be as the same velocity. Therefore the velocities are proportional to their differences, and therefore (by Lem. 1. Book 2.) con-

continually proportional. Therefore if out of an equal number of particles there be compounded any equal portions of time, the velocities at the beginning of those times will be as terms in a continued progression, which are taken by intervals, omitting every where an equal number of intermediate terms. But the ratio's of these terms are compounded of the equal ratio's of the intermediate terms equally repeated; and therefore are equal. Therefore the velocities, being proportional to those terms, are in geometrical progression. Let those equal particles of time be diminished, and their number increased *in infinitum*, so that the impulse of resistance may become continual; and the velocities at the beginnings of equal times, always continually proportional, will be also in this case continually proportional. *Q. E. D.*

Case 2. And, by division, the differences of the velocities, that is, the parts of the velocities lost in each of the times, are as the wholes: But the spaces described in each of the times are as the lost parts of the velocities, (by Prop. 1. Book 2.) and therefore are also as the wholes. *Q. E. D.*

Corol. Hence if to the rectangular asymptotes *AC*, *CH*, the Hyperbola *BG* is described, and *AB*, *DG* be drawn perpendicular to the asymptote *AC*, and both the velocity of the body, and the resistance of the medium, at the very beginning of the motion, be express'd by any given line *AC*, and after some time is elapsed, by the indefinite line *DC*; the time may be express'd by the area *ABGD*, and the space described in that time by the line *AD*. For if that area, by the motion of the point *D*, be uniformly increased in the same manner as the time, the right line *DC* will decrease in a geometrical ratio in the same manner as the velocity, and the parts of the right line *AC*, described in equal times, will decrease in the same ratio.

PROPOSITION III. PROBLEM I.

To define the motion of a body which, in a similar medium, ascends or descends in a right line, and is resisted in the ratio of its velocity, and acted upon by an uniform force of gravity.

The body ascending, let the gravity be expounded by any given rectangle *BACH*; and the resistance of the medium, at the beginning of the ascent, by the rectangle *BADE*, taken on the contrary side of the right line *AB*. Through the point *B*, with the rectangular asymptotes *AC*, *CH*, describe an Hyperbola, cutting the perpendiculars *DE*, *de*, in *G*, *g*; and the body ascending will in the time *DGgd* describe the space *EGge*; in the time *DGBA*, the space of the whole ascent *EGB*; in the time *ABKI*, the space of descent *BFK*; and in the time *IKki* the space of descent *KFfk*; and the velocities of the bodies (proportional to the resistance of the medium) in these periods of time, will be *ABED*, *ABed*, o, *ABFI*, *ABfi* respectively; and the greatest velocity which the body can acquire by descending, will be *BACH*.

For let the rectangle *BACH* be resolved into innumerable rectangles *Ak*, *Kl*, *Lm*, *Mn*, &c. which shall be as the increments of the velocities produced in so many equal times; then will o, *Ak*, *Al*, *Am*, *An*, &c. be as the whole velocities, and therefore (by supposition) as the resistances of the medium in the beginning of each of the equal times. Make *AC* to *AK*, or *ABHC* to *ABkK* as the force of gravity to the resistance in the beginning of the second time; then from the force of gravity subduct the resistances, and *ABHC*, *KkHC*, *LlHC*, *MmHC*, &c. will be as the absolute forces with which the body is acted upon in the beginning of each of the times, and therefore

fore (by Law 2) as the increments of the velocities, that is, as the rectangles *Ak, Kl, Lm, Mn,* &c. and therefore (by Lem. 1. Book 2.) in a geometrical progression. Therefore if the right lines *Kk, Ll, Mm, Nn,* &c. are produced so as to meet the Hyperbola in *q, r, s, t, &c.* the areas *ABqK, KqrL, LrsM, MstN,* &c. will be equal, and therefore analogous to the equal times and equal gravitating forces. But the area *ABqK* (by Corol. 3. Lem. 7 & 8. Book 1.) is to the area *Bkq* as *Kq* to $\frac{1}{2}$ *kq*, or *AC* to $\frac{1}{2}$ *AK*, that is as the force of gravity to the resistance in the middle of the first time. And by the like reasoning the areas *qKLr, rLMs, sMNt,* &c. are to the areas *qklr, rlms, smnt,* &c. as the gravitating forces to the resistances in the middle of the second, third, fourth time, and so on. Therefore since the equal areas *BAKq, qKLr, rLMs, sMNt,* &c. are analogous to the gravitating forces, the areas *Bkq, qklr, rlms, smnt,* &c. will be analogous to the resistances in the middle of each of the times, that is (by supposition) to the velocities, and so to the spaces described. Take the sums of the analogous quantities, and the areas *Bkq, Blr, Bms, Bnt,* &c. will be analogous to the whole spaces described; and also the areas *ABqK, ABrL, ABsM, ABtN,* &c. to the times. Therefore the body, in descending, will in any time *ABrL,* describe the space *Blr,* and in the time *LrtN* the space *rlnt.* *Q.E.D.* And the like demonstration holds in ascending motion.

Corol. 1. Therefore the greatest velocity that the body can acquire by falling, is to the velocity acquired in any given time, as the given force of gravity which perpetually acts upon it, to the resisting force which opposes it at the end of that time.

Corol. 2. But the time being augmented in an arithmetical progression, the sum of that greatest velocity and the velocity in the ascent, and also their difference in the descent, decreases in a geometrical progression.

Corol. 3. Alſo the differences of the ſpaces, which are deſcribed in equal differences of the times, decreaſe in the ſame geometrical progreſſion.

Corol. 4. The ſpace deſcribed by the body is the difference of two ſpaces, whereof one is as the time taken from the beginning of the deſcent, and the other as the velocity; which [ſpaces] alſo at the beginning of the deſcent are equal among themſelves.

Proposition IV. Problem II.

Suppoſing the force of gravity in any ſimilar medium to be uniform, and to tend perpendicularly to the plane of the horizon; to define the motion of a projectile therein, which ſuffers reſiſtance proportional to its velocity.

Let the projectile go from any place *D* in the direction of any right line *DP*, and let its velocity at the beginning of the motion be expounded by the length *DP*. From the point *P* let fall the perpendicular *PC* on the horizontal line *DC*, and cut *DC* in *A*, ſo that *DA* may be to *AC* as the reſiſtance of the medium ariſing from its motion upwards at the beginning, to the force of gravity: or (which comes to the ſame) ſo that the rectangle under *DA* and *DP* may be to that under *AC* and *CP*, as the whole reſiſtance at the beginning of the motion to the force of gravity. With the aſymptotes *DC*, *CP* deſcribe any Hyperbola *GTBS* cutting the perpendiculars *DG*, *AB* in *G* and *B*; compleat the parallelogram *DGKC*, and let its ſide *GK* cut *AB* in *Q*. Take a line N in the ſame ratio to *QB* as *DC* is in to *CP*; and from any point *R* of the right line *DC*, erect *RT* perpendicular to it, meeting the Hyperbola in *T*, and the right lines *EH*, *GK*, *DP* in *I*, *t*, and *V*; in that perpendicular

cular take Vr equal to $\frac{tGT}{N}$, or, which is the same thing, take Rr equal to $\frac{GTIE}{N}$; and the projectile in the time $DRTG$ will arrive at the point r, describing the curve line $DraF$, the locus of the point r; thence it will come to its greatest height a in the perpendicular AB; and afterwards ever approach to the asymptote PC. And its velocity in any point r will be as the tangent rL to the curve. *Q. E. I.*

For N is to QB as DC to CP or DR to RV, and therefore RV is equal to $\frac{DR \times QB}{N}$, and Rr (that is, $RV - Vr$, or $\frac{DR \times QB - tGT}{N}$) is equal to $\frac{DR \times AB - RDGT}{N}$. Now let the time be expounded by the area $RDGT$, and (by Laws Cor. 2.) distinguish the motion of the body into two others, one of ascent, the other lateral. And since the resistance is as the motion, let that also be distinguished into two parts proportional and contrary to the parts of the motion: and therefore the length described by the lateral motion, will be (by Prop. 2. Book 2.) as the line DR, and the height (by Prop. 3. Book 2.) as the area $DR \times AB - RDGT$, that is, as the line Rr. But in the very beginning of the motion the area $RDGT$ is equal to the rectangle $DR \times AQ$, and therefore that line Rr (or $\frac{DR \times AB - DR \times AQ}{N}$) will then be to DR as $AB - AQ$ or QB to N, that is, as CP to DC; and therefore as the motion upwards to the motion lengthwise at the beginning. Since therefore Rr is always as the height, and DR always as the length, and Rr is to DR at the beginning, as the height to the length: it follows, that Rr is always to DR as the height to the length; and therefore that the body will move in the line $DraF$, which is the locus of the point r. *Q.E.D.*

Cor. 1. Therefore Rr is equal to $\frac{DR \times AB}{N} - \frac{RDGT}{N}$; and therefore if RT be produced to X, so that RX may be equal to $\frac{DR \times AB}{N}$, that is, if the parallelogram $ACPY$ be compleated, and DY cutting CP in Z be drawn, and RT be produced till it meets DY in X; Xr will be equal to $\frac{RDGT}{N}$, and therefore proportional to the time.

Cor. 2. Whence if innumerable lines CR, or, which is the same, innumerable lines ZX, be taken in a geometrical progression; there will be as many lines Xr in an arithmetical progression. And hence the curve $DraF$ is easily delineated by the Table of Logarithms.

Cor. 3. If a Parabola be constructed to the vertex D, and the diameter DG, produced downwards, and its latus rectum is to $2\,DP$ as the whole resistance at the beginning of the motion to the gravitating force: the velocity with which the body ought to go from the place D, in the direction of the right line DP, so as in an uniform resisting medium to describe the curve $DraF$, will be the same as that with which it ought to go from the same place D, in the direction of the same right line DP, so as to describe a Parabola in a non-resisting medium. For the latus rectum of this Parabola, at the very beginning of the motion, is $\frac{DV^2}{Vr}$; and Vr is $\frac{tGT}{N}$ or $\frac{DR \times Tt}{2N}$. But a right

line,

line, which, if drawn, would touch the Hyperbola *GTS* in *G*, is parallel to *DK*, and therefore *Tt* is $\frac{CK \times DR}{DC}$, and N is $\frac{QB \times DC}{CP}$: And therefore *Vr* is equal to $\frac{DR^2 \times CK \times CP}{2\,DC^2 \times QB}$, that is, (because *DR* and *DC*, *DV* and *DP* are proportionals) to $\frac{DV^2 \times CK \times CP}{2\,DP^2 \times QB}$; and the latus rectum $\frac{DV^2}{Vr}$ comes out $\frac{2\,DP^2 \times QB}{CK \times CP}$, that is, (because *QB* and *CK*, *DA* and *AC* are proportional) $\frac{2\,DP^2 \times DA}{AC \times CP}$, and therefore is to 2 *DP*, as $DP \times DA$ to $CP \times AC$; that is, as the resistance to the gravity. *Q.E.D.*

Cor. 4. Hence if a body be projected from any place *D*, with a given velocity, in the direction of a right line *DP* given by position; and the resistance of the medium, at the beginning of the motion, be given: the curve *DraF*, which that body will describe, may be found. For the velocity being given, the latus rectum of the parabola is given, as is well known. And taking 2 *DP* to that latus rectum, as the force of gravity to the resisting force, *DP* is also given. Then cutting *DC* in *A*, so that $CP \times AC$ may be to $DP \times DA$ in the same ratio of the gravity to the resistance, the point *A* will be given. And hence the curve *DraF* is also given.

Cor. 5. And on the contrary, if the curve *DraF* be given, there will be given both the velocity of the body, and the resistance of the medium in each of the places *r*. For the ratio of $CP \times AC$ to $DP \times DA$ being given, there is given both the resistance of the medium at the beginning of the motion, and the latus rectum of the parabola; and thence the velocity at the begin-

beginning of the motion is given alſo. Then from the length of the tangent rL, there is given both the velocity proportional to it, and the reſiſtance proportional to the velocity in any place r.

Cor. 6. But ſince the length $2DP$ is to the latus rectum of the parabola as the gravity to the reſiſtance in D; and, from the velocity augmented, the reſiſtance is augmented in the ſame ratio, but the latus rectum of the parabola is augmented in the duplicate of that ratio; it is plain that the length $2DP$ is augmented in that ſimple ratio only; and is therefore always proportional to the velocity; nor will it be augmented or diminiſhed by the change of the angle CDP, unleſs the velocity be alſo changed.

Cor. 7. Hence appears the method of determining the curve $DraF$, nearly, from the phænomena, and thence collecting the reſiſtance and velocity with which the body is projected. Let two ſimilar and equal bodies be projected with the ſame velocity, from the place D, in different angles CDP, CDp; and let the places F, f, where they fall upon the horizontal plane DC, be known. Then taking any length for DP or Dp, ſuppoſe the reſiſtance in D to be to the gravity in any ratio whatſoever, and let that ratio be expounded by any length SM. Then by computation, from that aſſumed length DP, find the lengths DF, Df; and from the ratio $\frac{Ff}{DF}$, found by calculation, ſubduct the ſame ratio as found by experiment; and let the difference be expounded by the perpendicular MN. Repeat the ſame a ſecond and a third time, by aſſuming always a new ratio SM of the reſiſtance to the gravity, and collecting a new difference MN. Draw the affirmative differences on one ſide of the right line SM, and the negative on the other ſide; and through the points N, N, N draw a regular curve NNN, cutting the

the right line *SMMM* in *X*, and *SX* will be the true ratio of the reſiſtance to the gravity, which was to be found. From this ratio the length *DF* is to be collected by calculation; and a length, which is to the aſſumed length *DP*, as the length *DF* known by experiment to the length *DF* juſt now found, will be the true length *DP*. This being known, you will have both the curve line *DraF* which the body deſcribes, and alſo the velocity and reſiſtance of the body in each place.

SCHOLIUM.

But yet that the reſiſtance of bodies is in the ratio of the velocity, is more a mathematical hypotheſis than a phyſical one. In mediums void of all tenacity, the reſiſtances made to bodies are in the duplicate ratio of the velocities. For by the action of a ſwifter body, a greater motion, in proportion to a greater velocity, is communicated to the ſame quantity of the medium, in a leſs time; and in an equal time, by reaſon of a greater quantity of the diſturbed medium, a motion is communicated in the duplicate ratio greater; and the reſiſtance (by Law 2 and 3.) is as the motion communicated. Let us therefore ſee what motions ariſe from this law of reſiſtance.

SECTION II.

Of the Motion of Bodies that are resisted in the duplicate ratio of their Velocities.

PROPOSITION V. THEOREM III.

If a body is resisted in the duplicate ratio of its velocity, and moves by its innate force only through a similar medium; and the times be taken in a geometrical progression, proceeding from less to greater terms: I say that the velocities at the beginning of each of the times are in the same geometrical progression inversely; and that the spaces are equal, which are described in each of the times.

For since the resistance of the medium is proportional to the square of the velocity, and the decrement of the velocity is proportional to the resistance; if the time be divided into innumerable equal particles, the squares of the velocities at the beginning of each of the times will be proportional to the differences of the same velocities. Let those particles of time be *AK*, *KL*, *LM*, &c. taken in the right line *CD*; and erect the perpendiculars *AB*, *Kk*, *Ll*, *Mm*, &c. meeting the Hyperbola *BklmG*, described with the centre *C*, and the rectangular asymptotes *CD*, *CH*, in *B*, *k*, *l*, *m*, &c. then *AB* will be to *Kk*,

as

as CK to CA, and, by division, $AB - Kk$ to Kk as AK to CA, and, alternately, $AB - Kk$ to AK as Kk to CA, and therefore as $AB \times Kk$ to $AB \times CA$. Therefore since AK and $AB \times CA$ are given, $AB - Kk$ will be as $AB \times Kk$; and lastly, when AB and Kk coincide, as AB^2. And, by the like reasoning, $Kk - Ll$, $Ll - Mm$, &c. will be as Kk^2, Ll^2, &c. Therefore the squares of the lines AB, Kk, Ll, Mm, &c. are as their differences; and therefore, since the squares of the velocities were shewn above to be as their differences, the progression of both will be alike. This being demonstrated, it follows also that the areas described by these lines are in a like progression with the spaces described by these velocities. Therefore if the velocity at the beginning of the first time AK be expounded by the line AB, and the velocity at the beginning of the second time KL by the line Kk, and the length described in the first time by the area $AKkB$; all the following velocities will be expounded by the following lines Ll, Mm, &c. and the lengths described, by the areas Kl, Lm, &c. And, by composition, if the whole time be expounded by AM, the sum of its parts, the whole length described will be expounded by $AMmB$ the sum of its parts. Now conceive the time AM to be divided into the parts AK, KL, LM, &c. so that CA, CK, CL, CM, &c. may be in a geometrical progression; and those parts will be in the same progression, and the velocities AB, Kk, Ll, Mm, &c. will be in the same progression inversly, and the spaces described Ak, Kl, Lm, &c. will be equal. *Q.E.D.*

Cor. 1. Hence it appears, that if the time be expounded by any part AD of the asymptote, and the velocity in the beginning of the time by the ordinate AB; the velocity at the end of the time will be expounded by the ordinate DG; and the whole space described, by the adjacent hyperbolic area $ABGD$;

and

and the ſpace which any body can deſcribe in the ſame time *AD*, with the firſt velocity *AB*, in a non-reſiſting medium, by the rectangle *AB*×*AD*.

COR. 2. Hence the ſpace deſcribed in a reſiſting medium is given, by taking it to the ſpace deſcribed with the uniform velocity *AB* in a non-reſiſting medium, as the hyperbolic area *ABGD* to the rectangle *AB*×*AD*.

COR. 3. The reſiſtance of the medium is alſo given, by making it equal, in the very beginning of the motion, to an uniform centripetal force, which could generate, in a body falling thro' a non-reſiſting medium, the velocity *AB*, in the time *AC*. For if *BT* be drawn touching the hyperbola in *B*, and meeting the aſymptote in *T*; the right line *AT* will be equal to *AC*, and will expreſs the time, in which the firſt reſiſtance uniformly continued, may take away the whole velocity *AB*.

COR. 4. And thence is alſo given the proportion of this reſiſtance to the force of gravity, or any other given centripetal force.

COR. 5. And *vice verſa*, if there is given the proportion of the reſiſtance to any given centripetal force; the time *AC* is alſo given, in which a centripetal force equal to the reſiſtance may generate any velocity as *AB*; and thence is given the point *B*, through which the hyperbola, having *CH*, *CD* for its aſymptotes, is to be deſcribed; as alſo the ſpace *ABGD*, which a body, by beginning its motion with that velocity *AB*, can deſcribe in any time *AD*, in a ſimilar reſiſting medium.

PRO-

Proposition VI. Theorem IV.

Homogeneous and equal spherical bodies, oppos'd by resistances that are in the duplicate ratio of the velocities, and moving on by their innate force only, will, in times which are reciprocally as the velocities at the beginning, describe equal spaces, and lose parts of their velocities proportional to the wholes.

To the rectangular asymptotes *CD*, *CH* describe any hyperbola *BbEe*, cutting the perpendiculars *AB*, *ab*, *DE*, *de*, in *B*, *b*, *E*, *e*; let the initial velocities be expounded by the perpendiculars *AB*, *DE*, and the times by the lines *Aa*, *Dd*. Therefore as *Aa* is to *Dd*, so (by the hypothesis) is *DE* to *AB*, and so (from the nature of the hyperbola) is *CA* to *CD*; and, by composition, so is *Ca* to *Cd*. Therefore the areas *ABba*, *DEed*, that is, the spaces described, are equal among themselves, and the first velocities *AB*, *DE* are proportional to the last *ab*, *de*; and therefore, by division, proportional to the parts of the velocities lost, *AB*—*ab*, *DE*—*de*. *Q. E. D.*

Proposition VII. Theorem V.

If spherical bodies are resisted in the duplicate ratio of their velocities, in times which are as the first motions directly and the first resistances inversely, they will lose parts of their motions proportional to the wholes, and will describe spaces proportional to those times and the first velocities conjunctly.

For the parts of the motions lost are as the resistances and times conjunctly. Therefore, that those parts may

be

be proportional to the wholes, the resistance and time conjunctly ought to be as the motion. Therefore the time will be as the motion directly and the resistance inversely. Wherefore the particles of the times being taken in that ratio, the bodies will always lose parts of their motions proportional to the wholes, and therefore will retain velocities always proportional to their first velocities. And because of the given ratio of the velocities, they will always describe spaces, which are as the first velocities and the times conjunctly. *Q.E.D.*

Cor. 1. Therefore if bodies equally swift are resisted in a duplicate ratio of their diameters: Homogeneous globes moving with any velocities whatsoever, by describing spaces proportional to their diameters, will lose parts of their motions proportional to the wholes. For the motion of each globe will be as its velocity and mass conjunctly, that is, as the velocity and the cube of its diameter; the resistance (by supposition) will be as the square of the diameter and the square of the velocity conjunctly; and the time (by this proposition) is in the former ratio directly and in the latter inversely, that is, as the diameter directly and the velocity inversely; and therefore the space, which is proportional to the time and velocity, is as the diameter.

Cor. 2. If bodies equally swift are resisted in a sesquiplicate ratio of their diameters: Homogeneous globes, moving with any velocities whatsoever, by describing spaces that are in a sesquiplicate ratio of the diameters, will lose parts of their motions proportional to the wholes.

Cor. 3. And universally, if equally swift bodies are resisted in the ratio of any power of the diameters: the spaces, in which homogeneous globes, moving with any velocity whatsoever, will lose parts of their motions proportional to the wholes, will be as the cubes of the diameters applied to that power. Let those di-

ameters

ameters applied to that power. Let those diameters be D and E; and if the resistances, where the velocities are supposed equal, are as D^n and E^n: the spaces in which the globes, moving with any velocities whatsoever, will lose parts of their motions proportional to the wholes, will be as D^{3-n} and E^{3-n}. And therefore homogeneous globes, in describing spaces proportional to D^{3-n} and E^{3-n}, will retain their velocities in the same ratio to one another as at the beginning.

Cor. 4. Now if the globes are not homogeneous, the space described by the denser globe must be augmented in the ratio of the density. For the motion, with an equal velocity, is greater in the ratio of the density, and the time (by this Prop.) is augmented in the ratio of motion directly, and the space described in the ratio of the time.

Cor. 5. And if the globes move in different mediums, the space, in a medium which, *cæteris paribus*, resists the most, must be diminished in the ratio of the greater resistance. For the time (by this Prop.) will be diminished in the ratio of the augmented resistance, and the space in the ratio of the time.

Lemma II.

The moment of any Genitum *is equal to the moments of each of the generating sides drawn into the indices of the powers of those sides, and into their coefficients continually.*

I call any quantity a *Genitum*, which is not made by addition or subduction of divers parts, but is generated or produced in arithmetic by the multiplication, division, or extraction of the root of any terms whatsoever; in geometry by the invention of contents and sides, or of the extreams and means of proportionals. Quantities of this kind are products, quotients, roots, rectangles, squares, cubes, square and cubic sides, and the like. These quantities I here consider as variable and indetermined, and increasing or decreasing as it were by a perpetual motion or flux; and I understand their momentaneous increments or decrements by the name of Moments; so that the increments may be esteem'd as added, or affirmative moments; and the decrements as subducted, or negative ones. But take care not to look upon finite particles as such. Finite particles are not moments, but the very quantities generated by the moments. We are to conceive them as the just nascent principles of finite magnitudes. Nor do we in this Lemma regard the magnitude of the moments, but their first proportion as nascent. It will be the same thing, if, instead of moments, we use either the Velocities of the increments and decrements (which may also be called the motions, mutations, and fluxions of quantities) or any finite quantities proportional to those velocities. The coefficient of any generating side is the quantity which arises by applying the Genitum to that side.

Wherefore the sense of the Lemma is, that if the moments of any quantities A, B, C, &c. increasing or decreasing by a perpetual flux, or the velocities of the mutations which are proportional to them, be called a, b, c, &c. the moment or mutation of the generated rectangle AB will be $aB + bA$; the moment of the generated content ABC will be $aBC + bAC + cAB$: and the moments of the generated powers, A^2, A^3, A^4, $A^{\frac{1}{2}}$, $A^{\frac{3}{2}}$, $A^{\frac{1}{3}}$, $A^{\frac{2}{3}}$, A^{-1}, A^{-2}, $A^{-\frac{1}{2}}$ will be $2aA$, $3aA^2$, $4aA^3$, $\frac{1}{2}aA^{-\frac{1}{2}}$, $\frac{3}{2}aA^{\frac{1}{2}}$, $\frac{1}{3}aA^{-\frac{2}{3}}$, $\frac{2}{3}aA^{-\frac{1}{3}}$, $-aA^{-2}$, $-2aA^{-3}$, $-\frac{1}{2}aA^{-\frac{3}{2}}$ respectively. And in general, that the moment of any power $A^{\frac{n}{m}}$ will be $\frac{n}{m}aA^{\frac{n-m}{m}}$. Also that the moment of the generated quantity A^2B will be $2aAB + bA^2$; the moment of the generated quantity $A^3B^4C^2$ will be $3aA^2B^4C^2 + 4bA^3B^3C^2 + 2cA^3B^4C$; and the moment of the generated quantity $\frac{A^3}{B^2}$ or A^3B^{-2} will be $3aA^2B^{-2} - 2bA^3B^{-3}$; and so on. The Lemma is thus demonstrated.

Case 1. Any rectangle as AB augmented by a perpetual flux, when, as yet, there wanted of the sides A and B half their moments $\frac{1}{2}a$ and $\frac{1}{2}b$, was $A - \frac{1}{2}a$ into $B - \frac{1}{2}b$, or $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$; but as soon as the sides A and B are augmented by the other half moments; the rectangle becomes $A + \frac{1}{2}a$ into $B + \frac{1}{2}b$ or $AB + \frac{1}{2}aB + \frac{1}{2}bA + \frac{1}{4}ab$. From this rectangle subduct the former rectangle, and there will remain the excess $aB + bA$. Therefore with the whole increments a and b of the sides, the increment $aB + bA$ of the rectangle is generated. *Q. E. D.*

Case 2. Suppose AB always equal to G, and then the moment of the content ABC or GC (by Case 1.) will be $gC + cG$, that is, (putting AB and $aB + bA$ for G and g) $aBC + bAC + cAB$. And the reasoning is the same for contents under never so many sides. *Q. E. D.*

Case 3. Suppose the sides A, B, and C, to be always equal among themselves; and the moment $aB + bA$, of A^2, that is, of the rectangle AB, will be $2aA$; and the moment $aBC + bAC + cAB$ of A^3, that is, of the content ABC, will be $3aA^2$. And by the same reasoning the moment of any power A^n is naA^{n-1}. *Q. E. D.*

Case 4. Therefore since $\frac{1}{A}$ into A is 1, the moment of $\frac{1}{A}$ drawn into A, together with $\frac{1}{A}$ drawn into a, will be the moment of 1, that is, nothing. Therefore

fore the moment of $\frac{1}{A}$ or of A^{-1} is $\frac{-a}{A^2}$. And generally, since $\frac{1}{A^n}$ into A^n is 1, the moment of $\frac{1}{A^n}$ drawn into A^n together with $\frac{1}{A^n}$ into naA^{n-1} will be nothing. And therefore the moment of $\frac{1}{A^n}$ or A^{-n} will be $-\frac{na}{A^{n+1}}$. *Q. E. D.*

Case 5. And since $A^{\frac{1}{2}}$ into $A^{\frac{1}{2}}$ is A, the moment of $A^{\frac{1}{2}}$ drawn into $2A^{\frac{1}{2}}$ will be a, (by Case 3:) and therefore the moment of $A^{\frac{1}{2}}$ will be $\frac{a}{2A^{\frac{1}{2}}}$ or $\frac{1}{2}aA^{-\frac{1}{2}}$. And generally, putting $A^{\frac{m}{n}}$ equal to B, then A^m will be equal to B^n, and therefore maA^{m-1} equal to nbB^{n-1}, and maA^{-1} equal to nbB^{-1} or $nbA^{-\frac{m}{n}}$; and therefore $\frac{m}{n}aA^{\frac{m-n}{n}}$ is equal to b, that is, equal to the moment of $A^{\frac{m}{n}}$. *Q. E. D.*

Case 6. Therefore the moment of any generated quantity A^mB^n is the moment of A^m drawn into B^n, together with the moment of B^n drawn into A^m, that is, $maA^{m-1}B^n + nbB^{n-1}A^m$; and that whether the indices m and n of the powers be whole numbers or fractions, affirmative or negative. And the reasoning is the same for contents under more powers. *Q. E. D.*

Cor. 1. Hence in quantities continually proportional, if one term is given, the moments of the rest of the terms will be as the same terms multiplied by the number of intervals between them and the given term. Let A, B, C, D, E, F, be continually proportional; then if the term C is given, the moments of the rest of the terms will be among themselves, as —2A, —B, D, 2E, 3F.

Cor. 2. And if in four proportionals the two means are given, the moments of the extremes will be as those extremes. The same is to be understood of the sides of any given rectangle.

Cor. 3. And if the sum or difference of two squares is given, the moments of the sides will be reciprocally as the sides.

Scholium.

In a letter of mine to Mr. *J. Collins*, dated *December* 10. 1672. having described a method of Tangents, which I suspected to be the same with *Slusius*'s method, which at that time was not made publick; I subjoined these words; *This is one particular, or rather a corollary, of a general method, which extends itself, without any troublesome calculation, not only to the drawing of Tangents to any Curve lines, whether Geometrical or Mechanical, or any how respecting right lines or other Curves, but also to the resolving other abstruser kinds of Problems about the crookedness, areas, lengths, centres of gravity of Curves,* &c. *nor is it (as* Hudden's *method* de Maximis & Minimis) *limited to equations which are free from surd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite series.* So far that letter. And these last words relate to a Treatise I composed on that subject in the year 1671. The foundation of that general method is contained in the preceding Lemma.

PROPOSITION VIII. THEOREM VI.

If a body in an uniform medium, being uniformly acted upon by the force of gravity, ascends or descends in a right line; and the whole space described be distinguished into equal parts, and in the beginning of each of the parts, (by adding or subducting the resisting force of the medium to or from the force of gravity, when the body ascends or descends) you collect the absolute forces; I say that those absolute forces are in a geometrical progression. Pl. 2. Fig. 1.

For let the force of gravity be expounded by the given line *AC*; the force of resistance by the indefinite line *AK*; the absolute force in the descent of the body, by the difference *KC*; the velocity of the body by a line *AP*, which shall be a mean proportional between *AK* and *AC*, and therefore in a subduplicate ratio of the resistance; the increment of the resistance made in a given particle of time by the lineola *KL*, and the contemporaneous increment of the velocity by the lineola *PQ*; and with the centre *C*, and rectangular asymptotes *CA*, *CH*, describe any Hyperbola *BNS*, meeting the erected perpendiculars *AB*, *KN*, *LO* in *B*, *N*, and *O*. Because *AK* is as AP^2, the moment *KL* of the one will be as the moment 2 *APQ* of the other, that is, as $AP \times KC$; for the increment *PQ* of the velocity is (by Law 2.) proportional to the generating force *KC*. Let the ratio of *KL* be compounded with the ratio of *KN*, and the rectangle $KL \times KN$ will become as $AP \times KC \times KN$; that is, (because the rectangle $KC \times KN$ is given) as *AP*. But the ultimate ratio of the hyperbolic area *KNOL* to the rectangle $KL \times KN$ becomes, when the points *K* and *L* coincide, the ratio of of equality. Therefore that hyperbolic evanescent area is as *AP*. Therefore the whole hyperbolic area *ABOL* is composed of particles *KNOL* which are always proportional to the velocity *AP*; and therefore is itself proportional to the space described with that velocity. Let that area be now divided into equal parts, as *ABMI*, *IMNK*, *KNOL*, &c. and the absolute forces *AC*, *IC*, *KC*, *LC*, &c. will be in a geometrical progression. *Q.E.D.* And by a like reasoning, in the ascent of the body, taking, on the contrary side of the point *A*, the equal areas *ABmi*, *imnk*, *knol*, &c. it will appear that the absolute forces *AC*, *iC*, *kC*, *lC*, &c. are continually proportional. Therefore if all the spaces in the ascent and descent are taken equal; all the absolute forces *lC*, *kC*, *iC*, *AC*, *IC*, *KC*, *LC*, &c. will be continually proportional. *Q.E.D.*

Cor. 1. Hence if the space described be expounded by the hyperbolic area *ABNK*; the force of gravity, the velocity of the body, and the resistance of the medium, may be expounded by the lines *AC*, *AP*, and *AK* respectively; and *vice versa*.

Cor. 2. And the greatest velocity, which the body can ever acquire in an infinite descent, will be expounded by the line *AC*.

Cor. 3. Therefore if the resistance of the medium answering to any given velocity be known, the greatest velocity will be found, by taking it to that given velocity in a ratio subduplicate of the ratio which the force of gravity bears to that known resistance of the medium.

Proposition IX. Theorem VII.

Supposing what is above demonstrated, I say that if the tangents of the angles of the sector of a circle, and of an hyperbola, be taken proportional to the velocities, the radius being of a fit magnitude; all the time of the ascent to the highest place will be as the sector of the circle, and all the time of descending from the highest place as the sector of the hyperbola. Pl. 2. Fig. 2.

To the right line AC, which expresses the force of gravity, let AD be drawn perpendicular and equal. From the centre D with the semidiameter AD describe as well the quadrant AtE of a Circle; as the rectangular Hyperbola AVZ, whose axe is AX, principal vertex A, and asymptote DC. Let Dp, DP be drawn; and the circular sector AtD will be as all the time of the ascent to the highest place; and the hyperbolic sector ATD as all the time of descent from the highest place: If so be that the tangents Ap, AP of those sectors be as the velocities. *Fig.* 2.

Case 1. Draw Dvq cutting off the moments or least particles tDv and qDp, described in the same time, of the sector ADt and of the triangle ADp. Since those particles (because of the common angle D) are in a duplicate ratio of the sides, the particle tDv will be as $\frac{qDp \times tD^2}{pD^2}$, that is, (because tD is given) as $\frac{qDp}{pD^2}$. But pD^2 is $AD^2 + Ap^2$, that is, $AD^2 + AD \times Ak$, or $AD \times Ck$; and qDp is $\frac{1}{2} AD \times pq$. Therefore tDv, the particle of the sector, is as $\frac{pq}{Ck}$; that

is,

is, as the least decrement pq of the velocity directly, and the force Ck, which diminishes the velocity, inversely; and therefore as the particle of time answering to the decrement of the velocity. And, by composition, the sum of all the particles tDv in the sector ADt, will be as the sum of the particles of time answering to each of the lost particles pq, of the decreasing velocity Ap, till that velocity, being diminished into nothing, vanishes; that is, the whole sector ADt is as the whole time of ascent to the highest place. *Q.E.D.*

Case 2. Draw DQV cutting off the least particles TDV and PDQ of the sector DAV, and of the triangle DAQ; and these particles will be to each other as DT^2 to DP^2, that is, (if TX and AP are parallel) as DX^2 to DA^2 or TX^2 to AP^2; and, by division, as $DX^2 - TX^2$ to $DA^2 - AP^2$. But, from the nature of the hyperbola, $DX^2 - TX^2$ is AD^2; and, by the supposition, AP^2 is $AD \times AK$. Therefore the particles are to each other as AD^2 to $AD^2 - AD \times AK$; that is, as AD to $AD - AK$ or AC to CK: and therefore the particle TDV of the sector is $\frac{PDQ \times AC}{CK}$; and therefore (because AC and AD are given) as $\frac{PQ}{CK}$; that is, as the increment of the velocity directly, and as the force generating the increment inversely; and therefore as the particle of the time answering to the increment. And, by composition, the sum of the particles of time, in which all the particles PQ of the velocity AP are generated, will be as the sum of the particles of the sector ATD; that is, the whole time will be as the whole sector. *Q.E.D.*

Cor. 1. Hence if AB be equal to a fourth part of AC, the space which a body will describe by falling in any time will be to the space which the body could describe, by moving uniformly on in the same time with

with its greatest velocity *AC*, as the area *ABNK*, which expresses the space described in falling, to the area *ATD*, which expresses the time. For since *AC* is to *AP* as *AP* to *AK*, then (by Cor. 1. Lem. 2. of this Book) *LK* is to *PQ* as 2 *AK* to *AP*, that is, as 2 *AP* to *AC*, and thence *LK* is to $\frac{1}{2}$ *PQ* as *AP* to $\frac{1}{4}$ *AC* or *AB*; and *KN* is to *AC* or *AD* as *AB* to *CK*; and therefore, *ex æquo*, *LKNO* to *DPQ* as *AP* to *CK*. But *DPQ* was to *DTV* as *CK* to *AC*. Therefore, *ex æquo*, *LKNO* is to *DTV* as *AP* to *AC*; that is, as the velocity of the falling body to the greatest velocity which the body by falling can acquire. Since therefore the moments *LKNO* and *DTV* of the areas *ABNK* and *ATD* are as the velocities, all the parts of those areas generated in the same time, will be as the spaces described in the same time; and therefore the whole areas *ABNK* and *ADT* generated from the beginning, will be as the whole spaces described from the beginning of the descent. *Q. E. D.*

Cor. 2. The same is true also of the space described in the ascent. That is to say, that all that space is to the space described in the same time with the uniform velocity *AC*, as the area *ABnk* is to the sector *ADt*.

Cor. 3. The velocity of the body, falling in the time *ATD*, is to the velocity which it would acquire in the same time in a non-resisting space, as the triangle *APD* to the hyperbolic sector *ATD*. For the velocity in a non-resisting medium would be as the time *ATD*, and in a resisting medium is as *AP*, that is, as the triangle *APD*. And those velocities at the beginning of the descent, are equal among themselves, as well as those areas *ATD*, *APD*.

Cor. 4. By the same argument, the velocity in the ascent is to the velocity with which the body in the same time, in a non-resisting space, would lose all

all its motion of ascent, as the triangle *ApD* to the circular sector *AtD*; or as the right line *Ap* to the arc *At*.

Cor. 5. Therefore the time in which a body by falling in a resisting medium, would acquire the velocity *AP*, is to the time in which it would acquire its greatest velocity *AC* by falling in a non-resisting space, as the sector *ADT* to the triangle *ADC*: and the time in which it would lose its velocity *Ap* by ascending in a resisting medium, is to the time in which it would lose the same velocity by ascending in a non-resisting space, as the arc *At* to its tangent *Ap*.

Cor. 6. Hence from the given time there is given the space described in the ascent or descent. For the greatest velocity of a body descending *in infinitum* is given (by Corol. 2 and 3. Theor. 6. of this Book) and thence the time is given in which a body would acquire that velocity by falling in a non-resisting space. And taking the sector *ADT* or *ADt* to the triangle *ADC* in the ratio of the given time to the time just now found; there will be given both the velocity *AP* or *Ap*, and the area *ABNK* or *ABnk*, which is to the sector *ADT*, or *ADt*, as the space sought to the space which would, in the given time, be uniformly described with that greatest velocity found just before.

Cor. 7. And by going backward, from the given space of ascent or descent *ABnk* or *ABNK*, there will be given the time *ADt* or *ADT*.

Pro-

Proposition X. Problem III.

Suppose the uniform force of gravity to tend directly to the plane of the horizon, and the resistance to be as the density of the medium and the square of the velocity conjunctly: it is proposed to find the density of the medium in each place, which shall make the body move in any given curve line; the velocity of the body, and the resistance of the medium in each place. Pl. 2. Fig. 3.

Let *PQ* be a plane perpendicular to the plane of the scheme itself; *PFHQ* a curve line meeting that plane in the points *P* and *Q*; *G, H, I, K* four places of the body going on in this curve from *F* to *Q*; and *GB, HC, ID, KE* four parallel ordinates let fall from these points to the horizon, and standing on the horizontal line *PQ* at the points *B, C, D, E*; and let the distances *BC, CD, DE*, of the ordinates be equal among themselves. From the points *G* and *H* let the right lines *GL, HN*, be drawn touching the curve in *G* and *H*, and meeting the ordinates *CH, DI*, produced upwards, in *L* and *N*; and compleat the parallelogram *HCDM*. And the times, in which the body describes the arcs *GH, HI*, will be in a subduplicate ratio of the altitudes *LH, NI*, which the bodies would describe in those times, by falling from the tangents; and the velocities will be as the lengths described *GH, HI* directly and the times inversely. Let the times be expounded by T and *t*, and the velocities by $\frac{GH}{T}$ and $\frac{HI}{t}$; and the decrement of the velocity pro-

produced in the time t will be expounded by $\frac{GH}{T} - \frac{HI}{t}$. This decrement ariſes from the reſiſtance which retards the body, and from the gravity which accelerates it. Gravity, in a falling body, which in its fall deſcribes the ſpace NI, produces a velocity, with which it would be able to deſcribe twice that ſpace in the ſame time, as *Galileo* has demonſtrated; that is, the velocity $\frac{2NI}{t}$: but if the body deſcribes the arc HI, it augments that arc only by the length $HI - HN$ or $\frac{MI \times NI}{HI}$; and therefore generates only the velocity $\frac{2MI \times NI}{t \times HI}$. Let this velocity be added to the beforementioned decrement, and we ſhall have the decrement of the velocity ariſing from the reſiſtance alone, that is, $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$. Therefore ſince in the ſame time, the action of gravity generates, in a falling body, the velocity $\frac{2NI}{t}$; the reſiſtance will be to the gravity as $\frac{GH}{T} - \frac{HI}{t} + \frac{2MI \times NI}{t \times HI}$ to $\frac{2NI}{t}$, or as $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$ to $2NI$.

Now for the abſciſſa's CB, CD, CE put $-o$, o, $2o$. For the ordinate CH put P; and for MI put any ſeries $Qo + Ro^2 + So^3$ &c. And all the terms of the ſeries after the firſt, that is, $Ro^2 + So^3 +$ &c. will be NI; and the ordinates DI, EK and BG will be $P - Qo - Ro^2 - So^3 -$ &c. $P - 2Qo - 4Ro^2 - 8So^3 -$ &c. and $P + Qo - Ro^2 + So^3 -$ &c. reſpectively. And by ſquaring the dif-

differences of the ordinates $BG - CH$ and $CH - DI$, and to the squares thence produced adding the squares of BC and CD themselves, you will have $oo + QQoo - 2QRo^3 +$ &c. and $oo + QQoo + 2QRo^3 +$ &c. the squares of the arcs GH, HI; whose roots $o\sqrt{1+QQ} - \frac{QRoo}{\sqrt{1+QQ}}$, and $o\sqrt{1+QQ} + \frac{QRoo}{\sqrt{1+QQ}}$ are the arcs GH and HI. Moreover, if from the ordinate CH there be subducted half the sum of the ordinates BG and DI, and from the ordinate DI there be subducted half the sum of the ordinates CH and EK, there will remain Roo and $Roo + 3So^3$ the versed sines of the arcs GI and HK. And these are proportional to the lineolæ LH and NI, and therefore in the duplicate ratio of the infinitely small times T and t: and thence the ratio $\frac{t}{T}$ is $\sqrt{\frac{R + 3So}{R}}$ or $\frac{R + \frac{3}{2}So}{R}$; and $\frac{t \times GH}{T} - HI + \frac{2MI \times NI}{HI}$, by substituting the values of $\frac{t}{T}$, GH, HI, MI and NI just found, becomes $\frac{3Soo}{2R}\sqrt{1+QQ}$. And since $2NI$ is $2Roo$, the resistance will be now to the gravity as $\frac{3Soo}{2R}\sqrt{1+QQ}$ to $2Roo$, that is, as $3S\sqrt{1+QQ}$ to $4RR$.

And the velocity will be such, that a body going off therewith from any place H, in the direction of the tangent HN, would describe, in vacuo, a Parabola, whose diameter is HC, and its latus rectum $\frac{HN^2}{NI}$ or $\frac{1+QQ}{R}$.

And

And the resistance is as the density of the medium and the square of the velocity conjunctly; and therefore the density of the medium is as the resistance directly, and the square of the velocity inversely; that is, as $\frac{3S\sqrt{1+QQ}}{4RR}$ directly and $\frac{1+QQ}{R}$ inversely; that is, as $\frac{S}{R\sqrt{1+QQ}}$ *Q. E. I.*

Cor. 1. If the tangent *HN* be produced both ways, so as to meet any ordinate *AF* in *T*: $\frac{HT}{AC}$ will be equal to $\sqrt{1+QQ}$, and therefore in what has gone before may be put for $\sqrt{1+QQ}$. By this means the resistance will be to the gravity as $3S \times HT$ to $4RR \times AC$; the velocity will be as $\frac{HT}{AC\sqrt{R}}$, and the density of the medium will be as $\frac{S \times AC}{R \times HT}$.

Cor. 2. And hence, if the curve line *PFHQ* be defined by the relation between the base or abscissa *AC* and the ordinate *CH*, as is usual; and the value of the ordinate be resolved into a converg ng series: The problem will be expeditiously solved by the first terms of the series; as in the following examples.

Example 1. Let the line *PFHQ* be a semi-circle described upon the diameter *PQ*; to find the density of the medium that shall make a projectile move in that line.

Bisect the diameter *PQ* in *A*; and call *AQ*, n; *AC*, a; *CH*, e; and *CD*, o: then DI^2 or $AQ^2 - AD^2 = nn - aa - 2ao - oo$, or $ee - 2ao - oo$; and the root being extracted by our method, will give

$$DI = e - \frac{ao}{e} - \frac{oo}{2e} - \frac{aaoo}{2e^3} - \frac{ao^3}{2e^3} - \frac{a^3o^3}{2e^5} - \text{\&c.}$$

&c. Here put nn for $ee + aa$, and DI will become $= e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{anno^3}{2e^5} -$ &c.

Such ſeries I diſtinguiſh into ſucceſſive terms after this manner: I call that the firſt term, in which the infinitely ſmall quantity o is not found; the ſecond, in which that quantity is of one dimenſion only; the third, in which it ariſes to two dimenſions; the fourth, in which it is of three; and ſo *ad infinitum*. And the firſt term, which here is e, will always denote the length of the ordinate CH, ſtanding at the beginning of the indefinite quantity o. The ſecond term, which here is $\frac{ao}{e}$, will denote the difference between CH and DN; that is, the lineola MN which is cut off by compleating the parallelogram $HCDM$; and therefore always determines the poſition of the tangent HN; as, in this caſe, by taking MN to HM as $\frac{ao}{e}$ to o, or a to e. The third term, which here is $\frac{nnoo}{2e^3}$, will repreſent the lineola IN, which lies between the tangent and the curve; and therefore determines the angle of contact IHN, or the curvature which the curve line has in H. If that lineola IN is of a finite magnitude, it will be expreſs'd by the third term together with thoſe that follow *in infinitum*. But if that lineola be diminiſhed *in infinitum*, the terms following become infinitely leſs than the third term, and therefore may be neglected. The fourth term determines the variation of the curvature; the fifth, the variation of the variation; and ſo on. Whence, by the way, appears no contemptible uſe of theſe ſeries in the ſolution of problems that depend upon tangents, and the curvature of curves.

Now

Now compare the series $e - \frac{ao}{e} - \frac{nnoo}{2e^3} - \frac{anno^3}{2e^5}$ — &c. with the series $P - Qo - Roo - So^3 -$ &c. and for P, Q, R and S put e, $\frac{a}{e}$, $\frac{nn}{2e^3}$ and $\frac{ann}{2e^5}$, and for $\sqrt{1+QQ}$ put $\sqrt{1+\frac{aa}{ee}}$ or $\frac{n}{e}$; and the density of the medium will come out as $\frac{a}{ne}$, that is, (because n is given) as $\frac{a}{e}$, or $\frac{AC}{CH}$, that is, as that length of the tangent HT, which is terminated at the semidiameter AF standing perpendicularly on PQ: and the resistance will be to the gravity as $3a$ to $2n$, that is, as $3AC$ to the diameter PQ of the circle; and the velocity will be as $\sqrt{CH}$. Therefore if the body goes from the place F, with a due velocity, in the direction of a line parallel to PQ, and the density of the medium in each of the places H is as the length of the tangent HT, and the resistance also in any place H is to the force of gravity as $3AC$ to PQ, that body will describe the quadrant FHQ of a circle. *Q.E.I.*

But if the same body should go from the place P, in the direction of a line perpendicular to PQ, and should begin to move in an arc of the semi-circle PFQ, we must take AC or a on the contrary side of the centre A; and therefore its sign must be changed, and we must put $-a$ for $+a$. Then the density of the medium would come out as $-\frac{a}{e}$. But nature does not admit of a negative density, that is, a density which accelerates the motion of bodies; and therefore it cannot naturally come to pass, that a body by ascending from P should describe the quadrant PF of a circle. To produce such an effect, a body ought to be accelerated by an impelling medium, and not impeded by a resisting one.

EXAMPLE 2. Let the line PFQ be a Parabola, having its axis AF perpendicular to the horizon PQ; to find the density of the medium, which will make a projectile move in that line. *Fig.* 4.

From the nature of the Parabola, the rectangle PDQ is equal to the rectangle under the ordinate DI and some given right line: that is, if that right line be called b; PC, a; PQ, c; CH, e; and CD, o; the rectangle $a+o$ into $c-a-o$ or $ac-aa-2ao+co-oo$ is equal to the rectangle b into DI, and therefore DI is equal to $\frac{ac-aa}{b}+\frac{c-2a}{b}o-\frac{oo}{b}$. Now the second term $\frac{c-2a}{b}o$ of this series is to be put for Qo, and the third term $\frac{oo}{b}$ for Roo. But since there are no more terms, the coefficient S of the fourth term will vanish; and therefore the quantity $\frac{S}{R\sqrt{1+QQ}}$, to which the density of the medium is proportional, will be nothing. Therefore, where the medium is of no density, the projectile will move in a Parabola; as *Galileo* hath heretofore demonstrated. *Q.E.I.*

EXAMPLE 3. Let the line AGK be an Hyperbola, having its asymptote NX perpendicular to the horizontal plane AK; to find the density of the medium, that will make a projectile move in that line. *Fig.* 5.

Let MX be the other asymptote, meeting the ordinate DG produced in V; and from the nature of the Hyperbola, the rectangle of XV into VG will be given. There is also given the ratio of DN to VX, and therefore the rectangle of DN into VG is given. Let that be bb: and, compleating the parallelogram

2 $DNXZ$,

DNXZ; let *BN* be called a; *BD*, o; *NX*, c; and let the given ratio of *VZ* to *ZX* or *DN* be $\frac{m}{n}$. Then *DN* will be equal to $a - o$, *VG* equal to $\frac{bb}{a-o}$, *VZ* equal to $\frac{m}{n} \times \overline{a-o}$, and *GD* or *NX* — *VZ* — *VG* equal to $c - \frac{m}{n}a + \frac{m}{n}o - \frac{bb}{a-o}$. Let the term $\frac{bb}{a-o}$ be resolved into the converging series $\frac{bb}{a} + \frac{bb}{aa}o + \frac{bb}{a^3}oo + \frac{bb}{a^4}o^3$ &c. and *GD* will become equal to $c - \frac{m}{n}a - \frac{bb}{a} + \frac{m}{n}o - \frac{bb}{aa}o - \frac{bb}{a^3}o^2 - \frac{bb}{a^4}o^3$ &c. The second term $\frac{m}{n}o - \frac{bb}{aa}o$ of this series is to be used for Qo, the third $\frac{bb}{a^3}o^2$ with its sign changed for Ro^2, and the fourth $\frac{bb}{a^4}o^3$ with its sign changed also for So^3, and their coefficients $\frac{m}{n} - \frac{bb}{aa}$, $\frac{bb}{a^3}$ and $\frac{bb}{a^4}$ are to be put for Q, R and S in the former Rule. Which being done, the density of the medium will come out as

$$\frac{\frac{bb}{a^4}}{\frac{bb}{a^3}\sqrt{1 + \frac{mm}{nn} - \frac{2mbb}{naa} + \frac{b^4}{a^4}}}$$

or

$$\frac{1}{\sqrt{aa + \frac{mm}{nn}aa - \frac{2mbb}{n} + \frac{b^4}{aa}}},$$

that is, if in *VZ* you

you take VY equal to VG, as $\frac{1}{XY}$. For aa and $\frac{m^2}{n^2}a^2 - \frac{2mbb}{n} + \frac{b^4}{aa}$ are the squares of XZ and ZY. But the ratio of the resistance to gravity is found to be that of $3XY$ to $2YG$; and the velocity is that with which the body would describe a Parabola, whose vertex is G, diameter DG, latus rectum $\frac{XY^2}{VG}$. Suppose therefore that the densities of the medium in each of the places G are reciprocally as the distances XY, and that the resistance in any place G is to the gravity as $3XY$ to $2YG$; and a body let go from the place A, with a due velocity, will describe that Hyperbola AGK. *Q.E.I.*

Example 4. Suppose indefinitely, the line AGK to be an Hyperbola, described with the centre X, and the asymptotes MX, NX, so that, having constructed the rectangle $XZDN$, whose side ZD cuts the Hyperbola in G and its asymptote in V, VG may be reciprocally as any power DN^n of the line ZX or DN, whose index is the number n: To find the density of the medium in which a projected body will describe this curve. *Fig.* 5.

For BN, BD, NX put A, O, C respectively, and let VZ be to XZ or DN as d to e, and VG be equal to $\frac{bb}{DN^n}$; then DN will be equal to $A - O$, $VG = \frac{bb}{\overline{A-O}|^n}$, $VZ = \frac{d}{e}\overline{A - O}$, and GD or $NX - VZ - VG$ equal to $C - \frac{d}{e}A + \frac{d}{e}O - \frac{bb}{\overline{A-O}|^n}$. Let the term $\frac{bb}{\overline{A-O}|^n}$ be resolved into an infinite series $\frac{bb}{A^n}$

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$+ \frac{nbb}{A^{n+1}} \times O + \frac{nn+n}{2A^{n+2}} \times bbO^2 + \frac{n^3+3nn+2n}{6A^{n+3}}$ $\times bbO^3$ &c. and GD will be equal to $C - \frac{d}{e}A - \frac{bb}{A^n} + \frac{d}{e}O - \frac{nbb}{A^{n+1}}O - \frac{+nn+n}{2A^{n+2}}bbO^2 - \frac{+n^3+3nn+2n}{6A^{n+3}}bbO^3$ &c. The second term $\frac{d}{e}O - \frac{nbb}{A^{n+1}}O$ of this series is to be used for Qo, the third $\frac{nn+n}{2A^{n+2}}bbO^2$ for Roo, the fourth $\frac{n^3+3nn+2n}{6A^{n+3}}bbO^3$ for So^3. And thence the density of the medium $\frac{S}{R\sqrt{1+QQ}}$, in any place G, will be $\frac{n+2}{3\sqrt{A^2+\frac{dd}{ee}A^2-\frac{2dnbb}{eA^n}A+\frac{nnb^4}{A^{2n}}}}$, and therefore if in VZ you take VY equal to $n \times VG$, that density is reciprocally as XY. For A^2 and $\frac{dd}{ee}A^2 - \frac{2dnbb}{eA^n}A + \frac{nnb^4}{A^{2n}}$ are the squares of XZ and ZY. But the resistance in the same place G is to the force of gravity as $3S \times \frac{XY}{A}$ to $4RR$, that is, as XY to $\frac{2nn+2n}{n+2}VG$. And the velocity there, is the same wherewith the projected body would move in a Parabola, whose vertex is G, diameter GD, and latus rectum $\frac{1+QQ}{R}$ or $\frac{2XY^2}{nn+n \times VG}$. *Q.E.I.*

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SCHOLIUM.

In the ſame manner that the denſity of the medium comes out to be as $\frac{S \times AC}{R \times HT}$, in Corol. 1. if the reſiſtance is put as any power V^n of the velocity V, the denſity of the medium will come out to be as $\frac{S}{R^{\frac{4-n}{2}}} \times \overline{\frac{AC}{HT}}\Big|^{n-1}$. *Fig.* 3.

And therefore if a curve can be found, ſuch that the ratio of $\frac{S}{R^{\frac{4-n}{2}}}$ to $\overline{\frac{HT}{AC}}\Big|^{n-1}$, or of $\frac{S^2}{R^{4-n}}$ to $\overline{1+QQ}\Big|^{n-1}$ may be given: the body, in an uniform medium, whoſe reſiſtance is as the power V^n of the velocity V, will move in this curve. But let us return to more ſimple curves.

Becauſe there can be no motion in a Parabola except in a non-reſiſting medium, but in the Hyperbola's here deſcribed 'tis produced by a perpetual reſiſtance; it is evident that the line which a projectile deſcribes in an uniformly reſiſting medium, approaches nearer to theſe Hyperbola's than to a Parabola. That line is certainly of the hyperbolic kind, but about the vertex it is more diſtant from the aſymptotes, and in the parts remote from the vertex draws nearer to them, than theſe Hyperbola's here deſcribed. The difference however is not ſo great between the one and the other, but that theſe latter may be commodiouſly enough uſed in practice inſtead of the former. And perhaps theſe may prove more uſeful, than an Hyperbola that is more accurate, and at the ſame time more compounded. They may be made uſe of then in this manner. *Fig.* 5.

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Compleat the parallelogram *XYGT*, and the right line *GT* will touch the hyperbola in *G*, and therefore the density of the medium in *G* is reciprocally as the tangent *GT*, and the velocity there, as $\sqrt{\frac{GT^2}{GV}}$, and the resistance is to the force of gravity as *GT* to $\frac{2nn+2n}{n+2} \times GV$.

Therefore if a body projected from the place *A* in the direction of the right line *AH*, (*Fig.* 6.) describes the Hyperbola *AGK*, and *AH* produced meets the asymptote *NX* in *H*, and *AI* drawn parallel to it meets the other asymptote *MX* in *I*; the density of the medium in *A* will be reciprocally as *AH*, and the velocity of the body as $\sqrt{\frac{AH^2}{AI}}$, and the resistance there to the force of gravity as *AH* to $\frac{2nn+2n}{n+2} \times AI$. Hence the following rules are deduced.

Rule 1. If the density of the medium at *A*, and the velocity with which the body is projected remain the same, and the angle *NAH* be changed; the lengths *AH*, *AI*, *HX* will remain. Therefore if those lengths, in any one case, are found, the Hyperbola may afterwards be easily determined from any given angle *NAH*.

Rule 2. If the angle *NAH*, and the density of the medium at *A* remain the same, and the velocity with which the body is projected be changed, the length *AH* will continue the same; and *AI* will be changed in a duplicate ratio of the velocity reciprocally.

Rule 3. If the angle *NAH*, the velocity of the body at *A*, and the accelerative gravity remain the same, and the proportion of the resistance at *A* to the motive gravity be augmented in any ratio; the proportion of *AH* to *AI* will be augmented in the same ratio,

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the latus rectum of the abovementioned Parabola remaining the same, and also the length $\frac{AH^2}{AI}$ proportional to it; and therefore *AH* will be diminished in the same ratio, and *AI* will be diminished in the duplicate of that ratio. But the proportion of the resistance to the weight is augmented, when either the specific gravity is made less, the magnitude remaining equal, or when the density of the medium is made greater, or when, by diminishing the magnitude, the resistance becomes diminished in a less ratio than the weight.

Rule 4. Because the density of the medium is greater near the vertex of the Hyperbola, than it is in the place *A*; that a mean density may be preserv'd, the ratio of the least of the tangents *GT* to the tangent *AH* ought to be found, and the density in *A* augmented in a ratio a little greater than that of half the sum of those tangents to the least of the tangents *GT*.

Rule 5. If the lengths *AH*, *AI* are given, and the figure *AGK* is to be described: produce *HN* to *X*, so that *HX* may be to *AI* as $n+1$ to 1; and with the centre *X*, and the asymptotes *MX*, *NX* describe an Hyperbola thro' the point *A*, such that *AI* may be to any of the lines *VG* as XV^n to XI^n.

Rule 6. By how much the greater the number n is, so much the more accurate are these Hyperbola's in the ascent of the body from *A*, and less accurate in its descent to *K*; and the contrary. The Conic Hyperbola keeps a mean ratio between these, and is more simple than the rest. Therefore if the Hyperbola be of this kind, and you are to find the point *K*, where the projected body falls upon any right line *AN* passing thro' the point *A*: let *AN* produced meet the asymptotes *MX*, *NX* in *M* and *N*, and take *NK* equal to *AM*.

Rule 7. And hence appears an expeditious method of determining this Hyperbola from the phænomena. Let

Let two similar and equal bodies be projected with the same velocity, in different angles *HAK*, *hAk*, (*Fig.* 6.) and let them fall upon the plane of the horizon in *K* and *k*; and note the proportion of *AK* to *Ak*. Let it be as *d* to *e*. Then erecting a perpendicular *AI* of any length, assume any how the length *AH* or *Ah*, and thence graphically, or by scale and compass, collect the lengths *AK*, *Ak* (by Rule 6.) If the ratio of *AK* to *Ak* be the same with that of *d* to *e*, the length of *AH* was rightly assumed. If not, take on the indefinite right line *SM*, (*Fig.* 7.) the length *SM* equal to the assumed *AH*; and erect a perpendicular *MN*, equal to the difference $\frac{AK}{Ak} - \frac{d}{e}$ of the ratio's drawn into any given right line. By the like method, from several assumed lengths *AH*, you may find several points *N*; and draw thro' them all a regular curve *NNXN*, cutting the right line *SMMM* in *X*. Lastly, assume *AH* equal to the abscissa *SX*, and thence find again the length *AK*; and the lengths, which are to the assumed length *AI* and this last *AH*, as the length *AK* known by experiment, to the length *AK* last found, will be the true lengths *AI* and *AH*, which were to be found. But these being given, there will be given also the resisting force of the medium in the place *A*, it being to the force of gravity as *AH* to 2 *AI*. Let the density of the medium be increased by Rule 4. and if the resisting force just found be increased in the same ratio, it will become still more accurate.

Rule 8. The lengths *AH*, *HX* being found; let there be now required the position of the line *AH*, according to which a projectile thrown with that given velocity, shall fall upon any point *K*. At the points *A* and *K*, (*Fig* 6.) erect the lines *AC*, *KF* perpendicular to the horizon; whereof let *AC* be drawn downwards, and be equal to *AI* or $\frac{1}{2}$ *HX*. With the asymptotes *AK*, *KF*, describe an Hyperbola, whose con-

conjugate ſhall paſs thro' the point *C*; and from the centre *A*, with the interval *AH*, deſcribe a circle cutting that Hyperbola in the point *H*; then the projectile thrown in the direction of the right line *AH* will fall upon the point *K*. *Q. E. I.* For the point *H*, becauſe of the given length *AH*, muſt be ſomewhere in the circumference of the deſcribed circle. Draw *CH* meeting *AK* and *KF* in *E* and *F*; and becauſe *CH*, *MX* are parallel, and *AC*, *AI* equal, *AE* will be equal to *AM*, and therefore alſo equal to *KN*. But *CE* is to *AE* as *FH* to *KN*, and therefore *CE* and *FH* are equal. Therefore the point *H* falls upon the hyperbolic curve deſcribed with the aſymptotes *AK*, *KF*, whoſe conjugate paſſes thro' the point *C*; and is therefore found in the common interſection of this hyperbolic curve and the circumference of the deſcribed circle. *Q. E. D.* It is to be obſerved that this operation is the ſame, whether the right line *AKN* be parallel to the horizon, or inclined thereto in any angle; and that from two interſections *H*, *H*, there ariſe two angles *NAH*, *NAH*; and that in mechanical practice it is ſufficient once to deſcribe a circle, then to apply a ruler *CH*, of an indeterminate length, ſo to the point *C*, that its part *FH*, intercepted between the circle and the right line *FK*, may be equal to its part *CE* placed between the point *C* and the right line *AK*.

What has been ſaid of Hyperbola's may be eaſily applied to Parabola's. For if (*Fig.* 8.) a Parabola be repreſented by *XAGK*, touched by a right line *XV* in the vertex *X*; and the ordinates *IA*, *VG* be as any powers XI^n, XV^n of the abſciſſa's *XI*, *XV*; draw *XT*, *GT*, *AH*, whereof let *XT* be parallel to *VG*, and let *GT*, *AH* touch the Parabola in *G* and *A*: and a body projected from any place *A*, in the direction of the right line *AH*, with a due velocity, will deſcribe this Parabola, if the denſity of the medium in each

each of the places *G*, be reciprocally as the tangent *GT*. In that case the velocity in *G* will be the same as would cause a body, moving in a non-resisting space, to describe a Conic Parabola, having *G* for its vertex, *VG* produced downwards for its diameter, and $\frac{2GT^2}{nn - n \times VG}$ for its latus rectum. And the resisting force in *G* will be to the force of gravity, as *GT* to $\frac{2nn - 2n}{n - 2}VG$. Therefore if *NAK* represent an horizontal line, and, both the density of the medium at *A* and the velocity with which the body is projected, remaining the same, the angle *NAH* be any how alter'd; the lengths *AH*, *AI*, *HX* will remain; and thence will be given the vertex *X* of the Parabola, and the position of the right line *XI*, and by taking *VG* to *IA* as XV^n to XI^n, there will be given all the points *G* of the Parabola, thro' which the projectile will pass.

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SECTION III.

Of the Motions of Bodies which are resisted partly in the ratio of the Velocities, and partly in the duplicate of the same ratio.

PROPOSITION XI. THEOREM VIII.

If a body be resisted partly in the ratio, and partly in the duplicate ratio of its velocity, and moves in a similar medium by its innate force only; and the times be taken in arithmetical progression: then quantities reciprocally proportional to the velocities, increased by a certain given quantity, will be in geometrical progression. Pl. 3. Fig. 1.

With the centre *C*, and the rectangular asymptotes *CADd* and *CH* describe an Hyperbola *BEe*, and let *AB*, *DE*, *de*, be parallel to the asymptote *CH*. In the asymptote *CD* let *A*, *G* be given points: And if the time be expounded by the hyperbolic area *ABED* uniformly increasing; I say that the velocity may be express'd by the length *DF*, whose reciprocal *GD* together with the given line *CG*, compose the length *CD* increasing in a geometrical progression.

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For let the areola $DEed$ be the least given increment of the time, and Dd will be reciprocally as DE, and therefore directly as CD. Therefore the decrement of $\frac{1}{GD}$, which (by Lem. 2. Book 2.) is $\frac{Dd}{GD^2}$ will be also as $\frac{CD}{GD^2}$ or $\frac{CG + GD}{GD^2}$, that is, as $\frac{1}{GD} + \frac{CG}{GD^2}$. Therefore the time $ABED$ uniformly increasing by the addition of the given particles $EDde$, it follows that $\frac{1}{GD}$ decreases in the same ratio with the velocity. For the decrement of the velocity is as the resistance, that is, (by the supposition) as the sum of two quantities, whereof one is as the velocity, and the other as the square of the velocity; and the decrement of $\frac{1}{GD}$ is as the sum of the quantities $\frac{1}{GD}$ and $\frac{CG}{GD^2}$, whereof the first is $\frac{1}{GD}$ it self, and the last $\frac{CG}{GD^2}$ is as $\frac{1}{GD^2}$: therefore $\frac{1}{GD}$ is as the velocity, the decrements of both being analogous. And if the quantity GD, reciprocally proportional to $\frac{1}{GD}$, be augmented by the given quantity CG; the sum CD, the time $ABED$ uniformly increasing, will increase in a geometrical progression. *Q. E. D.*

Cor. 1. Therefore, if, having the points A and G given, the time be expounded by the hyperbolic area $ABED$, the velocity may be expounded by $\frac{1}{GD}$ the reciprocal of GD.

Cor. 2. And by taking GA to GD as the reciprocal of the velocity at the beginning, to the reciprocal of of the velocity at the end of any time *ABED*, the point *G* will be found. And that point being found, the velocity may be found from any other time given.

Proposition XII. Theorem IX.

The same things being supposed, I say, that if the spaces described are taken in arithmetical progression, the velocities augmented by a certain given quantity will be in geometrical progression. Pl. 3. Fig. 2.

In the asymptote *CD* let there be given the point *R*, and erecting the perpendicular *RS* meeting the Hyperbola in *S*, let the space described be expounded by the hyperbolic area *RSED*; and the velocity will be as the length *GD*, which, together with the given line *CG*, composes a length *CD* decreasing in a geometrical progression, while the space *RSED* increases in an arithmetical progression.

For, because the increment *EDde* of the space is given, the lineola *Dd*, which is the decrement of *GD*, will be reciprocally as *ED*, and therefore directly as *CD*; that is, as the sum of the same *GD* and the given length *CG*. But the decrement of the velocity, in a time reciprocally proportional thereto, in which the given particle of space *DdeE* is described, is as the resistance and the time conjunctly, that is, directly as the sum of two quantities, whereof one is as the velocity, the other as the square of the velocity, and inversely as the velocity; and therefore directly as the sum of two quantities, one of which is given, the other is as the velocity. Therefore the decrement both of the velocity and the line *GD*, is as a given quantity and a decreasing quantity conjunctly; and, because the decrements are analogous, the decreasing quantities will always

always be analogous; *viz.* the velocity, and the line *GD*. *Q.E.D.*

Cor. 1. If the velocity be expounded by the length *GD*, the space described will be as the hyperbolic area *DESR*.

Cor. 2. And if the point *R* be assumed any how, the point *G* will be found, by taking *GR* to *GD*, as the velocity at the beginning to the velocity after any space *RSED* is described. The point *G* being given, the space is given from the given velocity: and the contrary.

Cor. 3. Whence since (by Prop. 11.) the velocity is given from the given time, and (by this Prop.) the space is given from the given velocity; the space will be given from the given time: and the contrary.

Proposition XIII. Theorem X.

Supposing that a body attracted downwards by an uniform gravity ascends or descends in a right line; and that the same is resisted, partly in the ratio of its velocity, and partly in the duplicate ratio thereof: I say that, if right lines parallel to the diameters of a Circle and an Hyperbola be drawn thro' the ends of the conjugate diameters, and the velocities be as some segments of those parallels drawn from a given point; the times will be as the sectors of the areas, cut off by right lines drawn from the centre to the ends of the segments; and the contrary. Pl. 3. Fig. 3.

Case 1. Suppose first that the body is ascending, and from the centre *D*, with any semidiameter *DB*, describe a quadrant *BETF* of a circle, and thro' the end

end *B* of the ſemidiameter *DB* draw the indefinite line *BAP*, parallel to the ſemidiameter *DF*. In that line let there be given the point *A*, and take the ſegment *AP* proportional to the velocity. And ſince one part of the reſiſtance is as the velocity, and another part as the ſquare of the velocity; let the whole reſiſtance be as $AP^2 + 2BAP$. Join *DA*, *DP* cutting the circle in *E* and *T*, and let the gravity be expounded by DA^2, ſo that the gravity ſhall be to the reſiſtance in *P*, as DA^2 to $AP^2 + 2BAP$; and the time of the whole aſcent will be as the ſector *EDT* of the circle.

For draw *DVQ*, cutting off the moment *PQ* of the velocity *AP*, and the moment *DTV* of the ſector *DET* anſwering to a given moment of time; and that decrement *PQ* of the velocity will be as the ſum of the forces of gravity DA^2 and of reſiſtance $AP^2 + 2BAP$, that is, (by 12 Prop. 2 Book Elem.) as DP^2. Then the area *DPQ*, which is proportional to *PQ*, is as DP^2, and the area *DTV*, which is to the area *DPQ* as DT^2 to DP^2, is as the given quantity DT^2. Therefore the area *EDT* decreaſes uniformly according to the rate of the future time, by ſubduction of given particles *DTV*, and is therefore proportional to the time of the whole aſcent. *Q. E. D.*

Case 2. If the velocity in the aſcent of the body be expounded by the length *AP* as before, and the reſiſtance be made as $AP^2 + 2BAP$, and if the force of gravity be leſs than can be expreſſed by DA^2; take *BD* (*Fig.* 4.) of ſuch a length, that $AB^2 - BD^2$ may be proportional to the gravity, and let *DF* be perpendicular and equal to *DB*, and thro' the vertex *F* deſcribe the Hyperbola *FTVE*, whoſe conjugate ſemidiameters are *DB* and *DF*, and which cuts *DA* in *E*, and *DP*, *DQ* in *T* and *V*; and the time of the whole aſcent will be as the hyperbolic ſector *TDE*.

For the decrement *PQ* of the velocity produced in a given particle of time, is as the ſum of the reſiſtance

AP^2

$AP^2 + 2BAP$ and of the gravity $AB^2 - BD^2$, that is, as $BP^2 - BD^2$. But the area DTV is to the area DPQ as DT^2 to DP^2; and therefore, if GT be drawn perpendicular to DF, as GT^2 or $GD^2 - DF^2$ to BD^2, and as GD^2 to BP^2, and, by division, as DF^2 to $BP^2 - BD^2$. Therefore since the area DPQ is as PQ, that is, as $BP^2 - BD^2$; the area DTV will be as the given quantity DF^2. Therefore the area EDT decreases uniformly in each of the equal particles of time, by the subduction of so many given particles DTV, and therefore is proportional to the time. *Q. E. D.*

Case 3. Let AP be the velocity in the descent of the body, and $AP^2 + 2BAP$ the force of resistance, and $BD^2 - AB^2$ the force of gravity, the angle DBA being a right one. And if with the centre D, and the principal vertex B, there be described a rectangular Hyperbola $BETV$ (*Fig.* 5.) cutting DA, DP, and DQ produced in E, T, and V; the sector DET of this Hyperbola will be as the whole time of descent.

For the increment PQ of the velocity, and the area DPQ proportional to it, is as the excess of the gravity above the resistance, that is, as $BD^2 - AB^2 - 2BAP - AP^2$ or $BD^2 - BP^2$. And the area DTV is to the area DPQ, as DT^2 to DP^2; and therefore as GT^2 or $GD^2 - BD^2$ to BP^2, and as GD^2 to BD^2, and, by division, as BD^2 to $BD^2 - BP^2$. Therefore since the area DPQ is as $BD^2 - BP^2$, the area DTV will be as the given quantity BD^2. Therefore the area EDT increases uniformly in the several equal particles of time by the addition of as many given particles DTV, and therefore is proportional to the time of the descent. *Q. E. D.*

Cor. If with the centre D and the semidiameter DA there be drawn thro' the vertex A an arc At similar to the arc ET, and similarly subtending the angle

ADT: the velocity *AP* will be to the velocity, which the body in the time *EDT*, in a non-refifting fpace, can lofe in its afcent, or acquire in its defcent, as the area of the triangle *DAP* to the area of the fector *DAt*; and therefore is given from the time given. For the velocity in a non-refifting medium, is proportional to the time, and therefore to this fector; in a refifting medium it is as the triangle; and in both mediums, where it is leaft, it approaches to the ratio of equality, as the fector and triangle do.

SCHOLIUM.

One may demonftrate alfo that cafe in the afcent of the body, where the force of gravity is lefs than can be exprefs'd by DA^2 or $AB^2 + BD^2$, and greater than can be exprefs'd by $AB^2 - DB^2$, and muft be exprefs'd by AB^2. But I haften to other things.

PROPOSITION XIV. THEOREM XI.

The fame things being fuppofed, I fay, that the fpace defcribed in the afcent or defcent, is as the difference of the area by which the time is exprefs'd, and of fome other area which is augmented or diminifhed in an arithmetical progreffion; if the forces compounded of the refiftance and the gravity be taken in a geometrical progreffion. Pl. 3. Fig. 5, 6, 7.

Take *AC* (in the three laft figures) proportional to the gravity, and *AK* to the refiftance. But take them on the fame fide of the point *A*, if the body is defcending, otherwife on the contrary. Erect *Ab*, which make to *DB* as DB^2 to $4BAC$: and to the rectangular afymptotes *CK*, *CH*, defcribe the Hyperbola

bola bN, and erecting KN perpendicular to CK, the area $AbNK$ will be augmented or diminished in an arithmetical progression, while the forces CK are taken in a geometrical progression. I say therefore that the distance of the body from its greatest altitude is as the excess of the area $AbNK$ above the area DET.

For since AK is as the resistance, that is, as $AP^2 + 2BAP$; assume any given quantity Z, and put AK equal to $\frac{AP^2 + 2BAP}{Z}$; then (by Lem. 2. of this Book) the moment KL of AK will be equal to $\frac{2APQ + 2BA \times PQ}{Z}$ or $\frac{2BPQ}{Z}$, and the moment $KLON$ of the area $AbNK$, will be equal to $\frac{2BPQ \times LO}{Z}$ or $\frac{BPQ \times BD^3}{2Z \times CK \times AB}$.

Case 1. Now if the body ascends, and the gravity be as $AB^2 + BD^2$, BET, (in *Fig.* 5.) being a circle, the line AC, which is proportional to the gravity, will be $\frac{AB^2 + BD^2}{Z}$, and DP^2 or $AP^2 + 2BAP + AB^2 + BD^2$ will be $AK \times Z + AC \times Z$ or $CK \times Z$; and therefore the area DTV will be to the area DPQ as DT^2 or DB^2 to $CK \times Z$.

Case 2. If the body ascends, and the gravity be as $AB^2 - BD^2$, the line AC (in *Fig.* 6.) will be $\frac{AB^2 - BD^2}{Z}$ and DT^2 will be to DP^2 as DF^2 or DB^2 to $BP^2 - BD^2$ or $AP^2 + 2BAP + AB^2 - BD^2$, that is, to $AK \times Z + AC \times Z$ or $CK \times Z$. And therefore the area DTV will be to the area DPQ as DB^2 to $CK \times Z$.

Case 3. And by the same reasoning, if the body descends, and therefore the gravity is as $BD^2 - AB^2$, and the line AC (in *Fig.* 7.) becomes equal to

 BD^2

$\frac{BD^2 - AB^2}{Z}$; the area DTV will be to the area DPQ as DB^2 to $CK \times Z$: as above.

Since therefore these areas are always in this ratio; if for the area DTV, by which the moment of the time, always equal to itself, is expressed, there be put any determinate rectangle, as $BD \times m$, the area DPQ, that is, $\frac{1}{2} BD \times PQ$, will be to $BD \times m$ as $CK \times Z$ to BD^2. And thence $PQ \times BD^3$ becomes equal to $2BD \times m \times CK \times Z$, and the moment $KLON$ of the area $AbNK$, found before, becomes $\frac{BP \times BD \times m}{AB}$. From the area DET subduct its moment DTV or $BD \times m$, and there will remain $\frac{AP \times BD \times m}{AB}$. Therefore the difference of the moments, that is, the moment of the difference of the areas is equal to $\frac{AP \times BD \times m}{AB}$; and therefore (because of the given quantity $\frac{BD \times m}{AB}$) as the velocity AP; that is, as the moment of the space which the body describes in its ascent or descent. And therefore the difference of the areas, and that space, increasing or decreasing by proportional moments, and beginning together or vanishing together, are proportional. *Q. E. D.*

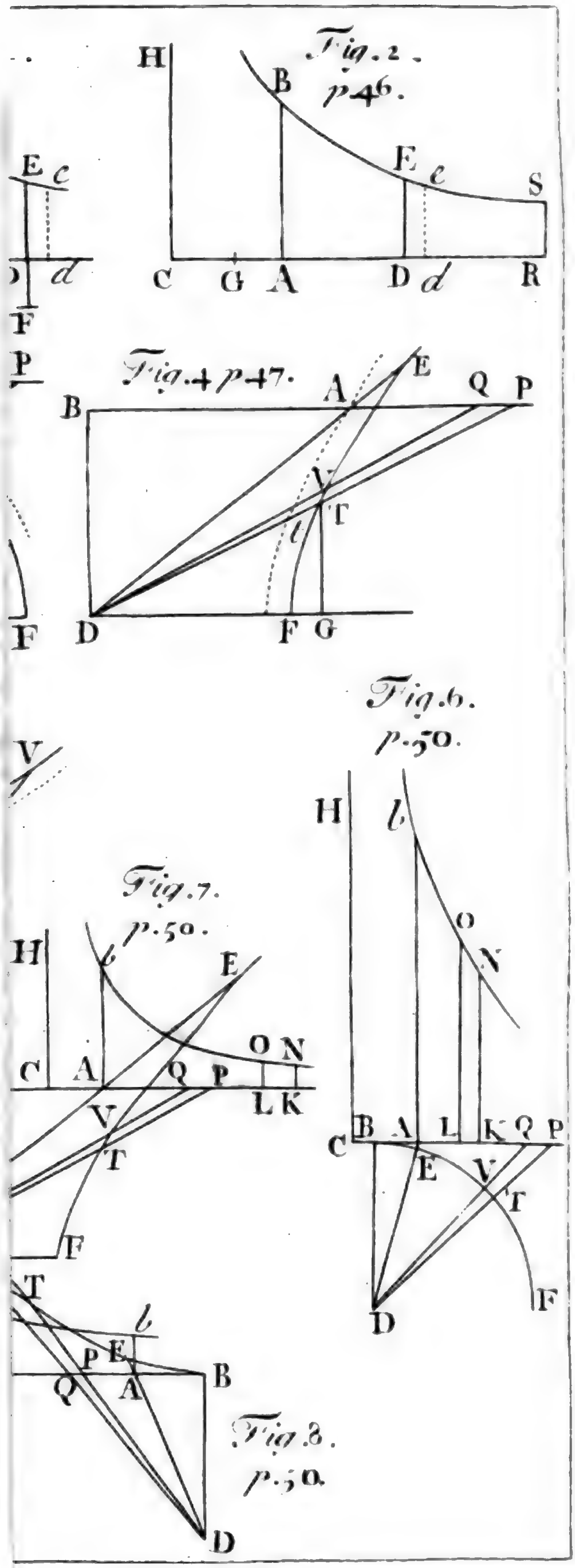
Cor. If the length, which arises by applying the area DET to the line BD, be called M; and another length V be taken in that ratio to the length M, which the line DA has to the line DE: the space which a body, in a resisting medium, describes in its whole ascent or descent, will be to the space, which a body, in a non-resisting medium, falling from rest can describe in the same time, as the difference of the aforesaid areas to $\frac{BD \times V^2}{AB}$: and therefore is given from the

the time given. For the space in a non-resisting medium is in a duplicate ratio of the time, or as V^2; and, because BD and AB are given, as $\frac{BD \times V^2}{AB}$. This area is equal to the area $\frac{DA^2 \times BD \times M^2}{DE^2 \times AB}$, and the moment of M is m; and therefore the moment of this area is $\frac{DA^2 \times BD \times 2M \times m}{DE^2 \times AB}$. But this moment is to the moment of the difference of the aforesaid areas DET and $AbNK$, *viz.* to $\frac{AP \times BD \times m}{AB}$, as $\frac{DA^2 \times BD \times M}{DE^2}$ to $\frac{1}{2} BD \times AP$, or as $\frac{DA^2}{DE^2}$ into DET to DAP; and therefore, when the areas DET and DAP are least, in the ratio of equality. Therefore the area $\frac{BD \times V^2}{AB}$ and the difference of the areas DET and $AbNK$, when all these areas are least, have equal moments; and are therefore equal. Therefore since the velocities, and therefore also the spaces in both mediums described together, in the beginning of the descent, or the end of the ascent, approach to equality, and therefore are then one to another as the area $\frac{BD \times V^2}{AB}$, and the difference of the areas DET and $AbNK$; and moreover since the space, in a non-resisting medium, is perpetually as $\frac{BD \times V^2}{AB}$, and the space, in a resisting medium, is perpetually as the difference of the areas DET and $AbNK$: it necessarily follows, that the spaces, in both mediums, described in any equal times, are one to another as that area $\frac{BD \times V^2}{AB}$, and the difference of the areas DET and $AbNK$. *Q.E.D.*

Scholium.

The resistance of sphærical bodies in fluids arises partly from the tenacity, partly from the attrition, and partly from the density of the medium. And that part of the resistance, which arises from the density of the fluid, is, as I said, in a duplicate ratio of the velocity, the other part, which arises from the tenacity of the fluid, is uniform, or as the moment of the time: and therefore we might now proceed to the motion of bodies, which are resisted partly by an uniform force, or in the ratio of the moments of the time, and partly in the duplicate ratio of the velocity. But it is sufficient to have cleared the way to this speculation in the 8th and 9th Prop. foregoing, and their Corollaries. For in those Propositions, instead of the uniform resistance made to an ascending body arising from its gravity, one may substitute the uniform resistance which arises from the tenacity of the medium, when the body moves by its *vis insita* alone; and when the body ascends in a right line, add this uniform resistance to the force of gravity, and subduct it when the body descends in a right line. One might also go on to the motion of bodies which are resisted in part uniformly, in part in the ratio of the velocity, and in part in the duplicate ratio of the same velocity. And I have opened a way to this in the 13th and 14th Prop. foregoing, in which the uniform resistance arising from the tenacity of the medium, may be substituted for the force of gravity, or be compounded with it as before. But I hasten to other things.

Plate III. Vol. II. P. 54.



SECTION IV.

Of the circular motion of bodies in resisting mediums.

LEMMA III.

Let P Q R *be a spiral cutting all the radii* S P, S Q, S R, *&c. in equal angles. Draw the right line* P T *touching the spiral in any point* P, *and cutting the radius* S Q *in* T; *draw* P O, Q O *perpendicular to the spiral, and meeting in* O, *and join* S O. *I say, that if the points* P *and* Q *approach and coincide, the angle* P S O *will become a right angle, and the ultimate ratio of the rectangle* $TQ \times 2PS$ *to* PQ^2 *will be the ratio of equality.* Pl. 4. Fig. 1.

For from the right angles *O P Q*, *O Q R*, subduct the equal angles *S P Q*, *S Q R*, and there will remain the equal angles *O P S*, *O Q S*. Therefore a circle which passes thro' the points *O*, *S*, *P*, will pass also thro' the point *Q*. Let the points *P* and *Q* coincide, and this circle will touch the spiral in the place of coincidence *P Q*, and will therefore cut the right line *O P* perpendicularly. Therefore *O P* will become a diameter of this circle, and the angle *O S P*, being in a semicircle, becomes a right one. *Q. E. D.*

Draw QD, SE perpendicular to OP, and the ultimate ratio's of the lines will be as follows; TQ to PD as TS or PS to PE, or $2PO$ to $2PS$; and PD to PQ as PQ to $2PO$; and, *ex æquo perturbatè*, TQ to PQ as PQ to $2PS$. Whence PQ^2 becomes equal to $TQ \times 2PS$. *Q. E. D.*

Proposition XV. Theorem XII.

If the density of a medium in each place thereof be reciprocally as the distance of the places from an immoveable centre, and the centripetal force be in the duplicate ratio of the density: I say, that a body may revolve in a spiral which cuts all the radii drawn from that centre in a given angle. Pl. 4. Fig. 2.

Suppose every thing to be as in the foregoing Lemma, and produce SQ to V, so that SV may be equal to SP. In any time let a body, in a resisting medium, describe the least arc PQ, and in double the time, the least arc PR; and the decrements of those arcs arising from the resistance, or their differences from the arcs which would be described in a non-resisting medium in the same times, will be to each other, as the squares of the times in which they are generated: Therefore the decrement of the arc PQ is the fourth part of the decrement of the arc PR. Whence also if the area QSr be taken equal to the area PSQ, the decrement of the arc PQ will be equal to half the lineola Rr; and therefore the force of resistance and the centripetal force are to each other as the lineola's $\frac{1}{2}Rr$ and TQ which they generate in the same time. Because the centripetal force with which the body is urged in P, is reciprocally as SP^2, and (by Lem. 10. Book 1.) the lineola TQ, which is generated by that force, is in a ratio

tio compounded of the ratio of this force and the duplicate ratio of the time in which the arc PQ is described, (for in this case I neglect the resistance, as being infinitely less than the centripetal force,) it follows, that $TQ \times SP^2$, that is, (by the last Lemma) $\frac{1}{2}PQ^2 \times SP$, will be in a duplicate ratio of the time, and therefore the time is as $PQ \times \sqrt{SP}$; and the velocity of the body, with which the arc PQ is described in that time, as $\frac{PQ}{PQ \times \sqrt{SP}}$ or $\frac{1}{\sqrt{SP}}$, that is, in the subduplicate ratio of SP reciprocally. And by a like reasoning, the velocity with which the arc QR is described, is in the subduplicate ratio of SQ reciprocally. Now those arcs PQ and QR are as the describing velocities to each other; that is, in the subduplicate ratio of SQ to SP, or as SQ to $\sqrt{SP \times SQ}$; and, because of the equal angles SPQ, SQr, and the equal areas PSQ, QSr, the arc PQ is to the arc Qr as SQ to SP. Take the differences of the proportional consequents, and the arc PQ will be to the arc Rr as SQ to $SP - \sqrt{SP \times SQ}$, or $\frac{1}{2}VQ$. For the points P and Q coinciding, the ultimate ratio of $SP - \sqrt{SP \times SQ}$ to $\frac{1}{2}VQ$ is the ratio of equality. Because the decrement of the arc PQ arising from the resistance, or its double Rr, is as the resistance and the square of the time conjunctly; the resistance will be as $\frac{Rr}{PQ^2 \times SP}$. But PQ was to Rr, as SQ to $\frac{1}{2}VQ$, and thence $\frac{Rr}{PQ^2 \times SP}$ becomes as $\frac{\frac{1}{2}VQ}{PQ \times SP \times SQ}$ or as $\frac{\frac{1}{2}OS}{OP \times SP^2}$. For the points P and Q coinciding, SP and SQ coincide also, and the angle PVQ becomes a right one; and, because of the similar triangles PVQ, PSO, PQ becomes to $\frac{1}{2}VQ$ as OP to $\frac{1}{2}OS$. Therefore $\frac{OS}{OP \times SP^2}$ is as the resistance, that is, in the ratio of the

the denſity of the medium in P and the duplicate ratio of the velocity conjunctly. Subduct the duplicate ratio of the velocity, namely the ratio $\frac{1}{SP}$, and there will remain the denſity of the medium in P as $\frac{OS}{OP \times SP}$. Let the ſpiral be given, and, becauſe of the given ratio of OS to OP, the denſity of the medium in P will be as $\frac{1}{SP}$. Therefore in a medium whoſe denſity is reciprocally as SP the diſtance from the centre, a body will revolve in this ſpiral. *Q.E.D.*

Cor. 1. The velocity in any place P, is always the ſame wherewith a body in a non-reſiſting medium with the ſame centripetal force would revolve in a circle, at the ſame diſtance SP from the centre.

Cor. 2. The denſity of the medium, if the diſtance SP be given, is as $\frac{OS}{OP}$, but if that diſtance is not given, as $\frac{OS}{OP \times SP}$. And thence a ſpiral may be fitted to any denſity of the medium.

Cor. 3. The force of the reſiſtance in any place P, is to the centripetal force in the ſame place as $\frac{1}{2}OS$ to OP. For thoſe forces are to each other as $\frac{1}{2}Rr$ and TQ or as $\frac{\frac{1}{4}VQ \times PQ}{SQ}$ and $\frac{\frac{1}{2}PQ^2}{SP}$, that is, as $\frac{1}{2}VQ$ and PQ, or $\frac{1}{2}OS$ and OP. The ſpiral therefore being given, there is given the proportion of the reſiſtance to the centripetal force; and *vice verſa*, from that proportion given the ſpiral is given.

Cor. 4. Therefore the body can't revolve in this ſpiral, except where the force of reſiſtance is leſs than half the centripetal force. Let the reſiſtance be made equal to half the centripetal force, and the ſpiral will coincide with the right line PS, and in that right line the

the body will descend to the centre with a velocity, that is to the velocity, with which it was proved before in the case of the Parabola, (Theor. 10. Book 1.) the descent would be made in a non-resisting medium, in the subduplicate ratio of unity to the number two. And the times of the descent will be here reciprocally as the velocities, and therefore given.

Cor. 5. And because at equal distances from the centre, the velocity is the same in the spiral *PQR* as it is in the right line *SP*, and the length of the spiral is to the length of the right line *PS*, in a given ratio, namely in the ratio of *OP* to *OS*; the time of the descent in the spiral will be to the time of the descent in the right line *SP* in the same given ratio, and therefore given.

Cor. 6. If from the centre *S* with any two given intervals, two circles are described; and these circles remaining, the angle which the spiral makes with the radius *PS* be any how changed; the number of revolutions which the body can compleat in the space between the circumferences of those circles, going round in the spiral from one circumference to another, will be as $\frac{PS}{OS}$, or as the tangent of the angle which the spiral makes with the radius *PS*; and the time of the same revolutions will be as $\frac{OP}{OS}$, that is, as the secant of the same angle, or reciprocally as the density of the medium.

Cor. 7. If a body, in a medium whose density is reciprocally as the distances of places from the centre, revolves in any curve *AEB* (*Fig.* 3.) about that centre, and cuts the first radius *AS* in the same angle in *B* as it did before in *A*, and that with a velocity, that shall be to its first velocity in *A* reciprocally in a subduplicate ratio of the distances from the centre (that is, as *AS* to a mean proportional between *AS* and *BS*) that

that body will continue to describe innumerable similar revolutions BFC, CGD, &c. and by its intersections will distinguish the radius AS into parts AS, BS, CS, DS, &c. that are continually proportional. But the times of the revolutions will be as the perimeters of the orbits AEB, BFC, CGD, &c. directly, and the velocities at the beginnings A, B, C of those orbits, inversely; that is, as $AS^{\frac{3}{2}}$, $BS^{\frac{3}{2}}$, $CS^{\frac{3}{2}}$. And the whole time in which the body will arrive at the centre, will be to the time of the first revolution, as the sum of all the continued proportionals $AS^{\frac{3}{2}}$, $BS^{\frac{3}{2}}$, $CS^{\frac{3}{2}}$, going on *ad infinitum*, to the first term $AS^{\frac{3}{2}}$; that is, as the first term $AS^{\frac{3}{2}}$ to the difference of the two first $AS^{\frac{3}{2}} - BS^{\frac{3}{2}}$, or as $\frac{2}{3}AS$ to AB very nearly. Whence the whole time may be easily found.

Cor. 8. From hence also may be deduced, near enough, the motions of bodies in mediums whose density is either uniform or observes any other assigned law. From the centre S, with intervals SA, SB, SC, &c. continually proportional, describe as many circles; and suppose the time of the revolutions between the perimeters of any two of those circles, in the medium whereof we treated, to be to the time of the revolutions between the same in the medium proposed, as the mean density of the proposed medium between those circles, to the mean density of the medium whereof we treated, between the same circles, nearly: And that the secant of the angle in which the spiral above determined, in the medium whereof we treated, cuts the radius AS, is in the same ratio to the secant of the angle in which the new spiral, in the proposed medium, cuts the same radius: And also that the number of all the revolutions between the same two circles is nearly as the tangents of those angles. If this be done every where between every two circles, the motion will

will be continued thro' all the circles. And by this means one may without difficulty conceive at what rate and in what time bodies ought to revolve in any regular medium.

COR. 9. And altho these motions becoming excentrical should be performed in spirals approaching to an oval figure; yet conceiving the several revolutions of those spirals to be at the same distances from each other, and to approach to the centre by the same degrees as the spiral above described, we may also understand how the motions of bodies may be performed in spirals of that kind.

PROPOSITION XVI. THEOREM XIII.

If the density of the medium in each of the places be reciprocally as the distance of the places from the immoveable centre, and the centripetal force be reciprocally as any power of the same distance, I say, that the body may revolve in a spiral intersecting all the radii drawn from that centre in a given angle. Pl. 4. Fig. 2.

This is demonstrated in the same manner as the foregoing proposition. For if the centripetal force in P be reciprocally as any power SP^{n+1} of the distance SP whose index is $n+1$: it will be collected as above, that the time in which the body describes any arc PQ will be as $PQ \times PS^{\frac{1}{2}n}$; and the resistance in P as $\frac{Rr}{PQ^2 \times SP^n}$, or as $\frac{\overline{1-\frac{1}{2}n} \times VQ}{PQ \times SP^n \times SQ}$, and therefore as $\frac{\overline{1-\frac{1}{2}n} \times OS}{OP \times SP^{n+1}}$, that is, (because $\frac{\overline{1-\frac{1}{2}n} \times OS}{OP}$ is a given

given quantity) reciprocally as SP^{n+1}. And therefore, since the velocity is reciprocally as $SP^{\frac{1}{2}n}$, the density in P will be reciprocally as SP.

Cor. 1. The resistance is to the centripetal force as $\overline{1 - \frac{1}{2}n} \times OS$ to OP.

Cor. 2. If the centripetal force be reciprocally as SP^3, $1 - \frac{1}{2}n$ will be $= 0$; and therefore the resistance and density of the medium will be nothing, as in Prop. 9. Book 1.

Cor. 3. If the centripetal force be reciprocally as any power of the radius SP, whose index is greater than the number 3, the affirmative resistance will be changed into a negative.

Scholium.

This Proposition and the former which relate to mediums of unequal density, are to be understood of the motion of bodies that are so small, that the greater density of the medium on one side of the body, above that on the other, is not to be consider'd. I suppose also the resistance, *cæteris paribus*, to be proportional to its density. Whence in mediums whose force of resistance is not as the density, the density must be so much augmented or diminished, that either the excess of the resistance may be taken away, or the defect supplied.

Proposition XVII. Problem IV.

To find the centripetal force and the resisting force of the medium, by which a body, the law of the velocity being given, shall revolve in a given spiral. Pl. 4. Fig. 4.

Let that spiral be PQR. From the velocity, with which the body goes over the very small arc PQ, the time

time will be given; and from the altitude *TQ*, which is as the centripetal force, and the square of the time, that force will be given. Then from the difference *RSr*, of the areas *PSQ* and *QSR* described in equal particles of time, the retardation of the body will be given; and from the retardation will be found the resisting force and density of the medium.

PROPOSITION XVIII. PROBLEM V.

The law of centripetal force being given, to find the density of the medium in each of the places thereof, by which a body may describe a given spiral.

From the centripetal force the velocity in each place must be found; then from the retardation of the velocity, the density of the medium is found, as in the foregoing Proposition.

But I have explain'd the method of managing these Problems in the tenth Proposition and second Lemma of this Book; and will no longer detain the reader in these perplex'd disquisitions. I shall now add some things relating to the forces of progressive bodies, and to the density and resistance of those mediums in which the motions hitherto treated of, and those akin to them, are performed.

SEC-

SECTION V.

Of the density and compression of fluids; and of Hydrostatics.

The Definition of a Fluid.

A fluid is any body whose parts yield to any force impressed on it, and, by yielding, are easily moved among themselves.

PROPOSITION XIX. THEOREM XIV.

All the parts of a homogeneous and unmoved fluid included in any unmoved vessel, and compressed on every side, (setting aside the consideration of condensation, gravity, and all centripetal forces) will be equally pressed on every side, and remain in their places without any motion arising from that pressure. Pl. 4. Fig. 5.

CASE 1. Let a fluid be included in the sphærical vessel *ABC* and uniformly compressed on every side: I say, that no part of it will be moved by that pressure. For if any part, as *D*, be moved, all such parts at the same distance from the centre on every side, must necessarily be moved at the same time by a like motion; because the pressure of them all is similar and equal; and all other motion is excluded that does not arise from that

that pressure. But if these parts come all of them nearer to the centre, the fluid must be condensed towards the centre, contrary to the supposition. If they recede from it, the fluid must be condensed towards the circumference; which is also contrary to the supposition. Neither can they move in any one direction retaining their distance from the centre, because for the same reason they may move in a contrary direction; but the same part cannot be moved contrary ways at the same time. Therefore no part of the fluid will be moved from its place. *Q.E.D.*

CASE 2. I say now, that all the sphærical parts of this fluid are equally pressed on every side. For let *EF* be a sphærical part of the fluid; if this be not pressed equally on every side, augment the lesser pressure till it be pressed equally on every side; and its parts (by Case 1.) will remain in their places. But before the increase of the pressure, they would remain in their places, (by Case 1.) and by the addition of a new pressure, they will be moved, by the definition of a fluid, from those places. Now these two conclusions contradict each other. Therefore it was false to say, that the sphere *EF* was not pressed equally on every side. *Q.E.D.*

CASE 3. I say besides, that different sphærical parts have equal pressures. For the contiguous sphærical parts press each other mutually and equally in the point of contact, (by Law 3.) But (by Case 2.) they are pressed on every side with the same force. Therefore any two sphærical parts not contiguous, since an intermediate sphærical part can touch both, will be pressed with the same force. *Q.E.D.*

CASE 4. I say now, that all the parts of the fluid are every where pressed equally. For any two parts may be touched by sphærical parts in any points whatever; and there they will equally press those sphærical parts, (by Case 3.) and are, reciprocally, equally pressed by them, (by Law 3.) *Q. E. D.*

Case 5. Since therefore any part *GHI* of the fluid is inclosed by the rest of the fluid as in a vessel, and is equally pressed on every side; and also its parts equally press one another, and are at rest among themselves; it is manifest that all the parts of any fluid as *GHI*, which is pressed equally on every side, do press each other mutually and equally, and are at rest among themselves. *Q. E. D.*

Case 6. Therefore if that fluid be included in a vessel of a yielding substance, or that is not rigid, and be not equally pressed on every side; the same will give way to a stronger pressure, by the definition of fluidity.

Case 7. And therefore in an inflexible or rigid vessel, a fluid will not sustain a stronger pressure on one side than on the other, but will give way to it, and that in a moment of time; because the rigid side of the vessel does not follow the yielding liquor. But the fluid, by thus yielding, will press against the opposite side, and so the pressure will tend on every side to equality. And because the fluid, as soon as it endeavours to recede from the part that is most pressed, is withstood by the resistance of the vessel on the opposite side; the pressure will on every side be reduced to equality, in a moment of time, without any local motion: and from thence the parts of the fluid, (by Case 5.) will press each other mutually and equally, and be at rest among themselves. *Q. E. D.*

Cor. Whence neither will a motion of the parts of the fluid among themselves, be changed by a pressure communicated to the external superficies, except so far as either the figure of the superficies may be somewhere alter'd, or that all the parts of the fluid, by pressing one another more intensely or remissly, may slide with more or less difficulty among themselves.

Pro-

PROPOSITION XX. THEOREM XV.

If all the parts of a sphærical fluid, homogeneous at equal distances from the centre, lying on a sphærical concentric bottom, gravitate towards the centre of the whole; the bottom will sustain the weight of a cylinder, whose base is equal to the superficies of the bottom, and whose altitude is the same with that of the incumbent fluid. Pl. 4. Fig. 6.

Let *DHM* be the superficies of the bottom, and *AEI* the upper superficies of the fluid. Let the fluid be distinguished into concentric orbs of equal thickness, by the innumerable sphærical superficies *BFK*, *CGL*; and conceive the force of gravity to act only in the upper superficies of every orb, and the actions to be equal on the equal parts of all the superficies. Therefore the upper superficies *AE* is pressed by the single force of its own gravity, by which all the parts of the upper orb, and the second superficies *BFK* will, (by Prop. 19.) according to its measure, be equally pressed. The second superficies *BFK* is pressed likewise by the force of its own gravity, which added to the former force, makes the pressure double. The third superficies *CGL* is, according to its measure, acted on by this pressure and the force of its own gravity besides, which makes its pressure triple. And in like manner the fourth superficies receives a quadruple pressure, the fifth superficies a quintuple, and so on. Therefore the pressure acting on every superficies, is not as the solid quantity of the incumbent fluid, but as the number of the orbs reaching to the upper surface of the fluid; and is equal to the gravity of the lowest orb multiplied by the number of orbs: that is, to the gravity of a solid whose ultimate ratio to the cylinder abovementioned (when the number of the orbs is increased and their thickness diminished *ad infinitum*, so that the action of gravity from the lowest superficies to the uppermost may become continued) is the ratio of equality. Therefore the lowest superficies sustains the weight of the cylinder above-determined. *Q.E.D.* And by a like reasoning the Proposition will be evident, where the gravity of the fluid decreases in any assigned ratio of the distance from the centre, and also where the fluid is more rare above and denser below. *Q.E.D.*

Cor. 1. Therefore the bottom is not pressed by the whole weight of the incumbent fluid, but only sustains that part of it which is described in the Proposition; the rest of the weight being sustained archwise by the sphærical figure of the fluid.

Cor. 2. The quantity of the pressure is the same always at equal distances from the centre, whether the superficies pressed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid, continued upwards from the compressed superficies, rises perpendicularly in a rectilinear direction, or creeps obliquely thro' crooked cavities and canals, whether those passages be regular or irregular, wide or narrow. That the pressure is not alter'd by any of these circumstances, may be collected by applying the demonstration of this Theorem to the several cases of fluids.

Cor. 3. From the same demonstration it may also be collected, (by Prop. 19.) that the parts of an heavy fluid acquire no motion among themselves, by the pressure of the incumbent weight; except that motion which arises from condensation.

Cor. 4. And therefore if another body of the same specific gravity, incapable of condensation, be immersed in this fluid, it will acquire no motion by the pressure of the incumbent weight: it will neither descend, nor ascend, nor change its figure. If it be

sphærical,

Plate IV. Vol. II. P. 68.

.1.

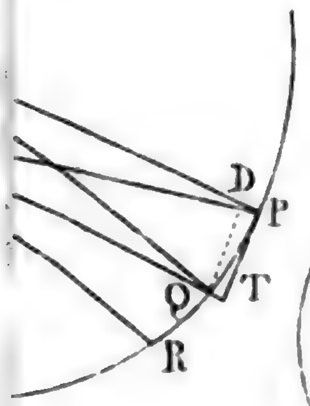
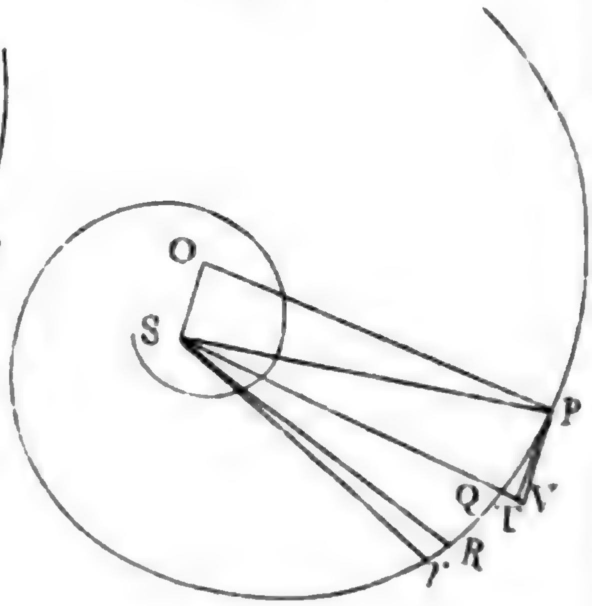


Fig. 2.
p. 56. 61.



. 3 .

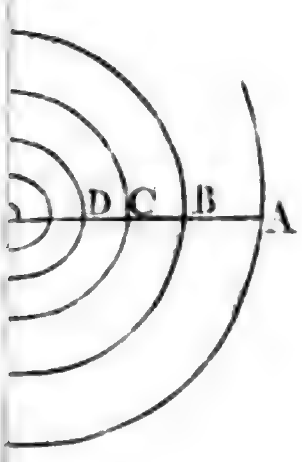
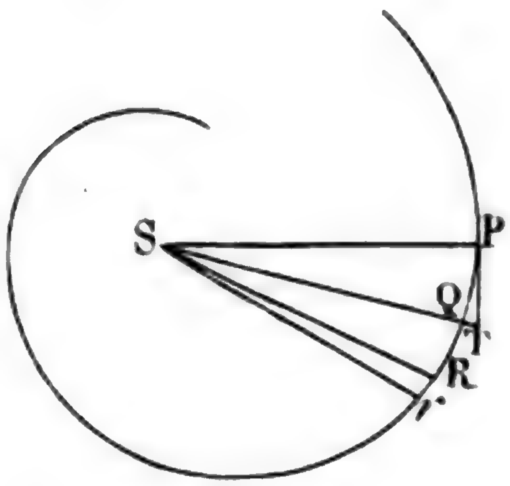


Fig. 4.
p. 62.



. 5 .
64 .

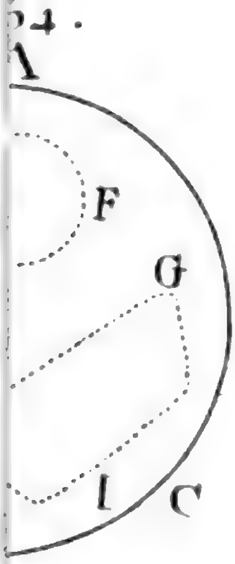
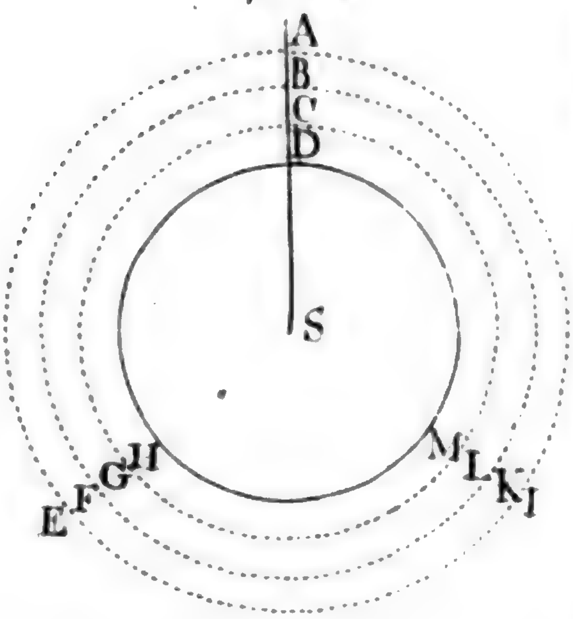


Fig. 6.
p. 67.



ſphærical, it will remain ſo notwithſtanding the preſſure; if it be ſquare, it will remain ſquare: and that whether it be ſoft, or fluid; whether it ſwims freely in the fluid, or lies at the bottom. For any internal part of a fluid is in the ſame ſtate with the ſubmerſed body; and the caſe of all ſubmerſed bodies that have the ſame magnitude, figure, and ſpecific gravity, is alike. If a ſubmerſed body retaining its weight, ſhould diſſolve and put on the form of a fluid, this body, if before it would have aſcended, deſcended, or from any preſſure aſſume a new figure, would now likewiſe aſcend, deſcend, or put on a new figure; and that becauſe its gravity and the other cauſes of its motion remain. But (by Caſe 5. Prop. 19.) it would now be at reſt and retain its figure. Therefore alſo in the former caſe.

Cor. 5. Therefore a body that is ſpecifically heavier than a fluid contiguous to it, will ſink, and that which is ſpecifically lighter will aſcend, and attain ſo much motion and change of figure, as that exceſs or defect of gravity is able to produce. For that exceſs or defect is the ſame thing as an impulſe, by which a body, otherwiſe *in equilibrio* with the parts of the fluid, is acted on; and may be compared with the exceſs or defect of a weight in one of the ſcales of a balance.

Cor. 6. Therefore bodies placed in fluids have a twofold gravity; the one true and abſolute, the other apparent, vulgar and comparative. Abſolute gravity is the whole force with which the body tends downwards: relative and vulgar gravity is the exceſs of gravity with which the body tends downwards more than the ambient fluid. By the firſt kind of gravity, the parts of all fluids and bodies gravitate in their proper places; and therefore their weights taken together, compoſe the weight of the whole. For the whole taken together is heavy, as may be experienced in veſſels full of liquor; and the weight of the whole is equal to the weights of all the parts, and is therefore compoſed of them. By the other kind of gravity bodies do not gravitate in their places, that is, compared with one another, they do not preponderate, but hindering one another's endeavours to descend, remain in their proper places, as if they were not heavy. Those things which are in the air and do not preponderate, are commonly looked on as not heavy. Those which do preponderate are commonly reckoned heavy, in as much as they are not sustained by the weight of the air. The common weights are nothing else but the excess of the true weights above the weight of the air. Hence also vulgarly those things are called light, which are less heavy; and by yielding to the preponderating air, mount upwards. But these are only comparatively light, and not truly so, because they descend *in vacuo*. Thus in water, bodies which, by their greater or less gravity, descend or ascend, are comparatively and apparently heavy or light, and their comparative and apparent gravity or levity is the excess or defect by which their true gravity either exceeds the gravity of the water or is exceeded by it. But those things which neither by preponderating descend, nor, by yielding to the preponderating fluid, ascend, altho' by their true weight they do increase the weight of the whole, yet comparatively, and in the sense of the vulgar, they do not gravitate in the water. For these cases are alike demonstrated.

Cor. 7. These things which have been demonstrated concerning gravity, take place in any other centripetal forces.

Cor. 8. Therefore if the medium in which any body moves be acted on either by its own gravity, or by any other centripetal force, and the body be urged more powerfully by the same force; the difference of the forces is that very motive force, which in the foregoing Propositions I have consider'd as a centripetal force. But if the body be more lightly urg'd by that

that force, the difference of the forces becomes a centrifugal force, and is to be consider'd as such.

COR. 9. But since fluids by pressing the included bodies do not change their external figures, it appears also, (by Cor. Prop. 19.) that they will not change the situation of their internal parts in relation to one another; and therefore if animals were immersed therein, and that all sensation did arise from the motion of their parts; the fluid will neither hurt the immersed bodies, nor excite any sensation, unless so far as those bodies may be condensed by the compression. And the case is the same of any system of bodies encompassed with a compressing fluid. All the parts of the system will be agitated with the same motions, as if they were placed in a vacuum, and would only retain their comparative gravity; unless so far as the fluid may somewhat resist their motions, or be requisite to conglutinate them by compression.

PROPOSITION XXI. THEOREM XVI.

Let the density of any fluid be proportional to the compression, and its parts be attracted downwards by a centripetal force reciprocally proportional to the distances from the centre: I say, that, if those distances be taken continually proportional, the densities of the fluid at the same distances will be also continually proportional. Pl. 5. Fig. 1.

Let *ATV* denote the sphærical bottom of the fluid, *S* the centre, *SA*, *SB*, *SC*, *SD*, *SE*, *SF*, &c. distances continually proportional. Erect the perpendiculars *AH*, *BI*, *CK*, *DL*, *EM*, *FN*, &c. which shall be as the densities of the medium in the places *A*, *B*, *C*, *D*, *E*, *F*; and the specific gravities in those places will be as $\frac{AH}{AS}, \frac{BI}{BS}, \frac{CK}{CS}$, &c. or, which is all one, as $\frac{AH}{AB}, \frac{BI}{BC}, \frac{CK}{CD}$, &c. Suppoſe firſt theſe gravities to be uniformly continued from *A* to *B*, from *B* to *C*, from *C* to *D*, &c. the decrements in the points *B*, *C*, *D*, &c. being taken by ſteps. And theſe gravities drawn into the altitudes *AB*, *BC*, *CD*, &c. will give the preſſures *AH*, *BI*, *CK*, &c. by which the bottom *ATV* is acted on, (by Theor. 15.) Therefore the particle *A* ſuſtains all the preſſures *AH*, *BI*, *CK*, *DL*, &c. proceeding *in infinitum*; and the particle *B* ſuſtains the preſſures of all but the firſt *AH*; and the particle *C* all but the two firſt *AH*, *BI*; and ſo on: and therefore the denſity *AH* of the firſt particle *A* is to the denſity *BI* of the ſecond particle *B* as the ſum of all $AH + BI + CK + DL$, *in infinitum*, to the ſum of all $BI + CK + DL$, &c. And *BI* the denſity of the ſecond particle *B* is to *CK* the denſity of the third *C*, as the ſum of all $BI + CK + DL$, &c. to the ſum of all $CK + DL$, &c. Therefore theſe ſums are proportional to their differences *AH*, *BI*, *CK*, &c. and therefore continually proportional, (by Lem. 1. of this Book) and therefore the differences *AH*, *BI*, *CK*, &c. proportional to the ſums, are alſo continually proportional. Wherefore ſince the denſities in the places *A*, *B*, *C*, &c. are as *AH*, *BI*, *CK*, &c. they will alſo be continually proportional. Proceed intermiſſively, and, *ex æquo*, at the diſtances *SA*, *SC*, *SE* continually proportional, the denſities *AH*, *CK*, *EM* will be continually proportional. And by the ſame reaſoning, at any diſtances *SA*, *SD*, *SG* continually proportional, the denſities *AH*, *DL*, *GO* will be continually proportional. Let now the points *A*, *B*, *C*, *D*, *E*, &c. coincide, ſo that the progreſſion of the ſpecific gravities from the bottom *A* to the top of the fluid may be made continual; and at any diſtances *SA*, *SD*, *SG*

SG continually proportional, the densities *AH*, *DL*, *GQ* being all along continually proportional, will still remain continually proportional. *Q. E. D.*

COR Hence if the density of the fluid in two places as *A* and *E* be given, its density in any other place *Q* may be collected. With the centre *S*, and the rectangular asymptotes *SQ*, *SX* describe (*Fig.* 2.) an Hyperbola cutting the perpendiculars *AH*, *EM*, *QT* in *a*, *e*, and *q*, as also the perpendiculars *HX*, *MY*, *TZ* let fall upon the asymptote *SX* in *h*, *m*, and *t*. Make the area *YmtZ* to the given area *YmhX* as the given area *EeqQ* to the given area *EeaA*; and the line *Zt* produced will cut off the line *QT* proportional to the density. For if the lines *SA*, *SE*, *SQ* are continually proportional, the areas *EeqQ*, *EeaA* will be equal, and thence the areas *YmtZ*, *XhmY* proportional to them will be also equal, and the lines *SX*, *SY*, *SZ*, that is, *AH*, *EM*, *QT* continually proportional as they ought to be. And if the lines *SA*, *SE*, *SQ* obtain any other order in the series of continued proportionals, the lines *AH*, *EM*, *QT*, because of the proportional hyperbolic areas, will obtain the same order in another series of quantities continually proportional.

PROPOSITION XXII. THEOREM XVII.

Let the density of any fluid be proportional to the compression, and its parts be attracted downwards by a gravitation reciprocally proportional to the squares of the distances from the centre: I say, that, if the distances be taken in harmonic progression, the densities of the fluid at those distances will be in a geometrical progression. Pl. 5. Fig. 3.

Let *S* denote the centre, and *SA*, *SB*, *SC*, *SD*, *SE*, the distances in Geometrical progression. Erect the

the perpendiculars AH, BI, CK, &c. which ſhall be as the denſities of the fluid in the places A, B, C, D, E, &c. and the ſpecific gravities thereof in thoſe places will be as $\frac{AH}{SA^2}$, $\frac{BI}{SB^2}$, $\frac{CK}{SC^2}$, &c. Suppoſe theſe gravities to be uniformly continued, the firſt from A to B, the ſecond from B to C, the third from C to D, &c. And theſe drawn into the altitudes AB, BC, CD, DE, &c. or, which is the ſame thing, into the diſtances SA, SB, SC, &c. proportional to thoſe altitudes, will give $\frac{AH}{SA}$, $\frac{BI}{SB}$, $\frac{CK}{SC}$, &c. the exponents of the preſſures. Therefore ſince the denſities are as the ſums of thoſe preſſures, the differences $AH - BI$, $BI - CK$, &c. of the denſities will be as the differences of thoſe ſums $\frac{AH}{SA}$, $\frac{BI}{SB}$, $\frac{CK}{SC}$, &c. With the centre S, and the aſymptotes SA, Sx, deſcribe any Hyperbola, cutting the perpendiculars AH, BI, CK, &c. in a, b, c, &c. and the perpendiculars Ht, Iu, Kw let fall upon the aſymptote Sx, in h, i, k; and the differences of the denſities tu, uw, &c. will be as $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. And the rectangles $tu \times th$, $uw \times ui$, &c. or tp, uq, &c. as $\frac{AH \times th}{SA}$, $\frac{BI \times ui}{SB}$, &c. that is, as Aa, Bb, &c. For, by the nature of the Hyperbola, SA is to AH or St, as th to Aa, and therefore $\frac{AH \times th}{SA}$ is equal to Aa. And, by a like reaſoning, $\frac{BI \times ui}{SB}$ is equal to Bb, &c. But Aa, Bb, Cc, &c. are continually proportional, and therefore proportional to their differences $Aa - Bb$, $Bb - Cc$, &c. and therefore the rectangles tp, uq, &c. are proportional

4 to

to those differences; as also the sums of the rectangles $tp+uq$ or $tp+uq+wr$ to the sums of the differences $Aa-Cc$ or $Aa-Dd$. Suppose several of these terms, and the sum of all the differences, as $Aa-Ff$, will be proportional to the sum of all the rectangles, as $zthn$. Increase the number of terms, and diminish the distances of the points A, B, C, &c. *in infinitum*, and those rectangles will become equal to the hyperbolic area $zthn$, and therefore the difference $Aa-Ff$ is proportional to this area. Take now any distances as SA, SD, SF in harmonic progression, and the differences $Aa-Dd$, $Dd-Ff$ will be equal; and therefore the areas $thlx$, $xlnz$ proportional to those differences will be equal among themselves, and the densities St, Sx, Sz, that is, AH, DL, FN continually proportional. *Q. E. D.*

Cor. 2. Hence if any two densities of the fluid, as AH and BI be given, the area $thiu$, answering to their difference tu will be given; and thence the density FN will be found at any height SF, by taking the area $thnz$ to that given area $thiu$ as the difference $Aa-Ff$ to the difference $Aa-Bb$.

Scholium.

By a like reasoning, it may be proved, that if the gravity of the particles of a fluid be diminished in a triplicate ratio of the distances from the centre; and the reciprocals of the squares of the distances SA, SB, SC, &c. (namely $\frac{SA^3}{SA^2}, \frac{SA^3}{SB^2}, \frac{SA^3}{SC^2}$) be taken in an Arithmetical progression, the densities AH, BI, CK, &c. will be in a Geometrical progression. And if the gravity be diminished in a quadruplicate ratio of the distances, and the reciprocals of the cubes of the distances (as $\frac{SA^4}{SA^3}, \frac{SA^4}{SB^3}, \frac{SA^4}{SC^3}$ &c.) be taken in

Arith-

Arithmetical progression, the densities *AH*, *BI*, *CK*, &c. will be in Geometrical progression. And so *in infinitum*. Again, if the gravity of the particles of the fluid be the same at all distances, and the distances be in Arithmetical progression, the densities will be in a Geometrical progression, as Dr. *Halley* has found. If the gravity be as the distance, and the squares of the distances be in Arithmetical progression, the densities will be in Geometrical progression. And so *in infinitum*. These things will be so, when the density of the fluid condensed by compression is as the force of compression, or, which is the same thing, when the space possessed by the fluid is reciprocally as this force. Other laws of condensation may be supposed, as that the cube of the compressing force may be as the biquadrate of the density; or the triplicate ratio of the force the same with the quadruplicate ratio of the density: In which case, if the gravity be reciprocally as the square of the distance from the centre, the density will be reciprocally as the cube of the distance. Suppose that the cube of the compressing force be as the quadrato-cube of the density; and if the gravity be reciprocally as the square of the distance, the density will be reciprocally in a sesquiplicate ratio of the distance. Suppose the compressing force to be in a duplicate ratio of the density, and the gravity reciprocally in a duplicate ratio of the distance, and the density will be reciprocally as the distance. To run over all the cases that might be offer'd, would be tedious. But as to our own air, this is certain from experiment, that its density is either accurately or very nearly at least as the compressing force; and therefore the density of the air in the atmosphere of the earth is as the weight of the whole incumbent air, that is, as the height of the mercury in the barometer.

PRO-

Proposition XXIII. Theorem XVIII.

If a fluid be composed of particles mutually flying each other, and the density be as the compression, the centrifugal forces of the particles will be reciprocally proportional to the distances of their centres. And vice versa, *particles flying each other with forces that are reciprocally proportional to the distances of their centres, compose an elastic fluid, whose density is as the compression.* Pl. 5. Fig. 4.

Let the fluid be supposed to be included in a cubic space *ACE*, and then to be reduced by compression into a lesser cubic space *ace*; and the distances of the particles retaining a like situation with respect to each other in both the spaces, will be as the sides *AB*, *ab* of the cubes; and the densities of the mediums will be reciprocally as the containing spaces AB^3, ab^3. In the plane side of the greater cube *ABCD* take the square *DP* equal to the plane side *db* of the lesser cube: and, by the supposition, the pressure with which the square *DP* urges the inclosed fluid, will be to the pressure with which that square *db* urges the inclosed fluid, as the densities of the mediums are to each other, that is, as ab^3 to AB^3. But the pressure with which the square *DB* urges the included fluid, is to the pressure with which the square *DP* urges the same fluid, as the square *DB* to the square *DP*, that is, as AB^2 to ab^2. Therefore, *ex æquo*, the pressure with which the square *DB* urges the fluid is to the pressure with which the square *db* urges the fluid; as *ab* to *AB*. Let the planes *FGH*, *fgh*, be drawn thro' the middles of the two cubes, and divide the fluid into two parts. These parts will

4. press

press each other mutually with the same forces with which they are themselves pressed by the planes *AC*, *ac*, that is, in the proportion of *ab* to *AB*: and therefore the centrifugal forces by which these pressures are sustained, are in the same ratio. The number of the particles being equal, and the situation alike, in both cubes, the forces which all the particles exert, according to the planes *FGH*, *fgh*, upon all, are as the forces which each exerts on each. Therefore the forces which each exerts on each according to the plane *FGH* in the greater cube, are to the forces which each exerts on each according to the plane *fgh* in the lesser cube, as *ab* to *AB*, that is, reciprocally as the distances of the particles from each other. *Q.E.D.*

And, *vice versa*, if the forces of the single particles are reciprocally as the distances, that is, reciprocally as the sides of the cubes *AB*, *ab*; the sums of the forces will be in the same ratio, and the pressures of the sides *DB*, *db* as the sums of the forces; and the pressure of the square *DP* to the pressure of the side *DB* as ab^2 to AB^2. And, *ex æquo*, the pressure of the square *DP* to the pressure of the side *db* as ab^3 to AB^3, that is, the force of compression in the one to the force of compression in the other, as the density in the former to the density in the latter. *Q.E.D.*

Scholium.

By a like reasoning, if the centrifugal forces of the particles are reciprocally in the duplicate ratio of the distances between the centres, the cubes of the compressing forces will be as the biquadrates of the densities. If the centrifugal forces be reciprocally in the triplicate or quadruplicate ratio of the distances, the cubes of the compressing forces will be as the quadrato-cubes, or cubo-cubes of the densities. And universally, if D be put for the distance, and E for the density

sity of the compressed fluid, and the centrifugal forces be reciprocally as any power D^n of the distance, whose index is the number n; the compressing forces will be as the cube roots of the power E^{n+2}, whose index is the number $n+2$: and the contrary. All these things are to be understood of particles whose centrifugal forces terminate in those particles that are next them, or are diffused not much further. We have an example of this in magnetical bodies. Their attractive virtue is terminated nearly in bodies of their own kind that are next them. The virtue of the magnet is contracted by the interposition of an iron plate; and is almost terminated at it. For bodies further off are not attracted by the magnet so much as by the iron plate. If in this manner particles repel others of their own kind that lie next them, but do not exert their virtue on the more remote, particles of this kind will compose such fluids as are treated of in this proposition. If the virtue of any particle diffuse itself every way *in infinitum*, there will be required a greater force to produce an equal condensation of a greater quantity of the fluid. But whether elastic fluids do really consist of particles so repelling each other, is a physical question. We have here demonstrated mathematically the property of fluids consisting of particles of this kind, that hence philosophers may take occasion to discuss that question.

SEC-

SECTION VI.

Of the motion and resistance of funependulous bodies.

PROPOSITION XXIV. THEOREM XIX.

The quantities of matter in funependulous bodies, whose centres of oscillation are equally distant from the centre of suspension, are in a ratio compounded of the ratio of the weights and the duplicate ratio of the times of the oscillations in vacuo.

For the velocity, which a given force can generate in a given matter in a given time, is as the force and the time directly, and the matter inversely. The greater the force or the time is, or the less the matter, the greater velocity will be generated. This is manifest from the second law of motion. Now if pendulums are of the same length, the motive forces in places equally distant from the perpendicular are as the weights: and therefore if two bodies by oscillating describe equal arcs, and those arcs are divided into equal parts; since the times in which the bodies describe each of the correspondent parts of the arcs are as the times of the whole oscillations, the velocities in the correspondent parts of the

the oscillations will be to each other, as the motive forces and the whole times of the oscillations directly, and the quantities of matter reciprocally: and therefore the quantities of matter are as the forces and the times of the oscillations directly and the velocities reciprocally. But the velocities reciprocally are as the times, and therefore the times directly and the velocities reciprocally are as the squares of the times; and therefore the quantities of matter are as the motive forces and the squares of the times, that is, as the weights and the squares of the times. *Q. E. D.*

Cor. 1. Therefore if the times are equal, the quantities of matter in each of the bodies are as the weights.

Cor. 2. If the weights are equal, the quantities of matter will be as the squares of the times.

Cor. 3. If the quantities of matter are equal, the weights will be reciprocally as the squares of the times.

Cor. 4. Whence since the squares of the times, *cæteris paribus*, are as the lengths of the pendulums; therefore if both the times and quantities of matter are equal, the weights will be as the lengths of the pendulums.

Cor. 5. And universally, the quantity of matter in the pendulous body is as the weight and the square of the time directly, and the length of the pendulum inversely.

Cor. 6. But in a non-resisting medium, the quantity of matter in the pendulous body is as the comparative weight and the square of the time directly, and the length of the pendulum inversely. For the comparative weight is the motive force of the body in any heavy medium, as was shewn above; and therefore does the same thing in such a non-resisting medium, as the absolute weight does in a vacuum.

Cor. 7. And hence appears a method both of comparing bodies one among another, as to the quantity of matter in each; and of comparing the weights of the ſame body in different places, to know the variation of its gravity. And by experiments made with the greateſt accuracy, I have always found the quantity of matter in bodies to be proportional to their weight.

Proposition XXV. Theorem XX.

Funipendulous bodies that are, in any medium, reſiſted in the ratio of the moments of time, and funipendulous bodies that move in a non-reſiſting medium of the ſame ſpecific gravity, perform their oſcillations in a cycloid in the ſame time, and deſcribe proportional parts of arcs together. Pl. 5. Fig. 5.

Let *AB* be an arc of a cycloid, which a body *D*, by vibrating in a non-reſiſting medium ſhall deſcribe in any time. Biſect that arc in *C*, ſo that *C* may be the loweſt point thereof; and the accelerative force with which the body is urged in any place *D* or *d* or *E* will be as the length of the arc *CD* or *Cd* or *CE*. Let that force be expreſſed by that ſame arc; and ſince the reſiſtance is as the moment of the time, and therefore given, let it be expreſs'd by the given part *CO* of the cycloidal arc, and take the arc *Od* in the ſame ratio to the arc *CD* that the arc *OB* has to the arc *CB*: and the force with which the body in *d* is urged in a reſiſting medium, being the exceſs of the force *Cd* above the reſiſtance *CO*, will be expreſſed by the arc *Od*, and will therefore be to the force with which the body *D* is urged in a non-reſiſting medium in the place *D*, as the arc *Od* to the arc *CD*; and therefore alſo in the place *B*, as the arc *OB* to the arc *CB*. Therefore if two bodies *D*, *d* go from the place *B*, and are urged by theſe forces; ſince the forces at the beginning are as the arcs *CB* and *OB*, the first velocities and arcs first described will be in the same ratio. Let those arcs be *BD* and *Bd*, and the remaining arcs *CD*, *Od*, will be in the same ratio. Therefore the forces, being proportional to those arcs *CD*, *Od*, will remain in the same ratio as at the beginning, and therefore the bodies will continue describing together arcs in the same ratio. Therefore the forces and velocities and the remaining arcs *CD*, *Od*, will be always as the whole arcs *CB*, *OB*, and therefore those remaining arcs will be described together. Therefore the two bodies *D* and *d* will arrive together at the places *C* and *O*; that which moves in the non-resisting medium, at the place *C*, and the other, in the resisting medium, at the place *O*. Now since the velocities in *C* and *O* are as the arcs *CB*, *OB*, the arcs which the bodies describe when they go farther, will be in the same ratio. Let those arcs be *CE* and *Oe*. The force with which the body *D* in a non-resisting medium is retarded in *E* is as *CE*, and the force with which the body *d* in the resisting medium is retarded in *e*, is as the sum of the force *Ce* and the resistance *CO*, that is, as *Oe*; and therefore the forces with which the bodies are retarded, are as the arcs *CB*, *OB*, proportional to the arcs *CE*, *Oe*; and therefore the velocities, retarded in that given ratio, remain in the same given ratio. Therefore the velocities and the arcs described with those velocities, are always to each other in that given ratio of the arcs *CB* and *OB*; and therefore if the entire arcs *AB*, *aB* are taken in the same ratio, the bodies *D* and *d* will describe those arcs together, and in the places *A* and *a* will lose all their motion together. Therefore the whole oscillations are isochronal, or are performed in equal times; and any parts of the arcs, as *BD*, *Bd*, or *BE*, *Be*, that are described together, are proportional to the whole arcs *BA*, *Ba*. *Q.E.D.*

Cor. Therefore the ſwifteſt motion in a reſiſting medium does not fall upon the loweſt point *C*, but is found in that point *O*, in which the whole arc deſcribed *Ba* is biſected. And the body proceeding from thence to *a*, is retarded at the ſame rate with which it was accelerated before in its deſcent from *B* to *O*.

Proposition XXVI. Theorem XXI.

Funipendulous bodies, that are reſiſted in the ratio of the velocity, have their oſcillations in a cycloid iſochronal.

For if two bodies, equally diſtant from their centres of ſuſpenſion, deſcribe, in oſcillating, unequal arcs, and the velocities in the correſpondent parts of the arcs be to each other as the whole arcs; the reſiſtances, proportional to the velocities, will be alſo to each other as the ſame arcs. Therefore if theſe reſiſtances be ſubducted from or added to the motive forces ariſing from gravity which are as the ſame arcs, the differences or ſums will be to each other in the ſame ratio of the arcs: and ſince the increments and decrements of the velocities are as theſe differences or ſums, the velocities will be always as the whole arcs: Therefore if the velocities are in any one caſe as the whole arcs, they will remain always in the ſame ratio. But at the beginning of the motion, when the bodies begin to deſcend and deſcribe thoſe arcs, the forces, which at that time are proportional to the arcs, will generate velocities proportional to the arcs. Therefore the velocities will be always as the whole arcs to be deſcribed, and therefore thoſe arcs will be deſcribed in the ſame time. *Q. E. D.*

Pro-

PROPOSITION XXVII. THEOREM XXII.

If funipendulous bodies are resisted in the duplicate ratio of their velocities, the differences between the times of the oscillations in a resisting medium, and the times of the oscillations in a non-resisting medium of the same specific gravity, will be proportional to the arcs described in oscillating nearly.

For let equal pendulums in a resisting medium describe the unequal arcs A, B; and the resistance of the body in the arc A will be to the resistance of the body in the correspondent part of the arc B in the duplicate ratio of the velocities, that is, as A A to B B nearly. If the resistance in the arc B were to the resistance in the arc A as A B to A A; the times in the arcs A and B would be equal (by the last Prop.) Therefore the resistance A A in the arc A, or A B in the arc B, causes the excess of the time in the arc A above the time in a non-resisting medium; and the resistance B B causes the excess of the time in the arc B above the time in a non-resisting medium. But those excesses are as the efficient forces A B and B B nearly, that is, as the arcs A and B. *Q.E.D.*

COR. 1. Hence from the times of the oscillations in unequal arcs in a resisting medium, may be known the times of the oscillations in a non-resisting medium of the same specific gravity. For the difference of the times will be to the excess of the time in the lesser arc above the time in a non-resisting medium, as the difference of the arcs to the lesser arc.

COR. 2. The shorter oscillations are more isochronal, and very short ones are performed nearly in the same

ſame times as in a non-reſiſting medium. But the times of thoſe which are performed in greater arcs are a little greater, becauſe the reſiſtance in the deſcent of the body, by which the time is prolonged, is greater, in proportion to the length deſcribed in the deſcent, than the reſiſtance in the ſubſequent aſcent, by which the time is contracted. But the time of the oſcillations, both ſhort and long, ſeems to be prolonged in ſome meaſure by the motion of the medium. For retarded bodies are reſiſted ſomewhat leſs, in proportion to the velocity, and accelerated bodies ſomewhat more, than thoſe that proceed uniformly forwards; becauſe the medium, by the motion it has received from the bodies, going forwards the ſame way with them, is more agitated in the former caſe, and leſs in the latter; and ſo conſpires more or leſs with the bodies moved. Therefore it reſiſts the pendulums in their deſcent more, and in their aſcent leſs, than in proportion to the velocity; and theſe two cauſes concurring prolong the time.

PROPOSITION XXVIII. THEOREM XXIII.

If a funipendulous body, oſcillating in a cycloid, be reſiſted in the ratio of the moments of the time, its reſiſtance will be to the force of gravity as the exceſs of the arc deſcribed in the whole deſcent above the arc deſcribed in the ſubſequent aſcent, to twice the length of the pendulum. Pl. 5. Fig. 5.

Let *BC* repreſent the arc deſcribed in the deſcent, *Ca* the arc deſcribed in the aſcent, and *Aa* the difference of the arcs: and things remaining as they were conſtructed and demonſtrated in Prop. 25. the force with which the oſcillating

oscillating body is urged in any place *D*, will be to the force of resistance as the arc *CD* to the arc *CO*, which is half of that difference *Aa*. Therefore the force with which the oscillating body is urged at the beginning or the highest point of the cycloid, that is, the force of gravity, will be to the resistance as the arc of the cycloid, between that highest point and lowest point *C*, is to the arc *CO*; that is, (doubling those arcs) as the whole cycloidal arc, or twice the length of the pendulum, to the arc *Aa*. *Q. E. D.*

Proposition XXIX. Problem VI.

Supposing that a body oscillating in a cycloid is resisted in a duplicate ratio of the velocity: to find the resistance in each place. Pl. 5. Fig. 6.

Let *Ba* be an arc described in one entire oscillation, *C* the lowest point of the cycloid, and *CZ* half the whole cycloidal arc, equal to the length of the pendulum; and let it be required to find the resistance of the body in any place *D*. Cut the indefinite right line *OQ* in the points *O*, *S*, *P*, *Q*, so that (erecting the perpendiculars *OK*, *ST*, *PI*, *QE*, and with the centre *O*, and the asymptotes *OK*, *OQ* describing the hyperbola *TIGE* cutting the perpendiculars *ST*, *PI*, *QE* in *T*, *I* and *E*, and thro' the point *I* drawing *KF*, parallel to the asymptote *OQ*, meeting the asymptote *OK* in *K*, and the perpendiculars *ST* and *QE* in *L* and *F*) the hyperbolic area *PIEQ* may be to the hyperbolic area *PITS* as the arc *BC*, described in the descent of the body, to the arc *Ca* described in the ascent; and that the area *IEF* may be to the area *ILT* as *OQ* to *OS*. Then with the perpendicular *MN* cut off the hyperbolic area *PINM*, and let that area be to the hyperbolic area *PIEQ* as the arc *CZ* to the arc

BC described in the descent. And if the perpendicular RG cut off the hyperbolic area $PIGR$, which shall be to the area $PIEQ$ as any arc CD to the arc BC described in the whole descent; the resistance in any place D will be to the force of gravity, as the area $\frac{OR}{OQ} IEF - IGH$ to the area $PINM$.

For since the forces arising from gravity with which the body is urged in the places Z, B, D, a, are as the arcs CZ, CB, CD, Ca, and those arcs are as the areas $PINM$, $PIEQ$, $PIGR$, $PITS$; let those areas be the exponents both of the arcs and of the forces respectively. Let Dd be a very small space described by the body in its descent; and let it be expressed by the very small area $RGgr$ comprehended between the parallels RG, rg; and produce rg to h, so that $GHhg$, and $RGgr$ may be the contemporaneous decrements of the areas IGH, $PIGR$. And the increment $GHhg - \frac{Rr}{OQ} IEF$, or $Rr \times HG - \frac{Rr}{OQ} IEF$, of the area $\frac{OR}{OQ} IEF - IGH$ will be to the decrement $RGgr$, or $Rr \times RG$, of the area $PIGR$, as $HG - \frac{IEF}{OQ}$ to RG; and therefore as $OR \times HG - \frac{OR}{OQ} IEF$ to $OR \times GR$ or $OP \times PI$, that is (because of the equal quantities $OR \times HG$, $OR \times HR - OR \times GR$, $ORHK - OPIK$, $PIHR$ and $PIGR + IGH$) as $PIGR + IGH - \frac{OR}{OQ} IEF$ to $OPIK$. Therefore if the area $\frac{OR}{OQ} IEF - IGH$ be called Y, and $RGgr$ the decrement of the area $PIGR$ be given, the increment of the area Y will be as $PIGR - Y$.

Then

Then if V represent the force arising from the gravity, proportional to the arc *CD* to be described, by which the body is acted upon in *D*, and R be put for the resistance; V — R will be the whole force with which the body is urged in *D*. Therefore the increment of the velocity is as V — R and the particle of time in which it is generated conjunctly. But the velocity itself is as the contemporaneous increment of the space described directly and the same particle of time inversely. Therefore, since the resistance is, by the supposition, as the square of the velocity, the increment of the resistance will (by Lem. 2.) be as the velocity and the increment of the velocity conjunctly, that is, as the moment of the space and V — R conjunctly; and therefore, if the moment of the space be given, as V — R; that is, if for the force V we put its exponent *PIGR*, and the resistance R be expressed by any other area Z, as *PIGR* — Z.

Therefore the area *PIGR* uniformly decreasing by the subduction of given moments, the area Y increases in proportion of *PIGR* — Y, and the area Z in proportion of *PIGR* — Z. And therefore if the areas Y and Z begin together, and at the beginning are equal, these, by the addition of equal moments, will continue to be equal; and in like manner decreasing by equal moments will vanish together. And, *vice versa*, if they together begin and vanish, they will have equal moments and be always equal: and that, because if the resistance Z be augmented, the velocity together with the arc *Ca*, described in the ascent of the body, will be diminished; and the point in which all the motion together with the resistance ceases, coming nearer to the point *C*, the resistance vanishes sooner than the area Y. And the contrary will happen when the resistance is diminished.

Now the area Z begins and ends where the resistance is nothing, that is, at the beginning of the motion where the arc *CD* is equal to the arc *CB*, and the right line

line RG falls upon the right line QE; and at the end of the motion where the arc CD is equal to the arc Ca, and RG falls upon the right line ST. And the area Y or $\frac{OR}{OQ}IEF - IGH$ begins and ends also where the resistance is nothing, and therefore where $\frac{OR}{OQ}IEF$ and IGH are equal; that is, (by the construction) where the right line RG falls successively upon the right lines QE and ST. Therefore those areas begin and vanish together, and are therefore always equal. Therefore the area $\frac{OR}{OQ}IEF - IGH$ is equal to the area Z, by which the resistance is expressed, and therefore is to the area $PINM$ by which the gravity is expressed as the resistance to the gravity. *Q.E.D.*

Cor. 1. Therefore the resistance in the lowest place C is to the force of gravity, as the area $\frac{OP}{OQ}IEF$ to the area $PINM$.

Cor. 2. But it becomes greatest, where the area $PIHR$ is to the area IEF as OR to OQ. For in that case its moment (that is, $PIGR$ — Y) becomes nothing.

Cor. 3. Hence also may be known the velocity in each place: as being in the subduplicate ratio of the resistance, and at the beginning of the motion equal to the velocity of the body oscillating in the same cycloid without any resistance.

However, by reason of the difficulty of the calculation by which the resistance and the velocity are found by this Proposition, we have thought fit to subjoin the Proposition following.

Pro-

Proposition XXX. Theorem XXIV.

If a right line aB (Pl. 6. Fig. 1.) *be equal to the arc of a cycloid which an oscillating body describes, and at each of its points* D *the perpendiculars* DK *be erected, which shall be to the length of the pendulum as the resistance of the body in the corresponding points of the arc to the force of gravity: I say, that the difference between the arc described in the whole descent and the arc described in the whole subsequent ascent drawn into half the sum of the same arcs, will be equal to the area* BKa *which all those perpendiculars take up.*

Let the arc of the cycloid, described in one entire oscillation, be expressed by the right line *aB*, equal to it, and the arc which would have been described *in vacuo*, by the length *AB*. Bisect *AB* in *C*, and the point *C* will represent the lowest point of the cycloid, and *CD* will be as the force arising from gravity, with which the body in *D* is urged in the direction of the tangent of the cycloid, and will have the same ratio to the length of the pendulum as the force in *D* has to the force of gravity. Let that force therefore be expressed by that length *CD*, and the force of gravity by the length of the pendulum, and if in *DE* you take *DK* in the same ratio to the length of the pendulum as the resistance has to the gravity, *DK* will be the exponent of the resistance. From the centre *C* with the interval *CA* or *CB* describe a semi-circle *BEeA*. Let the body describe, in the least time, the space *Dd*, and erecting the perpendiculars *DE*, *de*, meeting the circumference in *E* and *e*, they will be as the velocities which

which the body deſcending *in vacuo* from the point *B* would acquire in the places *D* and *d*. This appears by Prop. 52. Book 1. Let therefore theſe velocities be expreſſed by thoſe perpendiculars *DE*, *de*; and let *DF* be the velocity which it acquires in *D* by falling from *B* in the reſiſting medium. And if from the centre *C* with the interval *CF* we deſcribe the circle *FfM* meeting the right lines *de* and *AB* in *f* and *M*, then *M* will be the place to which it would thenceforward, without farther reſiſtance, aſcend, and *df* the velocity it would acquire in *d*. Whence alſo if *Fg* repreſent the moment of the velocity which the body *D*, in deſcribing the leaſt ſpace *Dd*, loſes by the reſiſtance of the medium; and *CN* be taken equal to *Cg*: then will *N* be the place to which the body, if it met no farther reſiſtance, would thenceforward aſcend, and *MN* will be the decrement of the aſcent ariſing from the loſs of that velocity. Draw *Fm* perpendicular to *df*, and the decrement *Fg* of the velocity *DF* generated by the reſiſtance *DK* will be to the increment *fm* of the ſame velocity generated by the force *CD*, as the generating force *DK* to the generating force *CD*. But becauſe of the ſimilar triangles *Fmf*, *Fhg*, *FDC*, *fm* is to *Fm* or *Dd* as *CD* to *DF*; and, *ex æquo*, *Fg* to *Dd* as *DK* to *DF*. Alſo *Fh* is to *Fg* as *DF* to *CF*; and, *ex æquo perturbatè*, *Fh* or *MN* to *Dd* as *DK* to *CF* or *CM*; and therefore the ſum of all the $MN \times CM$ will be equal to the ſum of all the $Dd \times DK$. At the moveable point *M* ſuppoſe always a rectangular ordinate erected equal to the indeterminate *CM*, which by a continual motion is drawn into the whole length *Aa*; and the trapezium deſcribed by that motion, or its equal, the rectangle $Aa \times \frac{1}{2} aB$, will be equal to the ſum of all the $MN \times CM$, and therefore to the ſum of all the $Dd \times DK$, that is, to the area *BKVTa*. *Q.E.D.*

Cor.

COR. Hence from the law of resistance and the difference *Aa* of the arcs *Ca*, *CB* may be collected the proportion of the resistance to the gravity nearly.

For if the resistance *DK* be uniform, the figure *BKTa* will be a rectangle under *Ba* and *DK*; and thence the rectangle under $\frac{1}{2}$ *Ba* and *Aa* will be equal to the rectangle under *Ba* and *DK*, and *DK* will be equal to $\frac{1}{2}$ *Aa*. Wherefore since *DK* is the exponent of the resistance, and the length of the pendulum the exponent of the gravity, the resistance will be to the gravity as $\frac{1}{2}$ *Aa* to the length of the pendulum; altogether as in Prop. 28. is demonstrated.

If the resistance be as the velocity, the figure *BKTa* will be nearly an ellipsis. For if a body, in a non-resisting medium, by one entire oscillation, should describe the length *BA*, the velocity in any place *D* would be as the ordinate *DE* of the circle described on the diameter *AB*. Therefore since *Ba* in the resisting medium, and *BA* in the non-resisting one, are described nearly in the same times; and therefore the velocities in each of the points of *Ba*, are to the velocities in the correspondent points of the length *BA* nearly as *Ba* is to *BA*; the velocity in the point *D* in the resisting medium will be as the ordinate of the circle or ellipsis described upon the diameter *Ba*; and therefore the figure *BKVTa* will be nearly an ellipsis. Since the resistance is supposed proportional to the velocity, let *OV* be the exponent of the resistance in the middle point *O*; and an ellipsis *BRVSa* described with the centre *O*, and the semiaxes *OB*, *OV* will be nearly equal to the figure *BKVTa*, and to its equal the rectangle $Aa \times BO$. Therefore $Aa \times BO$ is to $OV \times BO$ as the area of this ellipsis to $OV \times BO$; that is, *Aa* is to *OV* as the area of the semicircle to the square of the radius, or as 11 to 7 nearly; and therefore $\frac{7}{11}$ *Aa* is to the length of the pendulum, as the resistance of the oscillating body in *O* to its gravity.

Now

Now if the resistance *DK* be in the duplicate ratio of the velocity, the figure *BKVTa* will be almost a Parabola having *V* for its vertex and *OV* for its axis, and therefore will be nearly equal to the rectangle under $\frac{2}{3}$ *Ba* and *OV*. Therefore the rectangle under $\frac{1}{2}$ *Ba* and *Aa* is equal to the rectangle $\frac{2}{3}$ $Ba \times OV$, and therefore *OV* is equal to $\frac{3}{4}$ *Aa*: and therefore the resistance in *O* made to the oscillating body is to its gravity as $\frac{3}{4}$ *Aa* to the length of the pendulum.

And I take these conclusions to be accurate enough for practical uses. For since an Ellipsis or Parabola *BRVSa* falls in with the figure *BKVTa* in the middle point *V*, that figure, if greater towards the part *BRV* or *VSa* than the other, is less towards the contrary part, and is therefore nearly equal to it.

Proposition XXXI. Theorem XXV.

If the resistance made to an oscillating body in each of the proportional parts of the arcs described be augmented or diminished in a given ratio; the difference between the arc described in the descent and the arc described in the subsequent ascent, will be augmented or diminished in the same ratio.

For that difference arises from the retardation of the pendulum by the resistance of the medium, and therefore is as the whole retardation, and the retarding resistance proportional thereto. In the foregoing Proposition the rectangle under the right line $\frac{1}{2}$ *aB* and the difference *Aa* of the arcs *CB*, *Ca* was equal to the area *BKTa*. And that area, if the length *aB* remains, is augmented or diminished in the ratio of the ordinates *DK*; that is, in the ratio of the resistance, and is therefore as the length *aB* and the resistance conjunctly.

junctly. And therefore the rectangle under *Aa* and $\frac{1}{2}$ *aB* is as *aB* and the resistance conjunctly, and therefore *Aa* is as the resistance. *Q.E.D.*

Cor. 1. Hence if the resistance be as the velocity, the difference of the arcs in the same medium will be as the whole arc described: and the contrary.

Cor. 2. If the resistance be in the duplicate ratio of the velocity, that difference will be in the duplicate ratio of the whole arc: and the contrary.

Cor. 3. And universally, if the resistance be in the triplicate or any other ratio of the velocity, the difference will be in the same ratio of the whole arc: and the contrary.

Cor. 4. If the resistance be partly in the simple ratio of the velocity, and partly in the duplicate ratio of the same, the difference will be partly in the ratio of the whole arc, and partly in the duplicate ratio of it: and the contrary. So that the law and ratio of the resistance will be the same for the velocity, as the law and ratio of that difference for the length of the arc.

Cor. 5. And therefore if a pendulum describe successively unequal arcs, and we can find the ratio of the increment or decrement of this difference for the length of the arc described; there will be had also the ratio of the increment or decrement of the resistance for a greater or less velocity.

General Scholium.

From these Propositions, we may find the resistance of mediums by pendulums oscillating therein. I found the resistance of the air by the following experiments. I suspended a wooden globe or ball weighing $57\frac{7}{22}$ ounces Averdupois, its diameter $6\frac{7}{8}$ *London* inches, by a fine thread on a firm hook, so that the distance between the hook and the centre of oscillation of the globe was $10\frac{1}{2}$ foot. I marked on the thread a point 10 foot and

1 inch diſtant from the centre of ſuſpenſion; and even with that point I placed a ruler divided into inches, by the help whereof I obſerved the lengths of the arcs deſcribed by the pendulum. Then I number'd the oſcillations, in which the globe would loſe $\frac{1}{8}$ part of its motion. If the pendulum was drawn aſide from the perpendicular to the diſtance of 2 inches, and thence let go, ſo that in its whole deſcent it deſcribed an arc of two inches, and in the firſt whole oſcillation, compounded of the deſcent and ſubſequent aſcent, an arc of almoſt four inches: the ſame in 164 oſcillations loſt $\frac{1}{8}$ part of its motion, ſo as in its laſt aſcent to deſcribe an arc of $1\frac{3}{4}$ inches. If in the firſt deſcent it deſcribed an arc of 4 inches; it loſt $\frac{1}{8}$ part of its motion in 121 oſcillations, ſo as in its laſt aſcent to deſcribe an arc of $3\frac{1}{2}$ inches. If in the firſt deſcent it deſcribed an arc of 8, 16, 32, or 64 inches; it loſt $\frac{1}{8}$ part of its motion in 69, $35\frac{1}{2}$, $18\frac{1}{2}$, $9\frac{2}{3}$ oſcillations, reſpectively. Therefore the difference between the arcs deſcribed in the firſt deſcent and the laſt aſcent, was in the 1ſt, 2d, 3d, 4th, 5th, 6th caſe, $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8 inches, reſpectively. Divide thoſe differences by the number of oſcillations in each caſe, and in one mean oſcillation, wherein an arc of $3\frac{3}{4}$, $7\frac{1}{2}$, 15, 30, 60, 120 inches was deſcribed, the difference of the arcs deſcribed in the deſcent and ſubſequent aſcent will be $\frac{1}{656}$, $\frac{1}{242}$, $\frac{1}{69}$, $\frac{4}{71}$, $\frac{8}{37}$, $\frac{24}{29}$ parts of an inch, reſpectively. But theſe differences in the greater oſcillations are in the duplicate ratio of the arcs deſcribed nearly, but in leſſer oſcillations ſomething greater than in that ratio; and therefore (by Cor. 2. Prop. 31. of this Book) the reſiſtance of the globe, when it moves very ſwift, is in the duplicate ratio of the velocity, nearly; and when it moves ſlowly, ſomewhat greater than in that ratio.

Now let V represent the greatest velocity in any oscillation, and let A, B, and C be given quantities, and let us suppose the difference of the arcs to be $AV + BV^{\frac{3}{2}} + CV^2$. Since the greatest velocities are in the cycloid as $\frac{1}{2}$ the arcs described in oscillating, and in the circle as $\frac{1}{2}$ the chords of those arcs; and therefore in equal arcs are greater in the cycloid than in the circle, in the ratio of $\frac{1}{2}$ the arcs to their chords; but the times in the circle are greater than in the cycloid, in a reciprocal ratio of the velocity; it is plain that the differences of the arcs (which are as the resistance and the square of the time conjunctly) are nearly the same, in both curves: for in the cycloid those differences must be on the one hand augmented, with the resistance, in about the duplicate ratio of the arc to the chord, because of the velocity augmented in the simple ratio of the same; and on the other hand diminished, with the square of the time, in the same duplicate ratio. Therefore to reduce these observations to the cycloid, we must take the same differences of the arcs as were observed in the circle, and suppose the greatest velocities analogous to the half, or the whole arcs, that is, to the numbers $\frac{1}{2}$, 1, 2, 4, 8, 16. Therefore in the 2d, 4th, and 6th case, put 1, 4 and 16 for V; and the difference of the arcs in the 2d case will become $\frac{\frac{1}{2}}{121} = A + B + C$; in the 4th case $\frac{2}{35\frac{1}{2}} = 4A + 8B + 16C$; in the 6th case $\frac{8}{9\frac{2}{3}} = 16A + 64B + 256C$. These equations reduced give $A = 0,0000916$, $B = 0,0010847$, and $C = 0,0029558$. Therefore the difference of the arcs is as $0,0000916\,V + 0,0010847\,V^{\frac{3}{2}} + 0,0029558\,V^2$: and therefore since (by Cor. Prop. 30. applied to this case) the resistance of the globe in the middle

middle of the arc described in oscillating, where the velocity is V, is to its weight as $\frac{2}{11}AV + \frac{7}{10}BV^{\frac{3}{2}} + \frac{3}{4}CV^2$ to the length of the pendulum; if for A, B, and C you put the numbers found, the resistance of the globe will be to its weight, as $0{,}0000583V + 0{,}0007593V^{\frac{3}{2}} + 0{,}0022169V^2$ to the length of the pendulum between the centre of suspension and the ruler, that is, to 121 inches. Therefore since V in the 2^d case represents 1, in the 4^th case 4, and in the 6^th case 16: the resistance will be to the weight of the globe, in the 2^d case as 0,0030345 to 121, in the 4^th as 0,041748 to 121, in the 6^th as 0,61705 to 121.

The arc which the point marked in the thread described in the 6^th case, was of $120 - \frac{8}{9\frac{2}{3}}$ or $119\frac{5}{29}$ inches. And therefore since the radius was 121 inches, and the length of the pendulum between the point of suspension and the centre of the globe was 126 inches, the arc which the centre of the globe described was $124\frac{3}{31}$ inches. Because the greatest velocity of the oscillating body, by reason of the resistance of the air, does not fall on the lowest point of the arc described, but near the middle place of the whole arc: this velocity will be nearly the same as if the globe in its whole descent in a non-resisting medium should describe $62\frac{3}{62}$ inches the half of that arc, and that in a cycloid, to which we have above reduced the motion of the pendulum: and therefore that velocity will be equal to that which the globe would acquire by falling perpendicularly from a height equal to the versed sine of that arc. But that versed sine in the cycloid is to that arc $62\frac{3}{62}$ as the same arc to twice the length of the pendulum 252, and therefore equal to 15,278 inches. Therefore the velocity of the pendulum is the same which a body would acquire by falling, and in its fall de-

describing a space of 15,278 inches. Therefore with such a velocity the globe meets with a resistance, which is to its weight as 0,61705 to 121, or (if we take that part only of the resistance which is in the duplicate ratio of the velocity) as 0,56752 to 121.

I found by an hydrostatical experiment, that the weight of this wooden globe was to the weight of a globe of water of the same magnitude as 55 to 97: and therefore since 121 is to 213,4 in the same ratio, the resistance made to this globe of water moving forwards with the abovementioned velocity, will be to its weight as 0,56752 to 213,4, that is, as 1 to $376\frac{1}{50}$. Whence since the weight of a globe of water, in the time in which the globe with a velocity uniformly continued describes a length of 30,556 inches, will generate all that velocity in the falling globe; it is manifest that the force of resistance uniformly continued in the same time will take away a velocity, which will be less than the other in the ratio of 1 to $376\frac{1}{50}$, that is, the $\frac{1}{376\frac{1}{50}}$ part of the whole velocity. And therefore in the time that the globe, with the same velocity uniformly continued, would describe the length of its semi-diameter, or $3\frac{7}{16}$ inches, it would lose the $\frac{1}{3342}$ part of its motion.

I also counted the oscillations in which the pendulum lost $\frac{1}{4}$ part of its motion. In the following table the upper numbers denote the length of the arc described in the first descent, expressed in inches and parts of an inch; the middle numbers denote the length of the arc described in the last ascent; and in the lowest place are the numbers of the oscillations. I give an account of this experiment, as being more accurate than that in which only $\frac{1}{8}$ part of the motion was lost. I leave the calculation to such as are disposed to make it.

First descent	2	4	8	16	32	64
Last ascent	$1\frac{1}{2}$	3	6	12	24	48
Numb. of oscill.	374	272	$162\frac{1}{2}$	$83\frac{1}{3}$	$41\frac{2}{3}$	$22\frac{2}{3}$

I afterwards suspended a leaden globe of 2 inches in diameter, weighing $26\frac{1}{4}$ ounces Averdupois by the same thread, so that between the centre of the globe and the point of suspension there was an interval of $10\frac{1}{2}$ feet, and I counted the oscillations in which a given part of the motion was lost. The first of the following tables exhibits the number of oscillations in which $\frac{1}{8}$ part of the whole motion was lost; the second the number of oscillations in which there was lost $\frac{1}{4}$ part of the same.

First descent	1	2	4	8	16	32	64
Last ascent	$\frac{7}{8}$	$\frac{7}{4}$	$3\frac{1}{2}$	7	14	28	56
Numb. of oscill.	226	228	193	140	$90\frac{1}{2}$	53	30

First descent	1	2	4	8	16	32	64
Last ascent	$\frac{3}{4}$	$1\frac{1}{2}$	3	6	12	24	48
Numb. of oscill.	510	518	420	318	204	121	70

Selecting in the first table the 3^{d}, 5th, and 7th observation, and expressing the greatest velocities in these observations particularly by the numbers 1, 4, 16 respectively, and generally by the quantity V as above: there will come out in the 3^{d} observation $\frac{\frac{1}{2}}{193} = A + B + C$, in the 5th observation $\frac{2}{90\frac{1}{2}} = 4A + 8B + 16C$, in the 7^{h} observation $\frac{8}{30} = 16A + 64B + 256C$. These equations reduced give $A = 0{,}001414$, $B = 0{,}000297$, $C = 0{,}000879$. And thence the resistance of the globe moving with the velocity V will be to its weight $26\frac{1}{4}$ ounces, in the same ratio as $0{,}9009V + 0{,}000208V^{\frac{3}{2}} + 0{,}000659V^{2}$ to 121 inches

inches the length of the pendulum. And if we regard that part only of the resistance which is in the duplicate ratio of the velocity, it will be to the weight of the globe as $0,000659V^2$ to 121 inches. But this part of the resistance in the 1st experiment was to the weight of the wooden globe of $57\frac{7}{22}$ ounces as $0,002217V^2$ to 121; and thence the resistance of the wooden globe is to the resistance of the leaden one (their velocities being equal) as $57\frac{7}{22}$ into 0,002217 to $26\frac{1}{4}$ into 0,000659, that is, as $7\frac{1}{3}$ to 1. The diameters of the two globes were $6\frac{7}{8}$ and 2 inches, and the squares of these are to each other as $47\frac{1}{4}$ and 4, or $11\frac{13}{16}$ and 1, nearly. Therefore the resistances of these equally swift globes were in less than a duplicate ratio of the diameters. But we have not yet consider'd the resistance of the thread, which was certainly very considerable, and ought to be subducted from the resistance of the pendulums here found. I could not determine this accurately, but I found it greater than a third part of the whole resistance of the lesser pendulum; and thence I gathered that the resistances of the globes, when the resistance of the thread is subducted, are nearly in the duplicate ratio of their diameters. For the ratio of $7\frac{1}{3}-\frac{1}{3}$ to $1-\frac{1}{3}$, or $10\frac{1}{2}$ to 1 is not very different from the duplicate ratio of the diameters, $11\frac{13}{16}$ to 1.

Since the resistance of the thread is of less moment in greater globes, I tried the experiment also with a globe whose diameter was $18\frac{3}{4}$ inches. The length of the pendulum between the point of suspension and the centre of oscillation was $122\frac{1}{2}$ inches, and between the point of suspension and the knot in the thread $109\frac{1}{2}$ inches. The arc described by the knot at the first descent of the pendulum was 32 inches. The arc described by the same knot in the last ascent after five oscillations was 28 inches. The sum of the arcs or the whole arc described in one mean oscillation was 60 inches. The difference of the arcs 4 inches. The $\frac{1}{10}$ part of this, or the difference between the descent and ascent in one mean oscillation is $\frac{2}{5}$ of an inch. Then as the radius $109\frac{1}{2}$ to the radius $122\frac{1}{2}$ so is the whole arc of 60 inches described by the knot in one mean oscillation to the whole arc of $67\frac{1}{8}$ inches described by the centre of the globe in one mean oscillation; and so is the difference $\frac{2}{5}$ to a new difference 0,4475. If the length of the arc described were to remain, and the length of the pendulum should be augmented in the ratio of 126 to $122\frac{1}{2}$; the time of the oscillation would be augmented, and the velocity of the pendulum would be diminished in the subduplicate of that ratio; so that the difference 0,4475 of the arcs described in the descent and subsequent ascent would remain. Then if the arc described be augmented in the ratio of $124\frac{3}{31}$ to $67\frac{1}{8}$, that difference 0,4475 would be augmented in the duplicate of that ratio, and so would become 1,5295. These things would be so upon the supposition, that the resistance of the pendulum were in the duplicate ratio of the velocity. Therefore if the pendulum describe the whole arc of $124\frac{3}{31}$ inches, and its length between the point of suspension and the centre of oscillation be 126 inches, the difference of the arcs described in the descent and subsequent ascent would be 1,5295 inches. And this difference multiplied into the weight of the pendulous globe, which was 208 ounces, produces 318,136. Again in the pendulum abovementioned, made of a wooden globe, when its centre of oscillation, being 126 inches from the point of suspension, described the whole arc of $124\frac{3}{31}$ inches, the difference of the arcs described in the descent and ascent was $\frac{126}{121}$ into $\frac{8}{9\frac{2}{3}}$. This multiplied into the weight of the globe, which was $57\frac{7}{22}$ ounces, produces 49,396. But I multiply these differences into the weights of the globes, in order to find their resistances. For the diffe-

differences arise from the resistances, and are as the resistances directly and the weights inversely. Therefore the resistances are as the numbers 318,136 and 49,396. But that part of the resistance of the lesser globe, which is in the duplicate ratio of the velocity, was to the whole resistance as 0,56752 to 0,61675, that is, as 45,453 to 49,396; whereas that part of the resistance of the greater globe is almost equal to its whole resistance; and so those parts are nearly as 318,136 and 45,453, that is, as 7 and 1. But the diameters of the globes are $18\frac{3}{4}$ and $6\frac{7}{8}$; and their squares $351\frac{9}{16}$ and $47\frac{17}{64}$ are as 7,438 and 1, that is, as the resistances of the globes 7 and 1, nearly. The difference of these ratio's is scarce greater than may arise from the resistance of the thread. Therefore those parts of the resistances which are, when the globes are equal, as the squares of the velocities; are also, when the velocities are equal, as the squares of the diameters of the globes.

But the greatest of the globes, I used in these experiments, was not perfectly sphærical, and therefore in this calculation I have, for brevity's sake, neglected some little niceties; being not very sollicitous for an accurate calculus, in an experiment that was not very accurate. So that I could wish, that these experiments were tried again with other globes, of a larger size, more in number, and more accurately formed; since the demonstration of a vacuum depends thereon. If the globes be taken in a geometrical proportion, as suppose whose diameters are 4, 8, 16, 32 inches; one may collect from the progression observed in the experiments what would happen if the globes were still larger.

In order to compare the resistances of different fluids with each other, I made the following trials. I procured a wooden vessel 4 feet long, 1 foot broad, and 1 foot high. This vessel, being uncover'd, I fill'd with spring-water, and having immersed pendulums therein, I made them oscillate in the water. And I found that a leaden globe weighing $166\frac{1}{6}$ ounces, and in diameter $3\frac{5}{8}$ inches, moved therein as it is set down in the following table; the length of the pendulum from the point of suspension to a certain point marked in the thread being 126 inches, and to the centre of oscillation $134\frac{3}{8}$ inches.

The arc described in the first descent by a point marked in the thread, was inches	64	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
The arc described in the last ascent, was inches	48	24	12	6	3	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$
The difference of the arcs proportional to the motion lost, was inches	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
The number of the oscillations in water			$\frac{29}{60}$	$1\frac{1}{5}$	3	7	$11\frac{1}{4}$	$12\frac{2}{3}$	$13\frac{1}{3}$
The number of the oscillations in air.	$85\frac{1}{2}$		287	535					

In the experiments of the 4th column, there were equal motions lost in 535 oscillations made in the air, and $1\frac{1}{5}$ in water. The oscillations in the air were indeed a little swifter than those in the water. But if the oscillations in the water were accelerated in such a ratio that the motions of the pendulums might be equally swift in both mediums, there would be still the same number $1\frac{1}{5}$ of oscillations in the water, and by these the same quantity of motion would be lost as before; because the resistance is increased and the square of the time diminished in the same duplicate ratio. The pendulums therefore being of equal velocities, there were equal

equal motions lost in 535 oscillations in the air, and $1\frac{1}{5}$ in the water; and therefore the resistance of the pendulum in the water is to its resistance in the air as 535 to $1\frac{1}{5}$. This is the proportion of the whole resistances in the case of the 4th column.

Now let $AV + CV^2$ represent the difference of the arcs described in the descent and subsequent ascent by the globe moving in air with the greatest velocity V; and since the greatest velocity is in the case of the 4th column to the greatest velocity in the case of the 1st column as 1 to 8; and that difference of the arcs in the case of the 4th column to the difference in the case of the 1st column, as $\frac{2}{535}$ to $\frac{16}{85\frac{1}{2}}$, or as $85\frac{1}{2}$ to 4280: put in these cases 1 and 8 for the velocities, and $85\frac{1}{2}$ and 4280 for the differences of the arcs, and $A + C$ will be $= 85\frac{1}{2}$, and $8A + 64C = 4280$ or $A + 8C = 535$; and then, by reducing these equations, there will come out $7C = 449\frac{1}{2}$ and $C = 64\frac{3}{14}$ and $A = 21\frac{2}{7}$: and therefore the resistance, which is as $\frac{7}{11}AV + \frac{3}{4}CV^2$, will become as $13\frac{6}{11}V + 48\frac{9}{56}V^2$. Therefore in the case of the 4th column, where the velocity was 1, the whole resistance is to its part proportional to the square of the velocity, as $13\frac{6}{11} + 48\frac{9}{56}$ or $61\frac{12}{17}$ to $48\frac{9}{56}$; and therefore the resistance of the pendulum in water is to that part of the resistance in air, which is proportional to the square of the velocity, and which in swift motions is the only part that deserves consideration, as $61\frac{12}{17}$ to $48\frac{9}{56}$ and 535 to $1\frac{1}{5}$ conjunctly, that is, as 571 to 1. If the whole thread of the pendulum oscillating in the water had been immersed, its resistance would have been still greater; so that the resistance of the pendulum oscillating in the water, that is, that part which is proportional to the square of the velocity, and which only needs to be consider'd in swift bodies, is to the resistance of the same whole pendulum, oscillating in air with

with the same velocity, as about 850 to 1, that is, as the density of water to the density of air, nearly.

In this calculation, we ought also to have taken in that part of the resistance of the pendulum in the water, which was as the square of the velocity, but I found (which will perhaps seem strange) that the resistance in the water was augmented in more than a duplicate ratio of the velocity. In searching after the cause, I thought upon this, that the vessel was too narrow for the magnitude of the pendulous globe, and by its narrowness obstructed the motion of the water as it yielded to the oscillating globe. For when I immersed a pendulous globe, whose diameter was one inch only; the resistance was augmented nearly in a duplicate ratio of the velocity. I tried this by making a pendulum of two globes, of which the lesser and lower oscillated in the water, and the greater and higher was fastened to the thread just above the water, and by oscillating in the air, assisted the motion of the pendulum, and continued it longer. The experiments made by this contrivance proved according to the following table.

Arc descr. in first descent	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
Arc descr. in last ascent	12	6	3	$1\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{16}$
Diff. of arcs, proport. to mot. lost	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
Number of oscillations	$3\frac{3}{8}$	$6\frac{1}{2}$	$12\frac{1}{12}$	$21\frac{1}{5}$	34	53	$62\frac{1}{5}$

In comparing the resistances of the mediums with each other, I also caused iron pendulums to oscillate in quicksilver. The length of the iron wire was about 3 feet, and the diameter of the pendulous globe about $\frac{1}{3}$ of an inch. To the wire, just above the quicksilver, there was fixed another leaden globe of a bigness sufficient to continue the motion of the pendulum for some time. Then a vessel, that would hold about 3 pounds of quicksilver, was filled by turns with quicksilver and common

common water, that by making the pendulum oscillate successively in these two different fluids, I might find the proportion of their resistances: and the resistance of the quicksilver proved to be to the resistance of water as about 13 or 14 to 1; that is, as the density of quicksilver to the density of water. When I made use of a pendulous globe something bigger, as of one whose diameter was about $\frac{1}{2}$ or $\frac{2}{3}$ of an inch, the resistance of the quicksilver proved to be to the resistance of the water as about 12 or 10 to 1. But the former experiment is more to be relied on, because in the latter the vessel was too narrow in proportion to the magnitude of the immersed globe: For the vessel ought to have been enlarged together with the globe. I intended to have repeated these experiments with larger vessels, and in melted metals, and other liquors both cold and hot: but I had not leisure to try all; and besides, from what is already described, it appears sufficiently that the resistance of bodies moving swiftly is nearly proportional to the densities of the fluids in which they move. I don't say accurately. For more tenacious fluids, of equal density, will undoubtedly resist more than those that are more liquid, as cold oil more than warm, warm oil more than rain-water, and water more than spirit of wine. But in liquors, which are sensibly fluid enough, as in air, in salt and fresh water, in spirit of wine, of turpentine and salts, in oil cleared of its fæces by distillation and warmed, in oil of vitriol and in mercury, and melted metals, and any other such like, that are fluid enough to retain for some time the motion impressed upon them by the agitation of the vessel, and which being poured out are easily resolv'd into drops: I doubt not but the rule already laid down may be accurate enough, especially if the experiments be made with larger pendulous bodies, and more swiftly moved.

Lastly, since it is the opinion of some, that there is a certain æthereal medium extremely rare and subtile,

which

which freely pervades the pores of all bodies; and from ſuch a medium ſo pervading the pores of bodies, ſome reſiſtance muſt needs ariſe: in order to try whether the reſiſtance, which we experience in bodies in motion, be made upon their outward ſuperficies only, or whether their internal parts meet with any conſiderable reſiſtance upon their ſuperficies; I thought of the following experiment. I ſuſpended a round deal box by a thread 11 feet long, on a ſteel hook by means of a ring of the ſame metal, ſo as to make a pendulum of the aforeſaid length. The hook had a ſharp hollow edge on its upper part, ſo that the upper arc of the ring preſſing on the edge might move the more freely: and the thread was faſtened to the lower arc of the ring. The pendulum being thus prepared, I drew it aſide from the perpendicular to the diſtance of about 6 feet, and that in a plane perpendicular to the edge of the hook, leſt the ring, while the pendulum oſcillated, ſhould ſlide to and fro on the edge of the hook: For the point of ſuſpenſion, in which the ring touches the hook, ought to remain immoveable. I therefore accurately noted the place, to which the pendulum was brought, and letting it go, I marked three other places, to which it returned at the end of the 1ſt, 2d, and 3d oſcillation. This I often repeated, that I might find thoſe places as accurately as poſſible. Then I filled the box with lead and other heavy metals, that were near at hand. But firſt I weighed the box when empty, and that part of the thread that went round it, and half the remaining part extended between the hook and the ſuſpended box. For the thread ſo extended always acts upon the pendulum, when drawn aſide from the perpendicular, with half its weight. To this weight I added the weight of the air contained in the box. And this whole weight was about $\frac{1}{78}$ of the weight of the box when filled with the metals. Then becauſe the box when full of the metals, by extending the thread with its weight, increased the length of the pendulum, I shortened the thread so as to make the length of the pendulum, when oscillating, the same as before. Then drawing aside the pendulum to the place first marked, and letting it go, I reckoned about 77 oscillations, before the box returned to the second mark, and as many afterwards before it came to the third mark, and as many after that, before it came to the fourth mark. From whence I conclude that the whole resistance of the box, when full, had not a greater proportion to the resistance of the box, when empty, than 78 to 77. For if their resistances were equal, the box, when full, by reason of its *vis insita,* which was 78 times greater than the *vis insita* of the same when empty, ought to have continued its oscillating motion so much the longer, and therefore to have returned to those marks at the end of 78 oscillations. But it returned to them at the end of 77 oscillations.

Let therefore A represent the resistance of the box upon its external superficies, and B the resistance of the empty box on its internal superficies; and if the resistances to the internal parts of bodies equally swift be as the matter, or the number of particles that are resisted: then 78B will be the resistance made to the internal parts of the box, when full; and therefore the whole resistance A + B of the empty box will be to the whole resistance A + 78B of the full box as 77 to 78, and, by division, A + B to 77B, as 77 to 1, and thence A + B to B as 77 × 77 to 1, and, by division again, A to B as 5928 to 1. Therefore the resistance of the empty box in its internal parts will be above 5000 times less than the resistance on its external superficies. This reasoning depends upon the supposition that the greater resistance of the full box arises, not from any other latent cause, but only from the action of some subtile fluid upon the included metal.

This

This experiment is related by memory, the paper being loſt, in which I had deſcribed it; ſo that I have been obliged to omit ſome fractional parts, which are ſlipt out of my memory. And I have no leiſure to try it again. The firſt time I made it, the hook being weak, the full box was retarded ſooner. The cauſe I found to be, that the hook was not ſtrong enough to bear the weight of the box; ſo that as it oſcillated to and fro, the hook was bent ſometimes this and ſometimes that way. I therefore procured a hook of ſufficient ſtrength, ſo that the point of ſuſpenſion might remain unmoved, and then all things happened as is above deſcribed.

SECTION VII.
Of the motion of fluids and the resistance made to projected bodies.

PROPOSITION XXXII. THEOREM XXVI.

Suppose two similar systems of bodies consisting of an equal number of particles, and let the correspondent particles be similar and proportional, each in one system to each in the other, and have a like situation among themselves, and the same given ratio of density to each other; and let them begin to move among themselves in proportional times, and with like motions, (that is, those in one system among one another, and those in the other among one another.) And if the particles that are in the same system do not touch one another, except in the moments of reflexion; nor attract, nor repel each other, except with accelerative forces that are as the diameters of the correspondent particles inversely, and the squares of the velocities directly: I say, that the particles of those systems will continue to move among themselves with like motions and in proportional times.

Like bodies in like situations are said to be moved among themselves with like motions and in proportional times,

times, when their ſituations at the end of thoſe times are always found alike in reſpect of each other: as ſuppoſe we compare the particles in one ſyſtem with the correſpondent particles in the other. Hence the times will be proportional, in which ſimilar and proportional parts of ſimilar figures will be deſcribed by correſpondent particles. Therefore if we ſuppoſe two ſyſtems of this kind, the correſpondent particles, by reaſon of the ſimilitude of the motions at their beginning, will continue to be moved with like motions, ſo long as they move without meeting one another. For if they are acted on by no forces, they will go on uniformly in right lines by the 1ſt law. But if they do agitate one another, with ſome certain forces, and thoſe forces are as the diameters of the correſpondent particles inverſely and the ſquares of the velocities directly; then becauſe the particles are in like ſituations, and their forces are proportional, the whole forces with which correſpondent particles are agitated, and which are compounded of each of the agitating forces, (by Corol. 2. of the Laws) will have like directions, and have the ſame effect as if they reſpected centres placed alike among the particles; and thoſe whole forces will be to each other as the ſeveral forces which compoſe them, that is, as the diameters of the correſpondent particles inverſely, and the ſquares of the velocities directly: and therefore will cauſe correſpondent particles to continue to deſcribe like figures. Theſe things will be ſo (by Cor. 1 and 8. Prop. 4. Book 1.) if thoſe centres are at reſt. But if they are moved, yet by reaſon of the ſimilitude of the tranſlations, their ſituations among the particles of the ſyſtem will remain ſimilar; ſo that the changes introduced into the figures deſcribed by the particles will ſtill be ſimilar. So that the motions of correſpondent and ſimilar particles will continue ſimilar till their firſt meeting with each other; and thence will ariſe ſimilar colliſions, and ſimilar reflexions; which will again beget

ſimilar

similar motions of the particles among themselves (by what was just now shewn) till they mutually fall upon one another again, and so on *ad infinitum*.

Cor. 1. Hence if any two bodies, which are similar and in like situations to the correspondent particles of the systems, begin to move amongst them in like manner and in proportional times, and their magnitudes and densities be to each other as the magnitudes and densities of the corresponding particles: these bodies will continue to be moved in like manner and in proportional times. For the case of the greater parts of both systems and of the particles is the very same.

Cor. 2. And if all the similar and similarly situated parts of both systems be at rest among themselves: and two of them, which are greater than the rest, and mutually correspondent in both systems, begin to move in lines alike posited, with any similar motion whatsoever; they will excite similar motions in the rest of the parts of the systems, and will continue to move among those parts in like manner and in proportional times; and will therefore describe spaces proportional to their diameters.

Proposition XXXIII. Theorem XXVII.

The same things being supposed, I say that the greater parts of the systems are resisted in a ratio compounded of the duplicate ratio of their velocities, and the duplicate ratio of their diameters, and the simple ratio of the density of the parts of the systems.

For the resistance arises partly from the centripetal or centrifugal forces with which the particles of the system mutually act on each other, partly from the collisions and reflexions of the particles and the greater parts.

The reſiſtances of the firſt kind are to each other as the whole motive forces from which they ariſe, that is, as the whole accelerative forces and the quantities of matter in correſponding parts; that is, (by the ſuppoſition) as the ſquares of the velocities directly, and the diſtances of the correſponding particles inverſely, and the quantities of matter in the correſpondent parts directly: and therefore ſince the diſtances of the particles in one ſyſtem are to the correſpondent diſtances of the particles of the other, as the diameter of one particle or part in the former ſyſtem to the diameter of the correſpondent particle or part in the other, and ſince the quantities of matter are as the denſities of the parts and the cubes of the diameters; the reſiſtances are to each other as the ſquares of the velocities and the ſquares of the diameters and the denſities of the parts of the ſyſtems. *Q.E.D.* The reſiſtances of the latter ſort are as the number of correſpondent reflexions and the forces of thoſe reflexions conjunctly. But the number of the reflexions are to each other as the velocities of the correſponding parts directly and the ſpaces between their reflexions inverſely. And the forces of the reflexions are as the velocities and the magnitudes and the denſities of the correſponding parts conjunctly; that is, as the velocities and the cubes of the diameters and the denſities of the parts. And joining all theſe ratio's, the reſiſtances of the correſponding parts are to each other as the ſquares of the velocities and the ſquares of the diameters and the denſities of the parts conjunctly. *Q.E.D.*

Cor. 1. Therefore if thoſe ſyſtems are two elaſtic fluids, like our air, and their parts are at reſt among themſelves; and two ſimilar bodies proportional in magnitude and denſity to the parts of the fluids and ſimilarly ſituated among thoſe parts, be any how projected in the direction of lines ſimilarly poſited; and the accelerative forces with which the particles of the fluids mutually

mutually act upon each other, are as the diameters of the bodies projected inversely and the squares of their velocities directly: those bodies will excite similar motions in the fluids in proportional times, and will describe similar spaces and proportional to their diameters.

Cor. 2. Therefore in the same fluid a projected body that moves swiftly meets with a resistance that is in the duplicate ratio of its velocity, nearly. For if the forces, with which distant particles act mutually upon one another, should be augmented in the duplicate ratio of the velocity, the projected body would be resisted in the same duplicate ratio accurately; and therefore in a medium, whose parts when at a distance do not act mutually with any force on one another, the resistance is in the duplicate ratio of the velocity accurately. Let there be therefore three mediums *A, B, C,* consisting of similar and equal parts regularly disposed at equal distances. Let the parts of the mediums *A* and *B* recede from each other with forces that are among themselves as *T* and *V*; and let the parts of the medium *C* be entirely destitute of any such forces. And if four equal bodies *D*, *E*, *F*, *G* move in these mediums, the two first *D* and *E* in the two first *A* and *B*, and the other two *F* and *G* in the third *C*; and if the velocity of the body *D* be to the velocity of the body *E*, and the velocity of the body *F* to the velocity of the body *G* in the subduplicate ratio of the force *T* to the force *V*: the resistance of the body *D* to the resistance of the body *E*, and the resistance of the body *F* to the resistance of the body *G* will be in the duplicate ratio of the velocities; and therefore the resistance of the body *D* will be to the resistance of the body *F*, as the resistance of the body *E* to the resistance of the body *G*. Let the bodies *D* and *F* be equally swift, as also the bodies *E* and *G*; and augmenting the velocities of the bodies *D* and *F* in any ratio, and diminishing the forces of the particles of the medium *B* in the duplicate of the same ratio, the medium *B* will approach to the form and condition of the medium *C* at pleaſure; and therefore the reſiſtances of the equal and equally ſwift bodies *E* and *G* in theſe mediums will perpetually approach to equality, ſo that their difference will at laſt become leſs than any given. Therefore ſince the reſiſtances of the bodies *D* and *F* are to each other as the reſiſtances of the bodies *E* and *G*, thoſe will alſo in like manner approach to the ratio of equality. Therefore the bodies *D* and *F*, when they move with very great ſwiftneſs, meet with reſiſtances very nearly equal; and therefore ſince the reſiſtance of the body *F* is in a duplicate ratio of the velocity, the reſiſtance of the body *D* will be nearly in the ſame ratio.

Cor. 3. The reſiſtance of a body moving very ſwift in an elaſtic fluid is almoſt the ſame as if the parts of the fluid were deſtitute of their centrifugal forces, and did not fly from each other: if ſo be that the elaſticity of the fluid ariſe from the centrifugal forces of the particles, and the velocity be ſo great as not to allow the particles time enough to act.

Cor. 4. Therefore ſince the reſiſtances of ſimilar and equally ſwift bodies, in a medium whoſe diſtant parts do not fly from each other, are as the ſquares of the diameters; the reſiſtances made to bodies moving with very great and equal velocities in an elaſtic fluid, will be as the ſquares of the diameters, nearly.

Cor. 5. And ſince ſimilar, equal, and equally ſwift bodies, moving thro' mediums of the ſame denſity, whoſe particles do not fly from each other mutually, will ſtrike againſt an equal quantity of matter in equal times, whether the particles of which the medium conſiſts be more and ſmaller, or fewer and greater, and therefore impreſs on that matter an equal quantity of motion, and in return (by the 3^{d} law of motion) ſuffer an equal re-action from the ſame, that is, are equally reſiſted: it is manifeſt alſo, that in elaſtic fluids of the

the same density, when the bodies move with extreme swiftness, their resistances are nearly equal; whether the fluids consist of gross parts, or of parts never so subtile. For the resistance of projectiles moving with exceeding great celerities, is not much diminished by the subtilty of the medium.

Cor. 6. All these things are so in fluids, whose elastic force takes its rise from the centrifugal forces of the particles. But if that force arise from some other cause, as from the expansion of the particles after the manner of wool, or the boughs of trees, or any other cause, by which the particles are hindered from moving freely among themselves; the resistance, by reason of the lesser fluidity of the medium, will be greater than in the corollaries above.

Proposition XXXIV. Theorem XXVIII.

If in a rare medium, consisting of equal particles freely disposed at equal distances from each other, a globe and a cylinder described on equal diameters move with equal velocities, in the direction of the axis of the cylinder: the resistance of the globe will be but half so great as that of the cylinder.

For since the action of the medium upon the body is the same (by Cor. 5. of the laws) whether the body move in a quiescent medium, or whether the particles of the medium impinge with the same velocity upon the quiescent body: let us consider the body as if it were quiescent, and see with what force it would be impelled by the moving medium. Let therefore *ABKI* (*Pl.* 6. *Fig.* 2.) represent a sphærical body described from the centre *C* with the semidiameter *CA*, and let the particles of the medium impinge with a given velocity upon that ſphærical body, in the directions of right lines parallel to *AC*; and let *FB* be one of thoſe right lines. In *FB* take *LB* equal to the ſemidiameter *CB*, and draw *BD* touching the ſphere in *B*. Upon *KC* and *BD* let fall the perpendiculars *BE*, *LD*, and the force with which a particle of the medium, impinging on the globe obliquely in the direction *FB*, would ſtrike the globe in *B*, will be to the force with which the ſame particle, meeting the cylinder *ONGQ* deſcribed about the globe with the axis *ACI*, would ſtrike it perpendicularly in *b*, as *LD* to *LB* or *BE* to *BC*. Again, the efficacy of this force to move the globe according to the direction of its incidence *FB* or *AC*, is to the efficacy of the ſame to move the globe according to the direction of its determination, that is, in the direction of the right line *BC* in which it impels the globe directly, as *BE* to *BC*. And joining theſe ratio's the efficacy of a particle, falling upon the globe obliquely in the direction of the right line *FB*, to move the globe in the direction of its incidence, is to the efficacy of the ſame particle falling in the ſame line perpendicularly on the cylinder, to move it in the ſame direction, as BE^2 to BC^2. Therefore if in *bE*, which is perpendicular to the circular baſe of the cylinder *NAO*, and equal to the radius *AC*, we take *bH* equal to $\frac{BE^2}{CB}$: then *bH* will be to *bE* as the effect of the particle upon the globe to the effect of the particle upon the cylinder. And therefore the ſolid which is formed by all the right lines *bH* will be to the ſolid formed by all the right lines *bE* as the effect of all the particles upon the globe to the effect of all the particles upon the cylinder. But the former of theſe ſolids is a paraboloid whoſe vertex is *C*, its axis *CA* and latus rectum *CA*; and the latter ſolid is a cylinder circumſcribing the paraboloid: and

and it is known that a paraboloid is half its circumscribed cylinder. Therefore the whole force of the medium upon the globe is half of the entire force of the same upon the cylinder. And therefore if the particles of the medium are at rest, and the cylinder and globe move with equal velocities, the resistance of the globe will be half the resistance of the cylinder. *Q. E. D.*

Scholium.

By the same method other figures may be compared together as to their resistance; and those may be found which are most apt to continue their motions in resisting mediums. As if upon the circular base *CEBH* (*Pl.* 6. *Fig.* 3.) from the centre *O*, with the radius *OC*, and the altitude *OD*, one would construct a frustum *CBGF* of a cone, which should meet with less resistance than any other frustum constructed with the same base and altitude, and going forwards towards *D* in the direction of its axis: bisect the altitude *OD* in *Q*, and produce *OQ* to *S*, so that *QS* may be equal to *QC*, and *S* will be the vertex of the cone whose frustum is sought.

Whence by the bye, since the angle *CSB* is always acute, it follows, that if the solid *ADBE* (*Pl.* 6. *Fig.* 4.) be generated by the convolution of an elliptical or oval figure *ADBE* about its axe *AB*, and the generating figure be touched by three right lines *FG*, *GH*, *HI* in the points *F*, *B*, and *I*, so that *GH* shall be perpendicular to the axe in the point of contact *B*, and *FG*, *HI* may be inclined to *GH* in the angles *FGB*, *BHI* of 135 degrees; the solid arising from the convolution of the figure *ADFGHIE* about the same axe *AB*, will be less resisted than the former solid; if so be that both move forward in the direction of their axe *AB*, and that the extremity *B* of each go foremost. Which proposition I conceive may be of use in the building of ships.

If the figure *DNFG* be such a curve, that if from any point thereof as *N* the perpendicular *NM* be let fall on the axe *AB*, and from the given point *G* there be drawn the right line *GR* parallel to a right line touching the figure in *N*, and cutting the axe produced in *R*, *MN* becomes to *GR* as GR^3 to $4BR \times GB^2$; the solid described by the revolution of this figure about its axe *AB*, moving in the beforementioned rare medium from *A* towards *B*, will be less resisted than any other circular solid whatsoever, described of the same length and breadth.

The demonstration of these curious Theorems being omitted by the author, the analysis thereof, communicated by a friend, is added at the end of this volume.

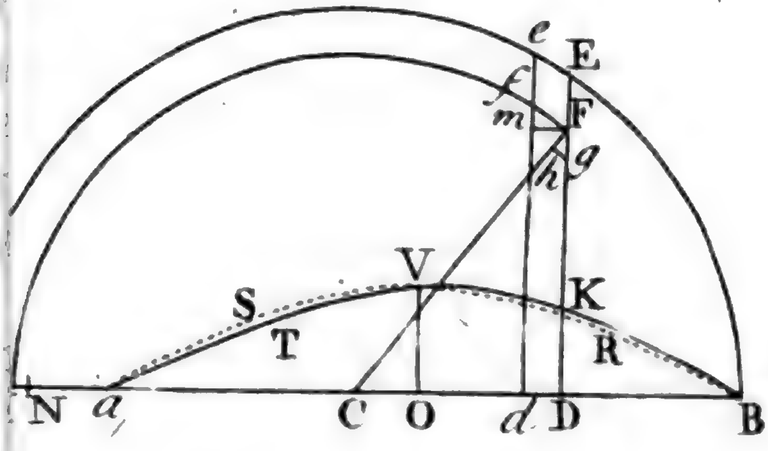
Proposition XXXV. Problem VII.

If a rare medium consist of very small quiescent particles of equal magnitudes and freely disposed at equal distances from one another: to find the resistance of a globe moving uniformly forwards in this medium.

Case 1. Let a cylinder described with the same diameter and altitude be conceived to go forward with the same velocity in the direction of its axis, thro' the same medium. And let us suppose that the particles of the medium, on which the globe or cylinder falls, fly back with as great a force of reflexion as possible. Then since the resistance of the globe (by the last Proposition) is but half the resistance of the cylinder, and since the globe is to the cylinder as 2 to 3, and since the cylinder by falling perpendicularly on the particles, and reflecting them with the utmost force communicates to them a velocity double to its own: it follows that the cylinder, in moving forward uniformly half the length of its axis, will communicate a motion to the particles, which is to the whole motion of the cylinder

as

Fig. 1. p. 91.



ig. 2. p. 117.

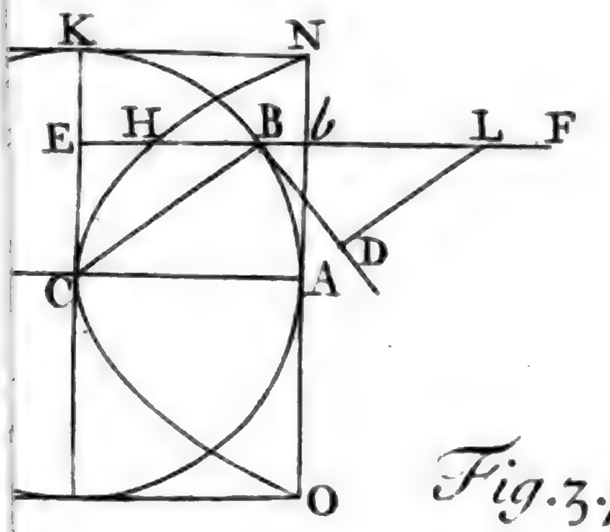
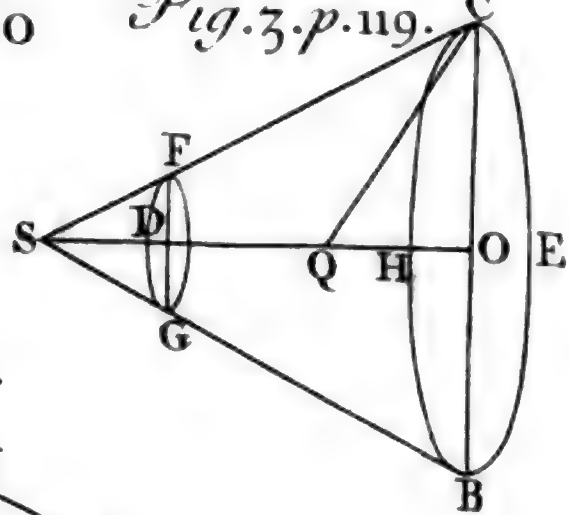
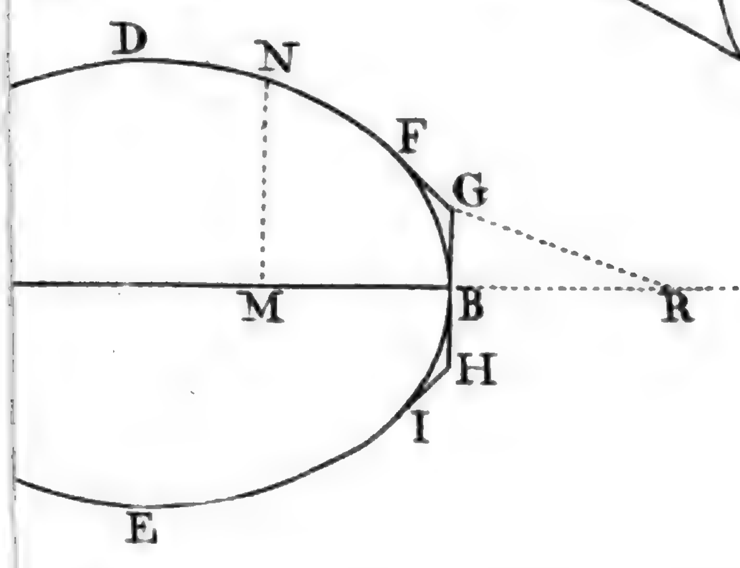


Fig. 3. p. 119.



ig. 4. ibid.



as the density of the medium to the density of the cylinder; and that the globe, in the time it describes one length of its diameter in moving uniformly forwards, will communicate the same motion to the particles; and in the time that it describes two thirds of its diameter, will communicate a motion to the particles, which is to the whole motion of the globe as the density of the medium to the density of the globe. And therefore the globe meets with a resistance, which is to the force by which its whole motion may be either taken away or generated in the time in which it describes two thirds of its diameter moving uniformly forwards, as the density of the medium to the density of the globe.

CASE 2. Let us suppose that the particles of the medium incident on the globe or cylinder are not reflected; and then the cylinder falling perpendicularly on the particles will communicate its own simple velocity to them, and therefore meets a resistance but half so great as in the former case, and the globe also meets with a resistance but half so great.

CASE 3. Let us suppose the particles of the medium to fly back from the globe with a force which is neither the greatest nor yet none at all, but with a certain mean force; then the resistance of the globe will be in the same mean ratio between the resistance in the first case and the resistance in the second. *Q.E.I.*

COR. 1. Hence if the globe and the particles are infinitely hard, and destitute of all elastic force, and therefore of all force of reflexion: the resistance of the globe will be to the force by which its whole motion may be destroyed or generated, in the time that the globe describes four third parts of its diameter, as the density of the medium to the density of the globe.

COR. 2. The resistance of the globe, *cæteris paribus*, is in the duplicate ratio of the velocity.

COR. 3. The resistance of the globe, *cæteris paribus*, is in the duplicate ratio of the diameter.

Cor. 4. The resistance of the globe is, *cæteris paribus*, as the density of the medium.

Cor. 5. The resistance of the globe is in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the density of the medium.

Cor. 6. The motion of the globe and its resistance may be thus expounded. Let *AB* (*Pl.* 7. *Fig.* 1.) be the time in which the globe may, by its resistance uniformly continued, lose its whole motion. Erect *AD*, *BC* perpendicular to *AB*. Let *BC* be that whole motion, and thro' the point *C*, the asymptotes being *AD*, *AB*, describe the hyperbola *CF*. Produce *AB* to any point *E*. Erect the perpendicular *EF* meeting the hyperbola in *F*. Compleat the parallelogram *CBEG*, and draw *AF* meeting *BC* in *H*. Then if the globe in any time *BE*, with its first motion *BC* uniformly continued, describes in a non-resisting medium the space *CBEG* expounded by the area of the parallelogram, the same in a resisting medium will describe the space *CBEF* expounded by the area of the hyperbola; and its motion at the end of that time will be expounded by *EF* the ordinate of the hyperbola; there being lost of its motion the part *FG*. And its resistance at the end of the same time will be expounded by the length *BH*; there being lost of its resistance the part *CH*. All these things appear by Cor. 1 and 3. Prop. 5. Book 2.

Cor. 7. Hence if the globe in the time T by the resistance R uniformly continued, lose its whole motion M: the same globe in the time *t* in a resisting medium, wherein the resistance R decreases in a duplicate ratio of the velocity, will lose out of its motion M the part $\frac{tM}{T+t}$, the part $\frac{TM}{T+t}$ remaining; and will describe a space which is to the space described in the same

ſame time t with the uniform motion M, as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 2,302585092994 is to the number $\frac{t}{T}$, becauſe the hyperbolic area *BCFE* is to the rectangle *BCGE* in that proportion.

SCHOLIUM.

I have exhibited in this Propoſition the reſiſtance and retardation of ſphærical projectiles in mediums that are not continued, and ſhewn that this reſiſtance is to the force by which the whole motion of the globe may be deſtroyed or produced in the time in which the globe can deſcribe two thirds of its diameter, with a velocity uniformly continued, as the denſity of the medium to the denſity of the globe, if ſo be the globe and the particles of the medium be perfectly elaſtic, and are indued with the utmoſt force of reflexion: and that this force, where the globe and particles of the medium are infinitely hard and void of any reflecting force, is diminiſhed one half. But in continued mediums, as water, hot oil, and quickſilver, the globe as it paſſes thro them does not immediately ſtrike againſt all the particles of the fluid that generate the reſiſtance made to it, but preſſes only the particles that lie next to it, which preſs the particles beyond, which preſs other particles, and ſo on; and in theſe mediums the reſiſtance is diminiſhed one other half. A globe in theſe extremely fluid mediums meets with a reſiſtance that is to the force by which its whole motion may be deſtroyed or generated in the time wherein it can deſcribe, with that motion uniformly continued, eight third parts of its diameter, as the denſity of the medium to the denſity of the globe. This I ſhall endeavour to ſhew in what follows.

PRO-

PROPOSITION XXXVI. PROBLEM VIII.

To define the motion of water running out of a cylindrical vessel thro' a hole made at the bottom.

Let *ACDB* (*Pl.* 7. *Fig.* 2.) be a cylindrical vessel, *AB* the mouth of it, *CD* the bottom parallel to the horizon, *EF* a circular hole in the middle of the bottom, *G* the centre of the hole, and *GH* the axis of the cylinder perpendicular to the horizon. And suppose a cylinder of ice *APQB* to be of the same breadth with the cavity of the vessel, and to have the same axis, and to descend perpetually with an uniform motion, and that its parts as soon as they touch the superficies *AB* dissolve into water, and flow down by their weight into the vessel, and in their fall compose the cataract or column of water *ABNFEM*, passing thro' the hole *EF*, and filling up the same exactly. Let the uniform velocity of the descending ice and of the contiguous water in the circle *AB* be that which the water would acquire by falling thro' the space *IH*; and let *IH* and *HG* lie in the same right line, and thro' the point *I* let there be drawn the right line *KL* parallel to the horizon, and meeting the ice on both the sides thereof in *K* and *L*. Then the velocity of the water running out at the hole *EF* will be the same that it would acquire by falling from *I* thro' the space *IG*. Therefore, by *Galileo*'s Theorems, *IG* will be to *IH* in the duplicate ratio of the velocity of the water that runs out at the hole to the velocity of the water in the circle *AB*, that is, in the duplicate ratio of the circle *AB* to the circle *EF*; those circles being reciprocally as the velocities of the water which in the same time and in equal quantities passes severally thro' each of them, and compleatly fills them both. We are now considering the velocity with which the water tends to the plane of the horizon. But the motion parallel to the same by which the parts of the falling water approach to each other, is not here taken notice of; since it is neither produced by gravity, nor at all changes the motion perpendicular to the horizon which the gravity produces. We suppose indeed that the parts of the water cohere a little, that by their cohesion they may in falling approach to each other with motions parallel to the horizon, in order to form one single cataract, and to prevent their being divided into several: but the motion parallel to the horizon arising from this cohesion does not come under our present consideration.

Case I. Conceive now the whole cavity in the vessel, which encompasses the falling water *ABNFEM*, to be full of ice, so that the water may pass thro' the ice as thro' a funnel. Then if the water pass very near to the ice only, without touching it; or, which is the same thing, if, by reason of the perfect smoothness of the surface of the ice, the water, tho' touching it, glides over it with the utmost freedom, and without the least resistance; the water will run thro' the hole *EF* with the same velocity as before, and the whole weight of the column of water *ABNFEM* will be all taken up as before in forcing out the water, and the bottom of the vessel will sustain the weight of the ice encompassing that column.

Let now the ice in the vessel dissolve into water; yet will the efflux of the water remain, as to its velocity, the same as before. It will not be less, because the ice now dissolved will endeavour to descend; it will not be greater, because the ice now become water cannot descend without hindering the descent of other water equal to its own descent. The same force ought always to generate the same velocity in the effluent water.

But

But the hole at the bottom of the vessel, by reason of the oblique motions of the particles of the effluent water, must be a little greater than before. For now the particles of the water do not all of them pass thro' the hole perpendicularly; but flowing down on all parts from the sides of the vessel, and converging towards the hole, pass thro' it with oblique motions; and in tending downwards meet in a stream whose diameter is a little smaller below the hole than at the hole itself, its diameter being to the diameter of the hole as 5 to 6, or as $5\frac{1}{2}$ to $6\frac{1}{2}$, very nearly, if I took the measures of those diameters right. I procured a very thin flat plate having a hole pierced in the middle, the diameter of the circular hole being $\frac{5}{8}$ parts of an inch. And that the stream of running water might not be accelerated in falling, and by that acceleration become narrower, I fixed this plate, not to the bottom, but to the side of the vessel, so as to make the water go out in the direction of a line parallel to the horizon. Then when the vessel was full of water, I opened the hole to let it run out; and the diameter of the stream, measured with great accuracy at the distance of about half an inch from the hole, was $\frac{21}{40}$ of an inch. Therefore the diameter of this circular hole was to the diameter of the stream very nearly as 25 to 21. So that the water in passing thro' the hole, converges on all sides, and after it has run out of the vessel, becomes smaller by converging in that manner, and by becoming smaller is accelerated till it comes to the distance of half an inch from the hole, and at that distance flows in a smaller stream and with greater celerity than in the hole itself, and this in the ratio of 25×25 to 21×21 or 17 to 12 very nearly, that is, in about the subduplicate ratio of 2 to 1. Now it is certain from experiments, that the quantity of water, running out in a given time thro' a circular hole made in the bottom of a vessel is equal to the quantity, which, flowing with the aforesaid velocity,

city, would run out in the same time, thro' another circular hole, whose diameter is to the diameter of the former as 21 to 25. And therefore that running water in passing thro' the hole itself has a velocity downwards equal to that which a heavy body would acquire in falling thro' half the height of the stagnant water in the vessel, nearly. But then after it has run out, it is still accelerated by converging, till it arrives at a distance from the hole that is nearly equal to its diameter, and acquires a velocity greater than the other in about the subduplicate ratio of 2 to 1; which velocity a heavy body would nearly acquire, by falling thro' the whole height of the stagnant water in the vessel.

Therefore in what follows let the diameter of the stream be represented by that lesser hole which we called *EF*. And imagine another plane *VW* above the hole *EF*, (*Pl.* 7. *Fig.* 3.) and parallel to the plane thereof, to be placed at a distance equal to the diameter of the same hole, and to be pierced thro' with a greater hole *ST*, of such a magnitude that a stream which will exactly fill the lower hole *EF* may pass thro' it; the diameter of which hole will therefore be to the diameter of the lower hole as 25 to 21, nearly. By this means the water will run perpendicularly out at the lower hole; and the quantity of the water running out will be, according to the magnitude of this last hole, the same, very nearly, which the solution of the problem requires. The space included between the two planes and the falling stream may be consider'd as the bottom of the vessel. But to make the solution more simple and mathematical, it is better to take the lower plane alone for the bottom of the vessel, and to suppose that the water which flowed thro' the ice as thro' a funnel, and ran out of the vessel thro' the hole *EF* made in the lower plane, preserves its motion continually, and that the ice continues at rest. Therefore in what follows let *ST* be the diameter of a circular hole described from the centre *Z*, and let the stream run out

 of

of the veſſel thro' that hole when the water in the veſſel is all fluid. And let *EF* be the diameter of the hole which the ſtream, in falling thro', exactly fills up, whether the water runs out of the veſſel by that upper hole *ST*, or flows thro' the middle of the ice in the veſſel, as thro' a funnel. And let the diameter of the upper hole *ST* be to the diameter of the lower *EF* as about 25 to 21, and let the perpendicular diſtance between the planes of the holes be equal to the diameter of the leſſer hole *EF*. Then the velocity of the water downwards in running out of the veſſel thro' the hole *ST*, will be in that hole the ſame that a body may acquire by falling from half the height *IZ*: and the velocity of both the falling ſtreams will be, in the hole *EF*, the ſame which a body would acquire by falling from the whole height *IG*.

Case 2. If the hole *EF* be not in the middle of the bottom of the veſſel, but in ſome other part thereof, the water will ſtill run out with the ſame velocity as before, if the magnitude of the hole be the ſame. For tho an heavy body takes a longer time in deſcending to the ſame depth, by an oblique line, than by a perpendicular line; yet in both caſes it acquires in its deſcent the ſame velocity, as *Galileo* has demonſtrated.

Case 3. The velocity of the water is the ſame when it runs out thro' a hole in the ſide of the veſſel. For if the hole be ſmall, ſo that the interval between the ſuperficies *AB* and *KL* may vaniſh as to ſenſe, and the ſtream of water horizontally iſſuing out may form a parabolic figure: from the latus rectum of this parabola may be collected, that the velocity of the effluent water is that which a body may acquire by falling the height *IG* or *HG* of the ſtagnant water in the veſſel. For by making an experiment, I found that if the height of the ſtagnant water above the hole were 20 inches, and the height

height of the hole above a plane parallel to the horizon were also 20 inches, a stream of water springing out from thence would fall upon the plane, at the distance of 37 inches, very nearly, from a perpendicular let fall upon that plane from the hole. For without resistance the stream would have fallen upon the plane at the distance of 40 inches, the latus rectum of the parabolic stream being 80 inches.

Case 4. If the effluent water tend upwards, it will still issue forth with the same velocity. For the small stream of water springing upwards, ascends with a perpendicular motion to *GH* or *GI* the height of the stagnant water in the vessel; excepting in so far as its ascent is hindered a little by the resistance of the air; and therefore it springs out with the same velocity that it would acquire in falling from that height. Every particle of the stagnant water is equally pressed on all sides, (by Prop. 19. Book 2.) and yielding to the pressure, tends all ways with an equal force, whether it descends thro' the hole in the bottom of the vessel, or gushes out in an horizontal direction thro' an hole in the side, or passes into a canal, and springs up from thence thro' a little hole made in the upper part of the canal. And it may not only be collected from reasoning, but is manifest also from the well-known experiments just mentioned, that the velocity with which the water runs out is the very same that is assigned in this Proposition.

Case 5. The velocity of the effluent water is the same, whether the figure of the hole be circular, or square, or triangular, or any other figure equal to the circular. For the velocity of the effluent water does not depend upon the figure of the hole, but arises from its depth below the plane *KL*.

Case 6. If the lower part of the vessel *ABDC* be immersed into stagnant water, and the height of the stagnant water above the bottom of the vessel be *GR*; the velocity with which the water that is in the vessel and *EF* to the sum of the same circles, (by Cor. 4.) and the weight of the whole water in the vessel is to the weight of the whole water perpendicularly incumbent on the bottom as the circle *AB* to the difference of the circles *AB* and *EF*. Therefore, *ex æquo perturbatè*, that part of the weight which presses upon the bottom is to the weight of the whole water perpendicularly incumbent thereon as the circle *AB* to the sums of the circles *AB* and *EF*, or the excess of twice the circle *AB* above the bottom.

Cor. 7. If in the middle of the hole *EF* there be placed the little circle *PQ* described about the centre *G*, and parallel to the horizon; the weight of water which that little circle sustains is greater than the weight of a third part of a cylinder of water whose base is that little circle and its height *GH*. For let *ABNFEM* (*Pl.* 7. *Fig.* 4.) be the cataract or column of falling water whose axis is *GH* as above, and let all the water, whose fluidity is not requisite for the ready and quick descent of the water, be supposed to be congealed; as well round about the cataract, as above the little circle. And let *PHQ* be the column of water, congealed above the little circle, whose vertex is *H*, and its altitude *GH*. And suppose this cataract to fall with its whole weight downwards, and not in the least to lie against or to press *PHQ*, but to glide freely by it without any friction, unless perhaps just at the very vertex of the ice where the cataract at the beginning of its fall may tends to a concave figure. And as the congealed water *AMEC*, *BNFD* lying round the cataract, is convex in its internal superficies *AME*, *BNF* towards the falling cataract, so this column *PHQ* will be convex towards the cataract also, and will therefore be greater than a cone whose base is that little circle *PQ* and its altitude *GH*, that is, greater than a third part of a cylinder described with the same base and altitude. Now that little circle sustains the weight of this column,

 that

that is, a weight greater than the weight of the cone or a third part of the cylinder.

Cor. 8. The weight of water which the circle *PQ*, when very small, sustains, seems to be less than the weight of two thirds of a cylinder of water whose base is that little circle, and its altitude *HG*. For, things standing as above supposed, imagine the half of a sphæroid described whose base is that little circle, and its semi-axis or alitude *HG*. This figure will be equal to two thirds of that cylinder, and will comprehend within it the column of congealed water *PHQ*, the weight of which is sustained by that little circle. For tho' the motion of the water tends directly downwards, the external superficies of that column must yet meet the base *PQ* in an angle somewhat acute, because the water in its fall is perpetually accelerated, and by reason of that acceleration becomes narrower. Therefore, since that angle is less than a right one, this column in the lower parts thereof will lie within the hemi-sphæroid. In the upper parts also it will be acute or pointed; because, to make it otherwise, the horizontal motion of the water must be at the vertex infinitely more swift than its motion towards the horizon. And the less this circle *PQ* is, the more acute will the vertex of this column be; and the circle being diminished in infinitum, the angle *PHQ* will be diminished in infinitum, and therefore the column will lie within the hemi-sphæroid. Therefore that column is less than that hemi-sphæroid, or than two third parts of the cylinder whose base is that little circle, and its altitude *GH*. Now the little circle sustains a force of water equal to the weight of this column, the weight of the ambient water being employed in causing its efflux out at the hole.

Cor. 9. The weight of water which the little circle *PQ* sustains when it is very small, is very nearly equal to the weight of a cylinder of water whose base is that little circle, and its altitude $\frac{1}{2}GH$. For this weight is an arithmetical mean between the weights of the cone and the hemi-ſphæroid abovementioned. But if that little circle be not very ſmall, but on the contrary increaſed till it be equal to the hole EF; it will ſuſtain the weight of all the water lying perpendicularly above it, that is, the weight of a cylinder of water whoſe baſe is that little circle and its altitude GH.

Cor. 10. And (as far as I can judge) the weight which this little circle ſuſtains is always to the weight of a cylinder of water whoſe baſe is that little circle and its altitude $\frac{1}{2}GH$, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$, or as the circle EF to the exceſs of this circle above half the little circle PQ, very nearly.

Lemma IV.

If a cylinder move uniformly forwards in the direction of its length, the reſiſtance made thereto is not at all changed by augmenting or diminiſhing that length; and is therefore the ſame with the reſiſtance of a circle, deſcribed with the ſame diameter, and moving forwards with the ſame velocity in the direction of a right line perpendicular to its plane.

For the ſides are not at all oppoſed to the motion; and a cylinder becomes a circle when its length is diminiſhed in infinitum.

Pro-

PROPOSITION XXXVII. THEOREM XXIX.

If a cylinder move uniformly forwards in a compressed, infinite, and non-elastic fluid, in the direction of its length; the resistance arising from the magnitude of its transverse section, is to the force by which its whole motion may be destroyed or generated, in the time that it moves four times its length, as the density of the medium to the density of the cylinder, nearly.

For let the vessel *ABDC* (*Pl.* 7. *Fig.* 5.) touch the surface of stagnant water with its bottom *CD*, and let the water run out of this vessel into the stagnant water thro' the cylindric canal *EFTS* perpendicular to the horizon; and let the little circle *PQ* be placed parallel to the horizon any where in the middle of the canal; and produce *CA* to *K*, so that *AK* may be to *CK* in the duplicate of the ratio, which the excess of the orifice of the canal *EF* above the little circle *PQ*, bears to the circle *AB*. Then 'tis manifest (by Case 5. Case 6. and Cor. 1. Prop. 36.) that the velocity of the water passing thro' the annular space between the little circle and the sides of the vessel, will be the very same which the water would acquire by falling, and in its fall describing the altitude *KC* or *IG*.

And (by Cor. 10. Prop. 36.) if the breadth of the vessel be infinite, so that the lineola *HI* may vanish, and the altitudes *IG*, *HG* become equal; the force of the water that flows down, and presses upon the circle will be to the weight of a cylinder whose base is that little circle and the altitude $\frac{1}{2}IG$, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$ very nearly. For the force of the water flowing downwards uniformly thro' the whole canal will be the same upon the little circle PQ in whatsoever part of the canal it be placed.

Let now the orifices of the canal EF, ST be closed, and let the little circle ascend in the fluid compressed on every side, and by its ascent let it oblige the water that lies above it to descend thro' the annular space between the little circle and the sides of the canal. Then will the velocity of the ascending little circle be to the velocity of the descending water as the difference of the circles EF and PQ is to the circle PQ; and the velocity of the ascending little circle will be to the sum of the velocities, that is, to the relative velocity of the descending water with which it passes by the little circle in its ascent, as the difference of the circles EF and PQ to the circle EF, or as $EF^2 - PQ^2$ to EF^2. Let that relative velocity be equal to the velocity with which it was shewn above that the water would pass thro' the annular space if the circle were to remain unmoved, that is, to the velocity which the water would acquire by falling, and in its fall describing the altitude IG; and the force of the water upon the ascending circle will be the same as before, (by cor. 5. of the laws of motion) that is, the resistance of the ascending little circle will be to the weight of a cylinder of water whose base is that little circle and its altitude $\frac{1}{2}IG$, as EF^2 to $EF^2 - \frac{1}{2}PQ^2$ nearly. But the velocity of the little circle will be to the velocity which the water acquires by falling, and in its fall describing the altitude IG, as $EF^2 - PQ^2$ to EF^2.

Let the breadth of the canal be increased in infinitum; and the ratio's between $EF^2 - PQ^2$ and EF^2, and between EF^2 and $EF^2 - \frac{1}{2}PQ^2$ will become at last ratio's of equality. And therefore the velocity of the little circle will now be the same which the water would acquire in falling, and in its fall describing the altitude IG; and the resistance will become equal to the weight of a cylinder whose base is that little circle,

and

and its altitude half the altitude *IG*, from which the cylinder must fall to acquire the velocity of the ascending circle. And with this velocity the cylinder in the time of its fall will describe four times its length. But the resistance of the cylinder moving forwards with this velocity in the direction of its length, is the same with the resistance of the little circle, (by Lem. 4.) and is therefore nearly equal to the force by which its motion may be generated while it describes four times its length.

If the length of the cylinder be augmented or diminished, its motion, and the time in which it describes four times its length, will be augmented or diminished in the same ratio; and therefore the force by which the motion, so increased or diminished, may be destroyed or generated, will continue the same; because the time is increased or diminished in the same proportion; and therefore that force remains still equal to the resistance of the cylinder, because (by Lem. 4.) that resistance will also remain the same.

If the density of the cylinder be augmented or diminished, its motion, and the force by which its motion may be generated or destroyed in the same time, will be augmented or diminished in the same ratio. Therefore the resistance of any cylinder whatsoever will be to the force by which its whole motion may be generated or destroyed in the time during which it moves four times its length, as the density of the medium to the density of the cylinder, nearly. *Q.E.D.*

A fluid must be compressed to become continued; it must be continued and non-elastic, that all the pressure arising from its compression may be propagated in an instant; and so acting equally upon all parts of the body moved, may produce no change of the resistance. The pressure arising from the motion of the body is spent in generating a motion in the parts of the fluid, and this creates the resistance. But the pressure arising from

from the compression of the fluid, be it never so forcible, if it be propagated in an instant, generates no motion in the parts of a continued fluid, produces no change at all of motion therein; and therefore neither augments nor lessens the resistance. This is certain, that the action of the fluid arising from the compression cannot be stronger on the hinder parts of the body moved than on its fore parts, and therefore cannot lessen the resistance described in this Proposition. And if its propagation be infinitely swifter than the motion of the body pressed, it will not be stronger on the fore parts than on the hinder parts. But that action will be infinitely swifter and propagated in an instant, if the fluid be continued and non-elastic.

COR. 1. The resistances made to cylinders going uniformly forwards in the direction of their lengths thro' continued infinite mediums, are in a ratio compounded of the duplicate ratio of the velocities and the duplicate ratio of the diameters, and the ratio of the density of the mediums.

COR. 2. If the breadth of the canal be not infinitely increased, but the cylinder go forwards in the direction of its length through an included quiescent medium, its axis all the while coinciding with the axis of the canal; its resistance will be to the force by which its whole motion in the time in which it describes four times its length, may be generated or destroyed, in a ratio compounded of the ratio of EF^2 to $EF^2 - \frac{1}{2}PQ^2$ once, and the ratio of EF^2 to $EF^2 - PQ^2$ twice, and the ratio of the density of the medium to the density of the cylinder.

COR. 3. The same things supposed, and that a length L is to the quadruple of the length of the cylinder in a ratio compounded of the ratio $EF^2 - \frac{1}{2}PQ^2$ to EF^2 once, and the ratio of $EF^2 - PQ^2$ to EF^2 twice; the resistance of the cylinder will be to the force by which its whole motion, in the time during which it describes

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the length L, may be destroyed or generated, as the density of the medium to the density of the cylinder.

SCHOLIUM.

In this proposition we have investigated that resistance alone which arises from the magnitude of the transverse section of the cylinder, neglecting that part of the same which may arise from the obliquity of the motions. For as in Case 1. of Prop. 36. the obliquity of the motions with which the parts of the water in the vessel converged on every side to the hole *EF*, hindered the efflux of the water thro' the hole; so in this proposition, the obliquity of the motions, with which the parts of the water, pressed by the antecedent extremity of the cylinder, yield to the pressure and diverge on all sides, retards their passage, thro' the places that lie round that antecedent extremity, towards the hinder parts of the cylinder, and causes the fluid to be moved to a greater distance; which increases the resistance, and that in the same ratio almost in which it diminished the efflux of the water out of the vessel, that is, in the duplicate ratio of 25 to 21, nearly. And as in Case 1. of that Proposition, we made the parts of the water pass thro' the hole *EF* perpendicularly and in the greatest plenty, by supposing all the water in the vessel lying round the cataract to be frozen, and that part of the water whose motion was oblique and useless to remain without motion; so in this proposition, that the obliquity of the motions may be taken away, and the parts of the water may give the freest passage to the cylinder, by yielding to it with the most direct and quick motion possible, so that only so much resistance may remain as arises from the magnitude of the transverse section, and which is incapable of diminution, unless by diminishing the diameter of the cylinder; we must conceive those parts of the fluid whose motions

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are oblique and uselefs, and produce refiftance, to be at reft among themfelves at both extremities of the cylinder, and there to cohere, and be joined to the cylinder. Let *ABCD* (*Pl.* 7. *Fig.* 6.) be a rectangle, and let *AE* and *BE* be two parabolic arcs, defcribed with the axis *AB*, and with a latus rectum that is to the fpace *HG*, which muft be defcribed by the cylinder in falling in order to acquire the velocity with which it moves, as *HG* to $\frac{1}{2}AB$. Let *CF* and *DF* be two other parabolic arcs defcribed with the axis *CD*, and a latus rectum quadruple of the former; and by the convolution of the figure about the axis *EF* let there be generated a folid, whofe middle part *ABDC* is the cylinder we are here fpeaking of, and the extreme parts *ABE* and *CDF* contain the parts of the fluid, at reft among themfelves, and concreted into two hard bodies, adhering to the cylinder at each end like a head and tail. Then if this folid *EACFDB* move in the direction of the length of its axis *FE* towards the parts beyond *E*, the refiftance will be the fame which we have here determined in this propofition, nearly; that is, it will have the fame ratio to the force with which the whole motion of the cylinder may be deftroyed or generated in the time that it is defcribing the length 4*AC* with that motion uniformly continued, as the denfity of the fluid has to the denfity of the cylinder, nearly. And (by Cor. 7. Prop. 36.) the refiftance muft be to this force in the ratio of 2 to 3, at the leaft.

LEMMA

LEMMA V.

If a cylinder, a ſphere, and a ſphæroid, of equal breadths be placed ſucceſſively in the middle of a cylindric canal, ſo that their axes may coincide with the axis of the canal; theſe bodies will equally hinder the paſſage of the water thro' the canal.

For the ſpaces, lying between the ſides of the canal, and the cylinder, ſphere, and ſphæroid, thro' which the water paſſes, are equal; and the water will paſs equally thro' equal ſpaces.

This is true upon the ſuppoſition that all the water above the cylinder, ſphere, or ſphæroid, whoſe fluidity is not neceſſary to make the paſſage of the water the quickeſt poſſible, is congealed, as was explained above in Cor. 7. Prop. 36.

LEMMA VI.

The ſame ſuppoſition remaining, the forementioned bodies are equally acted on by the water flowing thro' the canal.

This appears by Lem. 5. and the third law. For the water and the bodies act upon each other mutually, and equally.

LEMMA VII.

If the water be at reſt in the canal, and theſe bodies move with equal velocity and the contrary way thro' the canal, their reſiſtances will be equal among themſelves.

This appears from the laſt Lemma, for the relative motions remain the ſame among themſelves.

SCHO-

Scholium.

The caſe is the ſame of all convex and round bodies, whoſe axes coincide with the axis of the canal. Some difference may ariſe from a greater or leſs friction; but in theſe *lemmata* we ſuppoſe the bodies to be perfectly ſmooth, and the medium to be void of all tenacity and friction; and that thoſe parts of the fluid which by their oblique and ſuperfluous motions may diſturb, hinder, and retard the flux of the water thro' the canal, are at reſt amongſt themſelves; being fixed like water by froſt, and adhering to the fore and hinder parts of the bodies in the manner explained in the Scholium of the laſt Propoſition. For in what follows, we conſider the very leaſt reſiſtance that round bodies deſcribed with the greateſt given tranſverſe ſections can poſſibly meet with.

Bodies ſwimming upon fluids, when they move ſtraight forwards, cauſe the fluid to aſcend at their fore parts and ſubſide at their hinder parts, eſpecially if they are of an obtuſe figure; and thence they meet with a little more reſiſtance than if they were acute at the head and tail. And bodies moving in elaſtic fluids, if they are obtuſe behind and before, condenſe the fluid a little more at their fore parts, and relax the ſame at their hinder parts; and therefore meet alſo with a little more reſiſtance than if they were acute at the head and tail. But in theſe lemma's and propoſitions we are not treating of elaſtic, but non-elaſtic fluids; not of bodies floating on the ſurface of the fluid, but deeply immerſed therein. And when the reſiſtance of bodies in non-elaſtic fluids is once known, we may then augment this reſiſtance a little in elaſtic fluids, as our air; and in the ſurfaces of ſtagnating fluids, as lakes and ſeas.

PRO-

Proposition XXXVIII. Theorem XXX.

If a globe move uniformly forward in a compressed, infinite, and non-elastic fluid, its resistance is to the force by which its whole motion may be destroyed or generated in the time that it describes eight third parts of its diameter, as the density of the fluid to the density of the globe, very nearly.

For the globe is to its circumscribed cylinder as two to three; and therefore the force which can destroy all the motion of the cylinder while the same cylinder is describing the length of four of its diameters, will destroy all the motion of the globe while the globe is describing two thirds of this length, that is, eight third parts of its own diameter. Now the resistance of the cylinder is to this force very nearly as the density of the fluid to the density of the cylinder or globe (by Prop. 37.) and the resistance of the globe is equal to the resistance of the cylinder (by Lem. 5, 6, 7.) *Q.E.D.*

Cor. 1. The resistances of globes in infinite compressed mediums are in a ratio compounded of the duplicate ratio of the velocity, and the duplicate ratio of the diameter, and the ratio of the density of the mediums.

Cor. 2. The greatest velocity with which a globe can descend by its comparative weight thro' a resisting fluid, is the same which it may acquire by falling with the same weight, and without any resistance, and in its fall describing a space that is to four third parts of its diameter, as the density of the globe to the density of the fluid. For the globe in the time of its fall, moving with the velocity acquired in falling, will describe a space

ſpace that will be to eight third parts of its diameter as the denſity of the globe to the denſity of the fluid; and the force of its weight which generates this motion, will be to the force that can generate the ſame motion in the time that the globe deſcribes eight third parts of its diameter, with the ſame velocity as the denſity of the fluid to the denſity of the globe; and therefore (by this Propoſition) the force of weight will be equal to the force of reſiſtance, and therefore cannot accelerate the globe.

COR. 3. If there be given both the denſity of the globe and its velocity at the beginning of the motion, and the denſity of the compreſſed quieſcent fluid in which the globe moves; there is given at any time both the velocity of the globe and its reſiſtance, and the ſpace deſcribed by it, (by Cor. 7. Prop. 35.)

COR. 4. A globe moving in a compreſſed quieſcent fluid of the ſame denſity with itſelf, will loſe half its motion before it can deſcribe the length of two of its diameters, (by the ſame Cor. 7.)

PROPOSITION XXXIX. THEOREM XXXI.

If a globe move uniformly forward thro' a fluid incloſed and compreſſed in a cylindric canal, its reſiſtance is to the force by which its whole motion may be generated or deſtroyed in the time in which it deſcribes eight third parts of its diameter, in a ratio compounded of the ratio of the orifice of the canal, to the exceſs of that orifice above half the greateſt circle of the globe; and the duplicate ratio of the orifice of the canal, to the exceſs of that orifice above the greateſt circle of the globe; and the ratio of the denſity of the fluid to the denſity of the globe, nearly.

But

This appears by cor. 2. prop. 37. and the demonstration proceeds in the same manner as in the foregoing proposition.

SCHOLIUM.

In the two last propositions we suppose (as was done before in lem. 5.) that all the water which precedes the globe, and whose fluidity increases the resistance of the same, is congealed. Now if that water becomes fluid, it will somewhat increase the resistance. But in these propositions that increase is so small, that it may be neglected, because the convex superficies of the globe produces the very same effect almost as the congelation of the water.

PROPOSITION XL. PROBLEM IX.

To find by phænomena the resistance of a globe moving through a perfectly fluid compressed medium.

Let A be the weight of the globe in vacuo, B its weight in the resisting medium, D the diameter of the globe, F a space which is to $\frac{4}{3}$ D as the density of the globe to the density of the medium, that is, as A to A—B, G the time in which the globe falling with the weight B without resistance describes the space F, and H the velocity which the body acquires by that fall. Then H will be the greatest velocity with which the globe can possibly descend with the weight B in the resisting medium, by cor. 2. prop 38; and the resistance which the globe meets with, when descending with that velocity, will be equal to its weight B: and the resistance it meets with, in any other velocity, will be to the weight B in the duplicate ratio of that velocity to the greatest velocity H, by cor. 1. prop. 38.

This is the resistance that arises from the inactivity of the matter of the fluid. That resistance which arises from the elasticity, tenacity, and friction of its parts, may be thus investigated.

Let the globe be let fall so that it may descend in the fluid by the weight B; and let P be the time of falling, and let that time be expressed in seconds, if the time G be given in seconds. Find the absolute Number N agreeing to the logarithm $0,4342944819 \frac{2P}{G}$, and let L be the logarithm of the number $\frac{N+1}{N}$: and the velocity acquir'd in falling will be $\frac{N-1}{N+1}H$, and the height described will be $\frac{2PF}{G} - 1,3862943611F + 4,605170186LF$. If the fluid be of a sufficient depth, we may neglect the term $4,605170186LF$; and $\frac{2PF}{G} - 1,3862943611F$ will be the altitude described, nearly. These things appear by prop. 9. book 2. and its corollaries, and are true upon this supposition, that the globe meets with no other resistance but that which arises from the inactivity of matter. Now if it really meet with any resistance of another kind, the descent will be slower, and from the quantity of that retardation will be known the quantity of this new resistance.

That the velocity and descent of a body falling in a fluid might more easily be known, I have composed the following table; the first column of which denotes the times of descent, the second shews the velocities acquir'd in falling, the greatest velocity being 100000000, the third exhibits the spaces described by falling in those times, 2F being the space which the body describes in the time G with the greatest velocity,

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and the fourth gives the spaces described with the greatest velocity in the same times. The numbers in the fourth column are $\frac{2P}{G}$, and by subducting the number 1,3862944—4,6051702L, are found the numbers in the third column; and these numbers must be multiplied by the space F to obtain the spaces described in falling. A fifth column is added to all these, containing the spaces described in the same times by a body falling in vacuo with the force of B its comparative weight.

The Times P	*Velocities of the body falling in the fluid.*	*The spaces described in falling in the fluid.*	*The spaces described with the greatest motion.*	*The spaces described by falling in vacuo.*
0,001G	99999$\frac{29}{30}$	0,000001F	0,002F	0,000001F
0,01G	999967	0,0001F	0,02F	0,0001F
0,1G	9966799	0,0099834F	0,2F	0,01F
0,2G	19737532	0,0397361F	0,4F	0,04F
0,3G	29131261	0,0886815F	0,6F	0,09F
0,4G	37994896	0,1559070F	0,8F	0,16F
0,5G	46211716	0,2402290F	1,0F	0,25F
0,6G	53704957	0,3402706F	1,2F	0,36F
0,7G	60436778	0,4545405F	1,4F	0,49F
0,8G	66403677	0,5815071F	1,6F	0,64F
0,9G	71629787	0,7196609F	1,8F	0,81F
1G	76159416	0,8675617F	2F	1F
2G	96402758	2,6500055F	4F	4F
3G	99505475	4,6186570F	6F	9F
4G	99932930	6,6143765F	8F	16F
5G	99990920	8,6137964F	10F	25F
6G	99998771	10,6137179F	12F	36F
7G	99999834	12,6137073F	14F	49F
8G	99999980	14,6137059F	16F	64F
9G	99999997	16,6137057F	18F	81F
10G	99999999$\frac{3}{5}$	18,6137056F	20F	100F

SCHOLIUM.

In order to inveſtigate the reſiſtances of fluids from experiments, I procured a ſquare wooden veſſel, whoſe length and breadth on the inſide was 9 inches *Engliſh* meaſure, and its depth 9 foot $\frac{1}{2}$; this I filled with rain-water: and having provided globes made up of wax, and lead included therein, I noted the times of the deſcents of theſe globes, the height through which they deſcended being 112 inches. A ſolid cubic foot of *Engliſh* meaſure contains 76 pounds *Troy* weight of rain-water; and a ſolid inch contains $\frac{19}{36}$ ounces *Troy* weight or $253\frac{1}{3}$ grains; and a globe of water of one inch in diameter contains 132,645 grains in air, or 132,8 grains in vacuo; and any other globe will be as the exceſs of its weight in vacuo above its weight in water.

EXPER. 1. A globe whoſe weight was $156\frac{1}{4}$ grains in air, and 77 grains in water, deſcribed the whole height of 112 inches in 4 ſeconds. And, upon repeating the experiment, the globe ſpent again the very ſame time of 4 ſeconds in falling.

The weight of this globe in vacuo is $156\frac{13}{38}$ grains; and exceſs of this weight above the weight of the globe in water is $79\frac{13}{38}$ grains. Hence the diameter of the globe appears to be 0,84224 parts of an inch. Then it will be, as that exceſs to the weight of the globe in vacuo, ſo is the denſity of the water to the denſity of the globe; and ſo is $\frac{8}{3}$ parts of the diameter of the globe (*viz.* 2,24597 inches) to the ſpace 2F, which will be therefore 4,4256 inches. Now a globe falling in vacuo with its whole weight of $156\frac{13}{38}$ grains in one ſecond of time will deſcribe $193\frac{1}{3}$ inches; and falling in water in the ſame time with the weight of 77 grains without reſiſtance, will deſcribe 95,219 inches; and in the time G which is to one ſecond of time

time in the subduplicate ratio of the space F, or of 2,2128 inches to 95,219 inches, will describe 2,2128 inches, and will acquire the greatest velocity H with which it is capable of descending in water. Therefore the time G is 0,″15244. And in this time G with that greatest velocity H, the globe will describe the space 2F, which is 4,4256 inches; and therefore in 4 seconds will describe a space of 116,1245 inches. Subduct the space 1,3862944F or 3,0676 inches, and there will remain a space of 113,0569 inches, which the globe falling thro' water in a very wide vessel will describe in 4 seconds. But this space, by reason of the narrowness of the wooden vessel beforementioned, ought to be diminished in a ratio compounded of the subduplicate ratio of the orifice of the vessel to the excess of this orifice above half a great circle of the globe, and of the simple ratio of the same orifice to its excess above a great circle of the globe, that is, in a ratio of 1 to 0,9914. This done, we have a space of 112,08 inches, which a globe falling thro' the water in this wooden vessel in 4 seconds of time ought nearly to describe by this theory: but it described 112 inches by the experiment.

Exper. 2. Three equal globes, whose weights were severally 76 $\frac{1}{3}$ grains in air, and 5 $\frac{1}{16}$ grains in water, were let fall successively; and every one fell thro' the water in 15 seconds of time, describing in its fall a height of 112 inches.

By computation, the weight of each globe in vacuo is 76 $\frac{5}{12}$ grains; the excess of this weight above the weight in water, is 71 grains $\frac{17}{48}$; the diameter of the globe 0,81296 of an inch: $\frac{8}{3}$ parts of this diameter 2,16789 inches; the space 2F is 2,3217 inches; the space which a globe of 5 $\frac{1}{16}$ grains in weight would describe in one second without resistance, 12,808 inches, and the time G 0″,301056. Therefore the globe with the greatest velocity it is capable of receiving

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from a weight of 5 $\frac{1}{16}$ grains in its descent thro' water, will describe in the time 0″,301056 the space of 2,3217 inches; and in 15 seconds the space 115,678 inches. Subduct the space 1,3862944F or 1,609 inches, and there remains the space 114,069 inches; which therefore the falling globe ought to describe in the same time, if the vessel were very wide. But because our vessel was narrow, the space ought to be diminished by about 0,895 of an inch. And so the space will remain 113,174 inches, which a globe falling in this vessel ought nearly to describe in 15 seconds by the theory. But by the experiment it described 112 inches. The difference is not sensible.

EXPER. 3. Three equal globes, whose weights were severally 121 grains in air, and 1 grain in water, were successively let fall; and they fell thro' the water in the times 46″, 47″, and 50″, describing a height of 112 inches.

By the theory these globes ought to have fallen in about 40″. Now whether their falling more slowly were occasion'd from hence, that in slow motions the resistance arising from the force of inactivity, does really bear a less proportion to the resistance arising from other causes; or whether it is to be attributed to little bubbles that might chance to stick to the globes, or to the rarefaction of the wax by the warmth of the weather, or of the hand that let them fall; or, lastly, whether it proceeded from some insensible errors in weighing the globes in the water, I am not certain. Therefore the weight of the globe in water should be of several grains, that the experiment may be certain, and to be depended on.

EXPER. 4. I began the foregoing experiments to investigate the resistances of fluids, before I was acquainted with the theory laid down in the propositions immediately preceding. Afterwards, in order to examine the theory after it was discovered, I procured a wooden

wooden vessel, whose breadth on the inside was 8 $\frac{2}{3}$ inches, and its depth 15 feet and $\frac{1}{3}$. Then I made four globes of wax, with lead included, each of which weighed 139 $\frac{1}{4}$ grains in air, and 7 $\frac{1}{8}$ grains in water. These I let fall, measuring the times of their falling in the water with a pendulum oscillating to half seconds. The globes were cold, and had remained so some time, both when they were weighed and when they were let fall; because warmth rarefies the wax, and by rarefying it diminishes the weight of the globe in the water; and wax, when rarefied, is not instantly reduced by cold to its former density. Before they were let fall, they were totally immersed under water, lest, by the weight of any part of them that might chance to be above the water, their descent should be accelerated in its beginning. Then, when after their immersion they were perfectly at rest, they were let go with the greatest care, that they might not receive any impulse from the hand that let them down. And they fell successively in the times of 47$\frac{1}{2}$, 48$\frac{1}{2}$, 50 and 51 oscillations, describing a height of 15 feet and 2 inches. But the weather was now a little colder than when the globes were weighed, and therefore I repeated the experiment another day; and then the globes fell in the times of 49, 49 $\frac{1}{2}$, 50 and 53; and at a third trial in the times of 49 $\frac{1}{2}$, 50, 51 and 53 oscillations. And by making the experiment several times over, I found that the globes fell mostly in the times of 49 $\frac{1}{2}$ and 50 oscillations. When they fell slower, I suspect them to have been retarded by striking against the sides of the vessel.

Now, computing from the theory, the weight of the globe in vacuo is 139 $\frac{2}{5}$ grains. The excess of this weight above the weight of the globe in water 132 $\frac{11}{40}$ grains, the diameter of the globe 0,99868 of an inch, $\frac{8}{3}$ parts of the diameter 2,66315 inches, the space 2F 2,8066 inches, the space which a globe weighing 7 $\frac{1}{8}$ grains falling without resistance describes in a second of time 9,88164 inches, and the time G o″,376843. Therefore the globe with the greatest velocity with which it is capable of descending thro' the water by the force of a weight of 7 $\frac{1}{8}$ grains will in the time o″,376843 describe a space of 2,8066 inches, and in one second of time a space of 7,44766 inches, and in the time 25″, or in 50 oscillations the space 186,1915 inches. Subduct the space 1,386294F or 1,9454 inches, and there will remain the space 184,2461 inches, which the globe will describe in that time in a very wide vessel. Because our vessel was narrow, let this space be diminished in a ratio compounded of the subduplicate ratio of the orifice of the vessel to the excess of this orifice above half a great circle of the globe, and of the simple ratio of the same orifice to its excess above a great circle of the globe; and we shall have the space of 181,86 inches, which the globe ought by the theory to describe in this vessel in the time of 50 oscillations, nearly. But it described the space of 182 inches, by experiment, in 49$\frac{1}{2}$ or 50 oscillations.

Exper. 5. Four globes, weighing 154 $\frac{1}{8}$ grains in air, and 21 $\frac{1}{2}$ grains in water, being let fall several times, fell in the times of 28$\frac{1}{2}$, 29, 29$\frac{1}{2}$, and 30, and sometimes of 31, 32, and 33 oscillations, describing a height of 15 feet and 2 inches.

They ought by the theory to have fallen in the time of 29 oscillations, nearly.

Exper. 6. Five globes, weighing 212 $\frac{3}{8}$ grains in air, and 79 $\frac{1}{2}$ in water, being several times let fall, fell in the times of 15, 15 $\frac{1}{2}$, 16, 17, and 18 oscillations, describing a height of 15 feet and 2 inches.

By the theory they ought to have fallen in the time of 15 oscillations, nearly.

Exper. 7. Four globes weighing 293 $\frac{1}{8}$ grains in air, and 35 grains $\frac{7}{8}$ in water, being let fall several times, fell in the times of 29$\frac{1}{2}$, 30, 30$\frac{1}{2}$, 31, 32, and 33 oscillations, describing a height of 15 feet and 1 inch and $\frac{1}{2}$.

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By the theory they ought to have fallen in the time of 28 oscillations, nearly.

In searching for the cause that occasioned these globes of the same weight and magnitude to fall, some swifter and some slower, I hit upon this; that the globes, when they were first let go and began to fall, oscillated about their centres, that side which chanced to be the heavier descending first, and producing an oscillating motion. Now by oscillating thus, the globe communicates a greater motion to the water, than if it descended without any oscillations; and by this communication loses part of its own motion with which it should descend; and therefore as this oscillation is greater or less it will be more or less retarded. Besides the globe always recedes from that side of itself which is descending in the oscillation, and by so receding comes nearer to the sides of the vessel so as even to strike against them sometimes. And the heavier the globes are, the stronger this oscillation is; and the greater they are, the more is the water agitated by it. Therefore to diminish this oscillation of the globes, I made new ones of lead and wax, sticking the lead in one side of the globe very near its surface; and I let fall the globe in such a manner, that as near as possible, the heavier side might be lowest at the beginning of the descent. By this means the oscillations became much less than before, and the times in which the globes fell were not so unequal: as in the following experiments.

Exper. 8. Four globes weighing 139 grains in air and $6\frac{1}{2}$ in water, were let fall several times, and fell mostly in the time of 51 oscillations, never in more than 52, or in fewer than 50; describing a height of 182 inches.

By the theory they ought to fall in about the time of 52 oscillations.

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Exper. 9. Four globes weighing 273 $\frac{1}{4}$ grains in air, and 140 $\frac{3}{4}$ in water, being several times let fall, fell in never fewer than 12, and never more than 13 oscillations, describing a height of 182 inches.

These globes by the theory ought to have fallen in the time of 11 $\frac{1}{3}$ oscillations, nearly.

Exper. 10. Four globes, weighing 384 grains in air and 119 $\frac{1}{2}$ in water, being let fall several times, fell in the times of 17 $\frac{3}{4}$, 18, 18 $\frac{1}{2}$, and 19 oscillations, describing a height of 181 $\frac{1}{2}$ inches. And when they fell in the time of 19 oscillations, I sometimes heard them hit against the sides of the vessel before they reached the bottom.

By the theory they ought to have fallen in the time of 15 $\frac{5}{9}$ oscillations, nearly.

Exper. 11. Three equal globes, weighing 48 grains in the air, and 3 $\frac{29}{32}$ in water, being several times let fall, fell in the times of 43 $\frac{1}{2}$, 44, 44 $\frac{1}{2}$, 45 and 46 oscillations, and mostly in 44 and 45, describing a height of 182 inches $\frac{1}{2}$, nearly.

By the theory they ought to have fallen in the time of 46 oscillations and $\frac{5}{9}$, nearly.

Exper. 12. Three equal globes, weighing 141 grains in air and 4 $\frac{3}{8}$ in water, being let fall several times, fell in the times of 61, 62, 63, 64 and 65 oscillations, describing a space of 182 inches.

And by the theory they ought to have fallen in 64 $\frac{1}{2}$ oscillations, nearly.

From these experiments it is manifest, that when the globes fell slowly, as in the second, fourth, fifth, eighth, eleventh, and twelfth experiments, the times of falling are rightly exhibited by the theory; but when the globes fell more swiftly as in the sixth, ninth, and tenth experiments, the resistance was somewhat greater than in the duplicate ratio of the velocity. For the globes in falling oscillate a little; and this oscillation, in those globes that are light and fall slowly,

slowly, soon ceases by the weakness of the motion; but in greater and heavier globes, the motion being strong, it continues longer; and is not to be checked by the ambient water, till after several oscillations. Besides, the more swiftly the globes move, the less are they pressed by the fluid at their hinder parts; and if the velocity be perpetually increased, they will at last leave an empty space behind them, unless the compression of the fluid be increased at the same time. For the compression of the fluid ought to be increased (by Prop. 32 and 33.) in the duplicate ratio of the velocity, in order to preserve the resistance in the same duplicate ratio. But because this is not done, the globes that move swiftly are not so much pressed at their hinder parts as the others; and by the defect of this pressure it comes to pass that their resistance is a little greater than in a duplicate ratio of their velocity.

So that the theory agrees with the phænomena of bodies falling in water; it remains that we examine the phænomena of bodies falling in air.

Exper. 13. From the top of St. *Paul*'s Church in *London* in *June* 1710. there were let fall together two glass globes, one full of quicksilver, the other of air; and in their fall they described a height of 220 *English* feet. A wooden table was suspended upon iron hinges on one side, and the other side of the same was supported by a wooden pin The two globes lying upon this table were let fall together by pulling out the pin by means of an iron wire reaching from thence quite down to the ground; so that, the pin being removed, the table, which had then no support but the iron hinges, fell downwards; and turning round upon the hinges, gave leave to the globes to drop off from it. At the same instant, with the same pull of the iron wire that took out the pin, a pendulum oscillating to seconds was let go, and began to oscillate. The diameters and weights of the globes, and their times of falling, are exhibited in the following table.

The globes filled with mercury.			*The globes full of air.*		
Weights.	*Diameters.*	*Times in falling.*	*Weights.*	*Diameters.*	*Times in falling*
908 *Grains*	0,8 *of an inch*	4″	510 *Grains*	5,1 *inches*	8″$\frac{1}{2}$
983	0,8	4—	642	5,2	8
866	0,8	4	599	5,1	8
747	0,75	4+	515	5,0	8$\frac{1}{4}$
808	0,75	4	483	5,0	8$\frac{1}{2}$
784	0,75	4+	641	5,2	8

But the times observed must be corrected; for the globes of mercury (by *Galileo*'s theory) in 4 seconds of time, will describe 257 *English* feet, and 220 feet in only 3″ 42‴. So that the wooden table, when the pin was taken out, did not turn upon its hinges so quickly as it ought to have done; and the slowness of that revolution hindered the descent of the globes at the beginning. For the globes lay about the middle of the table, and indeed were rather nearer to the axis upon which it turned, than to the pin. And hence the times of falling were prolonged about 18‴; and therefore ought to be corrected by subducting that excess, especially in the larger globes, which, by reason of the largeness of their diameters, lay longer upon the revolving table than the others. This being done, the times in which the six larger globes fell, will come forth 8″ 12‴, 7″ 42‴, 7″ 42‴, 7″ 57‴, 8″ 12‴, and 7″ 42‴.

Therefore the fifth in order among the globes that were full of air, being 5 inches in diameter, and 483 grains in weight, fell in 8″ 12‴, describing a space of 220 feet. The weight of a bulk of water equal to this globe is 16600 grains; and the weight of an equal bulk of air is $\frac{16600}{860}$ grains, or 19$\frac{3}{10}$ grains; and therefore the weight of the globe in vacuo is 502$\frac{3}{10}$ grains; and

and this weight is to the weight of a bulk of air equal to the globe as 502 $\frac{3}{10}$ to 19 $\frac{3}{10}$, and so is 2F to $\frac{2}{3}$ of the diameter of the globe, that is, to 13 $\frac{1}{3}$ inches. Whence 2F becomes 28 feet 11 inches. A globe falling in vacuo with its whole weight of 502 $\frac{3}{10}$ grains, will in one second of time describe 193 $\frac{1}{3}$ inches as above; and with the weight of 483 grains will describe 185,905 inches; and with that weight 483 grains in vacuo will describe the space F or 14 feet 5 $\frac{1}{2}$ inches, in the time of 57‴ 58⁗, and acquire the greatest velocity it is capable of descending with in the air. With this velocity the globe in 8″ 12‴ of time will describe 245 feet and 5 $\frac{1}{3}$ inches. Subduct 1,3863F or 20 feet and $\frac{1}{2}$ an inch, and there remain 225 feet 5 inches. This space therefore the falling globe ought by the theory to describe in 8″ 12‴. But by the experiment it described a space of 220 feet. The difference is insensible.

By like calculations applied to the other globes full of air, I composed the following table.

The weights of the globes.	*The diameters.*	*The times of falling from a height of 220 feet.*		*The spaces which they would describe by the theory.*		*The Excesses.*	
510 *grains*	5,1 *inches*	8″	12‴	226 *feet*	11 *inches*	6 *feet*	11 *inches*
642	5,2	7	42	230	9	10	9
599	5,1	7	42	227	10	7	10
515	5	7	57	224	5	4	5
483	5	8	12	225	5	5	5
641	5,2	7	42	230	7	10	7

Exper. 14. *Anno* 1719. in the month of *July*, Dr. *Desaguliers* made some experiments of this kind again, by forming hogs bladders into sphærical orbs; which was done by means of a concave wooden sphere, which the bladders, being wetted well first, were put into. After that, being blown full of air, they were

obliged to fill up the ſphærical cavity that contained them; and then, when dry, were taken out. Theſe were let fall from the lantern on the top of the cupola of the ſame church; namely, from a height of 272 feet; and at the ſame moment of time there was let fall a leaden globe whoſe weight was about 2 pounds *Troy* weight. And in the mean time ſome perſons ſtanding in the upper part of the church where the globes were let fall, obſerved the whole times of falling; and others ſtanding on the ground obſerved the differences of the times between the fall of the leaden weight, and the fall of the bladder. The times were meaſured by pendulums oſcillating to half ſeconds. And one of thoſe that ſtood upon the ground had a machine vibrating four times in one ſecond; and another had another machine accurately made with a pendulum vibrating four times in a ſecond alſo. One of thoſe alſo who ſtood at the top of the church had a like machine. And theſe inſtruments were ſo contrived, that their motions could be ſtopped or renewed at pleaſure. Now the leaden globe fell in about four ſeconds and $\frac{1}{4}$ of time; and from the addition of this time to the difference of time above ſpoken of, was collected the whole time in which the bladder was falling. The times which the five bladders ſpent in falling after the leaden globe had reached the ground were the firſt time, $14\frac{3}{4}''$, $12\frac{3}{4}''$, $14\frac{5}{8}''$, $17\frac{3}{4}''$, and $16\frac{7}{8}''$; and the ſecond time $14\frac{1}{2}''$, $14\frac{1}{4}''$, $14''$, $19''$ and $16\frac{3}{4}''$. Add to theſe $4\frac{1}{4}''$, the time in which the leaden globe was falling, and the whole times in which the five bladders fell, were, the firſt time $19''$, $17''$, $18\frac{7}{8}''$, $22''$ and $21\frac{1}{8}''$; and the ſecond time, $18\frac{3}{4}''$, $18\frac{1}{2}''$, $18\frac{1}{4}''$, $23\frac{1}{4}''$ and $21''$. The times obſerved at the top of the church were, the firſt time, $19\frac{3}{8}''$, $17\frac{1}{4}''$, $18\frac{3}{4}''$, $22\frac{1}{8}''$ and $21\frac{5}{8}''$; and the ſecond time, $19''$, $18\frac{5}{8}''$, $18\frac{3}{8}''$, $24''$ and $21\frac{1}{4}''$. But the bladders did not always fall di-

directly down, but sometimes fluttered a little in the air, and waved to and fro as they were descending. And by these motions the times of their falling were prolonged, and increased by half a second sometimes, and sometimes by a whole second. The second and fourth bladder fell most directly the first time, and the first and third the second time. The fifth bladder was wrinkled, and by its wrinkles was a little retarded. I found their diameters by their circumferences measured with a very fine thread wound about them twice. In the following table I have compared the experiments with the theory; making the density of air to be to the density of rain-water as 1 to 860, and computing the spaces which by the theory the globes ought to describe in falling.

The weights of the bladders.	*The diameters.*	*The times of falling from a height of 272 foot.*	*The spaces which by the theory ought to have been described in those times.*		*The difference between the theory and the experiments.*	
128 *grains*	5,28 *inches*	19″	271 *foot*	11 *inches*	− 0 *foot*	1 *inch*
156	5,19	17	272	$0\frac{1}{2}$	+ 0	$0\frac{1}{2}$
$137\frac{1}{2}$	5,3	$18\frac{1}{2}$	272	7	+ 0	7
$97\frac{1}{2}$	5,26	22	277	4	+ 5	4
$99\frac{1}{8}$	5	$21\frac{1}{8}$	282	0	+ 10	0

Our theory therefore exhibits rightly, within a very little, all the resistance that globes moving either in air or in water meet with; which appears to be proportional to the densities of the fluids in globes of equal velocities and magnitudes.

In the scholium subjoined to the sixth section, we shewed by experiments of pendulums, that the resistances of equal and equally swift globes moving in air, water, and quicksilver, are as the densities of the fluids. We here prove the same more accurately by experiments of bodies falling in air and water. For pendulums at each oscillation excite a motion in the fluid al-

ways

ways contrary to the motion of the pendulum in its return; and the resistance arising from this motion, as also the resistance of the thread by which the pendulum is suspended, makes the whole resistance of a pendulum greater than the resistance deduced from the experiments of falling bodies. For by the experiments of pendulums described in that scholium, a globe of the same density as water in describing the length of its semidiameter in air would lose the $\frac{1}{3342}$ part of its motion. But by the theory delivered in this seventh section, and confirmed by experiments of falling bodies, the same globe in describing the same length would lose only a part of its motion equal to $\frac{1}{4586}$ supposing the density of water to be to the density of air as 860 to 1. Therefore the resistances were found greater by the experiments of pendulums (for the reasons just mentioned) than by the experiments of falling globes; and that in the ratio of about 4 to 3. But yet since the resistances of pendulums oscillating in air, water, and quicksilver, are alike increased by like causes, the proportion of the resistances in these mediums will be rightly enough exhibited by the experiments of pendulums, as well as by the experiments of falling bodies. And from all this it may be concluded, that the resistances of bodies, moving in any fluids whatsoever, tho' of the most extreme fluidity, are, *cæteris paribus*, as the densities of the fluids.

These things being thus established, we may now determine what part of its motion any globe projected in any fluid whatsoever would nearly lose in a given time. Let D be the diameter of the globe, and V its velocity at the beginning of its motion, and T the time in which a globe with the velocity V can describe in vacuo a space that is to the space $\frac{8}{3}$ D as the density of the globe to the density of the fluid; and the globe projected in that fluid will, in any other time

time t, lose the part $\frac{tV}{T+t}$, the part $\frac{TV}{T+t}$ remaining; and will describe a space, which may be to that described in the same time in vacuo with the uniform velocity V, as the logarithm of the number $\frac{T+t}{T}$ multiplied by the number 2,302585093 is to the number $\frac{t}{T}$, by cor. 7. prop. 35. In slow motions the resistance may be a little less, because the figure of a globe is more adapted to motion than the figure of a cylinder described with the same diameter. In swift motions the resistance may be a little greater, because the elasticity and compression of the fluid do not increase in the duplicate ratio of the velocity. But these little niceties I take no notice of.

And tho' air, water, quicksilver, and the like fluids, by the division of their parts in infinitum, should be subtilized and become mediums infinitely fluid; nevertheless, the resistance they would make to projected globes would be the same. For the resistance consider'd in the preceding propositions, arises from the inactivity of the matter; and the inactivity of matter is essential to bodies, and always proportional to the quantity of matter. By the division of the parts of the fluid, the resistance arising from the tenacity and friction of the parts may be indeed diminished; but the quantity of matter will not be at all diminished by this division; and if the quantity of matter be the same, its force of inactivity will be the same; and therefore the resistance here spoken of will be the same, as being always proportional to that force. To diminish this resistance, the quantity of matter in the spaces thro' which the bodies move must be diminished. And therefore the celestial spaces, thro' which the globes of the Planets and Comets are perpetually passing towards all parts, with the utmoſt freedom, and without the leaſt ſenſible diminution of their motion, muſt be utterly void of any corporeal fluid, excepting perhaps ſome extremely rare vapours, and the rays of light.

Projectiles excite a motion in fluids as they paſs thro' them; and this motion ariſes from the exceſs of the preſſure of the fluid at the fore-parts of the projectile above the preſſure of the ſame at the hinder parts; and cannot be leſs in mediums infinitely fluid, than it is in air, water, and quickſilver, in proportion to the denſity of matter in each. Now this exceſs of preſſure does, in proportion to its quantity, not only excite a motion in the fluid, but alſo acts upon the projectile ſo as to retard its motion: and therefore the reſiſtance in every fluid is as the motion excited by the projectile in the fluid; and cannot be leſs in the moſt ſubtile æther in proportion to the denſity of that æther, than it is in air, water, and quickſilver, in proportion to the denſities of thoſe fluids.

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SECTION VIII.
Of motion propagated thro' fluids.

PROPOSITION XLI. THEOREM XXXII.

A pressure is not propagated thro' a fluid in rectilinear directions, unless where the particles of the fluid lie in a right line. Pl. 8. Fig. 1.

If the particles *a*, *b*, *c*, *d*, *e*, lie in a right line, the pressure may be indeed directly propagated from *a* to *e*; but then the particle *e* will urge the obliquely posited particles *f* and *g* obliquely, and those particles *f* and *g* will not sustain this pressure, unless they be supported by the particles *h* and *k* lying beyond them; but the particles that support them, are also pressed by them; and those particles cannot sustain that pressure, without being supported by, and pressing upon, those particles that lie still farther, as *l* and *m*, and so on in infinitum. Therefore the pressure, as soon as it is propagated to particles that lie out of right lines, begins to deflect towards one hand and t'other, and will be propagated obliquely in infinitum; and after it has begun to be propagated obliquely, if it reaches more distant particles lying out of the right line, it will deflect again on each hand; and this it will do as often as it lights on particles that do not lie exactly in a right line. *Q. E. D.*

COR. If any part of a pressure, propagated thro' a fluid from a given point, be intercepted by any obstacle; the remaining part, which is not intercepted, will deflect into the spaces behind the obstacle. This may be demonstrated also after the following manner. Let a pressure be propagated from the point *A* (*Pl.* 8. *Fig.* 2.) towards any part, and, if it be possible, in rectilinear directions; and the obstacle *NBCK* being perforated in *BC*, let all the pressure be intercepted but the coniform part *APQ* passing thro' the circular hole *BC*. Let the cone *APQ* be divided into frustums by the transverse planes *de*, *fg*, *hi*. Then while the cone *ABC*, propagating the pressure, urges the conic frustum *degf* beyond it on the superficies *de*, and this frustum urges the next frustum *fgih* on the superficies *fg*, and that frustum urges a third frustum, and so in infinitum; it is manifest (by the third law) that the first frustum *defg* is, by the reaction of the second frustum *fghi*, as much urged and pressed on the superficies *fg*, as it urges and presses that second frustum. Therefore the frustum *degf* is compressed on both sides, that is, between the cone *Ade* and the frustum *fhig*; and therefore (by case 6. prop. 19.) cannot preserve its figure, unless it be compressed with the same force on all sides. Therefore with the same force with which it is pressed on the superficies *de*, *fg*, it will endeavour to break forth at the sides *df*, *eg*; and there (being not in the least tenacious or hard, but perfectly fluid) it will run out, expanding itself, unless there be an ambient fluid opposing that endeavour. Therefore, by the effort it makes to run out, it will press the ambient fluid, at its sides *df*, *eg*, with the same force that it does the frustum *fghi*; and therefore the pressure will be propagated as much from the sides *df*, *eg* into the spaces *NO*, *KL* this way and that way, as it

is

is propagated from the superficies *fg* towards *PQ. Q.E.D.*

Proposition XLII. Theorem XXXIII.

All motion propagated thro' a fluid, diverges from a rectilinear progress into the unmoved spaces. Pl. 8. Fig. 3.

Case 1. Let a motion be propagated from the point *A* thro' the hole *BC*, and, if it be possible, let it proceed in the conic space *BCQP* according to right lines diverging from the point *A*. And let us first suppose this motion to be that of waves in the surface of standing water; and let *de*, *fg*, *hi*, *kl*, &c. be the tops of the several waves, divided from each other by as many intermediate valleys or hollows. Then, because the water in the ridges of the waves is higher than in the unmoved parts of the fluid *KL*, *NO*, it will run down from off the tops of those ridges *e*, *g*, *i*, *l*, &c. *d*, *f*, *h*, *k*, &c. this way and that way towards *KL* and *NO*; and because the water is more depressed in the hollows of the waves than in the unmoved parts of the fluid *KL*, *NO*, it will run down into those hollows out of those unmoved parts. By the first deflux the ridges of the waves will dilate themselves this way and that way, and be propagated towards *KL* and *NO*. And because the motion of the waves from *A* towards *PQ* is carried on by a continual deflux from the ridges of the waves into the hollows next to them; and therefore cannot be swifter than in proportion to the celerity of the descent; and the descent of the water on each side towards *KL* and *NO* must be performed with the same velocity; it follows, that the dilatation of the waves on each side towards *KL* and *NO* will be propagated with the same velocity as the waves themſelves go forward with, directly from *A* to *PQ*. And therefore the whole ſpace this way and that way towards *KL* and *NO* will be filled by the dilated waves *rfgr*, *shis*, *tklt*, *vmnv*, &c. *Q.E.D.* That theſe things are ſo, any one may find by making the experiment in ſtill water.

Case 2. Let us ſuppoſe that *de*, *fg*, *hi*, *kl*, *mn*, repreſent pulſes ſucceſſively propagated from the point *A* thro' an elaſtic medium. Conceive the pulſes to be propagated by ſucceſſive condenſations and rarefactions of the medium, ſo that the denſeſt part of every pulſe may occupy a ſphærical ſuperficies deſcribed about the centre *A*, and that equal intervals intervene between the ſucceſſive pulſes. Let the lines *de*, *fg*, *hi*, *kl*, &c. repreſent the denſeſt parts of the pulſes, propagated thro' the hole *BC*; and becauſe the medium is denſer there, than in the ſpaces on either ſide towards *KL* and *NO*, it will dilate itſelf as well towards thoſe ſpaces *KL*, *NO* on each hand, as towards the rare intervals between the pulſes; and thence the medium becoming always more rare next the intervals, and more denſe next the pulſes, will partake of their motion. And becauſe the progreſſive motion of the pulſes ariſes from the perpetual relaxation of the denſer parts towards the antecedent rare intervals; and ſince the pulſes will relax themſelves on each hand towards the quieſcent parts of the medium *KL*, *NO*, with very near the ſame celerity; therefore the pulſes will dilate themſelves on all ſides into the unmoved parts *KL*, *NO*, with almoſt the ſame celerity with which they are propagated directly from the centre *A*; and therefore will fill up the whole ſpace *KLON*. *Q.E.D.* And we find the ſame by experience alſo in ſounds, which are heard tho' a mountain interpoſe; and if they come into a chamber thro' the window, dilate themſelves into all the parts of the room, and are heard in every corner; and not as reflected from the oppoſite walls, but directly

rectly propagated from the window, as far as our sense can judge.

Case 3. Let us suppose lastly, that a motion of any kind is propagated from *A* thro' the hole *BC*. Then since the cause of this propagation is, that the parts of the medium that are near the centre *A* disturb and agitate those which lie farther from it; and since the parts which are urged are fluid, and therefore recede every way towards those spaces where they are less pressed, they will by consequence recede towards all the parts of the quiescent medium; as well to the parts on each hand, as *KL* and *NO*, as to those right before as *PQ*: and by this means all the motion, as soon as it has passed thro' the hole *BC*, will begin to dilate itself, and from thence, as from its principle and centre, will be propagated directly every way. *Q.E.D.*

Proposition XLIII. Theorem XXXIV.

Every tremulous body in an elastic medium propagates the motion of the pulses on every side right forward; but in a non-elastic medium excites a circular motion.

Case 1. The parts of the tremulous body alternately going and returning, do in going urge and drive before them those parts of the medium that lie nearest, and by that impulse compress and condense them; and in returning suffer those compressed parts to recede again and expand themselves. Therefore the parts of the medium that lie nearest to the tremulous body, move to and fro by turns, in like manner as the parts of the tremulous body itself do; and for the same cause that the parts of this body agitate these parts of the medium, these parts being agitated by like tremors, will in their turn agitate others next to themſelves, and theſe others agitated in like manner, will agitate thoſe that lie beyond them, and ſo on in infinitum. And in the ſame manner as the firſt parts of the medium were condenſed in going, and relaxed in returning, ſo will the other parts be condenſed every time they go, and expand themſelves every time they return. And therefore they will not be all going and all returning at the ſame inſtant, (for in that caſe they would always preſerve determined diſtances from each other, and there could be no alternate condenſation and rarefaction;) but ſince in the places where they are condenſed, they approach to, and in the places where they are rarefied, recede from, each other; therefore ſome of them will be going while others are returning; and ſo on in infinitum. The parts ſo going, and in their going condenſed, are pulſes, by reaſon of the progreſſive motion with which they ſtrike obſtacles in their way; and therefore the ſucceſſive pulſes produced by a tremulous body, will be propagated in rectilinear directions; and that at nearly equal diſtances from each other, becauſe of the equal intervals of time in which the body, by its ſeveral tremors, produces the ſeveral pulſes. And tho' the parts of the tremulous body go and return in ſome certain and determinate direction, yet the pulſes propagated from thence thro' the medium, will dilate themſelves towards the ſides, by the foregoing propoſition; and will be propagated on all ſides from that tremulous body, as from a common centre, in ſuperficies nearly ſphærical and concentrical. An example of this we have in waves excited by ſhaking a finger in water, which proceed not only forwards and backwards agreeably to the motion of the finger, but ſpread themſelves in the manner of concentrical circles all round the finger, and are propagated on every ſide. For the gravity of the water ſupplies the place of elaſtic force.

CASE

CASE 2. If the medium be not elastic, then, because its parts cannot be condensed by the pressure arising from the vibrating parts of the tremulous body, the motion will be propagated in an instant towards the parts where the medium yields most easily, that is, to the parts which the tremulous body leaves for some time vacuous behind it. The case is the same with that of a body projected in any medium whatever. A medium yielding to projectiles does not recede in infinitum, but with a circular motion comes round to the spaces which the body leaves behind it. Therefore as often as a tremulous body tends to any part, the medium yielding to it comes round in a circle to the parts which the body leaves; and as often as the body returns to the first place, the medium will be driven from the place it came round to, and return to its original place. And tho' the tremulous body be not firm and hard, but every way flexible; yet if it continue of a given magnitude, since it cannot impel the medium by its tremors any where without yielding to it somewhere else; the medium receding from the parts where it is pressed, will always come round in a circle to the parts that yield to it. *Q.E.D.*

COR. 'Tis a mistake therefore to think, as some have done, that the agitation of the parts of flame conduces to the propagation of a pressure in rectilinear directions thro' an ambient medium. A pressure of that kind must be derived, not from the agitation only of the parts of flame, but from the dilatation of the whole.

PRO-

Proposition XLIV. Theorem XXXV.

If water ascend and descend alternately in the erected legs KL, MN *of a canal or pipe; and a pendulum be constructed, whose length between the point of suspension and the centre of oscillation is equal to half the length of the water in the canal: I say, that the water will ascend and descend in the same times in which the pendulum oscillates.* Pl. 8. Fig. 4.

I measure the length of the water along the axes of the canal and its legs, and make it equal to the sum of those axes; and take no notice of the resistance of the water, arising from its attrition by the sides of the canal. Let therefore *AB*, *CD* represent the mean height of the water in both legs; and when the water in the leg *KL* ascends to the height *EF*, the water will descend in the leg *MN* to the height *GH*. Let *P* be a pendulous body, *VP* the thread, *V* the point of suspension, *RPQS* the cycloid which the pendulum describes, *P* its lowest point, *PQ* an arc equal to the height *AE*. The force, with which the motion of the water is accelerated and retarded alternately, is the excess of the weight of the water in one leg above the weight in the other; and therefore, when the water in the leg *KL* ascends to *EF*, and in the other leg descends to *GH*, that force is double the weight of the water *EABF*, and therefore is to the weight of the whole water as *AE* or *PQ* to *VP* or *PR*. The force also with which the body *P* is accelerated or retarded in any place as *Q* of a cycloid, is (by cor. prop. 51.) to its whole weight, as its distance *PQ* from the lowest place *P* to the length *PR* of the cycloid. Therefore the motive forces of the water and pendulum, describing

bing

bing the equal spaces *AE*, *PQ* are as the weights to be moved; and therefore if the water and pendulum are quiescent at first, those forces will move them in equal times, and will cause them to go and return together with a reciprocal motion. *Q.E.D.*

Cor. 1. Therefore the reciprocations of the water in ascending and descending, are all performed in equal times, whether the motion be more or less intense or remiss.

Cor. 2. If the length of the whole water in the canal be of $6\frac{1}{9}$ feet of *French* measure, the water will descend in one second of time, and will ascend in another second, and so on by turns in infinitum; for a pendulum of $3\frac{1}{18}$ such feet in length will oscillate in one second of time.

Cor. 3. But if the length of the water be increased or diminished, the time of the reciprocation will be increased or diminished in the subduplicate ratio of the length.

Proposition XLV. Theorem XXXVI.

The velocity of waves is in the subduplicate ratio of the breadths.

This follows from the construction of the following proposition.

Proposition XLVI. Problem X.

To find the velocity of waves.

Let a pendulum be constructed, whose length between the point of suspension and the centre of oscillation is equal to the breadth of the waves; and in the time that the pendulum will perform one single oscillation,

lation, the waves will advance forward nearly a ſpace equal to their breadth.

That which I call the breadth of the waves is the tranſverſe meaſure lying between the deepeſt part of the hollows, or the tops of the ridges. Let *ABCDEF* (*Pl.* 8. *Fig.* 5.) repreſent the ſurface of ſtagnant water aſcending and deſcending in ſucceſſive waves; and let *A*, *C*, *E*, &c. be the tops of the waves; and let *B*, *D*, *F*, &c. be the intermediate hollows. Becauſe the motion of the waves is carried on by the ſucceſſive aſcent and deſcent of the water, ſo that the parts thereof, as *A*, *C*, *E*, &c. which are higheſt at one time, become loweſt immediately after; and becauſe the motive force, by which the higheſt parts deſcend and the loweſt aſcend, is the weight of the elevated water, that alternate aſcent and deſcent will be analogous to the reciprocal motion of the water in the canal, and obſerve the ſame laws as to the times of its aſcent and deſcent; and therefore (by prop. 44.) if the diſtances between the higheſt places of the waves *A*, *C*, *E*, and the loweſt *B*, *D*, *F*, be equal to twice the length of any pendulum, the higheſt parts *A*, *C*, *E*, will become the loweſt in the time of one oſcillation, and in the time of another oſcillation will aſcend again. Therefore between the paſſage of each wave, the time of two oſcillations will intervene; that is, the wave will deſcribe its breadth in the time that pendulum will oſcillate twice; but a pendulum of four times that length, and which therefore is equal to the breadth of the waves, will juſt oſcillate once in that time. *Q. E. I.*

Cor. 1. Therefore waves, whoſe breadth is equal to $3\frac{1}{18}$ *French* feet, will advance thro' a ſpace equal to their breadth in one ſecond of time; and therefore in one minute will go over a ſpace of $183\frac{1}{3}$ feet; and in an hour a ſpace of 11000 feet, nearly.

Cor. 2. And the velocity of greater or leſs waves will be augmented or diminiſhed in the ſubduplicate ratio of their breadth.

Theſe

These things are true upon the supposition, that the parts of water ascend or descend in a right line; but in truth, that ascent and descent is rather performed in a circle; and therefore I propose the time defined by this proposition as only near the truth.

Proposition XLVII. Theorem XXXVII.

If pulses are propagated thro' a fluid, the several particles of the fluid, going and returning with the shortest reciprocal motion, are always accelerated or retarded according to the law of the oscillating pendulum. Pl. 9. Fig. 1.

Let *AB*, *BC*, *CD*, &c. represent equal distances of successive pulses; *ABC* the line of direction of the motion of the successive pulses, propagated from *A* to *B*; *E*, *F*, *G* three physical points of the quiescent medium situate in the right line *AC* at equal distances from each other; *Ee*, *Ff*, *Gg* equal spaces of extreme shortness, thro' which those points go and return with a reciprocal motion in each vibration; ϵ, φ, γ any intermediate places of the same points; and *EF*, *FG* physical lineolæ, or linear parts of the medium lying between those points, and successively transfer'd into the places $\epsilon\varphi$, $\varphi\gamma$, and *ef*, *fg*. Let there be drawn the right line *PS* equal to the right line *Ee*. Bisect the same in *O*, and from the centre *O*, with the interval *OP*, describe the circle *SIPi*. Let the whole time of one vibration, with its proportional parts, be expounded by the whole circumference of this circle and its parts; in such sort, that when any time *PH* or *PHSh* is compleated, if there be let fall to *PS* the perpendicular *HL* or *hl*, and there be taken $E\epsilon$ equal to *PL* or *Pl*, the physical point *E* may be found in ϵ.

A

A point as E moving according to this law with a reciprocal motion, in its going from E thro' ε to e, and returning again thro' ε to E, will perform its several vibrations with the same degrees of acceleration and retardation with those of an oscillating pendulum. We are now to prove, that the several physical points of the medium will be agitated with such a kind of motion. Let us suppose then, that a medium hath such a motion excited in it from any cause whatsoever, and consider what will follow from thence.

In the circumference $PHSh$ let there be taken the equal arcs HI, IK, or hi, ik, having the same ratio to the whole circumference as the equal right lines EF, FG have to BC the whole interval of the pulses. Let fall the perpendiculars IM, KN or im, kn; then because the points E, F, G are successively agitated with like motions, and perform their entire vibrations composed of their going and return, while the pulse is transferr'd from B to C; if PH or $PHSh$ be the time elapsed since the beginning of the motion of the point E, then will PI or $PHSi$ be the time elapsed since the beginning of the motion of the point F, and PK or $PHSk$ the time elapsed since the beginning of the motion of the point G; and therefore $E\varepsilon$, $F\varphi$, $G\gamma$ will be respectively equal to PL, PM, PN, while the points are going, and to Pl, Pm, Pn, when the points are returning. Therefore $\varepsilon\gamma$ or $EG + G\gamma - E\varepsilon$ will, when the points are going, be equal to $EG - LN$, and in their return equal to $EG + ln$. But $\varepsilon\gamma$ is the breadth or expansion of the part EG of the medium in the place $\varepsilon\gamma$; and therefore the expansion of that part in its going, is to its mean expansion as $EG - LN$ to EG; and in its return as $EG + ln$ or $EG + LN$ to EG. Therefore since LN is to KH as IM to the radius OP, and KH to EG as the circumference $PHShP$ to BC; that is, if we put V for the radius of a circle whose circumference is equal to

BC

BC the interval of the pulses, as *OP* to V; and, *ex æquo*, *LN* to *EG* as *IM* to V; the expansion of the part *EG* or of the physical point *F* in the place $\epsilon\gamma$ to the mean expansion of the same part in its first place *EG*, will be as V—*IM* to V in going, and as V+*im* to V in its return. Hence the elastic force of the point *F* in the place $\epsilon\gamma$ to its mean elastic force in the place *EG*, is as $\frac{1}{V-IM}$ to $\frac{1}{V}$ in its going, and as $\frac{1}{V+im}$ to $\frac{1}{V}$ in its return. And by the same reasoning the elastic forces of the physical points *E* and *G* in going, are as $\frac{1}{V-HL}$ and $\frac{1}{V-KN}$ to $\frac{1}{V}$; and the difference of the forces to the mean elastic force of the medium, as $\frac{HL-KN}{VV-V\times HL-V\times KN+HL\times KN}$ to $\frac{1}{V}$; that is, as $\frac{HL-KN}{VV}$ to $\frac{1}{V}$, or as *HL*—*KN* to V; if we suppose (by reason of the very short extent of the vibrations) *HL* and *KN* to be indefinitely less than the quantity V. Therefore since the quantity V is given, the difference of the forces is as *HL*—*KN*; that is, (because *HL*—*KN* is proportional to *HK*, and *OM* to *OI* or *OP*; and because *HK* and *OP* are given) as *OM*; that is, if *Ff* be bisected in Ω, as $\Omega\phi$. And for the same reason the difference of the elastic forces of the physical points ϵ and γ in the return of the physical lineola $\epsilon\gamma$, is as $\Omega\phi$. But that difference (that is, the excess of the elastic force of the point ϵ above the elastic force of the point γ) is the very force by which the intervening physical lineola $\epsilon\gamma$ of the medium is accelerated in going, and retarded in returning; and therefore the accelerative force of the physical lineola $\epsilon\gamma$ is as its distance from Ω the middle place of the vibration. Therefore (by prop. 38. book 1.) the time is rightly expounded by the arc PI; and the linear part of the medium $\epsilon\gamma$ is moved according to the law abovementioned, that is, according to the law of a pendulum oscillating; and the case is the same of all the linear parts of which the whole medium is compounded. *Q.E.D.*

COR. Hence it appears that the number of the pulses propagated is the same with the number of the vibrations of the tremulous body, and is not multiplied in their progress. For the physical lineola $\epsilon\gamma$ as soon as it returns to its first place is at rest; neither will it move again, unless it receives a new motion, either from the impulse of the tremulous body, or of the pulses propagated from that body. As soon therefore as the pulses cease to be propagated from the tremulous body, it will return to a state of rest; and move no more.

PROPOSITION XLVIII. THEOREM XXXVIII.

The velocities of pulses propagated in an elastic fluid, are in a ratio compounded of the subduplicate ratio of the elastic force directly, and the subduplicate ratio of the density inversely; supposing the elastic force of the fluid to be proportional to its condensation.

CASE 1. If the mediums be homogeneous, and the distances of the pulses in those mediums be equal amongst themselves, but the motion in one medium is more intense than in the other: the contractions and dilatations of the correspondent parts will be as those motions. Not that this proportion is perfectly accurate. However, if the contractions and dilatations are not exceedingly intense, the error will not be sensible; and there-

therefore this proportion may be consider'd as physically exact. Now the motive elastic forces are as the contractions and dilatations; and the velocities generated in the same time in equal parts are as the forces. Therefore equal and corresponding parts of corresponding pulses will go and return together, thro' spaces proportional to their contractions and dilatations, with velocities that are as those spaces: and therefore the pulses, which in the time of one going and returning advance forwards a space equal to their breadth, and are always succeeding into the places of the pulses that immediately go before them, will, by reason of the equality of the distances, go forward in both mediums with equal velocity.

Case 2. If the distances of the pulses or their lengths are greater in one medium than in another; let us suppose that the correspondent parts describe spaces, in going and returning, each time proportional to the breadths of the pulses: then will their contractions and dilatations be equal. And therefore if the mediums are homogeneous, the motive elastic forces, which agitate them with a reciprocal motion, will be equal also. Now the matter to be moved by these forces is as the breadth of the pulses; and the space thro' which they move every time they go and return, is in the same ratio. And moreover, the time of one going and returning, is in a ratio compounded of the subduplicate ratio of the matter, and the subduplicate ratio of the space; and therefore is as the space. But the pulses advance a space equal to their breadths in the times of going once and returning once, that is, they go over spaces proportional to the times; and therefore are equally swift.

Case 3. And therefore in mediums of equal density and elastic force, all the pulses are equally swift. Now if the density or the elastic force of the medium were augmented, then because the motive force is increased in the ratio of the elastic force, and the matter to be moved is increased in the ratio of the density; the time which is necessary for producing the same motion as before, will be increased in the subduplicate ratio of the density, and will be diminished in the subduplicate ratio of the elastic force. And therefore the velocity of the pulses will be in a ratio compounded of the subduplicate ratio of the density of the medium inversely, and the subduplicate ratio of the elastic force directly. *Q. E. D.*

This proposition will be made more clear from the construction of the following problem.

Proposition XLIX. Problem XI.

The density and elastic force of a medium being given, to find the velocity of the pulses.

Suppose the medium to be press'd by an incumbent weight after the manner of our air; and let A be the height of a homogeneous medium, whose weight is equal to the incumbent weight and whose density is the same with the density of the compressed medium in which the pulses are propagated. Suppose a pendulum to be constructed, whose length between the point of suspension and the centre of oscillation is A: and in the time in which that pendulum will perform one entire oscillation composed of its going and returning, the pulse will be propagated right onwards, thro' a space equal to the circumference of a circle described with the radius A.

For letting those things stand which were constructed in Prop. 47. if any physical line as *EF* (*Pl.* 9. *Fig.* 1.) describing the space *PS* in each vibration, be acted on in the extremities *P* and *S* of every going and return that it makes by an elastic force that is equal to its weight; it will perform its several vibrations in the

the time in which the same might oscillate in a cycloid, whose whole perimeter is equal to the length *PS:* and that because equal forces will impel equal corpuscles thro' equal spaces in the same or equal times. Therefore since the times of the oscillations are in the subduplicate ratio of the lengths of the pendulums, and the length of the pendulum is equal to half the arc of the whole cycloid; the time of one vibration would be to the time of the oscillation of a pendulum, whose length is A, in the subduplicate ratio of the length $\frac{1}{2}PS$ or *PO* to the length A. But the elastic force, with which the physical lineola *EG* is urged, when it is found in its extreme places *P, S,* was (in the demonstration of prop. 47.) to its whole elastic force as *HL—KN* to V, that is, (since the point *K* now falls upon *P*) as *HK* to V: and all that force, or, which is the same thing, the incumbent weight by which the lineola *EG* is compress'd, is to the weight of the lineola as the altitude A of the incumbent weight to *EG* the length of the lineola; and therefore, *ex æquo,* the force with which the lineola *EG* is urged in the places *P* and *S,* is to the weight of that lineola as *HK*×A to V×*EG*; or as *PO*×A to VV; because *HK* was to *EG* as *PO* to V. Therefore since the times, in which equal bodies are impelled thro' equal spaces, are reciprocally in the subduplicate ratio of the forces, the time of one vibration, produced by the action of that elastic force, will be to the time of a vibration, produced by the impulse of the weight, in a subduplicate ratio of VV to *PO*×A, and therefore to the time of the oscillation of a pendulum whose length is A, in the subduplicate ratio of VV to *PO*×A, and the subduplicate ratio of *PO* to A conjunctly; that is, in the entire ratio of V to A. But in the time of one vibration composed of the going and returning of the pendulum, the pulse will be propagated right onwards thro' a space equal to its breadth *BC.* Therefore the time in which a pulse runs over the space

BC, is to the time of one oscillation composed of the going and returning of the pendulum, as V to A, that is, as *BC* to the circumference of a circle whose radius is A. But the time in which the pulse will run over the space *BC*, is to the time in which it will run over a length equal to that circumference, in the same ratio; and therefore in the time of such an oscillation, the pulse will run over a length equal to that circumference. *Q.E.D.*

Cor. 1. The velocity of the pulses is equal to that which heavy bodies acquire by falling with an equally accelerated motion, and in their fall describing half the altitude A. For the pulse will, in the time of this fall, supposing it to move with the velocity acquired by that fall, run over a space that will be equal to the whole altitude A; and therefore in the time of one oscillation composed of one going and return, will go over a space equal to the circumference of a circle described with the radius A: for the time of the fall is to the time of oscillation, as the radius of a circle to its circumference.

Cor. 2. Therefore since that altitude A is as the elastic force of the fluid directly, and the density of the same inversely; the velocity of the pulses will be in a ratio compounded of the subduplicate ratio of the density inversely, and the subduplicate ratio of the elastic force directly.

Proposition L. Problem XII.

To find the distances of the pulses.

Let the number of the vibrations of the body, by whose tremor the pulses are produced, be found to any given time. By that number divide the space which a pulse can go over in the same time, and the part found will be the breadth of one pulse. *Q.E.I.*

Scho-

Scholium.

The laſt propoſitions reſpect the motions of light and ſounds. For ſince light is propagated in right lines, it is certain that it cannot conſiſt in action alone, (by Prop. 41 and 42.) As to ſounds, ſince they ariſe from tremulous bodies, they can be nothing elſe but pulſes of the air propagated thro' it, (by Prop. 43.) And this is confirmed by the tremors, which ſounds, if they be loud and deep, excite in the bodies near them, as we experience in the ſound of drums. For quick and ſhort tremors are leſs eaſily excited. But it is well known, that any ſounds, falling upon ſtrings in uniſon with the ſonorous bodies, excite tremors in thoſe ſtrings. This is alſo confirmed from the velocity of ſounds. For ſince the ſpecific gravities of rain-water and quick-ſilver are to one another as about 1 to $13\frac{2}{3}$, and when the mercury in the barometer is at the height of 30 inches of our meaſure, the ſpecific gravities of the air and of rain-water are to one another as about 1 to 870: therefore the ſpecific gravity of air and quick-ſilver are to each other as 1 to 11890. Therefore when the height of the quick-ſilver is at 30 inches, a height of uniform air, whoſe weight would be ſufficient to compreſs our air to the denſity we find it to be of, muſt be equal to 356700 inches or 29725 feet of our meaſure. And this is that very height of the medium, which I have called A in the conſtruction of the foregoing propoſition. A circle whoſe radius is 29725 feet is 186768 feet in circumference. And ſince a pendulum $39\frac{1}{5}$ inches in length compleats one oſcillation, compoſed of its going and return, in two ſeconds of time, as is commonly known; it follows that a pendulum 29725 feet or 356700 inches in length will perform a like oſcillation in $190\frac{3}{4}$ ſeconds. Therefore in that time a sound will go right onwards 186768 feet, and therefore in one second 979 feet.

But in this computation we have made no allowance for the crassitude of the solid particles of the air, by which the sound is propagated instantaneously. Because the weight of air is to the weight of water as 1 to 870, and because salts are almost twice as dense as water; if the particles of air are supposed to be of near the same density as those of water or salt, and the rarity of the air arises from the intervals of the particles; the diameter of one particle of air will be to the interval between the centres of the particles, as 1 to about 9 or 10, and to the interval between the particles themselves as 1 to 8 or 9. Therefore to 979 feet, which, according to the above calculation, a sound will advance forward in one second of time, we may add $\frac{979}{9}$, or about 109 feet, to compensate for the crassitude of the particles of the air: and then a sound will go forward about 1088 feet in one second of time.

Moreover, the vapors floating in the air, being of another spring, and a different tone, will hardly, if at all, partake of the motion of the true air in which the sounds are propagated. Now if these vapors remain unmoved, that motion will be propagated the swifter thro' the true air alone, and that in the subduplicate ratio of the defect of the matter. So if the atmosphere consist of ten parts of true air and one part of vapors, the motion of sounds will be swifter in the subduplicate ratio of 11 to 10, or very nearly in the entire ratio of 21 to 20, than if it were propagated thro' eleven parts of true air: and therefore the motion of sounds above discovered must be encreased in that ratio. By this means the sound will pass thro' 1142 feet in one second of time.

These things will be found true in spring and autumn, when the air is rarefied by the gentle warmth of those seasons, and by that means its elastic force becomes

comes somewhat more intense. But in winter, when the air is condensed by the cold, and its elastic force is somewhat remitted, the motion of sounds will be slower in a subduplicate ratio of the density; and on the other hand, swifter in the summer.

Now by experiments it actually appears that sounds do really advance in one second of time about 1142 feet of *English* measure, or 1070 feet of *French* measure.

The velocity of sounds being known, the intervals of the pulses are known also. For M. *Sauveur*, by some experiments that he made, found that an open pipe about five *Paris* feet in length, gives a sound of the same tone with a viol-string that vibrates a hundred times in one second. Therefore there are near 100 pulses in a space of 1070 *Paris* feet, which a sound runs over in a second of time; and therefore one pulse fills up a space of about $10\frac{7}{10}$ *Paris* feet, that is, about twice the length of the pipe. From whence it is probable, that the breadths of the pulses, in all sounds made in open pipes, are equal to twice the length of the pipes.

Moreover, from the corollary of prop. 47. appears the reason, why the sounds immediately cease with the motion of the sonorous body, and why they are heard no longer when we are at a great distance from the sonorous bodies, than when we are very near them. And besides, from the foregoing principles it plainly appears how it comes to pass that sounds are so mightily encreased in speaking-trumpets. For all reciprocal motion uses to be encreased by the generating cause at each return. And in tubes hindering the dilatation of the sounds, the motion decays more slowly, and recurs more forcibly; and therefore is the more encreased by the new motion impressed at each return. And these are the principal phænomena of sounds.

SECTION IX.

Of the circular motion of fluids.

HYPOTHESIS.

The resistance, arising from the want of lubricity in the parts of a fluid, is, cæteris paribus, *proportional to the velocity with which the parts of the fluid are separated from each other.*

PROPOSITION LI. THEOREM XXXVIII.

If a solid cylinder infinitely long, in an uniform and infinite fluid, revolve with an uniform motion about an axis given in position, and the fluid be forced round by only this impulse of the cylinder, and every part of the fluid persevere uniformly in its motion; I say, that the periodic times of the parts of the fluid are as their distances from the axis of the cylinder.

Let *AFL* (*Pl.* 9. *Fig.* 2.) be a cylinder turning uniformly about the axis *S*, and let the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, &c. divide the fluid into innumerable concentric cylindric solid orbs of the same

same thickness. Then, because the fluid is homogeneous, the impressions which the contiguous orbs make upon each other mutually, will be (by the hypothesis) as their translations from each other, and as the contiguous superficies upon which the impressions are made. If the impression made upon any orb be greater or less on its concave, than on its convex side, the stronger impression will prevail, and will either accelerate or retard the motion of the orb, according as it agrees with, or is contrary to the motion of the same. Therefore, that every orb may persevere uniformly in its motion, the impressions made on both sides must be equal, and their directions contrary. Therefore since the impressions are as the contiguous superficies, and as their translations from one another; the translations will be inversely as the superficies, that is, inversely as the distances of the superficies from the axis. But the differences of the angular motions about the axis, are as those translations applied to the distances, or as the translations directly and the distances inversely; that is, joining these ratio's together, as the squares of the distances inversely. Therefore if there be erected the lines *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, &c. perpendicular to the several parts of the infinite right line *SABCDEQ* and reciprocally proportional to the squares of *SA*, *SB*, *SC*, *SD*, *SE*, &c. and thro' the extremities of those perpendiculars there be supposed to pass an hyperbolic curve; the sums of the differences, that is, the whole angular motions, will be as the correspondent sums of the lines *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, that is, (if to constitute a medium uniformly fluid, the number of the orbs be encreased and their breadth diminished in infinitum) as the hyperbolic area's *AaQ*, *BbQ*, *CcQ*, *DdQ*, *EeQ*, &c. analogous to the sums. And the times, reciprocally proportional to the angular motions, will be also reciprocally proportional to those areas. Therefore the periodic time of any particle as *D*, is reciprocally as the area *DdQ*, that is, (as appears

appears from the known methods of quadratures of curves) directly as the distance *SD.* *Q.E.D.*

Cor. 1. Hence the angular motions of the particles of the fluid are reciprocally as their distances from the axis of the cylinder, and the absolute velocities are equal.

Cor. 2. If a fluid be contained in a cylindric vessel of an infinite length, and contain another cylinder within, and both the cylinders revolve about one common axis, and the times of their revolutions be as their semidiameters, and every part of the fluid perseveres in its motion: the periodic times of the several parts will be as the distances from the axis of the cylinders.

Cor. 3. If there be added or taken away any common quantity of angular motion from the cylinder and fluid moving in this manner; yet because this new motion will not alter the mutual attrition of the parts of the fluid, the motion of the parts among themselves will not be changed. For the translations of the parts from one another depend upon the attrition. Any part will persevere in that motion, which, by the attrition made on both sides with contrary directions, is no more accelerated than it is retarded.

Cor. 4. Therefore if there be taken away from this whole system of the cylinders and the fluid, all the angular motion of the outward cylinder, we shall have the motion of the fluid in a quiescent cylinder.

Cor. 5. Therefore if the fluid and outward cylinder are at rest, and the inward cylinder revolve uniformly; there will be communicated a circular motion to the fluid, which will be propagated by degrees thro' the whole fluid; and will go on continually encreasing, till such time as the several parts of the fluid acquire the motion determined in cor. 4.

Cor. 6. And because the fluid endeavours to propagate its motion still farther, its impulse will carry the outmost cylinder also about with it, unless the cylinder be violently detained; and accelerate its motion till the periodic times of both cylinders become equal among themselves. But if the outward cylinder be violently detained, it will make an effort to retard the motion of the fluid; and unless the inward cylinder preserve that motion by means of some external force impressed thereon, it will make it cease by degrees.

All these things will be found true, by making the experiment in deep standing water.

Proposition LII. Theorem XL.

If a solid sphere, in an uniform and infinite fluid, revolves about an axis given in position with an uniform motion, and the fluid be forced round by only this impulse of the sphere; and every part of the fluid perseveres uniformly in its motion: I say, that the periodic times of the parts of the fluid are as the squares of their distances from the centre of the sphere.

Case 1. Let *AFL* be a sphere turning uniformly about the axis *S*, and let the concentric circles *BGM*, *CHN*, *DIO*, *EKP*, &c. divide the fluid into innumerable concentric orbs of the same thickness. Suppose those orbs to be solid; and because the fluid is homogeneous, the impressions which the contiguous orbs make one upon another, will be (by the supposition) as their translations from one another, and the contiguous superficies upon which the impressions are made. If the impression upon any orb be greater or less upon its concave than upon its convex side; the more forcible impression will prevail, and will either accelerate or retard the velocity of the orb, according as it is directed

rected with a conſpiring or contrary motion to that of the orb. Therefore that every orb may perſevere uniformly in its motion, it is neceſſary that the impreſſions made upon both ſides of the orb ſhould be equal, and have contrary directions. Therefore ſince the impreſſions are as the contiguous ſuperficies, and as their tranſlations from one another; the tranſlations will be inverſly as the ſuperficies, that is, inverſly as the ſquares of the diſtances of the ſuperficies from the centre. But the differences of the angular motions about the axis are as thoſe tranſlations applied to the diſtances, or as the tranſlations directly and the diſtances inverſly; that is, by compounding thoſe ratio's, as the cubes of the diſtances inverſly. Therefore, if upon the ſeveral parts of the infinite right line *SABCDEQ* there be erected the perpendiculars *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, &c. reciprocally proportional to the cubes of *SA*, *SB*, *SC*, *SD*, *SE*, &c. the ſums of the differences, that is, the whole angular motions, will be as the correſponding ſums of the lines *Aa*, *Bb*, *Cc*, *Dd*, *Ee*, &c. that is, (if to conſtitute an uniformly fluid medium the number of the orbs be encreaſed and their thickneſs diminiſhed in infinitum) as the hyperbolic areas *AaQ*, *BbQ*, *CcQ*, *DdQ*, *EeQ*, &c. analogous to the ſums; and the periodic times being reciprocally proportional to the angular motions, will be alſo reciprocally proportional to thoſe areas. Therefore the periodic time of any orb *DIO* is reciprocally as the area *DdQ*, that is, (by the known methods of quadratures) directly as the ſquare of the diſtance *SD*. Which was firſt to be demonſtrated.

Case 2. From the centre of the ſphere let there be drawn a great number of indefinite right lines, making given angles with the axis, exceeding one another by equal differences; and, by theſe lines revolving about the axis, conceive the orbs to be cut into innumerable annuli: then will every annulus have four annuli contiguous to it, that is, one on its inside, one on its outside, and two on each hand. Now each of these annuli cannot be impelled equally and with contrary directions by the attrition of the interior and exterior annuli unless the motion be communicated according to the law which we demonstrated in case 1. This appears from that demonstration. And therefore any series of annuli, taken in any right line extending itself in infinitum from the globe, will move according to the law of case 1. except we should imagine it hindered by the attrition of the annuli on each side of it. But now in a motion, according to this law, no such attrition is, and therefore cannot be any obstacle to the motion's persevering according to that law. If annuli at equal distances from the centre revolve either more swiftly or more slowly near the poles than near the ecliptic; they will be accelerated if slow, and retarded if swift, by their mutual attrition; and so the periodic times will continually approach to equality, according to the law of case 1. Therefore this attrition will not at all hinder the motion from going on according to the law of case 1. and therefore that law will take place; that is, the periodic times of the several annuli will be as the squares of their distances from the centre of the globe. Which was to be demonstrated in the second place.

Case 3. Let now every annulus be divided by transverse sections into innumerable particles constituting a substance absolutely and uniformly fluid; and because these sections do not at all respect the law of circular motion, but only serve to produce a fluid substance, the law of circular motion will continue the same as before. All the very small annuli will either not at all change their asperity and force of mutual attrition upon account of these sections, or else they will change the same equally. Therefore the proportion of the causes remaining the same, the proportion of the effects will

remain

remain the ſame alſo; that is, the proportion of the motions and the periodic times. *Q.E.D.* But now as the circular motion, and the centrifugal force thence ariſing, is greater at the ecliptic than at the poles, there muſt be ſome cauſe operating to retain the ſeveral particles in their circles; otherwiſe the matter that is at the ecliptic will always recede from the centre, and come round about to the poles by the outſide of the vortex, and from thence return by the axis to the ecliptic with a perpetual circulation.

Cor. 1. Hence the angular motions of the parts of the fluid about the axis of the globe, are reciprocally as the ſquares of the diſtances from the centre of the globe, and the abſolute velocities are reciprocally as the ſame ſquares applied to the diſtances from the axis.

Cor. 2. If a globe revolve with a uniform motion about an axis of a given poſition in a ſimilar and infinite quieſcent fluid with an uniform motion, it will communicate a whirling motion to the fluid like that of a vortex, and that motion will by degrees be propagated onwards in infinitum; and this motion will be encreaſed continually in every part of the fluid, till the periodical times of the ſeveral parts become as the ſquares of the diſtances from the centre of the globe.

Cor. 3. Becauſe the inward parts of the vortex are by reaſon of their greater velocity continually preſſing upon and driving forwards the external parts, and by that action are perpetually communicating motion to them, and at the ſame time thoſe exterior parts communicate the ſame quantity of motion to thoſe that lie ſtill beyond them, and by this action preſerve the quantity of their motion continually unchanged; it is plain that the motion is perpetually transferred from the centre to the circumference of the vortex, till it is quite ſwallowed up and loſt in the boundleſs extent of that circumference. The matter between any two ſphærical ſuperficies concentrical to the vortex will never be accelerated,

celerated; becauſe that matter will be always transferring the motion it receives from the matter nearer the centre to that matter which lies nearer the circumference.

Cor. 4. Therefore in order to continue a vortex in the ſame ſtate of motion, ſome active principle is required, from which the globe may receive continually the ſame quantity of motion which it is always communicating to the matter of the vortex. Without ſuch a principle it will undoubtedly come to paſs that the globe and the inward parts of the vortex, being always propagating their motion to the outward parts, and not receiving any new motion, will gradually move ſlower and ſlower, and at laſt be carried round no longer.

Cor. 5. If another globe ſhould be ſwimming in the ſame vortex at a certain diſtance from its centre, and in the mean time by ſome force revolve conſtantly about an axis of a given inclination; the motion of this globe will drive the fluid round after the manner of a vortex; and at firſt this new and ſmall vortex will revolve with its globe about the centre of the other; and in the mean time its motion will creep on, farther and farther, and by degrees be propagated in infinitum, after the manner of the firſt vortex. And for the ſame reaſon that the globe of the new vortex was carried about before by the motion of the other vortex, the globe of this other will be carried about by the motion of this new vortex, ſo that the two globes will revolve about ſome intermediate point, and by reaſon of that circular motion mutually fly from each other, unleſs ſome force reſtrains them. Afterwards, if the conſtantly impreſſed forces, by which the globes perſevere in their motions, ſhould ceaſe, and every thing be left to act according to the laws of mechanics, the motion of the globes will languiſh by degrees, (for the reaſon aſſigned in cor. 3 and 4.) and the vortices at laſt will quite ſtand ſtill.

Cor.

Cor. 6. If several globes in given places should constantly revolve with determined velocities about axes given in position, there would arise from them as many vortices going on in infinitum. For upon the same account that any one globe propagates its motion in infinitum, each globe apart will propagate its own motion in infinitum also; so that every part of the infinite fluid will be agitated with a motion resulting from the actions of all the globes. Therefore the vortices will not be confined by any certain limits, but by degrees run mutually into each other; and by the mutual actions of the vortices on each other, the globes will be perpetually moved from their places, as was shewn in the last corollary; neither can they possibly keep any certain position among themselves, unless some force restrains them. But if those forces, which are constantly impressed upon the globes to continue these motions, should cease; the matter (for the reason assigned in cor. 3 and 4.) will gradually stop, and cease to move in vortices.

Cor. 7. If a similar fluid be inclosed in a sphærical vessel, and by the uniform rotation of a globe in its centre, is driven round in a vortex; and the globe and vessel revolve the same way about the same axis, and their periodical times be as the squares of the semidiameters; the parts of the fluid will not go on in their motions without acceleration or retardation, till their periodical times are as the squares of their distances from the centre of the vortex. No constitution of a vortex can be permanent but this.

Cor. 8. If the vessel, the inclosed fluid, and the globe, retain this motion, and revolve besides with a common angular motion about any given axis; because the mutual attrition of the parts of the fluid is not changed by this motion, the motions of the parts among each other will not be changed. For the translations of the parts among themselves depend upon this attrition.

Any

Any part will persevere in that motion, in which its attrition on one side retards it just as much as its attrition on the other side accelerates it.

Cor. 9. Therefore if the vessel be quiescent, and the motion of the globe be given, the motion of the fluid will be given. For conceive a plane to pass thro' the axis of the globe, and to revolve with a contrary motion; and suppose the sum of the time of this revolution and of the revolution of the globe to be to the time of the revolution of the globe, as the square of the semidiameter of the vessel to the square of the semidiameter of the globe; and the periodic times of the parts of the fluid in respect of this plane will be as the squares of their distances from the centre of the globe.

Cor. 10. Therefore if the vessel move about the same axis with the globe, or with a given velocity about a different one, the motion of the fluid will be given. For if from the whole system we take away the angular motion of the vessel, all the motions will remain the same among themselves as before, by cor. 8. and those motions will be given by cor. 9.

Cor. 11. If the vessel and the fluid are quiescent, and the globe revolves with an uniform motion, that motion will be propagated by degrees through the whole fluid to the vessel, and the vessel will be carried round by it, unless violently detained; and the fluid and the vessel will be continually accelerated till their periodic times become equal to the periodic times of the globe. If the vessel be either withheld by some force, or revolve with any constant and uniform motion, the medium will come by little and little to the state of motion defined in cor. 8. 9. 10. nor will it ever persevere in any other state. But if then the forces, by which the globe and vessel revolve with certain motions, should cease, and the whole system be left to act according to the mechanical laws, the vessel and globe, by means of the intervening fluid, will act upon each other, and will continue to propagate their motions through the fluid to each other, till their periodic times become equal among themselves, and the whole syſtem revolves together like one ſolid body.

Scholium.

In all theſe reaſonings, I ſuppoſe the fluid to conſiſt of matter of uniform denſity and fluidity. I mean that the fluid is ſuch, that a globe placed any where therein may propagate with the ſame motion of its own, at diſtances from it ſelf continually equal, ſimilar and equal motions in the fluid, in the ſame interval of time. The matter by its circular motion endeavours to recede from the axis of the vortex; and therefore preſſes all the matter that lies beyond. This preſſure makes the attrition greater, and the ſeparation of the parts more difficult; and by conſequence diminiſhes the fluidity of the matter. Again, if the parts of the fluid are in any one place denſer or larger than in the others, the fluidity will be leſs in that place, becauſe there are fewer ſuperficies where the parts can be ſeparated from each other. In theſe caſes I ſuppoſe the defect of the fluidity to be ſupplied by the ſmoothneſs or ſoftneſs of the parts, or ſome other condition; otherwiſe the matter where it is leſs fluid, will cohere more, and be more ſluggiſh, and therefore will receive the motion more ſlowly, and propagate it farther than agrees with the ratio above aſſigned. If the veſſel be not ſphærical, the particles will move in lines, not circular, but anſwering to the figure of the veſſel, and the periodic times will be nearly as the ſquares of the mean diſtances from the centre. In the parts between the centre and the circumference, the motions will be ſlower where the ſpaces are wide, and ſwifter where narrow; but yet the particles will not tend to the circumference

cumference at all the more for their greater swiftness. For they then describe arcs of less curvity, and the conatus of receding from the centre is as much diminished by the diminution of this curvature, as it is augmented by the increase of the velocity. As they go out of narrow into wide spaces they recede a little farther from the centre, but in doing so are retarded; and when they come out of wide into narrow spaces they are again accelerated; and so each particle is retarded and accelerated by turns for ever. These things will come to pass in a rigid vessel. For the state of vortices in an infinite fluid is known by cor. 6. of this proposition.

I have endeavoured in this proposition to investigate the properties of vortices, that I might find whether the celestial phænomena can be explained by them. For the phænomenon is this, that the periodic times of the Planets revolving about Jupiter, are in the sesquiplicate ratio of their distances from Jupiter's centre; and the same rule obtains also among the Planets that revolve about the Sun. And these rules obtain also with the greatest accuracy, as far as has been yet discovered by astronomical observation. Therefore, if those Planets are carried round in vortices revolving about Jupiter and the Sun, the vortices must revolve according to that law. But here we found the periodic times of the parts of the vortex to be in the duplicate ratio of the distances from the centre of motion; and this ratio cannot be diminished and reduced to the sesquiplicate, unless either the matter of the vortex be more fluid, the farther it is from the centre, or the resistance arising from the want of lubricity in the parts of the fluid, should, as the velocity with which the parts of the fluid are separated goes on increasing, be augmented with it in a greater ratio than that in which the velocity increases. But neither of these suppositions seem reasonable. The more gross and less fluid parts will tend to the circumference, unleſs they are heavy towards the centre. And tho', for the ſake of demonſtration, I propoſed, at the beginning of this Section, an hypotheſis that the reſiſtance is proportional to the velocity, nevertheleſs, 'tis in truth probable that the reſiſtance is in a leſs ratio than that of the velocity. Which granted, the periodic times of the parts of the vortex will be in a greater than the duplicate ratio of the diſtances from its centre. If, as ſome think, the vortices move more ſwiftly near the centre, then ſlower to a certain limit, then again ſwifter near the circumference, certainly neither the ſeſquiplicate, nor any other certain and determinate ratio can obtain in them. Let philoſophers then ſee how that phænomenon of the ſeſquiplicate ratio can be accounted for by vortices.

Proposition LIII. Theorem XLI.

Bodies, carried about in a vortex and returning in the ſame orb, are of the ſame denſity with the vortex, and are moved according to the ſame law with the parts of the vortex, as to velocity and direction of motion.

For if any ſmall part of the vortex, whoſe particles or phyſical points preſerve a given ſituation among each other, be ſuppoſed to be congealed; this particle will move according to the ſame law as before, ſince no change is made either in its denſity, *vis inſita*, or figure. And again, if a congealed or ſolid part of the vortex be of the ſame denſity with the reſt of the vortex, and be reſolved into a fluid, this will move according to the ſame law as before, except in ſo far as its particles now become fluid, may be moved among themſelves. Neglect therefore the motion of the particles

ticles among themselves, as not at all concerning the progressive motion of the whole, and the motion of the whole will be the same as before. But this motion will be the same with the motion of other parts of the vortex at equal distances from the centre; because the solid, now resolved into a fluid, is become perfectly like to the other parts of the vortex. Therefore a solid, if it be of the same density with the matter of the vortex, will move with the same motion as the parts thereof, being relatively at rest in the matter that surrounds it. If it be more dense, it will endeavour more than before to recede from the centre; and therefore overcoming that force of the vortex, by which, being as it were kept in equilibrio, it was retained in its orbit, it will recede from the centre, and in its revolution describe a spiral, returning no longer into the same orbit. And by the same argument, if it be more rare it will approach to the centre. Therefore it can never continually go round in the same orbit, unless it be of the same density with the fluid. But we have shewn in that case, that it would revolve according to the same law with those parts of the fluid that are at the same or equal distances from the centre of the vortex.

Cor. 1. Therefore a solid revolving in a vortex, and continually going round in the same orbit, is relatively quiescent in the fluid that carries it.

Cor. 2. And if the vortex be of an uniform density, the same body may revolve at any distance from the centre of the vortex.

Scholium.

Hence it is manifest, that the Planets are not carried round in corporeal vortices. For according to the *Copernican* hypothesis, the Planets going round the Sun, revolve in ellipses, having the Sun in their common focus; and by radii drawn to the sun describe areas proportional to the times. But now the parts of a vortex can never revolve with such a motion. Let *AD, BE, CF, (Pl.9.Fig.3.)* represent three orbits described about the Sun *S*, of which let the utmost circle *CF* be concentric to the Sun; and let the aphelia of the two innermost be *A, B*; and their perihelia *D, E*. Therefore a body revolving in the orb *CF*, describing, by a radius drawn to the Sun, areas proportional to the times, will move with an uniform motion. And according to the laws of astronomy, the body revolving in the orb *BE* will move slower in its aphelion *B*, and swifter in its perihelion *E*; whereas, according to the laws of mechanics, the matter of the vortex ought to move more swiftly in the narrow space between *A* and *C*, than in the wide space between *D* and *F*; that is, more swiftly in the aphelion than in the perihelion. Now these two conclusions contradict each other. So at the beginning of the sign of Virgo, where the aphelion of Mars is at present, the distance between the orbits of Mars and Venus is to the distance between the same orbits at the beginning of the sign of Pisces, as about 3 to 2; and therefore the matter of the vortex between those orbits ought to be swifter at the beginning of Pisces, than at the beginning of Virgo, in the ratio of 3 to 2. For the narrower the space is, thro' which the same quantity of matter passes in the same time of one revolution, the greater will be the velocity with which it passes thro' it. Therefore if the Earth being relatively at rest in this celestial matter should be carried round by it, and revolve together with it about the Sun, the velocity of the Earth at the beginning of Pisces would be to its velocity at the beginning of Virgo in a sesquialteral ratio. Therefore the Sun's apparent diurnal motion at the beginning of Virgo, ought to be above 70 minutes; and at the beginning of Pisces less than 48

minutes,

Plate IX. Vol. II. P. 198.

Fig. 1 p. 173. 78.

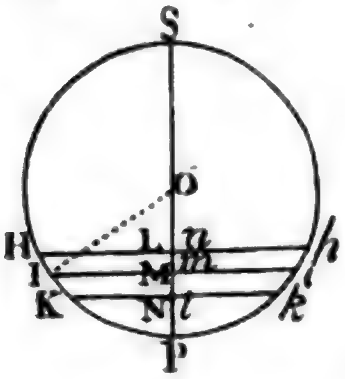


Fig. 2.
p. 184. 87.

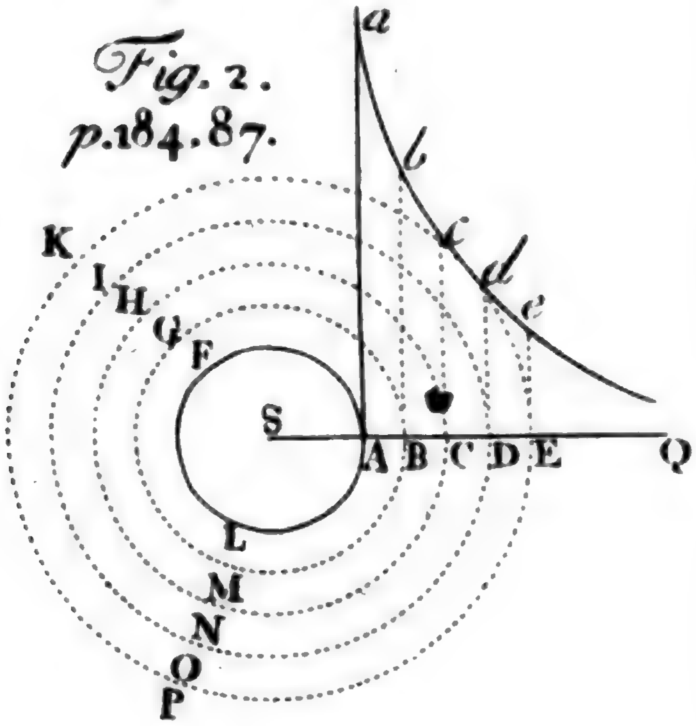
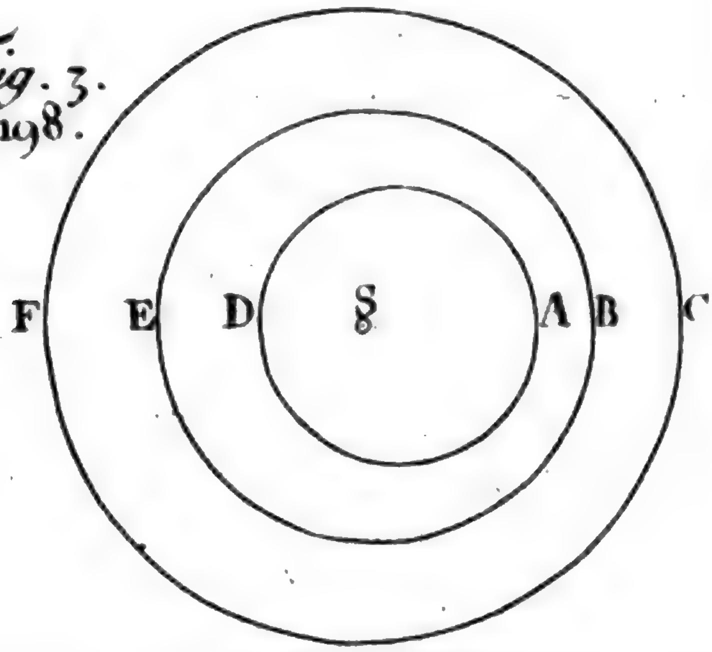


Fig. 3.
p. 198.



minutes. Whereas on the contrary that apparent motion of the Sun is really greater at the beginning of Pisces than at the beginning of Virgo, as experience testifies; and therefore the earth is swifter at the beginning of Virgo than at the beginning of Pisces. So that the hypothesis of vortices is utterly irreconcileable with astronomical phænomena, and rather serves to perplex than explain the heavenly motions. How these motions are performed in free spaces without vortices, may be understood by the first book; and I shall now more fully treat of it in the following book *of the System of the World.*

OF THE

SYSTEM

OF THE

WORLD.

BOOK III.

IN the preceding books I have laid down the principles of philoſophy; principles, not philoſophical, but mathematical; ſuch, to wit, as we may build our reaſonings upon in philoſophical enquiries. Theſe principles are, the laws and conditions of certain motions, and powers or forces, which chiefly have reſpect to philoſophy. But leſt they ſhould have appeared of themſelves dry and barren, I have illuſtrated them here and there, with ſome philoſophical ſcholiums, giving an account of ſuch things, as are of more general nature, and which philoſophy ſeems chiefly to be founded on; ſuch as the denſity and the reſiſtance of bodies,

bodies, ſpaces void of all bodies, and the motion of light and ſounds. It remains, that from the ſame principles, I now demonſtrate the frame of the Syſtem of the World. Upon this ſubject, I had indeed compos'd the third book in a popular method, that it might be read by many. But afterwards conſidering that ſuch as had not ſufficiently enter'd into the principles, could not eaſily diſcern the ſtrength of the conſequences, nor lay aſide the prejudices to which they had been many years accuſtomed; therefore to prevent the diſputes which might be rais'd upon ſuch accounts, I choſe to reduce the ſubſtance of that book into the form of propoſitions (in the mathematical way) which ſhould be read by thoſe only, who had firſt made themſelves maſters of the principles eſtabliſh'd in the preceding books. Not that I would adviſe any one to the previous ſtudy of every propoſition of thoſe books. For they abound with ſuch as might coſt too much time, even to readers of good mathematical learning. It is enough if one carefully reads the definitions, the laws of motion, and the firſt three ſections of the firſt book. He may then paſs on to this book, of the Syſtem of the World, and conſult ſuch of the remaining propoſitions of the firſt two books, as the references in this, and his occaſions, ſhall require.

THE

THE RULES OF REASONING in PHILOSOPHY.

RULE I.

We are to admit no more causes of natural things, than such as are both true and sufficient to explain their appearances.

To this purpose the philosophers say, that Nature do's nothing in vain, and more is in vain, when less will serve; For Nature is pleas'd with simplicity, and affects not the pomp of superfluous causes.

RULE II.

Therefore to the same natural effects we must, as far as possible, assign the same causes.

As to respiration in a man, and in a beast; the descent of stones in *Europe* and in *America*; the light of our culinary fire and of the Sun; the reflection of light in the Earth, and in the Planets.

RULE

RULE III.

The qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.

For since the qualities of bodies are only known to us by experiments, we are to hold for universal, all such as universally agree with experiments; and such as are not liable to diminution, can never be quite taken away. We are certainly not to relinquish the evidence of experiments for the sake of dreams and vain fictions of our own devising; nor are we to recede from the analogy of Nature, which uses to be simple, and always consonant to it self. We no otherways know the extension of bodies, than by our senses, nor do these reach it in all bodies; but because we perceive extension in all that are sensible, therefore we ascribe it universally to all others also. That abundance of bodies are hard we learn by experience. And because the hardness of the whole arises from the hardness of the parts, we therefore justly infer the hardness of the undivided particles not only of the bodies we feel but of all others. That all bodies are impenetrable, we gather not from reason, but from sensation. The bodies which we handle we find impenetrable, and thence conclude impenetrability to be an universal property of all bodies whatsoever. That all bodies are moveable, and endow'd with certain powers (which we call the *vires inertiæ*) of persevering in their motion or in their rest, we only infer from the like properties observ'd in the bodies which we have seen. The extension, hardness, impenetrability, mobility, and *vis inertiæ* of the whole, result from the extension

ſion, hardneſs, impenetrability, mobility, and *vires inertiæ* of the parts: and thence we conclude the leaſt particles of all bodies to be alſo all extended, and hard, and impenetrable, and moveable, and endow'd with their proper *vires inertiæ*. And this is the foundation of all philoſophy. Moreover, that the divided but contiguous particles of bodies may be ſeparated from one another, is matter of obſervation; and, in the particles that remain undivided, our minds are able to diſtinguiſh yet leſſer parts, as is mathematically demonſtrated. But whether the parts ſo diſtinguiſh'd, and not yet divided, may, by the powers of nature, be actually divided and ſeparated from one another, we cannot certainly determine. Yet had we the proof of but one experiment, that any undivided particle, in breaking a hard and ſolid body, ſuffer'd a diviſion, we might by virtue of this rule, conclude, that the undivided as well as the divided particles, may be divided and actually ſeparated to infinity.

Laſtly, If it univerſally appears, by experiments and aſtronomical obſervations, that all bodies about the Earth, gravitate towards the Earth; and that in proportion to the quantity of matter which they ſeverally contain; that the Moon likewiſe, according to the quantity of its matter, gravitates towards the Earth; that on the other hand our Sea gravitates towards the Moon; and all the Planets mutually one towards another; and the Comets in like manner towards the Sun; we muſt, in conſequence of this rule, univerſally allow, that all bodies whatſoever are endow'd with a principle of mutual gravitation. For the argument from the appearances concludes with more force for the univerſal gravitation of all bodies, than for their impenetrability; of which among thoſe in the celeſtial regions, we have no experiments, nor any manner of obſervation. Not that I affirm gravity to be eſſential to bodies. By their *vis inſita* I mean nothing but their *vis inertiæ*. This is immutable. Their gravity is diminished as they recede from the Earth.

RULE IV.

In experimental philosophy we are to look upon propositions collected by general induction from phænomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phænomena occur, by which they may either be made more accurate, or liable to exceptions.

This rule we must follow that the argument of induction may not be evaded by hypotheses.

THE

THE PHÆNOMENA or APPEARANCES.

PHÆNOMENON I.

That the circumjovial planets, by radij drawn to Jupiter's center, describe areas proportional to the times of description, and that their periodic times, the fixed Stars being at rest, are in the sesquiplicate proportion of their distances from its center.

THIS we know from astronomical observations. For the orbits of these planets differ but insensibly from circles concentric to Jupiter; and their motions in those circles are found to be uniform. And all astronomers agree, that their periodic times are in the sesquiplicate proportion of the semidiameters of their orbits: and so it manifestly appears from the following table.

The periodic times of the Satellites of Jupiter.

$1^{d}.\ 18^{h}.\ 27'.\ 34''.$ $3^{d}.\ 13^{h}.\ 13'.\ 42''.$ $7^{d}.\ 3^{h}.\ 42'.\ 36''.$ $16^{d}.\ 16^{h}.\ 32'.\ 9''.$

The

The distances of the Satellites from Jupiter's center.

From the observations of	1	2	3	4	
Borelli	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	*semidiam. of Jupiter.*
Townley *by the Microm.*	5,52	8,78	13,47	24,72	
Cassini *by the Telescope.*	5	8	13	23	
Cassini *by the eclip. of the satel.*	$5\frac{2}{3}$	9	$14\frac{23}{60}$	$25\frac{3}{10}$	
From the periodic times.	5,667	9,017	14,384	25,299	

Mr. *Pound* has determined by the help of excellent micrometers, the diameters of Jupiter and the elongation of its satellites after the following manner. The greatest heliocentric elongation of the fourth satellite from Jupiter's centre was taken with a micrometer in a 15 foot telescope, and at the mean distance of Jupiter from the Earth was found about 8'. 16". The elongation of the third satellite was taken with a micrometer in a telescope of 123 feet, and at the same distance of Jupiter from the Earth was found 4'. 42". The greatest elongations of the other satellites at the same distance of Jupiter from the Earth, are found from the periodic times to be 2'. 56". 47". and 1'. 51". 6"'.

The diameter of Jupiter taken with the micrometer in an 123 foot telescope several times, and reduced to Jupiter's mean distance from the Earth, proved always less than 40", never less than 38", generally 39". This diameter in shorter telescopes is 40", or 41". For Jupiter's light is a little dilated by the unequal refrangibility of the rays, and this dilatation bears a less ratio to the diameter of Jupiter in the longer and more perfect telescopes, than in those which are shorter and less perfect. The times in which two satellites, the first and the third, passed over Jupiter's body, were observed, from the beginning of the ingress to the beginning of the egress, and from the complete ingress to the complete egress, with the long telescope. And from the transit of the first satellite, the diameter of Jupiter at its mean distance

diſtance from the Earth, came forth $37\frac{1}{8}''$, and from the tranſit of the third $37\frac{1}{8}''$. There was obſerved alſo the time in which the ſhadow of the firſt ſatellite paſs'd over Jupiter's body, and thence the diameter of Jupiter at its mean diſtance from the Earth came out about $37''$. Let us ſuppoſe its diameter to be $37\frac{1}{4}''$ very nearly, and then the greateſt elongations of the firſt, ſecond, third and fourth ſatellite will be reſpectively equal to 5,965, 9,494, 15,141, and 26,63 ſemidiameters of Jupiter.

PHÆNOMENON. II.

That the circumſaturnal planets, by radij drawn to Saturn's center, deſcribe areas proportional to the times of deſcription, and that their periodic times, the fixed Stars being at reſt, are in the ſeſquiplicate proportion of their diſtances from its centre.

For as *Caſſini* from his own obſervations has determin'd, their diſtances from Saturn's centre, and their periodic times are as follow.

The periodic times of the ſatellites of Saturn.

$1^d.\ 21^h.\ 18'.\ 27''.$ $2^d.\ 17^h.\ 41'.\ 22''.$ $4^d.\ 12^h.\ 25'.\ 12''.$ $15^d.\ 22^h.\ 41'.\ 14''.$ $79^d.\ 7^h.\ 48'.\ 00''.$

The diſtances of the ſatellites from Saturn's center, in ſemidiameters of its Ring.

From obſervations	$1\frac{19}{20}$.	$2\frac{1}{2}$.	$3\frac{1}{2}$.	8.	24.
From the periodic times.	1, 93.	2, 47.	3, 45.	8.	23, 35.

The greateſt elongation of the fourth ſatellite from Saturn's centre is commonly determined from the obſervations to be eight of thoſe ſemidiameters very nearly.

ly. But the greatest elongation of this satellite from Saturn's centre, when taken with an excellent micrometer in M. *Huygens*'s telescope of 123 feet, appeared to be eight semidiameters and $\frac{7}{10}$ of a semidiameter. And from this observation and the periodic times, the distances of the satellites from Saturn's centre in semidiameters of the Ring are 2,1. 2,69. 3,75. 8,7. and 25,35. The diameter of Saturn observed in the same telescope was found to be to the diameter of the Ring as 3 to 7, and the diameter of the Ring, *May* 28, 29. 1719. was found to be 43″. And thence the diameter of the Ring when Saturn is at its mean distance from the Earth is 42″, and the diameter of Saturn 18″. These things appear so in very long and excellent telescopes, because in such telescopes the apparent magnitudes of the heavenly bodies bear a greater proportion to the dilatation of light in the extremities of those bodies, than in shorter telescopes. If we then reject all the spurious light, the diameter of Saturn will not amount to more than 16″.

Phænomenon III.

That the five primary Planets, Mercury, Venus, Mars, Jupiter and Saturn, with their several orbits, encompass the Sun.

That Mercury and Venus revolve about the Sun, is evident from their moon-like appearances. When they shine out with a full face, they are in respect of us, beyond or above the Sun; when they appear half-full, they are about the same height on one side or other of the Sun; when horn'd, they are below or between, us and the Sun, and they are sometimes, *when directly under*, seen like spots traversing the Sun's disk. That Mars surrounds the Sun, is as plain from its full face when near its conjunction with the Sun, and from the gibbose figure which it shews in its quadratures. And the

same

ſame thing is demonſtrable of Jupiter and Saturn, from their appearing full in all ſituations; for the ſhadows of their ſatellites that appear ſometimes upon their disks make it plain that the light they ſhine with, is not their own, but borrowed from the Sun.

PHÆNOMENON IV.

That the fixed Stars being at reſt, the periodic times of the five primary Planets, and (whether of the Sun about the Earth, or) of the Earth about the Sun, are in the ſeſquiplicate proportion of their mean diſtances from the Sun.

This proportion, firſt obſerv'd by *Kepler*, is now receiv'd by all aſtronomers. For the periodic times are the ſame, and the dimenſions of the orbits are the ſame, whether the Sun revolves about the Earth, or the Earth about the Sun. And as to the meaſures of the periodic times, all aſtronomers are agreed about them. But for the dimenſions of the orbits, *Kepler* and *Bullialdus*, above all others, have determin'd them from obſervations with the greateſt accuracy: and the mean diſtances correſponding to the periodic times, differ but inſenſibly from thoſe which they have aſſign'd, and for the moſt part fall in between them; as we may ſee from the following Table.

The periodic times, with reſpect to the fixed Stars, of the Planets and Earth revolving *about the Sun, in days and decimal parts of a day.*

♄	♃	♂	♁	♀	☿
10759, 275.	4332, 514.	686, 9785.	365, 2565.	224, 6176.	87, 9692.

The

The mean distances of the Planets and of the Earth from the Sun.

	♄	♃	♂
According to *Kepler*	951000.	519650.	152350.
To *Bullialdus*	954198.	522520.	152350.
To the periodic Times	954006.	520096.	152369.

	♁	♀	☿
According to *Kepler*	100000.	72400.	38806.
To *Bullialdus*	100000.	72398.	38585.
To the periodic times	100000.	72333.	38710.

As to Mercury and Venus, there can be no doubt about their distances from the Sun; for they are determin'd by the elongations of those Planets from the Sun. And for the distances of the superior Planets, all dispute is cut off by the eclipses of the satellites of Jupiter. For, by those eclipses, the position of the shadow, which Jupiter projects, is determin'd; whence we have the heliocentric longitude of Jupiter. And from its heliocentric and geocentric longitudes compar'd together, we determine its distance.

Phænomenon V.

Then the primary Planets, by radij drawn to the Earth, describe areas no wise proportional to the times: But that the areas, which they describe by radij drawn to the Sun, are proportional to the times of description.

For to the Earth they appear sometimes direct, sometimes stationary, nay and sometimes retrograde. But from the Sun they are always seen direct, and to proceed with a motion nearly uniform, that is to say, a little swifter in the perihelion and a little slower in the

 aphelion

aphelion diſtances, ſo as to maintain an equality in the deſcription of the areas. This is a noted propoſition among aſtronomers, and particularly demonſtrable in Jupiter, from the eclipſes of his ſatellites; by the help of which eclipſes, as we have ſaid, the heliocentric longitudes of that Planet, and its diſtances from the Sun are determined.

Phænomenon VI.

That the Moon by a radius drawn to the Earth's centre, deſcribes an area proportional to the time of deſcription.

This we gather from the apparent motion of the Moon, compar'd with its apparent diameter. It is true that the motion of the Moon is a little diſturb'd by the action of the Sun. But in laying down theſe phænomena, I neglect thoſe ſmall and inconſiderable errors.

THE

THE

PROPOSITIONS.

PROPOSITION I. THEOREM I.

That the forces by which the circumjovial Planets are continually drawn off from rectilinear motions, and retain'd in their proper orbits, tend to Jupiter's centre; and are reciprocally as the squares of the distances of the places of those Planets *from that centre.*

THE former part of this proposition appears from phæn. 1. and prop. 2. or 3. book 1. The latter from phæn. 1. and cor. 6. prop. 4. of the same book.

The same thing we are to understand of the Planets which encompass Saturn, by phæn. 2.

Proposition II. Theorem II.

That the forces by which the primary Planets are continually drawn off from rectilinear motions, and retain'd in their proper orbits, tend to the Sun; and are reciprocally as the squares of the distances of the places of those Planets *from the Sun's centre.*

The former part of the proposition is manifest from phæn. 5. and prop. 2. book 1. The latter from phæn. 4. and *cor.* 6. prop. 4. of the same book. But this part of the proposition is, with great accuracy, demonstrable from the quiescence of the aphelion points. For a very small aberration from the *reciprocal* duplicate proportion, would (by cor. 1. prop. 45. book 1.) produce a motion of the apsides, sensible enough in every single revolution, and in many of them enormously great.

Proposition III. Theorem III.

That the force by which the Moon is retain'd in its orbit, tends to the Earth; and is reciprocally as the square of the distance of its place from the Earth's centre.

The former part of the proposition is evident from phæn. 6. and prop. 2. or 3. book 1. The latter from the very slow motion of the Moon's Apogee; which in every single revolution amounting but to 3° 3'. *in consequentia*, may be neglected. For (by cor. 1. prop. 45. book 1.) it appears, that if the distance of the Moon from the Earth's centre, is to the semidiameter of the Earth,

Earth, as D to 1; the force, from which such a motion will result, is reciprocally as $D^{2\frac{4}{243}}$ *i, e.* reciprocally as the power of D, whose exponent is $2\frac{4}{243}$, that is to say, in the proportion of the distance something greater than reciprocally duplicate, but which comes $59\frac{3}{4}$ times nearer to the duplicate than to the triplicate proportion. But in regard that this motion is owing to the action of the Sun, (as we shall afterwards shew) it is here to be neglected. The action of the Sun, attracting the Moon from the Earth, is nearly as the Moon's distance from the Earth; and therefore (by what we have shewed in cor. 2. pr. 45. book 1.) is to the centripetal force of the Moon, as 2 to 357,45, or nearly so; that is, as 1 to $178\frac{29}{40}$. And if we neglect so inconsiderable a force of the Sun, the remaining force, by which the Moon is retained in its orb, will be reciprocally as D^2. This will yet more fully appear from comparing this force with the force of gravity, as is done in the next proposition.

Cor. If we augment the mean centripetal force by which the Moon is retained in its orb, first in the proportion of $177\frac{29}{40}$ to $178\frac{29}{40}$, and then in the duplicate proportion of the semidiameter of the Earth to the mean distance of the centres of the Moon and Earth; we shall have the centripetal force of the Moon at the surface of the Earth; supposing this force, in descending to the Earth's surface, continually to increase in the reciprocal duplicate proportion of the height.

Proposition IV. Theorem IV.

That the Moon gravitates towards the Earth; and, by the force of gravity is continually drawn off from a rectilinear motion, and retained in its orbit.

The mean distance of the Moon from the Earth in the syzygies in semidiameters of the Earth, is, according to *Ptolomy* and most Astronomers, 59, according to *Vendelin* and *Huygens* 60, to *Copernicus* 60 $\frac{1}{3}$, to *Street* 60 $\frac{2}{5}$, and to *Tycho* 56 $\frac{1}{2}$. But *Tycho*, and all that follow his tables of refraction, making the refractions of the Sun and Moon (altogether against the nature of light) to exceed the refractions of the fixt Stars, and that by four or five minutes *near the Horizon*, did thereby increase the Moon's *horizontal* parallax by a like number of minutes, that is, by a twelfth, or fifteenth part of the whole parallax. Correct this error, and the distance will become about 60 $\frac{1}{2}$ semidiameters of the Earth, near to what others have assigned. Let us assume the mean distance of 60 diameters in the syzygies; and suppose one revolution of the Moon, in respect of the fixt stars, to be completed in $27^d. 7^h. 43'$, as Astronomers have determined; and the circumference of the Earth to amount to 123249600 *Paris* feet, as the *French* have found by mensuration. And now if we imagine the Moon, deprived of all motion, to be let go, so as to descend towards the Earth with the impulse of all that force by which (by cor. prop. 3.) it is retained in its orb; it will, in the space of one minute of time, describe in its fall 15 $\frac{1}{12}$ *Paris* feet. This we gather by a calculus, founded either upon prop. 36. book 1. or (which comes to the same thing) upon cor. 9. prop. 4. of the same book. For the versed sine of that arc, which the Moon, in the space of one minute of time, would by its mean motion describe at the distance of 60 semidiameters of the Earth, is nearly 15 $\frac{1}{12}$ *Paris* feet, or more accurately 15 feet, 1 inch, and 1 line $\frac{4}{9}$. Wherefore, since that force, in approaching to the Earth, increases in the reciprocal duplicate proportion of the distance, and, upon that account, at the surface of the Earth, is 60×60 times greater, than at the Moon; a body in our regions, falling with that force, ought, in the space of one minute of time, to describe $60 \times 60 \times 15\frac{1}{12}$ *Paris* feet,

and,

and, in the space of one second of time, to describe 15 $\frac{1}{12}$ of those feet; or more accurately 15 feet, 1 inch, and 1 line $\frac{4}{9}$. And with this very force we actually find that bodies here upon Earth do really descend. For a pendulum oscillating seconds in the latitude of *Paris*, will be 3 *Paris* feet, and 8 lines $\frac{1}{2}$ in length, as Mr. *Huygens* has observed. And the space which a heavy body describes by falling in one second of time, is to half the length of this pendulum, in the duplicate ratio of the circumference of a circle to its diameter, (as Mr. *Huygens* has also shewn) and is therefore 15 *Paris* feet, 1 inch, 1 line $\frac{7}{9}$. And therefore the force by which the Moon is retained in its orbit becomes, at the very surface of the Earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by rule 1. & 2.) the force by which the Moon is retained in its orbit, is that very same force, which we commonly call gravity. For, were gravity another force different from that, then bodies descending to the Earth with the joint impulse of both forces would fall with a double velocity, and in the space of one second of time would describe 30 $\frac{1}{6}$ *Paris* feet; altogether against experience.

This calculus is founded on the hypothesis of the Earth's standing still. For, if both Earth and Moon move about the Sun, and at the same time about their common centre of gravity; the distance of the centres of the Moon and Earth from one another, will be 60 $\frac{1}{2}$ semidiameters of the Earth; as may be found by a computation from prop. 60. book 1.

SCHOLIUM.

The demonstration of this proposition may be more diffusely explained after the following manner. Suppose several moons to revolve about the Earth, as in the system

ſtem of Jupiter or Saturn; the periodic times of theſe moons (by the argument of induction) would obſerve the ſame law which *Kepler* found to obtain among the Planets; and therefore their centripetal forces would be reciprocally as the ſquares of the diſtances from the centre of the Earth, by prop. 1. of this book. Now if the loweſt of theſe were very ſmall, and were ſo near the Earth as almoſt to touch the tops of the higheſt mountains; the centripetal force thereof, retaining it in its orb, would be very nearly equal to the weights of any *terreſtrial* bodies that ſhould be found upon the tops of thoſe mountains, as may be known by the foregoing computation. Therefore if the ſame little moon ſhould be deſerted by its centrifugal force that carries it through its orb, and ſo be diſabled from going onwards therein, it would deſcend to the Earth; and that with the ſame velocity as heavy bodies do actually fall with, upon the tops of thoſe very mountains; becauſe of the equality of the forces that oblige them both to deſcend. And if the force by which that loweſt moon would deſcend, were different from gravity, and if that moon were to gravitate towards the Earth, as we find terreſtrial bodies do upon the tops of mountains, it would then deſcend with twice the velocity, as being impelled by both theſe forces conſpiring together. Therefore ſince both theſe forces, that is, the gravity of heavy bodies, and the centripetal forces of the moons, reſpect the centre of the Earth, and are ſimilar and equal between themſelves, they will (by rule 1. and 2.) have one and the ſame cauſe. And therefore the force which retains the Moon in its orbit, is that very force which we commonly call gravity; becauſe otherwiſe this little moon at the top of a mountain, muſt either be without gravity, or fall twice as ſwiftly as heavy bodies uſe to do.

PRO-

Proposition V. Theorem V.

That the circumjovial Planets gravitate towards Jupiter; the circumſaturnal towards Saturn; the circumſolar towards the Sun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilinear orbits.

For the revolutions of the circumjovial Planets about Jupiter, of the circumſaturnal about Saturn, and of Mercury and Venus, and the other circumſolar Planets about the Sun, are appearances of the ſame ſort with the revolution of the Moon about the Earth; and therefore by rule 2. muſt be owing to the ſame ſort of cauſes; eſpecially ſince it has been demonſtrated, that the forces, upon which thoſe revolutions depend, tend to the centres of Jupiter, of Saturn, and of the Sun; and that thoſe forces, in receding from Jupiter, from Saturn, and from the Sun, decreaſe in the ſame proportion, and according to the ſame law, as the force of gravity does in receding from the Earth.

Cor. 1. There is therefore a power of gravity tending to all the Planets. For doubtleſs Venus, Mercury, and the reſt, are bodies of the ſame ſort with Jupiter and Saturn. And ſince all attraction (by law 3.) is mutual, Jupiter will therefore gravitate towards all his own ſatellites, Saturn towards his, the Earth towards the Moon, and the Sun towards all the primary Planets.

Cor. 2. The force of gravity, which tends to any one Planet, is reciprocally as the ſquare of the diſtance of places from that Planet's centre.

Cor. 3. All the Planets do mutually gravitate towards one another, by cor. 1. and 2. And hence it it is, that Jupiter and Saturn, when near their conjunction, by their mutual attractions sensibly disturb each other's motions. So the Sun disturbs the motions of the Moon; and both Sun and Moon disturb our Sea, as we shall hereafter explain.

Scholium.

The force which retains the celestial bodies in their orbits, has been hitherto called centripetal force. But it being now made plain, that it can be no other than a gravitating force, we shall hereafter call it gravity. For the cause of that centripetal force, which retains the Moon in its orbit, will extend it self to all the Planets by rule 1. 2. and 4.

Proposition VI. Theorem VI.

That all bodies gravitate towards every Planet; and that the Weights of bodies towards any the same Planet, at equal distances from the centre of the Planet, are proportional to the quantities of matter which they severally contain.

It has been, now of a long time, observed by others, that all sorts of heavy bodies, (allowance being made for the inequality of retardation, which they suffer from a small power of resistance in the air) descend to the Earth *from equal heights* in equal times: and that equality of times we may distinguish to a great accuracy, by the help of pendulums. I tried the thing in gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I provided two wooden boxes, round and equal.

qual. I filled the one with wood, and suspended an equal weight of gold (as exactly as I could) in the centre of oscillation of the other. The boxes hanging by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally receiving the resistance of the air. And placing the one by the other, I observed them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by cor. 1. and 6. prop. 24. book 2.) was to the quantity of matter in the wood, as the action of the motive force (or *vis motrix)* upon all the gold, to the action of the same upon all the wood; that is, as the weight of the one to the weight of the other. And the like happened in the other bodies. By these experiments, in bodies of the same weight, I could manifestly have discovered a difference of matter less than the thousandth part of the whole, had any such been. But, without all doubt, the nature of gravity towards the Planets, is the same as towards the Earth. For, should we imagine our terrestrial bodies removed to the orb of the Moon, and there, together with the Moon, deprived of all motion, to be let go, so as to fall together towards the Earth: it is certain, from what we have demonstrated before, that, in equal times, they would describe equal spaces with the Moon, and of consequence are to the Moon, in quantity of matter, as their weights to its weight. Moreover, since the satellites of Jupiter perform their revolutions in times which observe the sesquiplicate proportion of their distances from Jupiter's centre, their accelerative gravities towards Jupiter will be reciprocally as the squares of their distances from Jupiter's centre; that is, equal, at equal distances. And therefore, these satellites, if supposed to fall *towards Jupiter* from equal heights, would describe equal spaces in equal times, in like manner as heavy bodies do on our Earth. And by the

same

ſame argument, if the circumſolar Planets were ſuppoſed to be let fall at equal diſtances from the Sun, they would, in their deſcent towards the Sun, deſcribe equal ſpaces in equal times. But forces, which equally accelerate unequal bodies, muſt be as thoſe bodies; that is to ſay, the weights of the Planets *towards the Sun* muſt be as their quantities of matter. Further, that the weights of Jupiter and of his ſatellites towards the Sun are proportional to the ſeveral quantities of their matter, appears from the exceeding regular motions of the ſatellites, (by cor. 3. prop. 65. book 1.) For if ſome of thoſe bodies were more ſtrongly attracted to the Sun in proportion to their quantity of matter, than others; the motions of the ſatellites would be diſturbed by that inequality of attraction (by cor. 2. prop. 65. book 1.) If, at equal diſtances from the Sun, any ſatellite in proportion to the quantity of its matter, did gravitate towards the Sun, with a force greater than Jupiter in proportion to his, according to any given proportion, ſuppoſe of *d* to *e*; then the diſtance between the centres of the Sun and of the ſateilite's orbit would be always greater than the diſtance between the centres of the Sun and of Jupiter, nearly in the ſubduplicate of that proportion; as by ſome computations I have found. And if the ſatellite did gravitate towards the Sun with a force, leſſer in the proportion of *e* to *d*, the diſtance of the centre of the ſatellite's orb from the Sun, would be leſs than the diſtance of the centre of Jupiter from the Sun, in the ſubduplicate of the ſame proportion. Therefore if, at equal diſtances from the Sun, the accelerative gravity of any ſatellite towards the Sun were greater or leſs than the accelerative gravity of Jupiter towards the Sun, but by one $\frac{1}{1000}$ part of the whole gravity; the diſtance of the centre of the ſatellite's orbit from the Sun would be greater or leſs than the diſtance of Jupiter from the Sun, by one $\frac{1}{2000}$ part of the whole diſtance; that is,

2 by

by a fifth part of the distance of the utmost satellite from the centre of Jupiter; an excentricity of the orbit, which would be very sensible. But the orbits of the satellites are concentric to Jupiter, and therefore the accelerative gravities of Jupiter, and of all its satellites towards the Sun, are equal among themselves. And by the same argument, the weights of Saturn and of his satellites towards the Sun, at equal distances from the Sun, are as their several quantities of matter: and the weights of the Moon and of the Earth towards the Sun, are either none, or accurately proportional to the masses of matter which they contain. But some they are by cor. 1. and 3. prop. 5.

But further, the weights of all the parts of every Planet towards any other Planet, are one to another as the matter in the several parts. For if some parts did gravitate more, others less, than for the quantity of their matter; then the whole Planet, according to the sort of parts with which it most abounds, would gravitate more or less, than in proportion to the quantity of matter in the whole. Nor is it of any moment, whether these parts are external or internal. For, if, for example, we should imagine the terrestrial bodies with us to be raised up to the orb of the Moon, to be there compared with its body: If the weights of such bodies were to the weights of the external parts of the Moon, as the quantities of matter in the one and in the other respectively; but to the weights of the internal parts, in a greater or less proportion, then likewise the weights of those bodies would be to the weight of the whole Moon, in a greater or less proportion; against what we have shewed above.

Cor. 1. Hence the weights of bodies do not depend upon their forms and textures. For if the weights could be altered with the forms, they would be greater or less, according to the variety of forms, in equal matter; altogether against experience.

Cor.

Cor. 2. Universally, all bodies about the Earth gravitate towards the Earth; and the weights of all, at equal distances from the Earth's centre, are as the quantities of matter which they severally contain. This is the quality of all bodies, within the reach of our experiments; and therefore, (by rule 3.) to be affirmed of all bodies whatsoever. If the *æther*, or any other body, were either altogether void of gravity, or were to gravitate less in proportion to its quantity of matter; then, because (according to *Aristotle*, *Des Cartes*, and others) there is no difference betwixt that and other bodies, but in *mere* form of matter, by a successive change from form to form, it might be changed at last into a body of the same condition with those which gravitate most in proportion to their quantity of matter; and, on the other hand, the heaviest bodies, acquiring the first form of that body, might by degrees, quite lose their gravity. And therefore the weights would depend upon the forms of bodies, and with those forms might be changed, contrary to what was proved in the preceding corollary.

Cor. 3. All spaces are not equally Full. For if all spaces were equally full, then the specific gravity of the fluid which fills the region of the air, on account of the extreme density of the matter, would fall nothing short of the specific gravity of quick-silver, or gold, or any other the most dense body; and therefore, neither gold, nor any other body, could descend in air. For bodies do not descend in fluids, unless they are specifically heavier than the fluids. And if the quantity of matter in a given space, can, by any rarefaction, be diminished, what should hinder a diminution to infinity?

Cor. 4. If all the solid particles of all bodies are of the same density, nor can be rarified without pores a void space or vacuum must be granted. By bodies of of the same density, I mean those, whose *vires inertiæ* are in the proportion of their bulks.

Cor. 5. The power of gravity is of a different nature from the power of magnetism. For the magnetic attraction is not as the matter attracted. Some bodies are attracted more by the magnet, others less; most bodies not at all. The power of magnetism, in one and the same body, may be increased and diminished; and is sometimes far stronger, for the quantity of matter, than the power of gravity; and in receding from the magnet, decreases not in the duplicate, but almost in the triplicate proportion of the distance, as nearly as I could judge from some rude observations.

Proposition VII. Theorem VII.

That there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain.

That all the Planets mutually gravitate one towards another, we have prov'd before; as well as that the force of gravity towards every one of them, consider'd apart, is reciprocally as the square of the distance of places from the centre of the planet. And thence (by prop. 69. book. 1. and its corollaries) it follows, that the gravity tending towards all the Planets, is proportional to the matter which they contain.

Moreover, since all the parts of any planet *A* gravitate towards any other planet *B*; and the gravity of every part is to the gravity of the whole, as the matter of the part to the matter of the whole; and (by law 3.) to every action corresponds an equal re-action: therefore the planet *B* will, on the other hand, gravitate towards all the parts of the planet *A*; and its gravity towards any one part will be to the gravity towards the whole, as the matter of the part to the matter of the whole. *Q. E. D.*

Cor. 1. Therefore the force of gravity towards any whole planet, ariſes from, and is compounded of, the forces of gravity towards all its parts. Magnetic and electric attractions afford us examples of this. For all attraction towards the whole ariſes from the attractions towards the ſeveral parts. The thing may be eaſily underſtood in gravity, if we conſider a greater planet, as form'd of a number of leſſer planets, meeting together in one globe. For *hence it would appear that* the force of the whole muſt ariſe from the forces of the component parts. If it is objected, that, according to this law, all bodies with us muſt mutually gravitate one towards another, whereas no ſuch gravitation any where appears: I anſwer, that ſince the gravitation towards theſe bodies is to the gravitation towards the whole Earth, as theſe bodies are to the whole Earth, the gravitation towards them muſt be far leſs than to fall under the obſervation of our ſenſes.

Cor. 2. The force of gravity towards the ſeveral equal particles of any body, is reciprocally as the ſquare of the diſtance of places from the particles; as appears from cor. 3. prop. 74. book 1.

Proposition VIII. Theorem VIII.

In two ſpheres mutually gravitating each towards the other, if the matter in places on all ſides round about and equidiſtant from the centres, is ſimilar; the weight of either ſphere towards the other, will be reciprocally as the ſquare of the diſtance between their centres.

After I had found that the force of gravity towards a whole planet did ariſe from, and was compounded of the

the forces of gravity towards all its parts; and towards every one part, was in the reciprocal proportion of the squares of the distances from the part: I was yet in doubt, whether that reciprocal duplicate proportion did accurately hold, or but nearly so, in the total force compounded of so many partial ones. For it might be that the proportion which accurately enough took place in greater distances, should be wide of the truth near the surface of the planet, where the distances of the particles are unequal, and their situation dissimilar. But by the help of prop. 75. and 76. book 1. and their corollaries, I was at last satisfy'd of the truth of the proposition, as it now lies before us.

Cor. 1. Hence we may find and compare together the weights of bodies towards different planets. For the weights of bodies revolving in circles about planets, are (by cor. 2. prop. 4. book 1.) as the diameters of the circles directly, and the squares of their periodic times reciprocally; and their weights at the surfaces of the planets, or at any other distances from their centres, are (by this prop.) greater or less, in the reciprocal duplicate proportion of the distances. Thus from the periodic times of Venus, revolving about the Sun, in $224^d. 16\frac{3}{4}^h$. of the utmost circumjovial satellite revolving about Jupiter, in $16^d. 16\frac{8}{15}^h$; of the Hugenian satellite about Saturn in $15^d. 22\frac{2}{3}^h$; and of the Moon about the Earth in $27^d. 7^h. 43'$; compared with the mean distance of Venus from the Sun, and with the greatest heliocentric elongations of the outmost circumjovial satellite from Jupiter's centre, $8', 16''$. of the Hugenian satellite from the centre of Saturn, $3'. 4''$, and of the Moon from the Earth, $10'. 33''$; by computation I found, that the weight of equal bodies, at equal distances from the centres of the Sun, of Jupiter, of Saturn, and of the Earth, towards the Sun, Jupiter, Saturn, and the Earth, were one to another, as 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$ respectively. Then because as the

the diſtances are increaſed or diminiſhed, the weights are diminiſhed or increaſed in a duplicate ratio; the weights of equal bodies towards the Sun, Jupiter, Saturn, and the Earth, at the diſtances 10000, 997, 791 and 109 from their centres, that is, at their very ſuperficies, will be as 10000, 943, 529 and 435 reſpectively. How much the weights of bodies are at the ſuperficies of the Moon, will be ſhewn hereafter.

COR. 2. Hence likewiſe we diſcover the quantity of matter in the ſeveral Planets. For their quantities of matter are as the forces of gravity at equal diſtances from their centres, that is, in the Sun, Jupiter, Saturn, and the Earth, as 1, $\frac{1}{1067}$, $\frac{1}{3021}$, and $\frac{1}{169282}$ reſpectively. If the parallax of the Sun be taken greater or leſs than 10″, 30‴, the quantity of matter in the Earth muſt be augmented or diminiſhed in the triplicate of that proportion.

COR. 3. Hence alſo we find the denſities of the Planets. For (by prop. 72. book 1.) the weights of equal and ſimilar bodies towards ſimilar ſpheres, are, at the ſurfaces of thoſe ſpheres, as the diameters of the ſpheres. And therefore the denſities of diſſimilar ſpheres are as thoſe weights applied to the diameters of the ſpheres. But the true diameters of the Sun, Jupiter, Saturn, and the Earth, were one to another as 10000, 997, 791 and 109; and the weights towards the ſame, as 10000, 943, 529, and 435 reſpectively; and therefore their denſities are as 100, 94½, 67 and 400. The denſity of the Earth, which comes out by this computation, does not depend upon the parallax of the Sun, but is determined by the parallax of the Moon, and therefore is here truly defin'd. The Sun therefore is a little denſer than Jupiter, and Jupiter than Saturn, and the Earth four times denſer than the Sun; for the Sun, by its great heat, is kept in a ſort of a rarefy'd ſtate. The Moon is denſer than the Earth, as ſhall appear afterwards.

COR.

Cor. 4. The smaller the Planets are, they are, *cæteris paribus*, of so much the greater density. For so the powers of gravity on their several surfaces, come nearer to equality. They are likewise, *cæteris paribus*, of the greater density, as they are nearer to the Sun. So Jupiter is more dense than Saturn, and the Earth than Jupiter. For the Planets were to be placed at different distances from the Sun, that according to their degrees of density, they might enjoy a greater or less proportion of the Sun's heat. Our water, if it were remov'd as far as the orb of Saturn, would be converted into ice, and in the orb of Mercury would quickly fly away in vapour. For the light of the Sun, to which its heat is proportional, is seven times denser in the orb of the Mercury than with us: and by the thermometer I have found, that a sevenfold heat of our summer-sun will make water boil. Nor are we to doubt, that the matter of Mercury is adapted to its heat, and is therefore more dense than the matter of our Earth; since, in a denser matter, the operations of nature require a stronger heat.

Proposition IX. Theorem IX.

That the force of gravity, consider'd downwards from the surface of the planets, decreases nearly in the proportion of the distances from their centres.

If the matter of the planet were of an uniform density, this proposition would be accurately true, (by prop. 73. book 1.) The error therefore can be no greater than what may arise from the inequality of the density.

Proposition X. Theorem X.

That the motions of the Planets in the heavens may ſubſiſt an exceeding long time.

In the ſcholium of prop. 40. book 2. I have ſhew'd that a globe of water, frozen into ice, and moving freely in our air, in the time that it would deſcribe the length of its ſemidiameter, would loſe by the reſiſtance of the air $\frac{1}{4586}$ part of its motion. And the ſame proportion holds nearly in all globes, (how great ſoever,) and mov'd with whatever velocity. But that our globe of earth is of greater denſity than it would be if the whole conſiſted of water only, I thus make out. If the whole conſiſted of water only, whatever was of leſs denſity than water, becauſe of its leſs ſpecific gravity, wou'd emerge and float above. And upon this account, if a globe of terreſtrial matter, cover'd on all ſides with water, was leſs denſe than water, it would emerge ſomewhere; and the ſubſiding water falling back, would be gathered to the oppoſite ſide. And ſuch is the condition of our Earth, which in a great meaſure is covered with ſeas. The Earth, if it was not for its greater denſity, would emerge from the ſeas, and, according to its degree of levity, would be raiſed more or leſs above their ſurface, the water of the ſeas flowing backwards to the oppoſite ſide. By the ſame argument, the ſpots of the Sun, which float upon the lucid matter thereof, are lighter than that matter. And however the Planets have been form'd, while they were yet in fluid maſſes, all the heavier matter ſubſided to the centre. Since therefore the common matter of our Earth on the ſurface thereof, is about twice as heavy as water, and a little lower, in mines, is found about three or four, or even five times more heavy; it is probable, that the quantity of the whole matter of the Earth may be be five or six times greater than if it consisted all of water; especially since I have before shew'd, that the Earth is about four times more dense than Jupiter. If therefore Jupiter is a little more dense than water, in the space of thirty days, in which that planet describes the length of 459 of its semidiameters, it would, in a medium of the same density with our air, lose almost a tenth part of its motion. But since the resistance of mediums decreases in proportion to their weight or density, so that water, which is $13\frac{2}{3}$ times lighter than quicksilver, resists less in that proportion; and air, which is 860 times lighter than water, resists less in the same proportion: Therefore in the heavens, where the weight of the medium, in which the Planets move, is immensely diminished, the resistance will almost vanish.

It is shewn in the scholium of prop. 22. book 2. that at the height of 200 miles above the Earth, the air is more rare than it is at the superficies of the Earth, in the ratio of 30 to 0,0000000000003998, or as 75000000000000 to 1 nearly. And hence the planet Jupiter, revolving in a medium of the same density with that superior air, would not lose by the resistance of the medium the 1000000th part of its motion in 1000000 years. In the spaces near the Earth, the resistance is produced only by the air, exhalations and vapours. When these are carefully exhausted by the air pump from under the receiver, heavy bodies fall within the receiver with perfect freedom, and without the least sensible resistance; gold itself and the lightest down, let fall together, will descend with equal velocity; and though they fall through a space of four, six, and eight feet, they will come to the bottom at the same time; as appears from experiments. And therefore the celestial regions being perfectly void of air and exhalations, the Planets and Comets meeting no sensible resistance in those ſpaces, will continue their motions through them for an immenſe tract of time.

HYPOTHESIS I.

That the centre of the ſyſtem of the world is immoveable.

This is acknowledg'd by all, while ſome contend that the Earth, others, that the Sun is fix'd in that centre. Let us ſee what may from hence follow.

PROPOSITION XI. THEOREM XI.

That the common centre of gravity of the Earth, the Sun, and all the Planets is immoveable.

For (by cor. 4. of the laws) that centre either is at reſt, or moves uniformly forward in a right line. But if that centre mov'd, the centre of the world would move alſo, againſt the hypotheſis.

PROPOSITION XII. THEOREM XII.

That the Sun is agitated by a perpetual motion, but never recedes far from the common centre of gravity of all the Planets.

For ſince (by cor. 2. prop. 8.) the quantity of matter in the Sun, is to the quantity of matter in Jupiter, as 1067 to 1; and the diſtance of Jupiter from the Sun, is to the ſemidiameter of the Sun, in a proportion but a ſmall matter greater; the common centre of gravity of Jupiter and the Sun, will fall upon a point a little without the ſurface of the Sun. By the ſame argument, ſince the quantity of matter in the Sun is to the quantity of matter in Saturn, as 3021 to 1; and the

the diſtance of Saturn from the Sun is to the ſemidiameter of the Sun in a proportion but a ſmall matter leſs; the common centre of gravity of Saturn and the Sun will fall upon a point a little within the ſurface of the Sun. And purſuing the principles of this computation, we ſhould find that tho' the Earth and all the Planets were plac'd on one ſide of the Sun, the diſtance of the common centre of gravity of all from the centre of the Sun would ſcarcely amount to one diameter of the Sun. In other caſes, the diſtances of thoſe centres is always leſs. And therefore, ſince that centre of gravity is in perpetual reſt, the Sun, according to the various poſitions of the Planets, muſt perpetually be moved every way, but will never recede far from that centre.

Cor. Hence the common centre of gravity of the Earth, the Sun, and all the Planets is to be eſteem'd the Centre of the World. For ſince the Earth, the Sun and all the Planets, mutually gravitate one towards another, and are therefore, according to their powers of gravity, in perpetual agitation, as the laws of motion require; it is plain that their moveable centres cannot be taken for the immoveable centre of the world. If that body were to be plac'd in the centre, towards which other bodies gravitate moſt, (according to common opinion) that privilege ought to be allow'd to the Sun. But ſince the Sun it ſelf is mov'd, a fixt point is to be choſen, from which the centre of the Sun recedes leaſt, and from which it would recede yet leſs, if the body of the Sun were denſer and greater, and therefore leſs apt to be mov'd.

Pro-

PROPOSITION XIII. THEOREM XIII.

The Planets move in ellipses which have their common focus in the centre of the Sun; and, by radij drawn to that centre, they describe areas proportional to the times of description.

We have discours'd above of these motions from the phænomena. Now that we know the principles on which they depend, from those principles we deduce the motions of the heavens *a priori*. Because the weights of the Planets towards the Sun, are reciprocally as the squares of their distances from the Sun's centre; if the Sun was at rest, and the other Planets did not mutually act one upon another, their orbits would be ellipses, having the Sun in their common focus; and they would describe areas proportional to the times *of description* by prop. 1 & 11. and cor. 1. prop. 13. book 1. But the mutual actions of the Planets one upon another, are so very small, that they may be neglected. And by prop. 66. book 1. they less disturb the motions of the Planets around the Sun in motion, than if those motions were perform'd about the Sun at rest.

It is true, that the action of Jupiter upon Saturn is not to be neglected. For the force of gravity towards Jupiter is to the force of gravity towards the Sun as 1 to 1067; and therefore in the conjunction of Jupiter and Saturn, because the distance of Saturn from Jupiter is to the distance of Saturn from the Sun, almost as 4 to 9; the gravity of Saturn towards Jupiter, will be to the gravity of Saturn towards the Sun, as 81 *to* 16×1067; or, as 1 to about 211. And hence arises a perturbation of the orb of Saturn in every conjunction of this Planet with Jupiter, so sensible that astronomers are puzled with it. As the Planet is differently situated in these conjunctions, its excentricity is sometimes augmented, sometimes diminish'd; its aphelion is sometimes carry'd forwards, sometimes backwards, and its mean motion is by turns accelerated and retarded. Yet the whole error in its motion about the Sun, tho' arising from so great a force, may be almost avoided (except in the mean motion) by placing the lower focus of its orbit in the common centre of gravity of Jupiter and the Sun, (according to prop. 67. book 1.) and therefore that error when it is greatest, scarcely exceeds two minutes. And the greatest error in the mean motion, scarcely exceeds two minutes yearly. But in the conjunction of Jupiter and Saturn, the accelerative forces of gravity of the Sun towards Saturn, of Jupiter towards Saturn, and of Jupiter toward the Sun, are almost as 16, 81 and $\frac{16\times81\times3021}{25}$ or 156609; and therefore the difference of the forces of gravity of the Sun towards Saturn, and of Jupiter towards Saturn, is to the force of gravity of Jupiter towards the Sun, as 65 to 156609, or as 1 to 2409. But the greatest power of Saturn to disturb the motion of Jupiter is proportional to this difference; and therefore the perturbation of the orbit of Jupiter is much less than that of Saturn's. The perturbations of the other orbits are yet far less, except that the orbit of the Earth is sensibly disturb'd by the Moon. The common centre of gravity of the Earth and Moon moves in an ellipse about the Sun in the focus thereof, and by a radius drawn to the Sun, describes areas proportional to the times of description. But the Earth in the mean time by a menstrual motion is revolv'd about this common centre.

PRO-

PROPOSITION XIV. THEOREM XIV.

The aphelions and nodes of the orbits of the Planets are fixt.

The aphelions are immoveable, by prop. 11. book 1. and ſo are the planes of the orbits by prop. 1. of the ſame book. And if the planes are fixt, the nodes muſt be ſo too. It is true, that ſome inequalities may ariſe from the mutual actions of the Planets and Comets in their revolutions. But theſe will be ſo ſmall that they may be here paſs'd by.

COR. 1. The fixt Stars are immoveable, ſeeing they keep the ſame poſition to the aphelions and nodes of the Planets.

COR. 2. And ſince theſe Stars are liable to no ſenſible parallax from the annual motion of the Earth, they can have no force, becauſe of their immenſe diſtance, to produce any ſenſible effect in our ſyſtem. Not to mention, that the fixt Stars, every where promiſcuouſly diſpers'd in the heavens, by their contrary attractions deſtroy their mutual actions, by prop. 70. book 1.

SCHOLIUM.

Since the Planets near the Sun (*viz.* Mercury, Venus, the Earth and Mars) are ſo ſmall that they can act but with little force upon each other; therefore their aphelions and nodes muſt be fixt, excepting in ſo far as they are diſturb'd by the actions of Jupiter and Saturn, and other higher bodies. And hence we may find, by the theory of gravity, that their aphelions move a little *in conſequentia*, in reſpect of the fixed Stars, and that in the ſeſquiplicate proportion of their ſeveral

ſeveral diſtances from the Sun. So that if the aphelion of Mars, in the ſpace of an hundred years, is carried 33′. 20″. *in conſequentia*, in reſpect of the fixed Stars; the aphelions of the Earth, of Venus, and of Mercury, will, in an hundred years be carried forwards 17′. 40″. 10′. 53″. and 4′. 16″. reſpectively. But theſe motions are ſo inconſiderable, that we have neglected them in this propoſition.

Proposition XV. Theorem I.

To find the principal diameters of the orbits of the Planets.

They are to be taken in the ſubſeſquiplicate proportion of the periodic times by prop. 15. book 1. and then to be ſeverally augmented in the proportion of the ſum of the maſſes of matter in the Sun and each Planet to the firſt of two mean proportionals betwixt that ſum and the quantity of matter in the Sun, by prop. 60. book 1.

Proposition XVI. Problem II.

To find the eccentricities and aphelions of the Planets.

This problem is reſolved by prop. 18. book 1.

Pro-

Proposition XVII. Theorem XV.

That the diurnal motions of the Planets are uniform, and that the libration of the Moon arises from its diurnal motion.

The proposition is prov'd from the first law of motion, and cor. 22. prop. 66. book 1. Jupiter, with respect to the fixed Stars, revolves in $9^h. 56'$. Mars in $24^h. 39'$. Venus in about 23^h. the Earth in $23^h. 56'$. the Sun in $25\frac{1}{2}$ days, and the Moon in 27 days 7 hours 43'. These things appear by the phænomena. The spots in the Sun's body return to the same situation on the Sun's disk, with respect to the Earth in $27\frac{1}{2}$ days; and therefore with respect to the fixed Stars the Sun revolves in about $25\frac{1}{2}$ days. But because the lunar day, arising from its uniform revolution about its axes is menstrual, *that is, equal to the time of its periodic revolution in its orb*, therefore the same face of the Moon will be always nearly turned to the upper focus of its orb; but, as the situation of that focus requires, will deviate a little, to one side and to the other, from the Earth in the lower focus; and this is the libration in longitude. For the libration in latitude arises from the Moon's latitude, and the inclination of its axis to the plane of the ecliptic. This theory of the libration of the Moon, Mr. *N. Mercator* in his astronomy, published at the beginning of the Year 1676, explained more fully out of the letters I sent him. The utmost satellite of Saturn seems to revolve about its axis with a motion like this of the Moon, respecting Saturn continually with the same face. For in its revolution round Saturn, as often as it comes to the eastern part of its orbit, it is scarcely visible, and generally quite disappears; which is like to be occasioned by by some spots in that part of its body, which is then turned toward the Earth, as M. *Cassini* has observed. So also the utmost satellite of Jupiter seems to revolve about its axis with a like motion, because in that part of its body which is turned from Jupiter, it has a spot, which always appears as if it were in Jupiter's own body, whenever the satellite passes between Jupiter and our eye.

PROPOSITION XVIII. THEOREM XVI.

That the axes of the Planets are less than the diameters drawn perpendicular to the axes.

The equal gravitation of the parts on all sides would give a sphærical figure to the Planets, if it was not for their diurnal revolution in a circle. By that circular motion it comes to pass that the parts receding from the axe endeavour to ascend about the equator. And therefore if the matter is in a fluid state, by its ascent towards the equator it will enlarge the diameters there, and by its descent towards the poles it will shorten the axe. So the diameter of Jupiter, (by the concurring observations of astronomers) is found shorter betwixt pole and pole, than from east to west. And by the same argument, if our Earth was not higher about the equator than at the poles, the Seas would subside about the poles, and rising towards the equator, would lay all things there under water.

PROPOSITION XIX. PROBLEM III.

To find the proportion of the axe of a Planet to the diameters perpendicular thereto.

Our countryman Mr. *Norwood*, measuring a distance of 905751 feet of *London* measure between *London* and

York

York in 1635, and observing the difference of latitudes to be 2°. 28′, determined the measure of one degree to be 367196 feet of *London* measure, that is 57300 *Paris* toises. M. *Picart* measuring an arc of one degree, and 22′. 55″. of the meridian between *Amiens* and *Malvoisine*, found an arc of one degree to be 57060 *Paris* toises. M. *Cassini* the father measured the distance upon the meridian from the town of *Collioure* in *Roussillon* to the observatory of *Paris*: And his son added the distance from the observatory to the citadel of *Dunkirk*. The whole distance was 486156 $\frac{1}{2}$ toises, and the difference of the latitudes of *Collioure* and *Dunkirk* was 8 degrees, and 31′. 11 $\frac{5}{6}$″. Hence an arc of one degree appears to be 57061 *Paris* toises. And from these measures we conclude, that the circumference of the Earth is 123249600, and its semidiameter 19615800 *Paris* feet, upon the supposition that the Earth is of a sphærical figure.

In the latitude of *Paris* a heavy body falling in a second of time, describes 15 *Paris* feet, 1 inch, 1 line as above, that is, 2173 lines $\frac{7}{9}$. The weight of the body is diminished by the weight of the ambient air. Let us suppose the weight lost thereby to be $\frac{1}{11000}$ part of the whole weight; then that heavy body falling *in vacuo* will describe a height of 2174 lines in one second of time.

A body in every sidereal day of 23^h. 56′. 4″. uniformly revolving in a circle at the distance of 19615800 feet from the centre, in one second of time describes an arc of 1433, 46 feet; the versed sine of which is 0,05236561 feet, or 7,54064 lines. And therefore the force with which bodies descend in the latitude of *Paris* is to the centrifugal force of bodies in the equator arising from the diurnal motion of the Earth, as 2174 to 7,54064.

The centrifugal force of bodies in the equator, is to the centrifugal force with which bodies recede directly from

from the Earth in the latitude of *Paris* 48°. 50′. 10″. in the duplicate proportion of the radius to the cosine of the latitude, that is, as 7,54064 to 3,267. Add this force to the force with which bodies descend by their weight in the latitude of *Paris*, and a body, in the latitude of *Paris*, falling by its whole undiminished force of gravity, in the time of one second, will describe 2177,267 lines, or 15 *Paris* feet, 1 inch, and 5,267 lines. And the total force of gravity in that latitude will be to the centrifugal force of bodies in the equator of the Earth, as 2177,267 to 7,54064, or as 289 to 1.

Wherefore if *APBQ* (*Pl.* 10. *Fig.* 1.) represent the figure of the Earth, now no longer sphærical, but generated by the rotation of an ellipsis about its lesser axe; and *ACQqca* a canal full of water, reaching from the pole *Qq* to the centre *Cc*, and thence rising to the equator *Aa*: The weight of the water in the leg of the canal *ACca*, will be to the weight of water in the other leg *QCcq*, as 289 to 288, because the centrifugal force, arising from the circular motion, sustains and takes off one of the 289 parts of the weight (in the one leg) and the weight of 288 in the other sustains the rest. But by computation (from cor. 2. prop. 91. book 1.) I find, that if the matter of the Earth was all uniform, and without any motion, and its axe *PQ* were to the diameter *AB*, as 100 to 101; the force of gravity in the place *Q*, towards the Earth, would be to the force of gravity in the same place *Q* towards a sphere describ'd about the centre *C* with the radius *PC*, or *QC*, as 126 to 125. And by the same argument, the force of gravity in the place *A* towards the sphæroid, generated by the rotation of the ellipse *APBQ* about the axe *AB*, is to the force of gravity in the same place *A*, towards the sphere describ'd about the centre *C* with the radius *AC*, as 125 to 126. But the force of gravity in the place *A*, towards the Earth, is a mean proportional betwixt the forces of gravity towards that ſphæroid and this ſphere; becauſe the ſphere, by having its diameter *PQ* diminiſhed, in the proportion of 101 to 100, is transformed into the figure of the Earth; and this figure, by having a third diameter perpendicular to the two diameters *AB* and *PQ* diminiſh'd in the ſame proportion, is converted into the ſaid ſphæroid; and the force of gravity in *A*, in either caſe, is diminiſh'd nearly in the ſame proportion. Therefore the force of gravity in *A*, towards the ſphere deſcrib'd about the centre *C*, with the radius *AC*, is to the force of gravity in *A*, towards the Earth, as 126 to $125\frac{1}{2}$. And the force of gravity in the place *Q*, towards the ſphere deſcrib'd about the centre *C* with the radius *QC*, is to the force of gravity in the place *A*, towards the ſphere deſcrib'd about the centre *C*, with the radius *AC*, in the proportion of the diameters, (by prop. 72. book 1.) that is, as 100 to 101. If therefore we compound thoſe three proportions 126 to 125, 126 to $125\frac{1}{2}$, and 100 to 101; into one: The force of gravity in the place *Q* towards the Earth, will be to the force of gravity in the place *A* towards the Earth, as $126 \times 126 \times 100$ to $125 \times 125\frac{1}{2} \times 101$; or as 501 to 500.

Now ſince (by cor. 3. prop. 91. book 1.) the force of gravity in either leg of the canal *ACca*, or *QCcq*, is as the diſtance of the places from the centre of the Earth, if thoſe legs are conceived to be divided by tranſverſe, parallel, and equidiſtant ſurfaces, into parts proportional to the wholes, the weights of any number of parts in the one leg *ACca*, will be to the weights of the ſame number of parts in the other leg, as their magnitudes and the accelerative forces of their gravity conjunctly, that is, as 101 to 100, and 500 to 501, or as 505 to 501. And therefore if the centrifugal force of every part in the leg *ACca*, ariſing from the diurnal motion, was to the weight of the ſame part, as

4 to

4 to 505, so that from the weight of every part, conceived to be divided into 505 parts, the centrifugal force might take off four of those parts, the weights would remain equal in each leg, and therefore the fluid would rest in an equilibrium. But the centrifugal force of every part is to the weight of the same part as 1 to 289; that is, the centrifugal force which should be $\frac{4}{505}$ parts of the weight, is only $\frac{1}{289}$ part thereof. And therefore, I say, by the rule of proportion, that if the centrifugal force $\frac{4}{505}$ make the height of the water in the leg *ACca* to exceed the height of the water in the leg *QCcq*, by one $\frac{1}{100}$ part of its whole height; the centrifugal force $\frac{1}{289}$ will make the excess of the height in the leg *ACca*, only $\frac{1}{229}$ part of the height of the water in the other leg *QCcq*. And therefore the diameter of the Earth at the equator, is to its diameter from pole to pole, as 230 to 229. And since the mean semidiameter of the Earth, according to *Picart*'s mensuration, is 19615800 *Paris* feet, or 3923,16 miles (reckoning 5000 feet to a mile) the Earth will be higher at the equator, than at the poles, by 85472 feet, or 17 $\frac{1}{10}$ miles. And its height at the equator will be about 19658600 feet, and at the poles 19573000 feet.

If, the density and periodic time of the diurnal revolution remaining the same, the Planet was greater or less than the Earth; the proportion of the centrifugal force to that of gravity, and therefore also of the diameter betwixt the poles to the diameter at the equator, would likewise remain the same. But if the diurnal motion was accelerated or retarded in any proportion, the centrifugal force would be augmented or diminished nearly in the same duplicate proportion; and therefore the difference of the diameters will be increased or diminished in the same duplicate ratio very nearly. And if the density of the Planet was augmented or diminished in any proportion, the force of gravity tending towards it would alſo be augmented or diminiſhed in the ſame proportion; and the difference of the diameters contrarywiſe would be diminiſhed in proportion as the force of gravity is augmented, and augmented in proportion as the force of gravity is diminiſhed. Wherefore, ſince the Earth, in reſpect of the fixt Stars, revolves in $23^h. 56'$, but Jupiter in $9^h. 56'$, and the ſquares of their periodic times are as 29 to 5, and their denſities as 400 to $94\frac{1}{2}$; the difference of the diameters of Jupiter will be to its leſſer diameter, as $\frac{29}{5} \times \frac{400}{94\frac{1}{2}} \times \frac{229}{1}$ to 1, or as 1 to $9\frac{1}{3}$ nearly. Therefore the diameter of Jupiter from eaſt to weſt, is to its diameter from pole to pole nearly as $10\frac{1}{3}$ to $9\frac{1}{3}$. Therefore ſince its greateſt diameter is $37''$, its leſſer diameter lying between the poles, will be $33''\ 25'''$. Add thereto about $3''$ for the irregular refraction of light, and the apparent diameters of this Planet will become $40''$ and $36''. 25'''$: which are to each other as $11\frac{1}{6}$ to $10\frac{1}{6}$ very nearly. Theſe things are ſo upon the ſuppoſition, that the body of Jupiter is uniformly denſe. But now if its body be denſer towards the plane of the equator than towards the poles, its diameters may be to each other as 12 to 11, or 13 to 12, or perhaps as 14 to 13.

And *Caſſini* obſerved in the year 1691, that the diameter of Jupiter reaching from eaſt to weſt, is greater by about a fifteenth part than the other diameter. Mr. *Pound* with his 123 foot teleſcope, and an excellent micrometer, meaſured the diameters of Jupiter in the year 1719, and found them as follows.

The

The times.			*Greatest diam.*	*Lesser diam.*	*The diam. to each other.*		
	day.	hours.	parts.	parts.		as	
Jan.	28	6	13,40	12,28	12	to	11
Mar.	6	7	13,12	12,20	$13\frac{1}{4}$	to	$12\frac{1}{4}$
Mar.	9	7	13,12	12,08	$12\frac{2}{3}$	to	$11\frac{1}{3}$
Apr.	9	9	12,32	11,48	$14\frac{1}{2}$	to	$13\frac{1}{2}$

So that the theory agrees with the phænomena. For the Planets are more heated by the Sun's rays towards their equators, and therefore are a little more condensed by that heat, than towards their poles.

Moreover, that there is a diminution of gravity occasioned by the diurnal rotation of the Earth, and therefore the Earth rises higher there than it does at the poles, (supposing that its matter is uniformly dense) will appear by the experiments of pendulums related under the following proposition.

Proposition XX. Problem IV.

To find and compare together the weights of bodies in the different regions of our Earth.

Because the weights of the unequal legs of the canal of water *ACQqca*, are equal; and the weights of the parts proportional to the whole legs, and alike situated in them, are one to another as the weights of the wholes, and therefore equal betwixt themselves; the weights of equal parts and alike situated in the legs, will be reciprocally as the legs, that is, reciprocally as 230 to 229. And the case is the same in all homogeneous equal bodies alike situated in the legs of the canal. Their weights are reciprocally as the legs, that is, reciprocally as the distances of the bodies from the centre of the Earth. Therefore if the bodies are situated in the uppermost parts of the canals, or on the surface of the Earth, their weights will be, one to another, reciprocally as their distances from the centre. And by the same argument, the weights in all other places round the whole surface of the Earth, are reciprocally as the distances of the places from the centre; and therefore, in the hypothesis of the Earth's being a sphæroid, are given in proportion.

Whence arises this theorem, that the increase of weight, in passing from the equator to the poles, is nearly as the versed sine of double the latitude, or, which comes to the same thing, as the square of the right sine of the latitude. And the arcs of the degrees of latitude in the meridian, increase nearly in the same proportion. And therefore, since the latitude of *Paris* is 48°. 50′, that of places under the equator, 00°. 00′. and that of places under the poles 90°; and the versed sines of double those arcs are 11334, 00000 and 20000, the radius being 10000; and the force of gravity at the pole is to the force of gravity at the equator, as 230 to 229, and the excess of the force of gravity at the pole, to the force of gravity at the equator, as 1 to 229, the excess of the force of gravity in the latitude of *Paris*, will be to the force of gravity at the equator as $1 \times \frac{11334}{20000}$ to 229, or as 5667 to 2290000. And therefore the whole forces of gravity in those places will be, one to the other, as 2295667 to 2290000. Wherefore, since the lengths of pendulums vibrating in equal times, are as the forces of gravity, and in the latitude of *Paris*, the length of a pendulum vibrating seconds, is 3 *Paris* feet, and 8 $\frac{1}{2}$ lines, or rather, because of the weight of the air 8 $\frac{5}{9}$ lines; the length of a pendulum vibrating in the same time under the equator, will be shorter by 1,087 lines. And by a like calculus the following table is made.

Latitude

Latitude of the place.	*Length of the pendulum.*		*Measure of one degree in the meridian.*
Deg.	Feet.	Lines.	Toises.
0	3 .	7,468	56637
5	3 .	7,482	56642
10	3 .	7,526	56659
15	3 .	7,596	56687
20	3 .	7,692	56724
25	3 .	7,812	56769
30	3 .	7,948	56823
35	3 .	8,099	56882
40	3 .	8,261	56945
1	3 .	8,294	56958
2	3 .	8,327	56971
3	3 .	8,361	56984
4	3 .	8,394	56997
45	3 .	8,428	57010
6	3 .	8,461	57022
7	3 .	8,494	57035
8	3 .	8,528	57048
9	3 .	8,561	57061
50	3 .	8,594	57074
55	3 .	8,756	57137
60	3 .	8,907	57196
65	3 .	9,044	57250
70	3 .	9,162	57295
75	3 .	9,258	57332
80	3 .	9,329	57360
85	3 .	9,372	57377
90	3 .	9,387	57382

By this table therefore it appears, that the inequality of degrees is so small, that the figure of the Earth, in geographical matters, may be conſidered as ſphærical; eſpecially if the Earth be a little denſer towards the plane of the equator than towards the poles.

Now ſeveral aſtronomers ſent into remote countries to make aſtronomical obſervations, have found that pendulum clocks do accordingly move ſlower near the equator than in our climates. And firſt of all in the year 1672, M. *Richer* took notice of it in the iſland of *Cayenne*. For when, in the month of *Auguſt*, he was obſerving the tranſits of the fixt Stars over the meridian, he found his clock to go ſlower than it ought in reſpect of the mean motion of the Sun, at the rate of 2′. 28″. a day. Therefore fitting up a ſimple pendulum to vibrate in ſeconds, which were meaſured by an excellent clock, he obſerved the length of that ſimple pendulum; and this he did over and over every week for ten months together. And upon his return to *France*, comparing the length of that pendulum, with the length of the pendulum at *Paris*, (which was 3 *Paris* feet and 8 $\frac{3}{5}$ lines) he found it ſhorter by 1 $\frac{1}{4}$ line.

Afterwards our friend Dr. *Halley*, about the year 1677, arriving at the iſland of St. *Helen*, found his pendulum-clock to go ſlower there than at *London*, without marking the difference. But he ſhortned the rod of his clock, by more than the $\frac{1}{8}$ of an inch, or 1 $\frac{1}{2}$ line. And to effect this, becauſe the length of the ſcrew at the lower end of the rod was not ſufficient, he interpoſed a wooden ring betwixt the nut and the ball.

Then in the year 1682. M. *Varin* and M. *des Hayes*, found the length of a ſimple pendulum vibrating in ſeconds at the royal obſervatory of *Paris* to be 3 feet and 8$\frac{5}{9}$ lines. And by the ſame method in the iſland of *Goree*, they found the length of an iſochronal pendulum to be 3 feet and 6$\frac{5}{9}$ lines, differing from the former by two lines. And in the ſame year, going to the iſlands

islands of *Guadaloupe* and *Martinico*, they found that the length of an isochronal pendulum in those islands was 3 feet and $6\frac{1}{2}$ lines.

After this M. *Couplet*, the son, in the month of *July* 1697, at the royal observatory of *Paris*, so fitted his pendulum clock to the mean motion of the Sun, that for a considerable time together, the clock agreed with the motion of the Sun. In *November* following, upon his arrival at *Lisbon*, he found his clock to go slower than before, at the rate of 2′. 13″. in 24 hours. And next *March* coming to *Paraiba* he found his clock to go slower there than at *Paris*, and at the rate of 4′. 12″. in 24 hours. And he affirms, that the pendulum vibrating in seconds was shorter at *Lisbon* by $2\frac{1}{2}$ lines, and at *Paraiba* by $3\frac{2}{3}$ lines, than at *Paris*. He had done better to have reckon'd those differences $1\frac{1}{3}$ and $2\frac{5}{9}$. For these differences correspond to the differences of the times 2′. 13″. and 4′. 12″. But this gentleman's observations are so gross, that we cannot confide in them.

In the following years 1699 and 1700. M. *des Hayes*, making another voyage to *America*, determin'd that in the islands of *Cayenne* and *Granada* the length of the pendulum vibrating in seconds was a small matter less than 3 feet and $6\frac{1}{2}$ lines; that in the island of St. *Christophers*, it was 3 feet and $6\frac{3}{4}$ lines; and in the island of St. *Domingo*, 3 feet and 7 lines.

And in the year 1704. P. *Feuillé* at *Puerto bello* in *America*, found that the length of the pendulum vibrating in seconds, was 3 *Paris* feet, and only $5\frac{7}{12}$ lines, that is, almost 3 lines shorter than at *Paris*; but the observation was faulty. For afterwards going to the island of *Martinico*, he found the length of the isochronal pendulum there, 3 *Paris* feet and $5\frac{10}{12}$ lines.

Now the latitude of *Paraiba* is 6°. 38′. south. That of *Puerto bello* 9°. 33′. north. And the latitudes of the islands *Cayenne*, *Goree*, *Guadaloupe*, *Martinico*, *Granada*, St. *Chriſtophers* and St. *Domingo*, are reſpectively 4°. 55′, 14°. 40″, 14°. 00′, 14°. 44′, 12°. 06′, 17°. 19′. and 19°. 48′, north. And the exceſſes of the length of the pendulum at *Paris* above the lengths of the iſochronal pendulums obſerv'd in thoſe latitudes, are a little greater than by the table of the lengths of the pendulum above computed. And therefore the Earth is a little higher under the equator than by the preceding calculus, and a little denſer at the centre than in mines near the ſurface, unleſs perhaps the heats of the torrid zone have a little extended the length of the pendulums.

For M. *Picart* has obſerv'd, that a rod of iron, which in froſty weather in the winter ſeaſon was one foot long, when heated by fire, was lengthen'd into 1 foot and ¼ line. Afterwards M. *de la Hire* found that a rod of iron, which in the like winter ſeaſon was 6 feet long; when expos'd to the heat of the ſummer Sun, was extended into 6 feet and ⅔ line. In the former caſe the heat was greater than in the latter. But in the latter it was greater than the heat of the external parts of an human body. For metals expos'd to the ſummer-ſun, acquire a very conſiderable degree of heat. But the rod of a pendulum-clock is never expos'd to the heat of the ſummer-ſun, nor ever acquires a heat equal to that of the external parts of an human body. And therefore though the 3 foot rod of a pendulum clock will indeed be a little longer in the ſummer than in the winter-ſeaſon; yet the difference will ſcarcely amount to ¼ line. Therefore the total difference of the lengths of iſochronal pendulums in different climates, cannot be aſcrib'd to the difference of heat. Nor indeed to the miſtakes of the *French* aſtronomers. For although there is not a perfect agreement betwixt their obſervations, yet the errors are ſo ſmall that they may be neglected; and in this they all agree, that iſochronal pendulums are ſhorter under the equator than at the royal obſervatory of *Paris*, by a difference not

leſs

leſs than 1 $\frac{1}{4}$ line, nor greater than 2 $\frac{2}{3}$ lines. By the obſervations of M. *Richer* in the iſland of *Cayenne*, the difference was 1$\frac{1}{4}$ line. That difference being corrected by thoſe of M. *des Hayes* becomes 1 $\frac{1}{2}$ line or 1 $\frac{3}{4}$ line. By the leſs accurate obſervations of others the ſame was made about two lines. And this diſagreement might ariſe partly from the errors of the obſervations, partly from the diſſimilitude of the internal parts of the Earth, and the height of mountains, partly from the different heats of the air.

I take an iron rod of 3 feet long to be ſhorter by a ſixth part of one line in winter time with us here in *England*, than in the ſummer. Becauſe of the great heats under the equator, ſubduct this quantity from the difference of one line and a quarter obſerv'd by M. *Richer*, and there will remain one line $\frac{1}{12}$, which agrees very well with 1$\frac{87}{1000}$ line collected by the theory a little before. M. *Richer* repeated his obſervations, made in the iſland of *Cayenne*, every week for 10 months together, and compared the lengths of the pendulum which he had there noted in the iron rods, with the lengths thereof which he obſerv'd in *France*. This diligence and care ſeems to have been wanting to the other obſervers. If this gentleman's obſervations are to be depended on, the Earth is higher under the equator than at the poles, and that by an exceſs of about 17 miles: as appeared above by the theory.

PROP.

Proposition XXI. Theorem XVII.

That the equinoctial points go backwards, and that the axe of the Earth, by a nutation in every annual revolution, twice vibrates towards the ecliptic, and as often returns to its former poſition.

The propoſition appears from cor. 20. prop. 66. book 1. But that motion of nutation muſt be very ſmall, and indeed ſcarce perceptible.

Proposition XXII. Theorem XVIII.

That all the motions of the Moon, and all the inequalities of thoſe motions, follow from the principles which we have laid down.

That the greater Planets, while they are carried about the Sun may, in the mean time, carry other leſſer Planets, revolving about them; and that thoſe leſſer Planets muſt move in ellipſes, which have their foci in the centres of the greater, appears from prop. 65. book 1. But then their motions will be ſeveral ways diſturb'd by the action of the Sun, and they will ſuffer ſuch inequalities as are obſerv'd in our Moon. Thus our Moon, (by cor. 2, 3, 4, and 5. prop. 66. book 1.) moves faſter, and, by a radius drawn to the Earth, deſcribes an area, greater for the time, and has its orbit leſs curv'd, and therefore approaches nearer to the Earth, in the ſyzygies than in the quadratures, excepting in ſo far as theſe effects are hinder'd by the motion of eccentricity. For (by cor. 9. prop. 66. book 1.) the

the eccentricity is greateſt, when the apogeon of the Moon is in the ſyzygies, and leaſt when the ſame is in the quadratures; and upon this account, the perigeon Moon is ſwifter, and nearer to us, but the apogeon Moon ſlower, and farther from us, in the ſyzygies than in the quadratures. Moreover the apogee goes forwards, and the nodes backwards: and this is done, not with a regular, but an unequal motion. For (by cor. 7 and 8. prop. 66. book 1.) the apogee goes more ſwiftly forwards in its ſyzygies, more ſlowly backwards in its quadratures; and, by the exceſs of its progreſs above its regreſs, advances yearly *in conſequentia*. But contrarywiſe the nodes (by cor. 11. prop. 66. book 1.) are quieſcent in their ſyzygies, and go faſteſt back in their quadratures. Further, the greateſt latitude of the Moon, (by cor. 10. prop. 66. book 1.) is greater in the quadratures of the Moon, than in its ſyzygies. And (by cor. 6. prop. 66. book 1.) the mean motion of the Moon is ſlower in the perihelion of the Earth, than in its aphelion. And theſe are the principal inequalities (of the Moon) taken notice of by aſtronomers.

But there are yet other inequalities, not obſerv'd by former aſtronomers; by which the motions of the Moon are ſo diſturb'd, that to this day we have not been able to bring them under any certain rule. For the velocities or horary motions of the apogee and nodes of the Moon, and their equations as well as the difference betwixt the greateſt eccentricity in the ſyzygies, and the leaſt eccentricity in the quadratures, and that inequality, which we call the variation, are (by cor. 14. prop. 66. book 1.) in the courſe of the year, augmented and diminiſh'd, in the triplicate proportion of the Sun's apparent diameter. And beſides (by cor. 1 and 2. lem. 10. and cor 16. prop. 66. book 1.) the variation is augmented and diminiſh'd, nearly in the duplicate proportion of the time between the

the quadratures. But in aſtronomical calculations, this inequality is commonly thrown into, and confounded with, the equation of the Moon's centre.

Proposition XXIII. Problem V.

To derive the unequal motions of the ſatellites of Jupiter and Saturn from the motions of our Moon.

From the motions of our Moon we deduce the correſponding motions of the moons or ſatellites of Jupiter, in this manner, by cor. 16. prop. 66. book 1. The mean motion of the nodes of the outmoſt ſatellite of Jupiter, is to the mean motion of the nodes of our Moon, in a proportion compounded of the duplicate proportion of the periodic time of the Earth about the Sun, to the periodic time of Jupiter about the Sun, and the ſimple proportion of the periodic time of the ſatellite about Jupiter to the periodic time of our Moon about the Earth: and therefore thoſe nodes, in the ſpace of an hundred years, are carried 8°. 24′. backwards, or *in antecedentia*. The mean motions of the nodes of the inner ſatellites, are to the mean motion of the nodes of the outmoſt, as their periodic times to the periodic time of the former, by the ſame corollary, and are thence given. And the motion of the apſis of every ſatellite *in conſequentia*, is to the motion of its nodes *in antecedentia*, as the motion of the apogee of our Moon, to the motion of its nodes (by the ſame corollary) and is thence given. But the motions of the apſides thus found, muſt be diminiſh'd in the proportion of 5 to 9, or of about 1 to 2, on account of a cauſe, which I cannot here deſcend to explain. The greateſt equations of the nodes, and of the apſis of every ſatellite, are to the greateſt equations of the nodes, and

and apogee of our Moon respectively, as the motions of the nodes and apsides of the satellites, in the time of one revolution of the former equations, to the motions of the nodes and apogee of our Moon, in the time of one revolution of the latter equations. The variation of a satellite, seen from Jupiter, is to the variation of our Moon, in the same proportion, as the whole motions of their nodes respectively, during the times, in which the satellite and our Moon, (after parting from) are revolv'd (again) to the Sun, by the same corollary; and therefore in the outmost satellite, the variation does not exceed 5″. 12‴.

PROPOSITION XXIV. THEOREM XIX.

That the flux and reflux of the Sea, arise from the actions of the Sun and Moon.

By cor. 19 and 20. prop. *66*. book 1. It appears that the waters of the sea ought twice to rise and twice to fall every day, as well lunar as solar; and that the greatest height of the waters in the open and deep seas, ought to follow the appulse of the luminaries to the meridian of the place, by a less interval than 6 hours; as happens in all that eastern tract of the *Atlantic* and *Æthiopic* seas between *France* and the *Cape of Good Hope*; and on the coasts of *Chili* and *Peru* in the *South-Sea*; in all which shoars the flood falls out about the second, third, or fourth hour, unless where the motion propagated from the deep ocean is by the shallowness of the channels, through which it passes to some particular places, retarded to the fifth, sixth, or seventh hour, and even later. The hours I reckon from the appulse of each luminary to the meridian of the place, as well under, as above the horizon; and by the hours of the lunar day, I understand the 24th parts of that time, which the Moon, by its apparent diurnal motion, employs

ploys to come about again to the meridian of the place which it left the day before. The force of the Sun or Moon in raising the sea, is greatest in the appulse of the luminary to the meridian of the place. But the force impressed upon the sea at that time continues a little while after the impression, and is afterwards encreas'd by a new, though less, force still acting upon it. This makes the sea rise higher and higher, till this new force becoming too weak to raise it any more, the sea rises to its greatest height. And this will come to pass perhaps in one or two hours, but more frequently near the shores in about three hours, or even more where the sea is shallow.

The two luminaries excite two motions, which will not appear distinctly, but between them will arise one mixt motion compounded out of both. In the conjunction or opposition of the luminaries, their forces will be conjoin'd, and bring on the greatest flood and ebb. In the quadratures the Sun will raise the waters which the Moon depresses, and depress the waters which the Moon raises, and from the difference of their forces, the smallest of all tides will follow. And because (as experience tells us) the force of the Moon is greater than that of the Sun, the greatest height of the waters will happen about the third lunar hour. Out of the syzygies and quadratures, the greatest tide, which by the single force of the Moon ought to fall out at the third lunar hour, and by the single force of the Sun at the third solar hour, by the compounded forces of both must fall out in an intermediate time, that approaches nearer to the third hour of the Moon, than to that of the Sun. And therefore while the Moon is passing from the syzygies to the quadratures, during which time the 3d hour of the Sun precedes the 3d hour of the Moon, the greatest height of the waters will also precede the 3d hour of the Moon; and that, by the greatest interval, a little after the octants of the

Moon;

Moon; and by like intervals, the greatest tide will follow the 3d lunar hour, while the Moon is passing from the quadratures to the syzygies. Thus it happens in the open sea. For in the mouths of rivers, the greater tides come later to their height.

But the effects of the luminaries depend upon their distances from the Earth. For when they are less distant, their effects are greater, and when more distant, their effects are less, and that in the triplicate proportion of their apparent diameter. Therefore it is, that the Sun, in the winter time, being then in its perigee, has a greater effect, and makes the tides in the syzygies something greater, and those in the quadratures something less than in the summer season; and every month the Moon, while in the perigee, raises greater tides than at the distance of 15 days before or after, when it is in its apogee. Whence it comes to pass, that two highest tides don't follow, one the other, in two immediately succeeding syzygies.

The effect of either luminary doth likewise depend upon its declination or distance from the equator. For, if the luminary was plac'd at the pole, it would constantly attract all the parts of the waters, without any intension or remission of its action, and could cause no reciprocation of motion. And therefore, as the luminaries decline from the equator towards either pole, they will, by degrees, lose their force, and on this account will excite lesser tides in the solstitial than in the equinoctial syzygies. But in the solstitial quadratures, they will raise greater tides than in the quadratures about the equinoxes; because the force of the Moon then situated in the equator, most exceeds the force of the Sun. Therefore the greatest tides fall out in those syzygies, and the least in those quadratures, which happen about the time of both equinoxes: and the greatest tide in the syzygies is always succeeded by the least tide in the quadratures, as we find by experience.

But, becauſe the Sun is leſs diſtant from the Earth in winter than in ſummer, it comes to paſs that the greateſt and leaſt tides more frequently appear before than after the vernal equinox, and more frequently after than before the autumnal.

Moreover, the effects of the luminaries depend upon the latitudes of places. Let *ApEP Pl.* 10. *Fig.* 2. repreſent the Earth cover'd with deep waters; *C* its centre; *P*, *p* its poles; *AE* the equator; *F*, any place without the equator; *Ff*, the parallel of the place; *Dd* the correſpondent parallel on the other ſide of the equator; *L*, the place of the Moon three hours before; *H*, the place of the Earth directly under it; *h*, the oppoſite place; *K*, *k* the places at 90 degrees diſtance; *CH*, *Ch*, the greateſt heights of the ſea from the centre of the Earth; and *CK*, *ck* its leaſt heights: and if with the axes *Hh*, *Kk*, an ellipſis is deſcrib'd, and by the revolution of that ellipſis about its longer axe *Hh*, a ſphæroid *HPKhpk*. is form'd, this ſphæroid will nearly repreſent the figure of the ſea; and *CF*, *Cf*, *CD*, *Cd*, will repreſent the heights of the ſea in the places *Ff*, *Dd*. But further, in the ſaid revolution of the ellipſis any point *N* deſcribes the circle *NM*, cutting the parallels *Ff*, *Dd*, in any places *RT*; and the equator *AE* in *S*; *CN* will repreſent the height of the ſea in all thoſe places *R*, *S*, *T*, ſituated in this circle. Wherefore in the diurnal revolution of any place *F*, the greateſt flood will be in *F*, at the 3d hour after the appulſe of the Moon to the meridian above the Horizon; and afterwards the greateſt ebb in *Q*, at the 3d hour after the ſetting of the Moon: and then the greateſt flood in *f*, at the 3d hour after the appulſe of the Moon to the meridian under the horizon, and laſtly, the greateſt ebb in *Q*, at the 3d hour after the riſing of the Moon; and the latter flood in *f*, will be leſs than the preceding flood in *F*. For the whole ſea is divided into two hemiſpherical floods, one in the hemiſphere *KHk* on the north ſide, the

the other in the opposite hemisphere *Khk* which we may therefore call the northern and the southern floods. These floods being always opposite the one to the other, come by turns to the meridians of all places, after an interval of 12 lunar hours. And seeing the northern countries partake more of the northern flood, and the southern countries more of the southern flood, thence arise tides, alternately greater and less in all places without the equator, in which the luminaries rise and set. But the greatest tide will happen, when the Moon declines towards the vertex of the place, about the 3d hour after the appulse of the Moon to the meridian above the horizon; and when the Moon changes its declination *to the other side of the equator*, that which was the greater tide will be chang'd into a lesser. And the greatest difference of the floods will fall out about the times of the solstices; especially if the ascending node of the Moon is about the first of Aries. So it is found by experience, that the morning tides in winter exceed those of the evening, and the evening tides in summer exceed those of the morning; at *Plymouth* by the height of one foot, but at *Bristol*, by the height of 15 inches, according to the observations of *Colepress* and *Sturmy*.

But the motions which we have been describing, suffer some alteration from that force of reciprocation, which the waters, being once moved, retain a little while *by their vis insita.* Whence it comes to pass that the tides may continue for some time, tho' the actions of the luminaries should cease. This power of retaining the impress'd motion lessens the difference of the alternate tides and makes those tides which immediately succeed after the syzygies greater, and those which follow next after the quadratures, less. And hence it is, that the alternate tides at *Plymouth* and *Bristol*, don't differ much more one from the other than by the height of a foot or 15 inches, and that the greatest tides of all at those ports are not the first but the third after the syzygies. And besides all the motions are retarded in their passage through shallow channels, so that the greatest tides of all in some streights and mouths of rivers, are the fourth or even the fifth after the syzygies.

Farther it may happen that the tide may be propagated from the ocean through different channels towards the same port, and may pass quicker through some channels than through others, in which case the same tide, divided into two or more succeeding one another, may compound new motions of different kinds. Let us suppose two equal tides flowing towards the same port from different places, the one preceding the other by 6 hours; and suppose the first tide to happen at the third hour of the appulse of the Moon to the meridian of the port. If the Moon at the time of the appulse to the meridian was in the equator, every 6 hours alternately there would arise equal floods, which meeting with as many equal ebbs would so ballance one the other, that for that day the water would stagnate and remain quiet. If the Moon then declined from the equator, the tides in the ocean would be alternately greater and less as was said. And from thence two greater and two lesser tides would be alternately propagated towards that port. But the two greater floods would make the greatest height of the waters to fall out in the middle time betwixt both; and the greater and lesser floods would make the waters to rise to a mean height in the middle time between them, and in the middle time between the two lesser floods the waters would rise to their least height. Thus in the space of 24 hours the waters would come, not twice, as commonly, but once only to their greatest, and once only to their least height; and their greatest height, if the Moon declined towards the elevated pole, would happen at the 6 or 30th hour after the appulse of the Moon to the meridian; and when the Moon changed its declination this flood

would

would be changed into an ebb. An example of all which Dr. *Halley* has given us, from the observations of seamen in the port of *Batsham* in the kingdom of *Tunquin* in the latitude of 20°. 50′. north. In that port, on the day which follows after the passage of the Moon over the equator, the waters stagnate: when the Moon declines to the north they begin to flow and ebb, not twice, as in other ports, but once only every day, and the flood happens at the setting, and the greatest ebb at the rising of the Moon. This tide encreases with the declination of the Moon till the 7th or 8th day; then for the 7 or 8 days following, it decreases at the same rate as it had increased before, and ceases when the Moon changes its declination, crossing over the equator to the south. After which the flood is immediatly chang'd into an ebb; and thenceforth the ebb happens at the setting, and the flood at the rising of the Moon; till the Moon again passing the equator changes its declination. There are two inlets to this port, and the neighbouring channels, one from the seas of *China*, between the continent and the island of *Luconia*, the other from the *Indian* sea, between the continent and the island of *Borneo*. But whether there be really two tides propagated through the said channels, one from the *Indian* sea in the space of 12 hours, and one from the sea of *China* in the space of 6 hours, which therefore happening at the 3d and 9th lunar hours, by being compounded together, produce those motions, or whether there be any other circumstances in the state of those seas, I leave to be determin'd by observations on the neighbouring shoars.

Thus I have explain'd the causes of the motions of the Moon and of the Sea. Now it is fit to subjoin something concerning the quantity of those motions.

PRO-

Proposition XXV. Problem VI.

To find the forces with which the Sun disturbs the motions of the Moon. Pl. 10. Fig. 3.

Let *S* represent the Sun, *T* the Earth, *P* the Moon, *CADB* the Moon's orbit. In *SP* take *SK* equal to *ST*; and let *SL* be to *SK*, in the duplicate proportion of *SK* to *SP*; draw *LM* parallell to *PT*; and if *ST* or *SK* is ſuppos'd to repreſent the accelerated force of gravity of the Earth towards the Sun, *SL* will repreſent the accelerative force of gravity of the Moon towards the Sun. But that force is compounded of the parts *SM* and *LM*, of which the force *LM*, and that part of *SM* which is repreſented by *TM*, diſturb the motion of the Moon, as we have ſhew'd in prop. 66. book 1. and its corollaries. Foraſmuch as the Earth and Moon are revolv'd about their common centre of gravity, the motion of the Earth about that centre will be alſo diſturb'd by the like forces, but we may conſider the ſums both of the forces and of the motions as in the Moon, and repreſent the ſum of the forces by the lines *TM* and *ML*, which are analogous to them both. The force *ML* (in its mean quantity) is, to the centripetal force by which the Moon may be retain'd in its orbit revolving about the Earth at reſt at the diſtance *PT*, in the duplicate proportion of the periodic time of the Moon about the Earth, to the periodic time of the Earth about the Sun (by cor. 17. prop. 66. book 1.) that is in the duplicate proportion of $27^d. 7^h. 43'$. to $365^d. 6^h. 9'$; or as 1000 to 178725; or as 1 to $178\frac{29}{40}$. But in the 4th prop. of this book we found, that if both Earth and Moon were revolv'd about their common centre of gravity, the mean diſtance of the one from the other would be nearly $60\frac{1}{2}$ mean ſemidiameters of the Earth. And the

the force, by which the Moon may be kept revolving in its orbit about the Earth in rest at the distance *PT* of $60\frac{1}{2}$ semidiameters of the Earth, is to the force by which it may be revolv'd in the same time at the distance of 60 semidiameters, as $60\frac{1}{2}$ to 60; and this force is to the force of gravity with us, very nearly as 1 to 60×60. Therefore the mean force *ML* is to the force of gravity on the surface of our Earth, as $1 \times 60\frac{1}{2}$ to $60 \times 60 \times 60 \times 178\frac{29}{40}$, or as 1 to 638092, 6. whence by the proportion of the lines *TM*, *ML*, the force *TM* is also given; and these are the forces with which the Sun disturbs the motions of the Moon. *Q. E. I.*

PROPOSITION XXVI. PROBLEM VII.

To find the horary increment of the area, which the Moon, by a radius drawn to the Earth, describes in a circular orbit.

We have above shew'd that the area, which the Moon describes by a radius drawn to the Earth, is proportional to the time of description; excepting in so far as the Moon's motion is disturb'd by the action of the Sun. And here we propose to investigate the inequality of the moment, or horary increment *of that area, or motion so disturb'd.* To render the calculus more easy, we shall suppose the orbit of the Moon to be circular, and neglect all inequalities, but that only which is now under consideration. And because of the immense distance of the Sun, we shall further suppose, that the lines *SP* and *ST*, are parallel. By this means, the force *LM* *Pl.* 10. *Fig.* 4. will be always reduc'd to its mean quantity *TP*, as well as the force *TM*, to its mean quantity 3 *PK*. These forces, (by cor. 2. of the laws of motion) compose the force *TL*; and this force by letting fall the perpendicular *LE* upon the radius *TP*, is resolv'd into the forces *TE*, *EL*; of which the force *TE*, acting constantly in the direction of the radius *TP*, neither accelerates or retards the description of the area *TPC*, made by that radius *TP*; but *EL acting on the radius TP* in a perpendicular direction, accelerates or retards *the description of the area* in proportion as it accelerates or retards the Moon. That acceleration of the Moon, in its passage from the quadrature *C*, to the conjunction *A*, is in every moment of time, as the *generating* accelerative force *EL*, that is, as $\frac{3PK \times TK}{TP}$ Let the time be represented by the mean motion of the Moon, or (which comes to the same thing) by the angle *CTP*, or even by the arc *CP*. At right angles upon *CT*, erect *CG* equal to *CT*. And supposing the quadrantal arc *AC* to be divided into an infinite number of equal parts *Pp* &c. these *parts* may represent the like *infinite* number of the equal parts of time. Let fall *pk* perpendicular on *CT*; and draw *TG* meeting with *KP*, *kp* produc'd, in *F* and *f*; then will *FK* be equal to *TK*, and *Kk* be to *PK* as *Pp* to *Tp*, that is, in a giv'n proportion; and therefore $FK \times Kk$, or the area *FKkf*, will be as $\frac{3PK \times TK}{TP}$, that is as *EL*; and compounding, the whole area *GCKF* will be as the sum of all the forces *EL* impress'd upon the Moon in the whole time *CP*; and therefore also as the velocity generated by that sum, that is, as the acceleration of the description of the area *CTP*, or as the increment of the moment *thereof*. The force by which the Moon may in its periodic time *CADB* of $27^d. 7^h. 43'$, be retain'd revolving about the Earth in rest at the distance *TP*, would cause a body, falling in the time *CT*, to describe the length $\frac{1}{2}CT$, and at the same time to acquire a velocity equal to that with which the Moon is is mov'd in its orbit. This appears from cor. 9. prop. 4. book 1. But since Kd, drawn perpendicular on TP, is *but* a third part of EL, and *equal to* the half of TP, or ML, in the octants, the force EL in the octants, where it is greatest, will exceed the force ML, in the proportion of 3 to 2; and therefore will be to that force by which the Moon in its periodic time may be retain'd revolving about the Earth at rest, as 100 to $\frac{2}{3} \times 17872\frac{1}{2}$, or 11915; and in the time CT will generate a velocity equal to $\frac{100}{11915}$ parts of the velocity of the Moon; but in the time CPA, will generate a greater velocity in the proportion of CA to CT or TP. Let the greatest force EL in the octants be represented by the area $FK \times Kk$, or by the rectangle $\frac{1}{2}\, TP \times Pp$, which is equal thereto. And the velocity which that greatest force can generate in any time CP, will be to the velocity which any other lesser force EL can generate in the same time, as the rectangle $\frac{1}{2}\, TP \times CP$ to the area $KCGF$; but the velocities generated in the whole time CPA, will be one to the other as the rectangle $\frac{1}{2}\, TP \times CA$ to the triangle TCG; or as the quadrantal arc CA to the radius TP. And therefore (by prop. 9. book 5. elem.) the latter velocity generated in the whole time, will be $\frac{100}{11915}$ parts of the velocity of the Moon. To this velocity of the Moon, which is proportional to the mean moment of the area (supposing this mean moment to be represented by the number 11915) we add and substract the half of the other velocity; the sum 11915 + 50, or 11965 will represent the greatest moment of the area in the syzygy A; and the difference 11915 — 50, or 11865, the least moment thereof in the quadratures. Therefore the areas, which in equal times, are described in the syzygies and quadratures, are, one to the other, as 11965 to 11865. And if to the least moment 11865, we add a moment which shall be

to

to 100, the difference of the two former moments as the trapezium *FKCG* to the triangle *TCG*, or which comes to the same thing, as the square of the sine *PK* to the square of the radius *TP*, (that is, as *Pd* to *TP*) the sum will represent the moment of the area, when the Moon is in any intermediate place *P*.

But these things take place, only in the hypothesis that the Sun and the Earth are at rest, and that the synodical revolution of the Moon is finished in $27^{d}. 7^{h}. 43'$. But since the Moon's synodical period is really $29^{d}. 12^{h}. 44'$, the increments of the moments must be inlarged, in the same proportion as the time is, that is, in the proportion of 1080853 to 1000000. Upon which account, the whole increment, which was $\frac{100}{11915}$ parts of the mean moment, will now become $\frac{100}{11023}$ parts thereof. And therefore the moment of the area, in the quadrature of the Moon, will be to the moment thereof in the syzygy, as 11023—50 to 11023+50; or as 10973 to 11073; and to the moment thereof when the Moon is in any intermediate place *P*, as 10973 to 10973+*Pd*; that is, supposing *TP*=100.

The area therefore, which the Moon, by a radius drawn to the Earth, describes in the several little equal parts of time, is nearly as the sum of the number 219,46, and the versed sine of the double distance of the Moon from the nearest quadrature, considered in a circle which hath unity for its radius. Thus it is, when the variation in the octants is in its mean quantity. But if the variation there is greater or less, that versed sine must be augmented or diminished in the same proportion.

PRO-

PROPOSITION XXVII. PROBLEM VIII.

From the horary motion of the Moon, to find its diſtance from the Earth.

The area which the Moon, by a radius drawn to the Earth, deſcribes in every moment of time, is as the horary motion of the Moon, and the ſquare of the diſtance of the Moon from the Earth conjunctly. And therefore the diſtance of the Moon from the Earth is in a proportion compounded of the ſubduplicate proportion of the area directly, and the ſubduplicate proportion of the horary motion inverſely. *Q. E. I.*

COR. 1. Hence the apparent diameter of the Moon is given. For it is reciprocally as the diſtance of the Moon from the Earth. Let aſtronomers try how accurately this rule agrees with the phænomena.

COR. 2. Hence alſo the orbit of the Moon may be more exactly defin'd from the phænomena than hitherto could be done.

PROPOSITION XXVIII. PROBLEM IX.

To find the Diameters of the orbit, in which, without eccentricity, the Moon would move.

The curvature of the orbit which a body deſcribes, if attracted in lines perpendicular to the orbit, is as the force of attraction directly, and the ſquare of the velocity inverſely. I eſtimate the curvatures of lines, compared one with another, according to the evaneſcent proportion of the ſines or tangents of their angles of contact to equal radij, ſuppoſing thoſe radij to be infinitely diminiſhed. But the attraction of the Moon towards the Earth in the ſyzygies, is the exceſs of its gravity

gravity towards the Earth above the force of the Sun $2PK$ (see *Fig.* prop. 25.) bywhich force, the accelerative gravity of the Moon towards the Sun exceeds the accelerative gravity of the Earth towards the Sun, or is exceeded by it. But in the quadratures that attraction is the sum of the gravity of the Moon towards the Earth, and the Sun's force KT, by which the Moon is attracted towards the Earth. And these attractions, putting N for $\frac{AT+CT}{2}$ are nearly as $\frac{178725}{AT^2} + \frac{2000}{CT \times N}$ and $\frac{178725}{CT^2} + \frac{1000}{AT \times N}$, or as $178725\, N \times CT^2 - 2000\, AT^2 \times CT$, and $178725\, N \times AT^2 + 1000\, CT^2 \times AT$. For if the accelerative gravity of the Moon towards the Earth be represented by the number 178725, the mean force ML, which in the quadratures is PT or TK, and draws the Moon towards the Earth, will be 1000; and the mean force TM, in the syzygies will be 3000; from which, if we substract the mean force ML, there will remain 2000, the force by which the Moon in the syzygies is drawn from the Earth, and which we above called $2PK$. But the velocity of the Moon in the syzygies A and B, is to its velocity in the quadratures C and D, as CT to AT, and the moment of the area, which the Moon by a radius drawn to the Earth describes in the syzygies, to the moment of that area *described* in the quadratures conjunctly; that is, as $11073\, CT$ to $10973\, AT$. Take this ratio twice inversely, and the former ratio once directly, and the curvature of the orb of the Moon in the syzygies will be to the curvature thereof in the quadratures, as $120406729 \times 178725\, AT^2 \times CT^2 \times N - 120406729 \times 2000\, AT^4 \times CT$, to $122611329 \times 178725\, AT^2 \times CT^2 \times N + 122611329 \times 1000\, CT^4 \times AT$, that is, as $2151969\, AT \times CT \times N - 24081\, AT^3$ to $2191371\, AT \times CT \times N + 12261\, CT^3$.

Because

Because the figure of the Moon's orbit is unknown, let us, in its stead, assume the ellipse *DBCA*, *Pl.* 10. *Fig.* 5. in the centre of which we suppose the Earth to be situated, and the greater axe *de* to lie between the quadratures, as the lesser *AB* between the syzygies. But since the plane of this ellipse is revolved about the Earth by an angular motion, and the orbit, whose curvature we now examine should be described in a plane void of such motion; we are to consider the figure which the Moon, while it is revolved in that ellipse, describes in this plane, that is to say the figure *Cpa*, the several points *p* of which are found by assuming any point *P* in the ellipse, which may represent the place of the Moon, and drawing *Tp* equal to *TP*, in such manner that the angle *PTp* may be equal to the apparent motion of the Sun from the time of the last quadrature in *C*; or (which comes to the same thing) that the angle *CTp* may be to the angle *CTP*, as the time of the synodic revolution of the Moon to the time of the periodic revolution thereof, or as $29^d. 12^h. 44'$, to $27^d. 7^h. 43'$. If therefore in this proportion we take the angle *CTa* to the right angle *CTA*, and make *Ta* of equal length with *TA*; we shall have *a* the lower, and *C* the upper apsis of this orbit. But by computation I find, that the difference betwixt the curvature of this orbit *Cpa* at the vertex *a*, and the curvature of a circle described about the centre *T*, with the interval *TA*, is to the difference betwixt the curvature of the ellipse at the vertex *A*, and the curvature of the same circle, in the duplicate proportion of the angle *CTP* to the angle *CTp*; and that the curvature of the ellipse in *A*, is to the curvature of that circle, in the duplicate proportion of *TA* to *TC*; and the curvature of that circle to the curvature of a circle described about the centre *T* with the interval *TC*, as *TC* to *TA*; but that the curvature of this *last arch* is to the curvature of the ellipse in *C*, in the duplicate

plicate proportion of TA to TC; and that the difference betwixt the curvature of the ellipse in the vertex C, and the curvature of this last circle, is to the difference betwixt the curvature of the figure Tpa, at the vertex C, and the curvature of this same *last* circle, in the duplicate proportion of the angle CTp to the angle CTP. All which proportions are easily drawn from the sines of the angles of contact, and of the differences of those angles. But by comparing those proportions together, we find the curvature of the figure Cpa at a, to be to its curvature at C, as $AT^3. \times \frac{16824}{100000} CT^2 AT$ to $CT^3 + \frac{16824}{100000} AT^2 \times CT$. Where the number $\frac{16824}{100000}$ represents the difference of the squares of the angles CTP and CTp, applied to the square of the lesser angle CTP; or (which is all one) the difference of the squares of the times $27^d. 7^h. 43'$, and $29^d. 12^h. 44'$. applied to the square of the time $27^d. 7^h. 43'$.

Since therefore a represents the syzygy of the Moon, and C its quadrature, the proportion now found must be the same with that proportion of the curvature of the Moon's orb in the syzygies, to the curvature thereof in the quadratures, which we found above. Therefore, in order to find the proportion of CT to AT, Let us multiply the extremes and the means, and the terms which come out applied to $AT \times CT$, become $2062{,}79\ CT^4 - 2151969\ N \times CT^3 + 368676\ N \times AT \times CT^2 + 36342\ AT^2 \times CT^2 - 362047\ N \times AT^2 \times CT + 2191371\ N \times AT^3 + 4051{,}4\ AT^4 = 0$. Now if for the half sum N of the terms AT and CT, we put 1, and x for their half difference, then CT will be $= 1 + x$, and $AT = 1 - x$. And substituting those values in the equation, after resolving thereof, we shall find $x = 0{,}00719$; and from thence the semidiameter $CT = 1{,}00719$, and the semidiameter $AT = 0{,}99281$, which numbers are nearly as $70\frac{1}{24}$, and $69\frac{1}{24}$. Therefore the Moon's distance from the Earth

Earth in the ſyzygies, is to its diſtance in the quadratures (ſetting aſide the conſideration of eccentricity) as $69\frac{1}{24}$ to $70\frac{1}{24}$; or in round numbers as 69 to 70.

Proposition XXIX. Problem X.

To find the variation of the Moon.

This inequality is owing partly to the elliptic figure of the Moon's orbit, partly to the inequality of the moments of the area which the Moon by a radius drawn to the Earth deſcribes. If the Moon *P* revolved in the ellipſe *DBCA*, about the Earth quieſcent in the centre of the ellipſe, and by the radius *TP*, drawn to the Earth, deſcribed the area *CTP*, proportional to the time *of deſcription*; and the greateſt ſemidiameter *CT* of the ellipſe was to the leaſt *TA* as 70 to 69; the tangent of the angle *CTP* would be to the tangent of the angle of the mean motion computed from the quadrature *C*, as the ſemidiameter *TA* of the ellipſe, to its ſemidiameter *TC*, or as 69 to 70. But the deſcription of the area *CTP*, as the Moon advances from the quadrature to the ſyzygy, ought to be in ſuch manner accelerated, that the moment of the area in the Moon's ſyzygy, may be to the moment thereof in its quadrature, as 11073 to 10973; and that the exceſs of the moment in any intermediate place *P*, above the moment in the quadrature, may be as the ſquare of the ſine of the angle *CTP*. Which we may effect with accuracy enough, if we diminiſh the tangent of the angle *CTP*, in the ſubduplicate proportion of the number 10973 to the number 11073, that is, in proportion of the number 68,6877 to the number 69. Upon which account the tangent of the angle *CTP*, will now be to the tangent of the mean motion, as 68,6877 to 70; and the angle *CTP*, in the octants, where the mean

mean motion is 45°, will be found 44°. 27'. 28". which subſtracted from 45°. the angle of the mean motion, leaves the greateſt variation 32'. 32". Thus it would be, if the Moon in paſſing from the quadrature to the ſyzygy, deſcribed an angle *CTA* of 90 degrees only. But becauſe of the motion of the Earth, by which the Sun is apparently transferr'd *in conſequentia*, the Moon, before it overtakes the Sun, deſcribes an angle *CTa*, greater than a right angle, in the proportion of the time of the ſynodic revolution of the Moon, to the time of its periodic revolution, that is, in the proportion of $29^d. 12^h. 44'$. to $27^d. 7^h. 43'$. Whence it comes to paſs, that all the angles about the centre *T*, are dilated in the ſame proportion, and the greateſt variation, which otherwiſe would be *but* 32'. 32", now augmented in the ſaid proportion becomes 35'. 10".

And this is its magnitude in the mean diſtance of the Sun from the Earth, neglecting the differences, which may ariſe from the curvature of the *orbis magnus*, and the ſtronger action of the Sun upon the Moon when horn'd and new, than when gibbous and full. In other diſtances, of the Sun from the Earth, the greateſt variation is in a proportion compounded of the duplicate proportion of the time of the ſynodic revolution of the Moon (the time of the year being given) directly, and the triplicate proportion of the diſtance of the Sun from the Earth, inverſely. And therefore, in the apogee of the Sun, the greateſt variation is 33'. 14", and in its perigee, 37'. 11", if the eccentricity of the Sun is to the tranſverſe ſemidiameter of the *orbis magnus*, as $16\frac{15}{16}$ to 1000.

Hitherto we have inveſtigated the variation in an orb not eccentric, in which, to wit, the Moon in its octants is always in its mean diſtance from the Earth. If the Moon, on account of its eccentricity, is more or leſs removed from the Earth, than if placed in this orb, the variation may be ſomething greater, or ſomething

thing leſs, than according to this rule. But I leave the exceſs or defect to the determination of aſtronomers from the phænomena.

Proposition XXX. Problem XI.

To find the horary motion of the nodes of the Moon in a circular orbit, Pl. 11. Fig. 1.

Let *S* repreſent the Sun, *T* the Earth, *P* the Moon. *NPn* the orbit of the Moon, *Npn* the orthographic projection of the orbit upon the plane of the ecliptic; *N*, *n* the nodes; *nTNm*, the line of the nodes produced indefinitely; *PI*, *PK* perpendiculars upon the lines *ST*, *Qq*; *Pp* a perpendicular upon the plane of the ecliptic; *A*, *B* the Moon's ſyzygies in the plane of the ecliptic; *AZ* a perpendicular let fall upon *Nn*, the line of the nodes; *Q*, *q* the quadratures of the Moon in the plane of the ecliptic, and *pK*, a perpendicular on the line *Qq* lying between the quadratures. The force of the Sun to diſturb the motion of the Moon (by prop. 25.) is twofold, one proportional to the line *LM*, the other to the line *MT*, in the ſcheme of that propoſition. And the Moon by the former force is drawn towards the Earth, by the latter towards the Sun, in a direction parallel to the right line *ST* joining the Earth and the Sun. The former force *LM* acts in the direction of the plane of the Moon's orbit, and therefore makes no change upon the ſituation thereof, and is upon that account to be neglected. The latter force *MT*, by which the plane of the Moon's orbit is diſturbed, is the ſame with the force 3 *PK* or 3 *IT*. And this force (by prop. 25.) is to the force, by which the Moon may, in its periodic time, be uniformly revolved in a circle about the Earth at reſt, as 3 *IT* to the radius of the circle multiplied by the number 178,725, or as *IT* to the radius thereof multiplied by 59,575. But in this calculus, and all that follows I consider all the lines drawn from the Moon to the Sun, as parallel to the line which joins the Earth and the Sun, because what inclination there is, almost as much diminishes all effects in some cases, as it augments them in others, and we are now enquiring after the mean motions of the nodes, neglecting such niceties as are of no moment, and would only serve to render the calculus more perplext.

Now suppose *PM* to represent an arc which the Moon describes in the least moment of time, and *ML* a little line, the half of which the Moon, by the impulse of the said force 3 *IT*, would describe in the same time. And joining *PL*, *MP*, let them be produced to *m* and *l*, where they cut the plane of the ecliptic, and upon *Tm* let fall the perpendicular *PH*. Now since the right line *ML* is parallel to the plane of the ecliptic, and therefore can never meet with the right line *ml* which lies in that plane, and yet both those right lines lye in one common plane *LMPml*, they will be parallel, and upon that account the triangles *LMP*, *lmP* will be similar. And seeing *MPm* lies in the plane of the orbit, in which the Moon did move while in the place *P*; the point *m* will fall upon the line *Nn*, which passes through the nodes *N*, *n*, of that orbit. And because the force by which the half of the little line *LM* is generated, if the whole had been together, and at once impressed in the point *P*, would have generated that whole line, and caused the Moon to move in the arc whose chord is *LP*; that is to say, would have transferred the Moon from the plane *MPmT* into the plane *LPlT*; therefore the angular motion of the nodes generated by that force, will be equal to the angle *mTl*. But *ml* is to *mP*, as *ML* to *MP*; and since *MP*, because of the time given, is also given, *ml* will be as the rectangle

tangle $ML \times mP$, that is, as the rectangle $IT \times mP$. And, if Tml is a right angle, the angle mTl will be as $\frac{ml}{Tm}$ and therefore as $\frac{IT \times Pm}{Tm}$, that is, (because Tm and mP, TP and PH are proportional) as $\frac{IT \times PH}{TP}$; and therefore, because TP is given, as $IT \times PH$. But if the angle Tml or STN is oblique, the angle mTl will be yet less, in proportion of the sine of the angle STN to the radius, or AZ to AT. And therefore the velocity of the nodes, is as $IT \times PH \times AZ$, or as the solid content of the sines of the three angles, TPI, PTN, and STN.

If these are right angles, as happens when the nodes are in the quadratures, and the Moon in the syzygy, the little line ml will be removed to an infinite distance, and the angle mTl will become equal to the angle mPl. But in this case the angle mPl is to the angle PTM, which the Moon in the same time by its apparent motion describes about the Earth, as 1 to 59,575. For the angle mPl is equal to the angle LPM, that is, to the angle of the Moon's deflexion from a rectilinear path, which angle, if the gravity of the Moon should have then ceased, the said force of the Sun $3\,IT$ would by it self have generated in that given time; and the angle PTM is equal to the angle of the Moon's deflexion from a rectilinear path, which angle, if the force of the Sun $3\,IT$ should have then ceased, the force alone by which the Moon is retained in its orbit would have generated in the same time. And these forces (as we have above shew'd) are, the one to the other, as 1 to 59,575. Since therefore, the mean horary motion of the Moon (in respect of the fixt Stars) is $32'.\ 56''.\ 27'''.\ 12\frac{1}{2}^{iv}$, the horary motion of the node in this case will be $33''.\ 10'''.\ 33^{iv}.\ 12^{v}$. But in other cases, the horary motion will be to $33''$.

$10'''. 33^{iv}. 12^{v}$. as the solid content of the sines of the three angles *TPI*, *PTN* and *STN* (or of the distances of the Moon from the quadrature, of the Moon from the node, and of the node from the Sun) to the cube of the radius. And as often as the sine of any angle is changed from positive to negative, and from negative to positive, so often must the regressive be changed into a progressive, and the progressive into a regressive motion. Whence it comes to pass, that the nodes are progressive, as often as the Moon happens to be placed between either quadrature, and the node nearest to that quadrature. In other cases, they are regressive, and by the excess of the regress above the progress, they are monthly transferred *in antecedentia*.

Cor. 1. Hence if from *P* and *M*, the extreme points of a least arc *PM*, *Pl.* 11. *Fig.* 2. on the line *Qq* joining the quadratures we let fall the perpendiculars *PK*, *Mk*, and produce the same till they cut the line of the nodes *Nn*, in *D* and *d*; the horary motion of the nodes will be as the area *MPDd*, and the square of the tine *AZ* conjunctly. For let *PK*, *PH* and *AZ* be the three said sines, *viz.* *PK* the sine of the distance of the Moon from the quadrature, *PH* the sine of the distance of the Moon from the node, and *AZ* the sine of the distance of the node from the Sun: and the velocity of the node will be as the solid content of $PK \times PH \times AZ$. But *PT* is to *PK*, as *PM* to *Kk*; and therefore, because *PT* and *PM* are given, *Kk* will be as *PK*. Likewise *AT* is to *PD*, as *AZ* to *PH*, and therefore *PH* is as the rectangle $PD \times AZ$, and by compounding those proportions, $PK \times PH$ is as the solid content $Kk \times PD \times AZ$, and $PK \times PH \times AZ$, as $Kk \times PD \times AZ^2$. that is, as the area *PDdM* and AZ^2 conjunctly. Q. E. D.

Cor.

COR. 2. In any given poſition of the nodes, their mean horary motion is half their horary motion in the Moon's ſyzygies; and therefore is to 16″. 35‴. 16^{iv}. 36^{v}, as the ſquare of the ſine of the diſtance of the nodes from the ſyzygies to the ſquare of the radius, or as AZ^2, to AT^2. For if the Moon, by an uniform motion deſcribes the ſemicircle *QAq*, the ſum of all the areas *PDdM* during the time of the Moon's paſſage from *Q* to *M*, will make up the area *QMdE*, terminating at the tangent *QE* of the circle. And by the time that the Moon has arrived at the point *n*, that ſum will make up the whole area *EQAn* deſcribed by the line *PD*; but when the Moon proceeds from *n* to *q*, the line *PD* will fall without the circle, and will deſcribe the area *nqe*, terminating at the tangent *qe* of the circle; which area, becauſe the nodes were before regreſſive, but are now progreſſive, muſt be ſubducted from the former area, and being itſelf equal to the area *QEN*, will leave the ſemicircle *NQAn*. While therefore the Moon deſcribes a ſemicircle, the ſum of all the areas *PDdM* will be the area of that ſemicircle; and while the Moon deſcribes a complete circle, the ſum of thoſe areas will be the area of the whole circle. But the area *PDdM*, when the Moon is in the ſyzygies is the rectangle of the arc *PM* into the radius *PT*; and the ſum of all the areas, *every one* equal to this area, in the time that the Moon deſcribes a complete circle is the rectangle of the whole circumference into the radius of the circle; and this rectangle, being double the area of the circle, will be double the quantity of the former ſum. If therefore the nodes went on with that velocity uniformly continued, which they acquire in the Moon's ſyzygies, they would deſcribe a ſpace double of that which they deſcribe in fact; and therefore the mean motion, by which, if uniformly continued, they would deſcribe the ſame ſpace with that which they do in fact deſcribe by an unequal motion, is *but* one half of that motion which they are possessed of in the Moon's syzygies. Wherefore since their greatest horary motion, if the nodes are in the quadratures, is $33''. 10'''. 33^{iv}. 12^{v}$, their mean horary motion in this case will be $16''. 35'''. 16^{iv}. 36^{v}$. And seeing the horary motion of the nodes is every where as AZ^2 and the area $PDdM$ conjunctly, and therefore in the Moon's syzygies, the horary motion of the nodes is as AZ^2 and the area $PDdM$ conjunctly, that is, (because the area $PDdM$ described in the syzygies is given) as AZ^2; therefore the mean motion also will be as AZ^2, and therefore when the nodes are without the quadratures, this motion will be to $16''. 35'''. 16^{iv}. 36^{v}$. as AZ^2 to AT^2. *Q. E. D.*

Proposition XXXI. Problem XII.

To find the horary motion of the nodes of the Moon in an elliptic orbit, Pl. 12. Fig. 1.

Let $Qpmaq$ represent an ellipse, described with the greater axe Qq, and the lesser axe ab; $QAqB$ a circle circumscribed; T the Earth in the common centre of both; S the Sun; p the Moon moving in this ellipse; and pm an arc which it describes in the least moment of time; N and n the nodes joined by the line Nn; pK and mk perpendiculars upon the axe Qq, produced both ways till they meet the circle in P and M, and the line of the nodes in D and d. And if the Moon, by a radius drawn to the Earth, describes an area proportional to the time *of description*, the horary motion of the node in the ellipse will be as the area $pDdm$, and AZ^2 conjunctly.

For let PF touch the circle in P, and produced meet TN in F; and pf touch the ellipse in p, and produced meet the same TN in f, and both tangents concur

Fig. 1. p. 273.

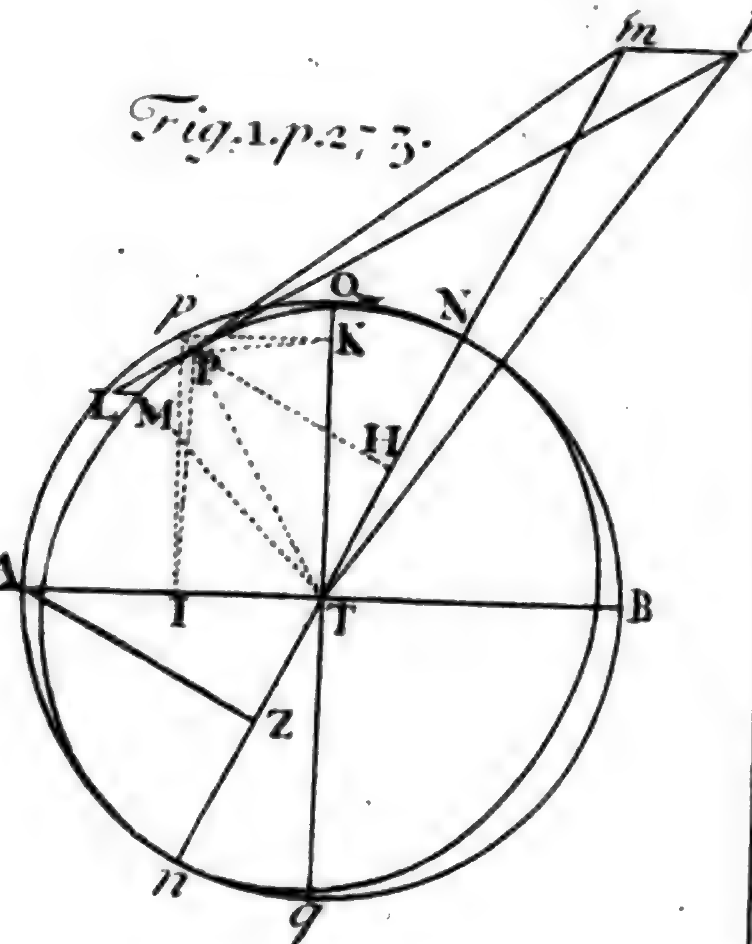
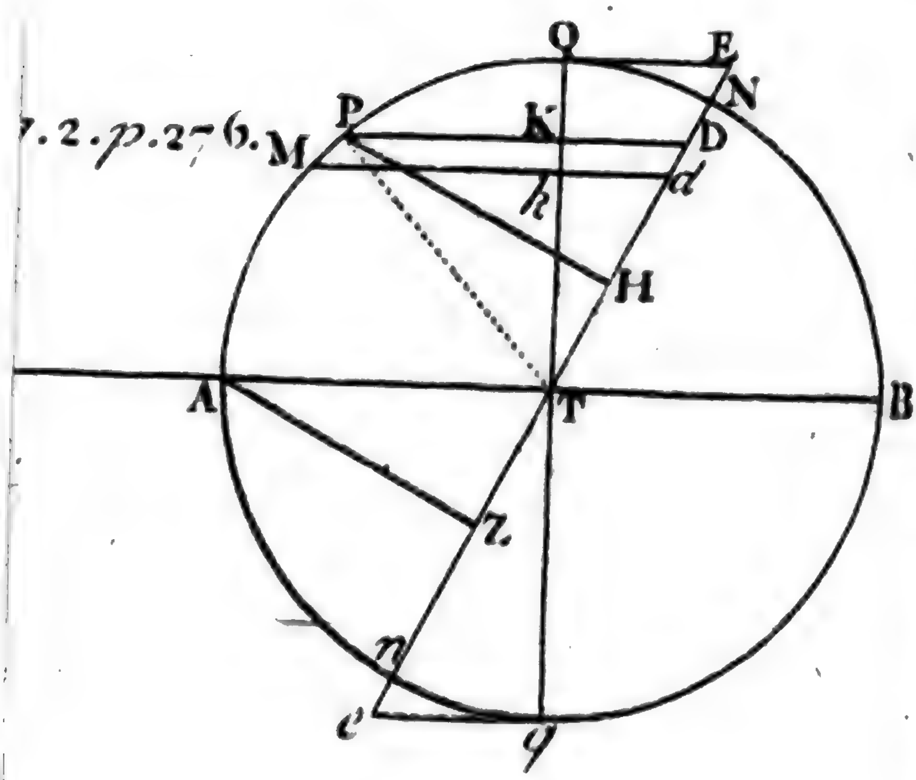


Fig. 2. p. 276.



in the axe *TQ* at *Y*. And let *ML* represent the ſpace which the Moon, by the impulſe of the abovementioned force 3 *IT* or 3 *PK*, would deſcribe with a transverſe motion, in the mean time while, revolving in the circle it deſcribes the arc *PM*; and *ml* denote the ſpace, which the Moon revolving in the ellipſe would deſcribe in the ſame time by the impulſe of the ſame force 3 *IT* or 3 *PK*; and let *LP* and *lp* be produced till they meet the plane of the ecliptic in *G* and *g*, and *FG* and *fg* be joined, of which *FG* produced may cut *pf*, *pg*, and *TQ* in *c*, *e* and *R* reſpectively; and *fg* produced may cut *TQ* in *r*. Becauſe the force 3 *IT* or 3 *PK* in the circle, is to the force 3 *IT* or 3 *pK* in the ellipſe, as *PK* to *pK*, or as *AT* to *aT*; the ſpace *ML*, generated by the former force, will be to the ſpace *ml* generated by the latter, as *PK* to *pK*, that is, becauſe of the ſimilar figures *PYKp*, and *FYRc*, as *FR* to *cR*. But (becauſe of the ſimilar triangles *PLM*, *PGF*) *ML* is to *FG*, as *PL* to *PG*, that is (on account of the parallels *Lk*, *PK*, *GR*) as *pl* to *pe*, that is, (becauſe of the ſimilar triangles *plm*, *cpe*) as *lm* to *ce*; and inverſely as *LM* is to *lm*, or as *FR* is to *cR*, ſo is *FG* to *ce*. And therefore if *fg* was to *ce*, as *fy* to *cY*, that is as *fr* to *cR*, (that is as *fr* to *FR* and *FR* to *cR* conjunctly, that is, as *fT* to *FT*, and *FG* to *ce* conjunctly) becauſe the ratio of *FG* to *ce*, expung'd on both ſides, leaves the ratios *fg* to *FG* and *fT* to *FT*, *fg* would be to *FG*, as *fT* to *FT*; and therefore the angles which *FG* and *fg* would ſubtend at the Earth *T* would be equal each to other. But theſe angles, (by what we have ſhew'd in the preceding propoſition) are the motions of the nodes, while the Moon deſcribes, in the circle the arc *PM*, in the ellipſe the arc *pm*: And therefore the motions of the nodes in the circle, and in the ellipſe, would be equal to each other. Thus I ſay it would be if *fg* was to *ce*, as *fY* to *cY*,] that is, if fg was equal to $\frac{ce \times f\Upsilon}{c\Upsilon}$. But because of the similar triangles fgp, cep, fg is to ce as fp to cp; and therefore fg is equal to $\frac{ce \times fp}{cp}$; and therefore the angle which fg subtends in fact, is to the former angle which FG subtends, that is to say, the motion of the nodes in the ellipse is to the motion of the same in the circle, as this fg or $\frac{ce \times fp}{cp}$, to the former fg or $\frac{ce \times f\Upsilon}{c\Upsilon}$, that is as $fp \times c\Upsilon$ to $f\Upsilon \times cp$, or as fp to $f\Upsilon$, and $c\Upsilon$ to cp, that is, if ph parallel to TN meet FP in h, as Fh to $F\Upsilon$ and $F\Upsilon$ to FP; that is, as Fh to FP or Dp to DP, and therefore as the area $Dpmd$ to the area $DPMd$. And therefore seeing (by corol. 1. prop. 30.) the latter area and AZ^2 conjunctly are proportional to the horary motion of the nodes in the circle, the former area and AZ^2 conjunctly will be proportional to the horary motion of the nodes in the ellipse. *Q. E. D.*

COR. Since therefore in any given position of the nodes, the sum of all the areas $pDdm$, in the time while the Moon is carried from the quadrature to any place m, is the area $mpQEd$ terminated at the tangent of the ellipse QE; and the sum of all those areas, in one entire revolution, is the area of the whole ellipse: the mean motion of the nodes in the ellipse will be to the mean motion of the nodes in the circle, as the ellipse to the circle; that is, as Ta to TA or 69 to 70. And therefore since (by corol. 2. prop. 30.) the mean horary motion of the nodes in the circle is to $16''. 35'''. 16^{iv}. 36^{v}$. as AZ^2 to AT^2, if we take the angle $16''. 21'''. 3^{iv}. 30^{v}$. to the angle $16''. 35'''. 16^{iv}. 36^{v}$. as 69 to 70, the mean horary motion of the nodes in the ellipse will be to $16''. 21'''. 3^{iv}. 30^{v}$. as AZ^2 to AT^2; that is, as the square of the sine of the distance of the node from the Sun to the square of the radius.

But

But the Moon, by a radius drawn to the Earth, describes the area in the syzygies with a greater velocity than it does that in the quadratures, and upon that account the time is contracted in the syzygies, and prolong'd in the quadratures; and together with the time the motion of the nodes is likewise augmented or diminish'd. But the moment of the area in the quadrature of the Moon, was to the moment thereof in the syzygies as 10973 to 11073; and therefore the mean moment in the octants is to the excess in the syzygies, and to the defect in the quadratures, as 11023, the half sum of those numbers, to their half difference 50. Wherefore since the time of the Moon's mora in the several little equal parts of its orbit, is reciprocally as its velocity; the mean time in the octants will be to the excess of the time in the quadratures, and to the defect *of the time* in the syzygies, arising from this cause, nearly as 11023 to 50. But reckoning from the quadratures to the syzygies, I find that the excess of the moments of the area, in the several places, above the least moment in the quadratures, is nearly as the square of the sine of the Moon's distance from the quadratures; and therefore the difference betwixt the moment in any place, and the mean moment in the octants, is as the difference betwixt the square of the sine of the Moon's distance from the quadratures, and the square of the sine of 45 degrees, or half the square of the radius; and the increment of the time in the several places between the octants and quadratures, and the decrement thereof between the octants and syzygies is in the same proportion. But the motion of the nodes while the Moon describes the several little equal parts of its orbit, is accelerated or retarded in the duplicate proportion of the time. For that motion while the Moon describes PM, is (*cæteris paribus*) as ML, and ML is in the duplicate proportion of the time. Wherefore the motion of the nodes in the syzygies, in the time while the Moon deſcribes giv'n little parts of its orbit, is diminiſh'd in the duplicate proportion of the number 11073 to the number 11023; and the decrement is to the remaining motion as 100 to 10973; but to the whole motion as 100 to 11073 nearly. But the decrement in the places between the octants and ſyzygies, and the increment in the places between the octants and quadratures, is to this decrement, nearly as the whole motion in theſe places to the whole motion in the ſyzygies, and the difference betwixt the ſquare of the ſine of the Moon's diſtance from the quadrature, and the half ſquare of the radius, to the half ſquare of the radius conjunctly. Wherefore, if the nodes are in the quadratures, and we take two places, one on one ſide, one on the other, equally diſtant from the octant and other two diſtant by the ſame interval, one from the ſyzygy, the other from the quadrature, and from the decrements of the motions in the two places between the ſyzygy and octant, we ſubtract the increments of the motions in the two other places between the octant and the quadrature; the remaining decrement will be equal to the decrement in the ſyzygy: as will eaſily appear by computation. And therefore the mean decrement, which ought to be ſubducted from the mean motion of the nodes, is the fourth part of the decrement in the ſyzygy. The whole horary motion of the nodes in the ſyzygies (when the Moon by a radius drawn to the Earth, was ſuppos'd to deſcribe an area proportional to the time) was $32''. 42'''. 7^{iv}$. And we have ſhew'd, that the decrement of the motion of the nodes, in the time while the Moon, now moving with greater velocity, deſcribes the ſame ſpace, was to this motion as 100 to 11073; and therefore this decrement is $17'''. 43^{iv}. 11^{v}$. The fourth part of which $4'''. 25^{iv}. 48^{v}$. ſubtracted from the mean horary motion above found $16''. 21'''. 3^{iv}. 30^{v}$. leaves $16''. 16'''. 37^{iv}. 42^{v}$. their correct mean horary motion.

If

If the nodes are without the quadratures, and two places are consider'd, one on one side, one on the other equally distant from the syzygies; the sum of the motions of the nodes when the Moon is in those places, will be to the sum of their motions, when the Moon is in the same places and the nodes in the quadratures, as AZ^2, to AT^2. And the decrements of the motions, arising from the causes but now explained, will be mutually as the motions themselves, and therefore the remaining motions will be mutually betwixt themselves as AZ^2. to AT^2. And the mean motions will be as the remaining motions. And therefore in any giv'n position of the nodes, their correct mean horary motion is to $16''. 16'''. 37^{iv}. 42^{v}$. as AZ^2, to AT^2. that is, as the square of the sine of the distance of the nodes from the syzygies to the square of the radius.

Proposition XXXIII. Problem XIII.

To find the mean motion of the nodes of the Moon. Pl. 12. Fig. 2.

The yearly mean motion is the sum of all the mean horary motions, throughout the course of the year. Suppose that the node is in *N*, and that after ev'ry hour is elaps'd, it is drawn back again to its former place; so that, notwithstanding its proper motion, it may constantly remain in the same situation, with respect to the fixt Stars; while in the mean time the Sun *S*, by the motion of the Earth, is seen to leave the node and to proceed till it compleats its apparent annual course by an uniform motion. Let *Aa* represent a given least arc, which the right line *TS* always drawn to the Sun, by its intersection with the circle *NAn*, describes in the least given moment of time; and the mean

mean horary motion (from what we have above ſhew'd) will be as AZ^2, that is (becauſe AZ and ZY are proportional) as the rectangle of AZ into ZY, that is, as the area $AZYa$. And the ſum of all the mean horary motions from the beginning will be as the ſum of all the areas $aYZA$, that is as the area NAZ. But the greateſt $AZYa$ is equal to the rectangle of the arc Aa into the radius of the circle; and therefore the ſum of all theſe rectangles in the whole circle, will be to the like ſum of all the greateſt rectangles, as the area of the whole circle to the rectangle of the whole circumference into the radius, that is, as 1 to 2. But the horary motion correſponding to that greateſt rectangle, was $16''. 16'''. 37^{iv}. 42^{v}$. and this motion in the complete courſe of the ſidereal year $365^d. 6^h. 9'$. amounts to $39°. 38'. 7''. 50'''$. and therefore the half thereof $19°. 49'. 3''. 55'''$. is the mean motion of the nodes correſponding to the whole circle. And the motion of the nodes, in the time while the Sun is carry'd from N to A is to $19°. 49'. 3''. 55'''$. as the area NAZ to the whole circle.

Thus it would be if the node was after every hour drawn back again to its former place, that ſo, after a compleat revolution, the Sun at the year's end would be found again in the ſame node which it had left when the year begun. But becauſe of the motion of the node in the mean time, the Sun muſt needs meet the node ſooner, and now it remains that we compute the abreviation of the time. Since then the Sun, in the courſe of the year, travels 360 degrees, and the node in the ſame time by its greateſt motion would be carried $39°. 38'. 7''. 50'''$, or 39,6355 degrees; and the mean motion of the node in any place N, is to its mean motion in its quadratures, as AZ^2 to AT^2: the motion of the Sun will be to the motion of the node in N, as $360\ AT^2$, to $39{,}6355\ AZ^2$; that is, as $9{,}0827646\ AT^2$ to AZ^2. Wherefore if we ſuppoſe the circum-

ference

ference *NAn* of the whole circle to be divided into little equal parts, such as *Aa*, the time in which the Sun would describe the little arc *Aa*, if the circle was quiescent, will be to the time of which it would describe the same arc, supposing the circle together with the nodes to be revolv'd about the centre *T*, reciprocally as $9,0827646\, AT^2$ to $9,0827646\, AT^2 + AZ^2$. For the time is reciprocally as the velocity with which the little arc is describ'd, and this velocity is the sum of the velocities of both Sun and node. If therefore the sector *NTA* represent the time in which the Sun by it self, without the motion of the node, would describe the arc *NA*, and the indefinitely small part *ATa* of the sector represent the little moment of the time, in which it would describe the least arc *Aa*; and (letting fall *aY* perpendicular upon *Nn*) if in *AZ* we take *dZ*, of such length, that the rectangle of *dZ* into *ZY*, may be to the least part *ATa* of the sector, as AZ^2 to $9,0827646\, AT^2 + AZ^2$, that is to say, that *dZ* may be to $\frac{1}{2}AZ$, as AT^2 to $9,0827646\, AT^2 + AZ^2$; the rectangle of *dZ* into *ZY* will represent the decrement of the time arising from the motion of the node, while the arc *Aa* is describ'd. And if the curve *NdGn* is the locus where the point *d* is always found, the curvilinear area *NdZ* will be as the whole decrement *of time* while the whole arc *NA* is describ'd. And therefore, the excess of the sector *NAT* above the area *NdZ* will be as the whole time. But because the motion of the node in a less time, is less in proportion of the time, the area *AaYZ* must also be diminish'd in the same proportion. Which may be done by taking in *AZ* the line *eZ* of such length, that it may be to the length of *AZ*, as AZ^2 to $9,0827646\, AT^2 + AZ^2$. For so the rectangle of *eZ* into *ZY*, will be to the area *AZYa*, as the decrement of the time in which the arc *Aa* is describ'd, to the whole time in which it would

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would have been deſcrib'd, if the node had been quieſcent. And therefore that rectangle will be as the decrement of the motion of the node. And if the curve *NeFn* is the locus of the point *e*, the whole area *NeZ*, which is the ſum of all the decrements *of that motion*, will be as the whole decrement *thereof* during the time in which the arc *AN* is deſcrib'd; and the remaining area *NAe* will be as the remaining motion, which is the true motion of the node, during the time in which the whole arc *NA* is deſcrib'd by the joint motions of both Sun and node. Now the area of the ſemicircle is to the area of the figure *NeFn* found by the method of infinite ſeries, nearly as 793 to 60. But the motion correſponding *or proportional* to the whole circle was $19^{\circ}\ 49'.\ 3''.\ 55'''$. and therefore the motion correſponding to double the figure *NeFn* is $1^{\circ}.\ 29'.\ 58''.\ 2'''$. which taken from the former motion leaves $18^{\circ}.\ 19'.\ 5''.\ 53'''$. the whole motion of the node with reſpect to the fixed Stars in the interval between two of its conjunctions with the Sun; and this motion ſubducted from the annual motion of the Sun 360°. leaves $341^{\circ}.\ 40'.\ 54''.\ 7'''$. the motion of the Sun in the interval between the ſame conjunctions. But as this motion is to the annual motion 360°. ſo is the motion of the node but juſt now found $18^{\circ}\ 19'.\ 5''.\ 53'''$. to its annual motion which will therefore be $19^{\circ}.\ 18'.\ 1''.\ 23'''$. And this is the mean motion of the nodes in the ſidereal year. By aſtronomical tables it is $19^{\circ}.\ 21'.\ 21''.\ 50'''$. The difference is leſs than $\frac{1}{300}$ part of the whole motion, and ſeems to ariſe from the eccentricity of the Moon's orbit, and its inclination to the plane of the ecliptic. By the eccentricity of this orbit, the motion of the nodes is too much accelerated, and on the other hand, by the inclination of the orbit, the motion of the nodes is ſomething retarded, and reduc'd to its juſt velocity.

PRO-

Proposition XXXIII. Problem XIV.

To find the true motion of the nodes of the Moon. Pl. 12. Fig. 3.

In the time which is as the area *NTA—NdZ* (in the preceding *Fig.*) that motion is as the area *NAe*, and is thence giv'n. But because the calculus is too difficult it will be better to use the following construction of the problem. About the centre *C*, with any interval *CD*, describe the circle *BEFD*, produce *DC* to *A*, so as *AB* may be to *AC*, as the mean motion to half the mean true motion when the nodes are in their quadratures (that is, as 19°. 18′. 1″. 23‴. to 19°. 49′. 3″. 55‴. and therefore *BC* to *AC*, as the difference of those motions 0°. 31′. 2″. 32‴. to the latter motion 19°. 49′. 3″. 55‴. that is, as 1 to $38\frac{3}{10}$). Then through the point *D*, draw the indefinite line *Gg*, touching the circle in *D*; and if we take the angle *BCE*, or *BCF*, equal to the double distance of the Sun from the place of the node, as found by the mean motion; and drawing *AE* or *AF*, cutting the perpendicular *DG* in *G*, we take another angle which shall be to the whole motion of the node, in the interval between its syzygies (that is to 9°. 11′. 3″.) as the tangent *DG* to the whole circumference of the circle *BED*; and add this *last* angle (for which the angle *DAG* may be us'd) to the mean motion of the nodes, while they are passing from the quadratures to the syzygies, and subtract it from their mean motion, while they are passing from the syzygies to the quadratures; we shall have their true motion. For the true motion so found will nearly agree with the true motion which comes out from assuming the time as the area *NTA—NdZ*, and the motion of the node as the area *NAe*,

as

as whoever will please to examine and make the computations will find. And this is the semi-menstrual equation of the motion of the nodes. But there is also a menstrual equation, but which is by no means necessary for finding of the Moon's latitude. For since the variation of the inclination of the Moon's orbit to the plane of the ecliptic is liable to a twofold inequality: the one semi-menstrual, the other menstrual: the menstrual inequality *of this variation*, and the menstrual equation of the nodes, so moderate and correct each other, that in computing the latitude of the Moon both may be neglected.

COR. From this and the preceding prop. it appears that the nodes are quiescent in their syzygies, but regressive in their quadratures, by an hourly motion of 16″. 19‴. 26^{iv}. And that the equation of the motion of the nodes in the octants is 1°. 30′. all which exactly agree with the phænomena of the heavens.

SCHOLIUM.

Mr. *Machin* Astron. Prof. Gresh. and Dr. *Henry Pemberton* separately found out the motion of the nodes by a different method. Mention has been made of this method in another place. Their several papers, both of which I have seen, contained two propositions, and exactly agreed with each other in both of them. Mr. *Machin*'s paper coming first to my hands, I shall here insert it.

OF

f the motion of the Moon's nodes.

PROPOSITION I.

mean motion of the Sun from the node, is 'efined by a geometric mean proportional, be- :ween the mean motion of the Sun, and that mean motion with which the Sun recedes with the greatest swiftness from the node in the quadratures.

Let *T* (*Pl.* 13. *Fig.* 1.) be the Earth's place, *Nn* the line of the Moon's nodes at any given time, *KTM* a perpendicular thereto, *TA* a right line revolving about the centre with the same angular velocity with which the Sun and the node recede from one another, in such sort that the angle between the quiescent right line *Nn*, and the revolving line *TA*, may " be always equal to the distance of the places of the " Sun and node. Now if any right line *TK* be divi- " ded into parts, *TS* and *SK*, and those parts be taken " as the mean horary motion of the Sun to the mean " horary motion of the node in the quadratures, and " there be taken the right line *TH*, a mean proportio- " nal between the part *TS* and the whole *TK*, this " right line will be proportional to the Sun's mean mo- " tion from the node.

" For let there be described the circle *NKnM* from " the centre *T* and with the radius *TK*, and about the " same centre, with the semi-axes *TH* and *TN*, let there " be described an ellipsis *NHnL*. And in the time in " which the Sun recedes from the node through the arc " *Na*, if there be drawn the right line *Tba*, the area of " the sector *NTa* will be the exponent of the sum of " the motions of the Sun and node in the same time. " Let therefore the extremely small arc *aA* be that " which the right line *Tba*, revolving according to the " abovesaid law, will uniformly describe in a given par-

“ ticle of time, and the extremely small sector *TAa* will
“ be as the sum of the velocities with which the Sun
“ and node are carried two different ways in that time.
“ Now the Sun's velocity is almost uniform, its inequa-
“ lity being so small as scarcely to produce the least in-
“ equality in the mean motion of the nodes. The other
“ part of this sum, namely the mean quantity of the ve-
“ locity of the node, is increased in the recess from the
“ syzygies in a duplicate ratio of the sine of its distance
“ from the Sun (by corol. prop. 31. of this book) and
“ being greatest in its quadratures with the Sun in *K*,
“ is in the same ratio to the Sun's velocity as *SK* to *TS*,
“ that is, as (the difference of the squares of *TK* and
“ *TH*, or) the rectangle *KHM* to TH^2. But the
“ ellipsis *NBH* divides the sector *ATa*, the exponent
“ of the sums of these two velocities, into two parts
“ *ABba* and *BTb*, proportional to the velocities. For
“ produce *BT* to the circle in β, and from the point
“ *B* let fall upon the greater axis the perpendicular *BG*,
“ which being produced both ways may meet the circle
“ in the points *F* and *f*; and because the space *ABba*
“ is to the sector *TBb* as the rectangle $AB\beta$ to BT^2,
“ (that rectangle being equal to the difference of the
“ squares of *TA* and *TB*, because the right line $A\beta$
“ is equally cut in *T*, and unequally in *B*;) therefore
“ when the space *ABba* is the greatest of all in *K*,
“ this ratio will be the same as the ratio of the rectangle
“ *KHM* to HT^2. But the greatest mean velocity of
“ the node was shewn above to be in that very ratio to
“ the velocity of the Sun; and therefore in the quadra-
“ tures the sector *ATa* is divided into parts proportio-
“ nal to the velocities. And because the rectangle *KHM*
“ is to HT^2, as *FBf* to BG^2, and the rectangle $AB\beta$
“ is equal to the rectangle *FBf*; therefore the little a-
“ rea *ABba*, where it is greatest, is to the remaining
“ sector *TBb*, as the rectangle $AB\beta$ to BG^2. But the
“ ratio of these little areas always was as the rectangle

“ $AB\beta$

" $AB\beta$ to BT^2, and therefore the little area $ABba$ in the place A is less than its correspondent little area in the quadratures, in the duplicate ratio of BG to BT, that is, in the duplicate ratio of the sine of the Sun's distance from the node. And therefore the sum of all the little areas $ABba$, to wit, the space ABN will be as the motion of the node in the time in which the Sun hath been going over the arc NA since he left the node. And the remaining space, namely the elliptic sector NTB, will be as the Sun's mean motion in the same time. And because the mean annual motion of the node is that motion which it performs in the time that the Sun completes one period of its course, the mean motion of the node from the Sun will be to the mean motion of the Sun it self, as the area of the circle to the area of the ellipsis; that is as the right line TK to the right line TH, which is a comean proportional between TK and TS; or which mes to the same, as the mean proportional TH to the right line TS.

Proposition II.

The mean motion of the Moon's nodes being given, to find their true motion.

" Let the angle A be the distance of the Sun from the mean place of the node, or the Sun's mean motion from the node. Then if we take the angle B, whose tangent is to the tangent of the angle A, as TH to TK, that is, in the subduplicate ratio of the mean horary motion of the Sun to the mean horary motion of the Sun from the node, when the node is in the quadrature, that angle B will be the distance of the Sun from the node's true place. For join FT, and by the demonstration of the last proportion, the angle FTN will be the distance of the Sun from the mean

" place of the node, and the angle *ATN* the distance " from the true place, and the tangents of these angles " are between themselves as *TK* to *TH*.

" COR. Hence the angle *FTA* is the equation of " the Moon's nodes, and the sine of this angle where " it is greatest in the octants, is to the radius as *KH* " to *TK* + *TH*. But the sine of this equation in a- " ny other place *A* is to the greatest sine, as the sine " of the sums of the angles *FTN* + *ATN* to the radius; " that is, nearly as the sine of double the distance of " the Sun from the mean place of the node (namely " 2 *FTN*) to the radius.

SCHOLIUM.

" If the mean horary motion of the nodes in the qua- " dratures be $16''. 16'''. 37^{iv}. 42^{v}$. that is in a whole " sidereal year $39°. 38'. 7''. 50'''$. *TH* will be to " *TK* in the sub-duplicate ratio of the number " 9,0827646 to the number 10, 827646, that is, as " 18, 6524761 to 19, 6524761. And therefore *TH* " is to *HK* as 18,6524761 to 1, that is, as the moti- " on of the Sun in a sidereal year to the mean motion " of the node $19°. 18'. 1''. 23\frac{2}{3}'''$.

" But if the mean motion of the Moon's nodes in " 20 Julian years is $386°. 50'. 15''$. as is collected from " the observations made use of in the theory of the " Moon, the mean motion of the nodes in one sidereal " year will be $19°. 20'. 31''. 58'''$. And *TH* will be " to *HK* as $360°$. to $19°. 20'. 31''. 58'''$. that is, " as 18, 61214 to 1, and from hence the mean horary " motion of the nodes in the quadratures will come out " $16''. 18'''. 48^{iv}$. And the greatest equation of the " nodes in the octants will be $1° 29'. 57''$.

PRO-

PROPOSITION XXXIV. PROBLEM XV.

To find the horary variation of the inclination of the Moon's orbit to the plane of the ecliptic.

Let *A* and *a*, (*Pl.* 13. *Fig.* 2.) represent the syzygies; *Q* and *q* the quadratures; *N* and *n* the nodes; *P* the place of the Moon in its orbit; *p* the orthographic projection of that place upon the plane of the ecliptic; and *mTl* the momentaneous motion of the nodes as above. If upon *Tm* we let fall the perpendicular *PG*, and joining *pG* we produce it till it meet *Tl* in *g*, and join also *Pg*; the angle *PGp* will be the inclination of the Moon's orbit to the plane of the ecliptic when the Moon is in *P*; and the angle *Pgp* will be the inclination of the same after a small moment of time is elaps'd; and therefore the angle *GPg* will be the momentaneous variation of the inclination. But this angle *GPg* is to the angle *GTg*, as *TG* to *PG* and *Pp* to *PG* conjunctly. And therefore if for the moment of time we assume an hour; since the angle *GTg* (by prop. 30.) is to the angle $33''.\ 10'''.\ 33^{iv}$. as $IT \times PG \times AZ$, to AT^3, the angle *GPg* (or the horary variation of the inclination) will be to the angle $33''.\ 10'''.$ 33^{iv}. as $IT \times AZ \times TG \times \frac{Pp}{PG}$ to AT^3. *Q. E. I.*

And thus it would be if the Moon was uniformly revolv'd in a circular orbit. But if the orbit is elliptical, the mean motion of the nodes will be diminish'd in proportion of the lesser axis to the greater, as we have shewn above. And the variation of the inclination will be also diminish'd in the same proportion.

COR. 1. Upon *Nn* erect the perpendicular *TF*, and let *pM* be the horary motion of the Moon in the plane of the ecliptic; upon QT let fall the perpendiculars pK, Mk, and produce them till they meet TF in H and h; then IT will be to AT, as Kk to Mp; and TG to Hp as TZ to AT; and therefore $IT \times TG$ will be equal to $\frac{Kk \times Hp \times TZ}{Mp}$, that is, equal to the area $HpMh$ multiplied into the ratio $\frac{TZ}{Mp}$: and therefore the horary variation of the inclination will be to $33''.\ 10'''.\ 33^{iv}.$ as the area $HpMh$ multiply'd into $AZ \times \frac{TZ}{Mp} \times \frac{Pp}{PG}$ to AT^3.

COR. 2. And therefore, if the Earth and nodes were after every hour drawn back from their new, and instantly restor'd to their old places, so as their situation might continue given for a whole periodic month together; the whole variation of the inclination during that month would be to $33''.\ 10'''.\ 33^{iv}$, as the aggregate of all the areas $HpMh$, generated in the time of one revolution of the point p, (with due regard in summing to their proper signs $+$ and $-$). multiply'd into $AZ \times TZ \times \frac{Pp}{PG}$ to $Mp \times AT^3$, that is, as the whole circle $QAqa$ multiply'd into $AZ \times TZ \times \frac{Pp}{PG}$ to $Mp \times AT^3$, that is, as the circumference $QAqa$ multiply'd into $AZ \times TZ \times \frac{Pp}{PG}$ to $2\ Mp \times AT^2$.

COR. 3. And therefore, in a giv'n position of the nodes, the mean horary variation, from which, if uniformly continu'd through the whole month, that menstrual variation might be generated, is to $33''.\ 10'''.\ 33^{iv}.$ as $AZ \times TZ \times \frac{Pp}{PG}$ to $2\ AT^2$, or as $Pp \times \frac{AZ \times TZ}{\frac{1}{2}AT}$ to $PG \times 4\ AT$, that is (because Pp is to PG, as the sine

sine of the aforesaid inclination to the radius; and $\frac{AZ \times TZ}{\frac{1}{2} AT}$ to 4 AT, as the sine of double the angle ATn to four times the radius) as the sine of the same inclination multiply'd into the sine of double the distance of the nodes from the Sun, to four times the square of the radius.

Cor. 4. Seeing the horary variation of the inclination, when the nodes are in the quadratures, is (by this prop.) to the angle 33″. 10‴. 33^{iv}, as $IT \times AZ \times TG \times \frac{Pp}{PG}$ to AT^3, that is, as $\frac{IT \times TG}{\frac{1}{2} AT} \times \frac{Pp}{PG}$, to 2 AT, that is, as the sine of double the distance of the Moon from the quadratures multiply'd into $\frac{Pp}{PG}$ to twice the radius: the sum of all the horary variations during the time that the Moon, in this situation of the nodes, passes from the quadrature to the syzygy (that is in the space of $177\frac{1}{6}$ hours) will be to the sum of as many angles 33″. 10‴. 33^{iv}. or 5878″, as the sum of all the sines of double the distance of the Moon from the quadratures multiply'd into $\frac{Pp}{PG}$, to the sum of as many diameters; that is, as the diameter multiplied into $\frac{Pp}{PG}$ to the circumference; that is, if the inclination be 5°. 1′, as $7 \times \frac{874}{10000}$ to 22, or as 278 to 10000. And therefore the whole variation, compos'd out of the sum of all the horary variations in the foresaid time, is 163″. or 2′. 43″.

Proposition XXXV. Problem XVI.

To a given time to find the inclination of the Moon's orbit to the plane of the ecliptic.

Let AD (*Pl.* 14. *Fig.* 1.) be the ſine of the greateſt inclination, and AB the ſine of the leaſt. Biſect BD in C; and round the centre C, with the interval BC, deſcribe the circle BGD. In AC take CE in the ſame proportion to EB as EB to twice BA. And if to the time giv'n we ſet off the angle AEG equal to double the diſtance of the nodes from the quadratures, and upon AD let fall the perpendicular GH; AH will be the ſine of the inclination requir'd.

For GE^2 is equal to $GH^2 + HE^2 = BHD + HE^2 = HBD + HE^2 - BH^2 = HBD + BE^2 - 2\,BH \times BE = BE^2 + 2\,EC \times BH = 2\,EC \times AB + 2\,EC \times BH = 2\,EC \times AH$. Wherefore ſince $2\,EC$ is giv'n, GE^2 will be as AH. Now let AEg repreſent double the diſtance of the nodes from the quadratures, in a given moment of time after, and the arc Gg, on account of the giv'n angle GEg, will be as the diſtance GE. But Hh is to Gg, as GH to GC, and therefore Hh is as the rectangle $GH \times Gg$, or $GH \times GE$, that is, as $\frac{GH}{GE} \times GE^2$ or $\frac{GH}{GE} \times AH$; that is, as AH and the ſine of the angle AEG conjunctly. If therefore in any one caſe, AH be the ſine of inclination, it will increaſe by the ſame increments as the ſine of inclination doth, by cor. 3. of the preceding prop. and therefore will always continue equal to that ſine. But when the point G falls upon either point B or D, AH is equal to this ſine, and therefore remains always equal thereto. *Q. E. D.*

In this demonſtration I have ſuppos'd, that the angle

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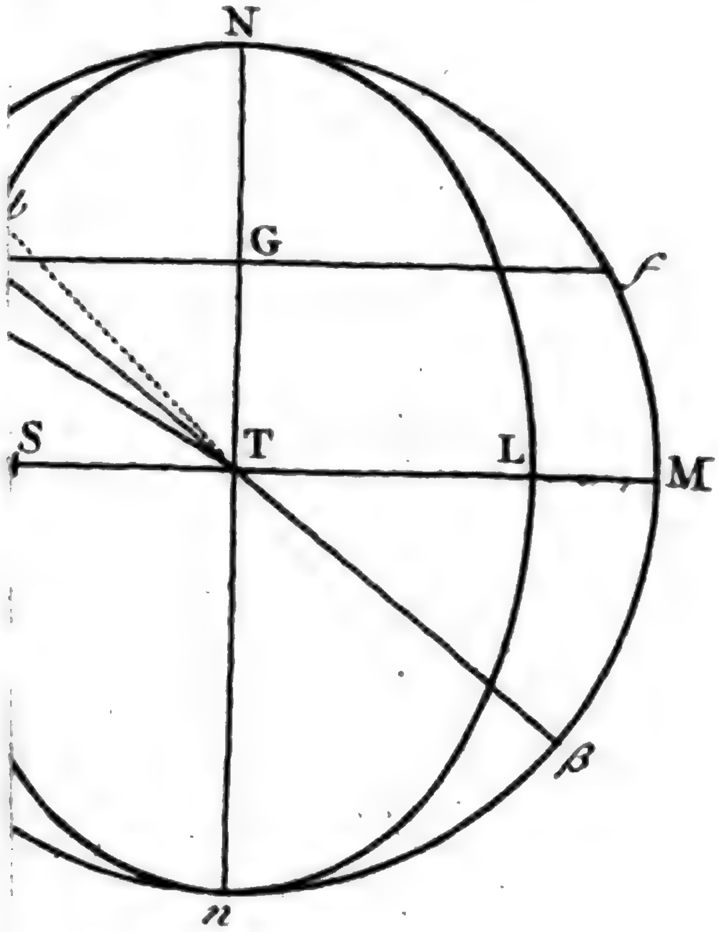

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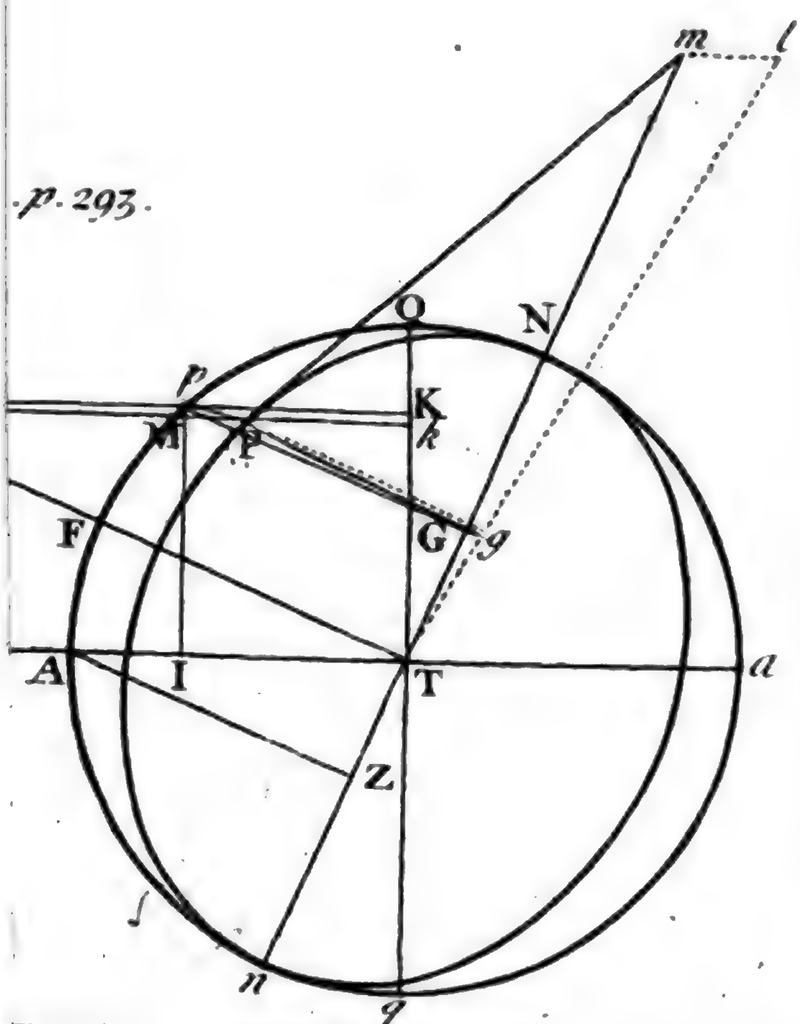

gle BEG representing double the distance of the nodes from the quadratures, increaseth uniformly. For I cannot descend to ev'ry minute circumstance of inequality. Now suppose that BEG is a right angle, and that Gg is in this case the horary increment of double the distance of the nodes from the Sun; then by cor. 3. of the last prop. the horary variation of the inclination in the same case, will be to $33''. 10'''. 33^{iv}.$ as the rectangle of AH the sine of the inclination into the sine of the right angle BEG, double the distance of the nodes from the Sun, to four times the square of the radius; that is, as AH the sine of the mean inclination to four times the radius, that is, seeing the mean inclination is about $5^{\circ}. 8\frac{1}{2}$, as its sine 896 to 40000, the quadruple of the radius, or as 224 to 10000. But the whole variation, corresponding to BD the difference of the sines, is to this horary variation, as the diameter BD to the arc Gg, that is, conjunctly as the diameter BD to the semi-circumference BGD, and as the time of $2079\frac{7}{10}$ hours, in which the node proceeds from the quadratures to the syzygies, to one hour, that is, as 7 to 11 and $2079\frac{7}{10}$ to 1. Wherefore compounding all these proportions, we shall have the whole variation BD to $33''. 10'''. 33^{iv}.$ as $224 \times 7 \times 2079\frac{7}{10}$ to 110000, that is, as 29645 to 1000; and from thence that variation BD will come out $16'. 23\frac{1}{2}''.$

And this is the greatest variation of the inclination, abstracting from the situation of the Moon in its orbit. For if the nodes are in the syzygies, the inclination suffers no change from the various positions of the Moon. But if the nodes are in the quadratures, the inclination is less when the Moon is in the syzygies than when it is in the quadratures, by a difference of $2'. 43''.$ as we shew'd in cor. 4. of the preceding prop. and the whole mean variation BD, diminish'd by $1'. 21\frac{1}{2}''.$ the half of this excess, becomes $15'. 2''.$ when

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the Moon is in the quadratures; and increas'd by the ſame, becomes 17′. 45″. when the Moon is in the ſyzygies. If therefore the Moon be in the ſyzygies, the whole variation in the paſſage of the nodes from the quadratures to the ſyzygies will be 17′. 45″. And therefore if the inclination be 5°. 17′. 20″. when the nodes are in the ſyzygies, it will be 4°. 59′. 35″. when the nodes are in the quadratures and the Moon in the ſyzygies. The truth of all which is confirm'd by obſervations.

Now if the inclination of the orbit ſhould be requir'd, when the Moon is in the ſyzygies, and the nodes any where between them and the quadratures; let *AB* be to *AD*, as the ſine of 4°. 59′. 35″. to the ſine of 5°. 17′. 20″. and take the angle *AEG*, equal to double the diſtance of the nodes from the quadratures; and *AH* will be the ſine of the inclination deſir'd. To this inclination of the orbit the inclination of the ſame is equal, when the Moon is 90°. diſtant from the nodes. In other ſituations of the Moon, this menſtrual inequality to which the variation of the inclination is obnoxious in the calculus of the Moon's latitude, is balanc'd and in a manner took off, by the menſtrual inequality of the motion of the nodes (as we ſaid before) and therefore may be neglected in the computation of the ſaid latitude.

SCHOLIUM.

By theſe computations of the lunar motions, I was willing to ſhew that by the theory of gravity the motions of the Moon could be calculated from their phyſical cauſes. By the ſame theory I moreover found, that the annual equation of the mean motion of the Moon ariſes from the various dilatation which the orbit of the Moon ſuffers from the action of the Sun, accord-

according to cor. 6. prop. 66. book 1. The force of this action is greater in the perigeon Sun, and dilates the Moon's orbit; in the apogeon Sun it is less, and permits the orbit to be again contracted. The Moon moves slower in the dilated, and faster in the contracted orbit; and the annual equation, by which this inequality is regulated, vanishes in the apogee and perigee of the Sun. In the mean distance of the Sun from the Earth it arises to about 11′. 50″. In other distances of the Sun, it is proportional to the equation of the Sun's centre, and is added to the mean motion of the Moon, while the Earth is passing from its aphelion to its perihelion, and subducted while the Earth is in the opposite semicircle. Taking for the radius of the *orbis magnus*, 1000, and $16\frac{7}{8}$ for the Earth's eccentricity, this equation when of the greatest magnitude, by the theory of gravity comes out 11′. 49″. But the eccentricity of the Earth seems to be something greater, and with the eccentricity this equation will be augmented in the same proportion. Suppose the eccentricity $16\frac{11}{12}$, and the greatest equation will be 11′. 51″.

Further, I found that the apogee and nodes of the Moon move faster in the perihelion of the Earth, where the force of the Sun's action is greater, than in the aphelion thereof, and that in the reciprocal triplicate proportion of the Earth's distance from the Sun. And hence arise annual equations of those motions proportional to the equation of the Sun's centre. Now the motion of the Sun is in the reciprocal duplicate proportion of the Earth's distance from the Sun, and the greatest equation of the centre, which this inequality generates, is 1°. 56′. 20″. corresponding to the abovemention'd eccentricity of the the Sun $16\frac{11}{12}$. But if the motion of the Sun had been in the reciprocal triplicate proportion of the distance, this inequality would have generated the greatest equation 2°. 54′. 30″.

30″. And therefore the greatest equations which the inequalities of the motions of the Moon's apogee and nodes do generate, are to 2°. 54′. 30″. as the mean diurnal motion of the Moon's apogee and the mean diurnal motion of its nodes are to the mean diurnal motion of the Sun. Whence the greatest equation of the mean motion of the apogee comes out 19′. 43″. and the greatest equation of the mean motion of the nodes 9′. 24″. The former equation is added, and the latter subducted, while the Earth is passing from its perihelion to its aphelion, and contrariwise when the Earth is in the opposite semicircle.

By the theory of gravity I likewise found, that the action of the Sun upon the Moon is something greater when the transverse diameter of the Moon's orbit passeth through the Sun, than when the same is perpendicular upon the line which joins the Earth and the Sun: And therefore the Moon's orbit is something larger in the former than in the latter case. And hence arises another equation of the Moon's mean motion, depending upon the situation of the Moon's apogee in respect of the Sun; which is in its greatest quantity, when the Moon's apogee is in the octants of the Sun, and vanishes when the apogee arrives at the quadratures or syzygies. And it is added to the mean motion, while the Moon's apogee is passing from the quadrature of the Sun to the syzygy, and subducted while the apogee is passing from the syzygy to the quadrature. This equation, which I shall call the semi-annual, when greatest in the octants of the apogee, arises to about 3′. 45″. so far as I could collect from the phænomena. And this is its quantity in the mean distance of the Sun from the Earth. But it is increased and diminished in the reciprocal triplicate proportion of the Sun's distance, and therefore is nearly 3′. 34″. when that distance is greatest, and 3′. 56″. when least. But when the Moon's apogee is without the octants, it becomes

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less, and is to its greatest quantity, as the sine of double the distance of the Moon's apogee from the nearest syzygy, or quadrature to the radius.

By the same theory of gravity, the action of the Sun upon the Moon is something greater, when the line of the Moon's nodes passes through the Sun, than when it is at right angles with the line which joins the Sun and the Earth. And hence arises another equation of the Moon's mean motion, which I shall call the second semi-annual, and this is greatest when the nodes are in the octants of the Sun, and vanishes when they are in the syzygies or quadratures; and in other positions of the nodes is proportional to the sine of double the distance of either node from the nearest syzygy or quadrature. And it is added to the mean motion of the Moon, if the Sun is *in antecedentiâ* to the node which is nearest to him, and subducted if *in consequentiâ*; and in the octants, where it is of the greatest magnitude, it arises to 47″. in the mean distance of the Sun from the Earth, as I find from the theory of gravity. In other distances of the Sun this equation, greatest in the octants of the nodes, is reciprocally as the cube of the Sun's distance from the Earth, and therefore in the Sun's perigee it comes to about 49″, and in its apogee to about 45″.

By the same theory of gravity, the Moon's apogee goes forward at the greatest rate, when it is either in conjunction with or in opposition to the Sun, but in its quadratures with the Sun it goes backward. And the eccentricity comes, in the former case, to its greatest quantity, in the latter to its least, by cor. 7. 8. and 9. prop. 66. book 1. And those inequalities by the corollaries we have nam'd, are very great, and generate the principal, which I call the semi-annual, equation of the apogee. And this semi-annual equation in its greatest quantity comes to about 12°. 18″. as nearly ly as I could collect from the phænomena. Our countryman *Horrox* was the first who advanced the theory of the Moon's moving in an ellipse about the Earth placed in its lower focus. Dr. *Halley* improved the notion, by putting the centre of the ellipse in an epicycle whose centre is uniformly revolved about the Earth. And from the motion in this epicycle the mentioned inequalities in the progress and regress of the apogee, and in the quantity of eccentricity do arise. Suppose the mean distance of the Moon from the Earth to be divided into 100000 parts, and let *T* (*Pl.* 14. *Fig.* 2.) represent the Earth, and *TC* the Moon's mean eccentricity of 5505 such parts. Produce *TC* to *B*, so as *CB* may be the sine of the greatest semi-annual equation 12°. 18′, to the radius *TC*; and the circle *BDA* described about the centre *C*, with the interval *CB*, will be the epicycle spoke of, in which the centre of the Moon's orbit is placed, and revolved according to the order of the letters *BDA*. Set off the angle *BCD* equal to twice the annual argument, or twice the distance of the Sun's true place from the place of the Moon's apogee once equated, and *CTD* will be the semi-annual equation of the Moon's apogee, and *TD* the eccentricity of its orbit, tending to the place of the apogee now twice equated. But having the Moon's mean motion, the place of its apogee, and its eccentricity, as well as the longer axe of its orbit 200000; from these *data* the true place of the Moon in its orbit, together with its distance from the Earth, may be determined by the methods commonly known.

In the perihelion of the Earth where the force of the Sun is greatest, the centre of the Moon's orbit moves faster about the centre *C*, than in the aphelion, and that in the reciprocal triplicate proportion of the Sun's distance from the Earth. But because the equation of the Sun's centre is included in the annual argument, the centre of the Moon's orbit moves faster in

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its epicycle *BDA*, in the reciprocal duplicate proportion of the Sun's distance from the Earth. Therefore that it may move yet faster in the reciprocal simple proportion of the distance; suppose that from *D* the centre of the orbit a right line *DE* is drawn, tending towards the Moon's apogee once equated, that is, parallel to *TC*; and set off the angle *EDF* equal to the excess of the foresaid annual argument above the distance of the Moon's apogee from the Sun's perigee *in consequentia*; or, which comes to the same thing, take the angle *CDF* equal to the complement of the Sun's true anomaly to 360°. And let *DF* be to *DC*, as twice the eccentricity of the *orbis magnus* to the Sun's mean distance from the Earth and the Sun's mean diurnal motion from the Moon's apogee to the Sun's mean diurnal motion from its own apogee conjunctly, that is, as $33\frac{7}{8}$ to 1000, and 52′. 27″. 16‴. to 59′. 8″. 10‴. conjunctly; or as 3 to 100. And imagine the centre of the Moon's orbit, placed in the point *F*, to be revolved in an epicycle whose centre is *D*, and radius *DF*, while the point *D* moves in the circumference of the circle *DABD*. For by this means the centre of the Moon's orbit comes to describe a certain curve line, about the centre *C*, with a velocity which will be almost reciprocally as the cube of the Sun's distance from the Earth, as it ought to be.

The calculus of this motion is difficult, but may be render'd more easy by the following approximation. Assuming as above the Moon's mean distance from the Earth of 100000 parts, and the eccentricity *TC* of 5505 such parts, the line *CB* or *CD* will be found $1172\frac{3}{4}$, and *DF* $35\frac{1}{5}$ of those parts. And this line *DF* at the distance *TC* subtends the angle at the Earth, which the removal of the centre of the orbit from the place *D* to the place *F* generates in the motion of this centre; and double this line *DF* in a parallel position, at the distance of the upper focus of the Moon's orbit from the Earth, subtends at the Earth the same angle as *DF* did before, which that removal generates in the motion of this upper focus; but at the distance of the Moon from the Earth this double line 2*DF* at the upper focus, in a parallel position to the first line *DF*, subtends an angle at the Moon which the said removal generates in the motion of the Moon, which angle may be therefore called the second equation of the Moon's centre. And this equation, in the mean distance of the Moon from the Earth, is nearly as the sine of the angle which that line *DF* contains with the line drawn from the point *F* to the Moon, and when in its greatest quantity amounts to 2′. 25″. But the angle which the line *DF* contains with the line drawn from the point *F* to the Moon, is found either by subtracting the angle *EDF* from the mean anomaly of the Moon, or by adding the distance of the Moon from the Sun, to the distance of the Moon's apogee from the apogee of the Sun. And as the radius to the sine of the angle thus found, so is 2′. 25″. to the second equation of the centre; to be added, if the forementioned sum be less than a semicircle, to be subducted if greater. And from the Moon's place in its orbit thus corrected, its longitude may be found in the syzygies of the luminaries.

The atmosphere of the Earth to the height of 35 or 40 miles refracts the Sun's light. This refraction scatters and spreads the light over the Earth's shadow; and the dissipated light near the limits of the shadow dilates the shadow. Upon which accounts, to the diameter of the shadow, as it comes out by the parallax, I add 1 or $1\frac{1}{3}$ minute in lunar eclipses.

But the theory of the Moon ought to be examined and proved from the phænomena, first in the syzygies; then in the quadratures; and last of all in the octants; and whoso pleases to undertake the work, will find it not amiss to assume the following mean motions of the Sun

Sun and Moon, at the royal observatory of *Greenwich* to the last day of *December* at noon, *anno* 1700, O. S. *viz.* The mean motion of the Sun ♑ 20°. 43′. 40″. and of its apogee ♋ 7°. 44′. 30″. the mean motion of the Moon ♒ 15°. 21′. 00″; of its apogee, ♓ 8°. 20′. 00″. and of its ascending node, ♌ 27°. 24′. 20″; and the difference of meridians betwixt the observatory at *Greenwich* and the royal observatory at *Paris*, 0ʰ. 9′. 20″. but the mean motion of the Moon and of its apogee, are not yet obtained with sufficient accuracy.

Proposition XXXVI. Problem XVII.

To find the force of the Sun to move the Sea.

The Sun's force *ML* or *PT* to disturb the motions of the Moon, was, (by prop. 25.) in the Moon's quadratures, to the force of gravity with us, as 1 to 638092,6. And the force *TM—LM*, or 2 *PK* in the Moon's syzygies, is double that quantity. But descending to the surface of the Earth, these forces are diminished in proportion of the distances from the centre of the Earth, that is, in the proportion of $60\frac{1}{2}$ to 1; and therefore the former force on the Earth's surface is to the force of gravity, as 1 to 38604600. And by this force the Sea is depressed in such places as are 90 degrees distant from the Sun. But by the other force which is twice as great, the Sea is rais'd; not only in the places directly under the Sun, but in those also which are directly opposed to it. And the sum of these forces is to the force of gravity, as 1 to 12868200. And because the same force excites the same motion, whether it depresses the waters in those places which are 90 degrees distant from the Sun; or raises them in the places which are directly under, and directly opposed to the Sun; the foresaid sum will be the total force of the Sun to disturb the Sea, and will have the same effect as if the whole was employed in raising the Sea in the places directly under and directly oppos'd to the Sun, and did not act at all in the places which are 90 degrees removed from the Sun.

And this is the force of the Sun to disturb the Sea in any given place, where the Sun is at the same time both vertical, and in its mean distance from the Earth. In other positions of the Sun, its force to raise the Sea is as the versed sine of double its altitude above the horizon of the place directly, and the cube of the distance from the Earth reciprocally.

Cor. Since the centrifugal force of the parts of the Earth, arising from the Earth's diurnal motion, which is to the force of gravity as 1 to 289, raises the waters under the equator to a height exceeding that under the poles by 85472 *Paris* feet, as above in prop. 19. the force of the Sun which we have now shewed to be to the force of gravity, as 1 to 12868200, and therefore is to that centrifugal force as 289 to 12868200, or as 1 to 44527, will be able to raise the waters in the places directly under and directly oppos'd to the Sun, to a height exceeding that in the places which are 90 degrees removed from the Sun, only by one *Paris* foot and $113\frac{1}{30}$ inches. For this measure is to the measure of 85472 feet, as 1 to 44527.

Proposition XXXVII. Problem XVIII.

To find the force of the Moon to move the Sea.

The force of the Moon to move the Sea is to be deduced from its proportion to the force of the Sun, and this proportion is to be collected from the proportion of the motions of the Sea, which are the effects of those forces. Before the mouth of the river *Avon*, three miles below *Bristol*, the height of the ascent of the

the water, in the vernal and autumnal syzygies of the luminaries, (by the observations of *Samuel Sturmy*) amounts to about 45 feet, but in the quadratures to 25 only. The former of those heights arises from the sum of the foresaid forces, the latter from their difference. If therefore S and L are supposed to represent respectively the forces of the Sun and Moon, while they are in the equator, as well as in their mean distances from the Earth, we shall have L+S to L—S as 45 to 25, or as 9 to 5.

At *Plymouth* (by the observations of *Samuel Colepress*) the tide in its mean height rises to about 16 feet, and in the spring and autumn the height thereof in the syzygies may exceed that in the quadratures by more than 7 or 8 feet. Suppose the greatest difference of those heights to be 9 feet, and L+S will be to L—S, as $20\frac{1}{2}$ to $11\frac{1}{2}$, or as 41 to 23; a proportion that agrees well enough with the former. But because of the great tide at *Bristol*, we are rather to depend upon the observations of *Sturmy*, and therefore till we procure something that is more certain, we shall use the proportion of 9 to 5.

But because of the reciprocal motions of the waters, the greatest tides do not happen at the times of the syzygies of the luminaries, but as we have said before, are the third in order after the syzygies; (or reckoning from the syzygies) follow next after the third appulse of the Moon to the meridian of the place after the syzygies; or rather (as *Sturmy* observes) are the third after the day of the new or full Moon, or rather nearly after the twelfth hour from the new or full Moon, and therefore fall nearly upon the forty third hour after the new or full of the Moon. But in this port they fall out about the seventh hour after the appulse of the Moon to the meridian of the place; and therefore follow next after the appulse of the Moon to the meridian, when the Moon is distant from the Sun, or from opposition with the Sun by about 18 or 19 degrees *in consequentia.* So the summer and winter seasons come not to their height in the solstices themselves, but when the Sun is advanced beyond the solstices by about a tenth part of its whole course, that is, by about 36 or 37 degrees. In like manner the greatest tide is raised after the appulse of the Moon to the meridian of the place, when the Moon has passed by the Sun, *or the opposition thereof*, by about a tenth part of the whole motion from *one greatest* tide to *the next following greatest* tide. Suppose that distance about $18\frac{1}{2}$ degrees. And the Sun's force in this distance of the Moon from the syzygies and quadratures, will be of less moment to augment and diminish that part of the motion of the Sea which proceeds from the motion of the Moon, than in the syzygies and quadratures themselves, in the proportion of the radius to the co-sine of double this distance, or of an angle of 37 degrees, that is, in proportion of 10000000 to 7986355. And therefore in the preceding analogy, in place of S we must put 0,7986355 S. But further, the force of the Moon in the quadratures must be diminished, on account of its declination from the equator. For the Moon in those quadratures, or rather in $18\frac{1}{2}$ degrees past the quadratures, declines from the equator by about 22°. 13′. And the force of either luminary to move the Sea is diminished as it declines from the equator, nearly in the duplicate proportion of the co-sine of the declination. And therefore the force of the Moon in those quadratures is only 0,8570327 L; whence we have L + 0,7986355 S, to 0,8570327 L — 0,7986355 S, as 9 to 5.

Further yet, the diameters of the orbit, in which the Moon should move, setting aside the consideration of eccentricity, are one to the other, as 69 to 70. And therefore the Moon's distance from the Earth in the syzygies, is to its distance in the quadratures, *cæteris paribus*,

paribus, as 69 to 70. And its diſtances, when $18\frac{1}{2}$ degrees advanced beyond the ſyzygies, where the greateſt tide was excited, and when $18\frac{1}{2}$ degrees paſſed by the quadratures, where the leaſt tide was produced, are to its mean diſtance as 69,098747 and 69,897345 to $69\frac{1}{2}$. But the force of the Moon to move the Sea is in the reciprocal triplicate proportion of its diſtance. And therefore its forces, in the greateſt and leaſt of thoſe diſtances, are to its force in its mean diſtance, as 0,9830427 and 1,017522 to 1. From whence we have 1,017522 L × 0,7986355 S to 0,9830427 × 0,8570327 L—0,7986355 S as 9 to 5. And S to L, as 1 to 4,4815. Wherefore ſince the force of the Sun is to the force of gravity as 1 to 12868200, the Moon's force will be to the force of gravity, as 1 to 2871400.

Cor. 1. Since the waters excited by the Sun's force riſe to the height of a foot and $11\frac{1}{30}$ inches, the Moon's force will raiſe the ſame to the height of 8 feet and $7\frac{5}{22}$ inches; and the joint forces of both will raiſe the ſame to the height of $10\frac{1}{2}$ feet; and when the Moon is in its perigee, to the height of $12\frac{1}{2}$ feet, and more, eſpecially when the wind ſets the ſame way as the tide. And a force of that quantity is abundantly ſufficient to excite all the motions of the Sea, and agrees well with the proportion of thoſe motions. For in ſuch Seas as lye free and open from eaſt to weſt, as in the *Pacific* Sea, and in thoſe tracts of the *Atlantic* and *Ethiopic* Seas which lye without the tropics, the waters commonly riſe to 6, 9, 12, or 15 feet. But in the *Pacific* Sea, which is of a greater depth as well as of a larger extent, the tides are ſaid to be greater than in the *Atlantic* and *Ethiopic* Seas. For to have a full tide raiſed, an extent of Sea from eaſt to weſt is required of no leſs than 90 degrees. In the *Ethiopic* Sea, the waters riſe to a leſs height within the tropics than in the temperate zones, because of the narrowness of the Sea between *Africa* and the southern parts of *America*. In the middle of the open Sea the waters cannot rise without falling, together and at the same time, upon both the eastern and western shoars; when notwithstanding in our narrow Seas, they ought to fall on those shores by alternate turns. Upon which account, there is commonly but a small flood and ebb in such islands, as lie far distant from the continent. On the contrary in some ports, where to fill and empty the bays alternately, the waters are with great violence forced in and out through shallow chanels, the flood and ebb must be greater than ordinary, as at *Plymouth* and *Chepstow-Bridge* in *England*, at the mountains of St. *Michael*, and the town of *Auranches* in *Normandy*, and at *Cambaia* and *Pegu* in the *East-Indies*. In these places the Sea is hurryed in and out with such violence, as sometimes to lay the shoars under water, sometimes to leave them dry, for many miles. Nor is this force of the influx and efflux to be broke, till it has raised and depressed the waters to 30, 40, or 50 feet and above. And a like account is to be given of long and shallow chanels or streights, such as the *Magellanic* streights and those chanels which environ *England*. The tide in such ports and streights, by the violence of the influx and efflux, is augmented above measure. But on such shoars as ly towards the deep and open Sea, with a steep descent, where the waters may freely rise and fall without that precipitation of influx and efflux, the proportion of the tides agrees with the forces of the Sun and Moon.

Cor. 2. Since the Moon's force to move the Sea is to the force of gravity, as 1 to 2871400, it is evident that this force is far less than to appear sensibly in statical or hydrostatical experiments, or even in those of pendulums. It is in the tides only that this force shews it self by any sensible effect.

Cor.

Cor. 3. Because the force of the Moon to move the Sea is to the like force of the Sun as 4,4815 to 1; and those forces (by cor. 14. prop. 66. book 1.) are as the densities of the bodies of the Sun and Moon and the cubes of their apparent diameters conjunctly; the density of the Moon will be to the density of the Sun as 4,4815 to 1 directly, and the cube of the Moon's diameter to the cube of the Sun's diameter inversely; that is, (seeing the mean apparent diameters of the Moon and Sun are 31′. 16½″. and 32′. 12″.) as 4891 to 1000. But the density of the Sun was to the density of the Earth, as 1000 to 4000; and therefore the density of the Moon is to the density of the Earth as 4891 to 4000, or as 11 to 9. Therefore the body of the Moon is more dense and more earthly, than the Earth it self.

Cor. 4. And since the true diameter of the Moon, (from the observations of astronomers) is to the true diameter of the Earth, as 100 to 365, the mass of matter in the Moon will be to the mass of matter in the Earth as 1 to 39,788.

Cor. 5. And the accelerative gravity on the surface of the Moon will be about three times less than the accelerative gravity on the surface of the Earth.

Cor. 6. And the distance of the Moon's centre from the centre of the Earth will be to the distance of the Moon's centre from the common centre of gravity of the Earth and Moon, as 40,788 to 39,788.

Cor. 7. And the mean distance of the centre of the Moon from the centre of the Earth will be (in the Moon's octants) nearly 60⅖ of the greatest semidiameters of the Earth. For the greatest semidiameter of the Earth was 19658600 *Paris* feet, and the mean distance of the centres of the Earth and Moon, consisting of 60⅖ such semidiameters, is equal to 1187379440 feet. And this distance (by the preceeding cor.) is to the distance of the Moon's centre from the common centre of gravity of the Earth and Moon, as 40,788 to 39,788; which latter distance therefore is 1158268534 feet. And since the Moon, in respect of the fixt Stars, performs its revolution in $27^{d}. 7^{h}. 43\frac{4}{9}'$. the versed-sine of that angle which the Moon in a minute of time describes is 12752341, to the radius 1000,000000,000000. And as the radius is to this versed-sine, so are 1158268534 feet to 14,7706353 feet. The Moon therefore falling towards the Earth, by that force which retains it in its orbit, would in one minute of time describe 14,7706353 feet. And if we augment this force in the proportion of $178\frac{29}{40}$ to $177\frac{29}{40}$, we shall have the total force of gravity at the orbit of the Moon, by cor. prop. 3. And the Moon falling by this force, in one minute of time would describe 14,8538067 feet. And at the 60^{th} part of the distance of the Moon from the Earth's centre. That is, at the distance of 197896573 feet from the centre of the Earth, a body falling by its weight, would, in one second of time, likewise describe 14,8538067 feet. And therefore at the distance of 19615800, which compose one mean semidiameter of the Earth, a heavy body would describe in falling 15,11175, or 15 feet, 1 inch and $4\frac{1}{11}$ lines in the same time. This will be the descent of bodies in the latitude of 45 degrees. And by the foregoing table to be found under prop. 20. the descent in the latitude of *Paris* will be a little greater by an excess of about $\frac{2}{3}$ parts of a line. Therefore by this computation heavy bodies in the latitude of *Paris* falling *in vacuo* will describe 15 *Paris* feet, 1 inch, $4\frac{25}{33}$ lines very nearly in one second of time. And if the gravity be diminished by taking away a quantity equal to the centrifugal force arising in that latitude from the Earth's diurnal motion; heavy bodies falling there will describe in one second of time 15 feet, 1 inch, and $1\frac{1}{2}$ line. And with this velocity heavy bodies

do

do really fall in the latitude of *Paris*, as we have ſhewn above in prop. 4. and 19.

COR. 8. The mean diſtance of the centres of the Earth and Moon in the ſyzygies of the Moon is equal to 60 of the greateſt ſemidiameters of the Earth, ſubducting only about one 30th part of a ſemidiameter. And in the Moon's quadratures the mean diſtance of the ſame centres is $60\frac{1}{6}$ ſuch ſemidiameters of the Earth. For theſe two diſtances are to the mean diſtance of the Moon in the octants, as 69 and 70 to $69\frac{1}{2}$, by prop. 28.

COR. 9. The mean diſtance of the centres of the Earth and Moon in the ſyzygies of the Moon is 60 mean ſemidiameters of the Earth, and a 10th part of one ſemidiameter; and in the Moon's quadratures the mean diſtance of the ſame centres is 61 mean ſemidiameters of the Earth, ſubducting one 30th part of one ſemidiameter.

COR. 10. In the Moon's ſyzygies its mean horizontal parallax in the latitudes of 0,30,38,45,52,60,90 degrees, is 57′. 20″. 57′. 16″. 57′. 14″. 57′. 12″. 57′ 10″. 57′. 8″. 57′. 4″. reſpectively.

In theſe computations I don't conſider the magnetic attraction of the Earth whoſe quantity is very ſmall and unknown. If this quantity ſhould ever be found out, and the meaſures of degrees upon the meridian, the lengths of iſochronous pendulums in different parallels, the laws of the motions of the Sea, and the Moon's parallax, with the apparent diameters of the Sun and Moon, ſhould be more exactly determined from phænomena; we ſhould then be inabled to bring this calculation to a greater accuracy.

PRO-

PROPOSITION XXXVII. PROBLEM XIX.

To find the figure of the Moon's body.

If the Moon's body were fluid like our Sea, the force of the Earth to raiſe that fluid, in the neareſt and remoteſt parts, would be to the force of the Moon, by which our Sea is raiſed in the places under and oppoſite to the Moon, as the accelerative gravity of the Moon towards the Earth, to the accelerative gravity of the Earth towards the Moon, and the diameter of the Moon to the diameter of the Earth conjunctly, that is, as 39,788 to 1, and 100 to 365 conjunctly, or as 1081 to 100. Wherefore, ſince our Sea, by the force of the Moon, is raiſed to $8\frac{1}{5}$ feet; the lunar fluid would be raiſed by the force of the Earth to 93 feet. And upon this account, the figure of the Moon would be a ſpheroid, whoſe greateſt diameter produced would paſs through the centre of the Earth, and exceed the diameters perpendicular thereto, by 186 feet. Such a figure therefore the Moon affects, and muſt have put on from the beginning. *Q. E. I.*

COR. Hence it is, that the ſame face of the Moon always reſpects the Earth; nor can the body of the Moon poſſibly reſt in any other poſition, but would return always by a libratory motion to this ſituation. But thoſe librations however muſt be exceeding ſlow, becauſe of the weakneſs of the forces which excite them; ſo that the face of the Moon which ſhould be always obverted to the Earth, may for the reaſon aſſigned in prop. 17. be turned towards the other focus of the Moon's orbit, without being immediately drawn back, and converted again towards the Earth.

LEM-

LEMMA I.

If APEp (Pl. 14. Fig. 3.) *represent the Earth uniformly dense, mark'd with the centre* C, *the poles* P, p, *and the equator* AE; *and if about the centre* C, *with the radius* CP, *we suppose the sphere* Pape *to be described, and* QR *to denote the plane on which a right line, drawn from the centre of the Sun to the centre of the Earth, insists at right angles, and farther suppose, that the several particles of the whole exterior Earth* Pap APepE, *without the height of the said sphere, endeavour to recede towards this side and that side from the plane* QR, *every particle by a force proportional to its distance from that plane; I say in the first place, that the whole force and efficacy of all the particles, that are situate in* AE *the circle of the equator, and disposed uniformly without the globe, encompassing the same after the manner of a ring, to wheel the Earth about its centre, is to the whole force and efficacy of as many particles, in that point* A *of the equator which is at the greatest distance from the plane* QR, *to wheel the Earth about its centre with a like circular motion, as 1 to 2. And that circular motion will be performed about an axis lying in the common section of the equator and the plane* QR.

For let there be described from the centre *K*, with the diameter *IL*, the semicircle *INLK*. Suppose the semicircumference *INL* to be divided into innumerable

rable equal parts, and from the ſeveral parts N to the diameter IL let fall the ſines NM. Then the ſums of the ſquares of all the ſines NM will be equal to the ſums of the ſquares of the ſines KM, and both ſums together will be equal to the ſums of the ſquares of as many ſemidiameters KN; and therefore the ſum of the ſquares of all the ſines NM will be but half ſo great as the ſum of the ſquares of as many ſemidiameters KN.

Suppoſe now the circumference of the circle AE to be divided into the like number of little equal parts, and from every ſuch part F a perpendicular FG to be let fall upon the plane QR, as well as the perpendicular AH from the point A. Then the force by which the particle F recedes from the plane QR, will (by ſuppoſition) be as that perpendicular FG, and this force multiplied by the diſtance CG will repreſent the power of the particle F to turn the Earth round its centre. And therefore the power of a particle in the place F, will be to the power of a particle in the place A, as $FG \times GC$ to $AH \times HC$; that is, as FC^2 to AC^2: and therefore the whole power of all the particles F, in their proper places F, will be to the power of the like number of particles in the place A, as the ſum of all the FC^2 to the ſum of all the AC^2, that is, (by what we have demonſtrated before) as 1 to 2. *Q. E. D.*

And becauſe the action of thoſe particles is exerted in the direction of lines perpendicularly receding from the plane QR, and that equally from each ſide of this plane, they will wheel about the circumference of the circle of the equator, together with the adherent body of the Earth, round an axe, which lies as well in the plane QR, as in that of the equator.

L E M-

LEMMA II.

The same things still supposed, I say in the second place, that the total force or power of all the particles situated every where about the sphere to turn the Earth about the said axe, is to the whole force of the like number of particles, uniformly dispos'd round the whole circumference of the equator AE *in the fashion of a ring, to turn the whole Earth about with the like circular motion, as* 2 *to* 5. Pl. 14. Fig. 4.

For, let IK be any lesser circle parallel to the equator AE, and let L, l be any two equal particles in this circle, situated without the sphere $Pape$. And if upon the plane QR, which is at right angles with a radius drawn to the Sun, we let fall the perpendiculars LM, lm; the total forces by which these particles recede from the plane QR, will be proportional to the perpendiculars LM, lm. Let the right line Ll be drawn parallel to the plane $Pape$, and bisect the same in X; and thro' the point X draw Nn, parallel to the plane QR, and meeting the perpendiculars LM, lm in N and n; and upon the plane QR let fall the perpendicular XY. And the contrary forces of the particles L and l, to wheel about the Earth contrary-wise, are as $LM \times MC$, and $lm \times mC$, that is, as $LN \times MC + NM \times MC$, and $ln \times mC - nm \times mC$; or $LN \times MC + NM \times MC$, and $LN \times mC - NM \times mC$, and $LN \times Mm - NM \times \overline{MC + mC}$, the difference of the two, is the force of both taken together to turn the Earth round. The affirmative part of this difference $LN \times Mm$, or $2LN \times NX$, is to $2AH \times HC$, the force of two particles of the same size situated

ated in *A*, as LX^2 to AC^2. And the negative part $NM \times MC + mC$, or $2\,XY \times CY$, is to $2\,AH \times HC$, the force of the same two particles situated in *A*, as CX^2 to AC^2. And therefore the difference of the parts, that is, the force of the two particles *L* and *l*, taken together, to wheel the Earth about, is to the force of two particles, equal to the former and situated in the place *A*, to turn in like manner the Earth round, as $LX^2 - CX^2$ to AC^2. But if the circumference *IK* of the circle *IK* is suppos'd to be divided into an infinite number of little equal parts *L*, all the LX^2 will be to the like number of IX^2, as 1 to 2 (by lem. 1.) and to the same number of AC^2, as IX^2 to $2\,AC^2$; and the same number of CX^2, to as many AC^2, as $2\,CX^2$ to $2\,AC^2$. Wherefore the united forces of all the particles in the circumference of the circle *IK*, are to the joint forces of as many particles in the place *A*, as $IX^2 - 2\,CX^2$ to $2\,AC^2$; and therefore (by lem. 1.) to the united forces of as many particles in the circumference of the circle *AE*, as $IX^2 - 2\,CX^2$ to AC^2.

Now if *Pp* the diameter of the sphere is conceiv'd to be divided into an infinite number of equal parts, upon which a like number of circles *IK* are suppos'd to insist, the matter in the circumference of every circle *IK* will be as IX^2. And therefore the force of that matter to turn the Earth about will be as IX^2 into $IX^2 - 2\,CX^2$. And the force of the same matter, if it was situated in the circumference of the circle *AE*, would be as IX^2 into AC^2. And therefore the force of all the particles of the whole matter, situated without the sphere in the circumferences of all the circles, is to the force of the like number of particles situated in the circumference of the greatest circle *AE*, as all the IX^2 into $IX^2 - 2\,CX^2$ to as many IX^2 into AC^2, that is, as all the $AC^2 - CX^2$ into $AC^2 - 3\,CX^2$ to as many $AC^2 - CX^2$ into

AC^2, that is, as all the $AC^4 - 4AC^2 \times CX^2 + 3CX^4$ to as many $AC^4 - AC^2 \times CX^2$, that is, as the whole fluent quantity whose fluxion is $AC^4 - 4AC^2 \times CX^2 + 3CX^4$, to the whole fluent quantity whose fluxion is $AC^4 - AC^2 \times CX^2$; and therefore by the method of fluxions, as $AC^4 \times CX - \frac{4}{3}AC^2 \times CX^3 + \frac{3}{5}CX^5$ to $AC^4 \times CX - \frac{1}{3}AC^2 \times CX^3$, that is, if for CX we write the whole Cp, or AC, as $\frac{4}{15}AC^5$ to $\frac{2}{3}AC^5$, that is, as 2 to 5. *Q. E. D.*

LEMMA III.

The same things still supposed, I say in the third place, that the motion of the whole Earth about the axe abovenamed, arising from the motions of all the particles, will be to the motion of the foresaid ring about the same axe, in a proportion compounded of the proportion of the matter in the Earth to the matter in the ring; and the proportion of three squares of the quadrantal arc of any circle, to two squares of its diameter, that is, in the proportion of the matter to the matter, and of the number 925275, *to the number* 1000000.

For the motion of a cylinder, revolv'd about its quiescent axe, is to the motion of the inscrib'd sphere revolv'd together with it, as any four equal squares to three circles inscrib'd in three of those squares: And the motion of this cylinder is to the motion of an exceeding thin ring, surrounding both sphere and cylinder in their common contact, as double the matter in the cylinder to triple the matter in the ring: And this motion of the ring, uniformly continued about

bout the axe of the cylinder, is to the uniform motion of the ſame about its own diameter perform'd in the ſame periodic time, as the circumference of a circle to double its diameter.

HYPOTHESIS II.

If the other parts of the Earth were took away, and the remaining ring was carried alone about the Sun in the orbit of the Earth by the annual motion, while by the diurnal motion it was in the mean time revolved about its own axe, inclined to the plane of the ecliptic by an angle of $23\frac{1}{2}$ degrees; the motion of the equinoctial points would be the ſame, whether the ring were fluid, or whether it conſiſted of a hard and rigid matter.

PROPOSITION XXXIX. PROBLEM XX.

To find the preceſſion of the equinoxes.

The middle horary motion of the Moon's nodes, in a circular orbit when the nodes are in the quadratures, was $16''.\ 35'''.\ 16^{iv}.\ 36^{v}.$ the half of which $8''.\ 17'''.\ 38^{iv}.\ 18^{v}.$ (for the reaſons above explain'd) is the mean horary motion of the nodes in ſuch an orbit, which motion in a whole ſidereal year becomes $20^{o}.\ 11'.\ 46''.$ Becauſe therefore the nodes of the Moon in ſuch an orbit would be yearly transfer'd $20^{o}.\ 11'.\ 46''.$ in *antecedentia*; and if there were more Moons, the motion of the nodes of every one, (by cor. 16. prop. 66. book 1.) would be as its periodic time; if upon the ſurface of the Earth, a Moon was revolv'd in the time of a ſidereal day, the annual motion of the nodes of this Moon would be to $20^{o}.\ 11'.\ 46''.$

as

as $23^h. 56'$. the sidereal day, to $27^d. 7^h. 43'$. the periodic time of our Moon, that is, as 1436 to 39343. And the same thing would happen to the nodes of a ring of Moons encompassing the Earth, whether these Moons did not mutually touch each the other, or whether they were molten and form'd into a continued ring, or whether that ring should become rigid and inflexible.

Let us then suppose that this ring is in quantity of matter equal to the whole exterior Earth *PapAPepE*, which lies without the sphere *Pape* (see *Fig. Lem.* 2.) and because this sphere is to that exterior Earth, as aC^2 to $AC^2 - aC^2$, that is, (seeing *PC* or *aC* the least semidiameter of the Earth is to *AC* the greatest semidiameter of the same as 229 to 230) as 52441 to 459; if this ring encompass'd the Earth round the equator, and both together were revolv'd about the diameter of the ring, the motion of the ring (by lem. 3.) would be to the motion of the inner sphere, as 459 to 52441 and 1000000 to 925275 conjunctly, that is, as 4590 to 485223; and therefore the motion of the ring would be to the sum of the motions of both ring and sphere, as 4590 to 489813. Wherefore if the ring adheres to the sphere, and communicates its motion to the sphere, by which its nodes or equinoctial points recede: the motion remaining in the ring will be to its former motion, as 4590 to 489813, upon which account the motion of the equinoctial points will be diminish'd in the same proportion. Wherefore the annual motion of the equinoctial points of the body, composed of both ring and sphere, will be to the motion, $20^o. 11'. 46''$. as 1436 to 39343 and 4590 to 489813 conjunctly, that is as 100 to 292369. But the forces by which the nodes of a number of Moons (as we explained above) and therefore by which the equinoctial points of the ring recede (that is the forces 3 *IT* in *Fig. prop.* 30) are in the several particles as the distances of those particles from the plane *QR*; and by these forces the particles recede from that plane: and therefore (by lem. 2.) if the matter of the ring was spread all over the surface of the sphere, after the fashion of the figure *PapAPepE*, in order to make up that exterior part of the Earth, the total force or power of all the particles to wheel about the Earth round any diameter of the equator, and therefore to move the equinoctial points, would become less than before, in the proportion of 2 to 5. Wherefore the annual regress of the equinoxes now would be to 20°. 11'. 46''. as 10 to 73092: that is, would be 9''. 56'''. 50^{iv}.

But because the plane of the equator is inclin'd to that of the ecliptic, this motion is to be diminish'd in the proportion of the sine 91706, (which is the co-sine of 23 $\frac{1}{2}$ deg.) to the radius 100000. And the remaining motion will now be 9''. 7'''. 20^{iv}. which is the annual precession of the equinoxes, arising from the force of the Sun.

But the force of the Moon to move the sea was to the force of the Sun nearly as 4,4815 to 1. And the force of the Moon to move the equinoxes is to that of the Sun in the same proportion. Whence the annual precession of the equinoxes, proceeding from the force of the Moon, comes out 40''. 52'''. 52^{iv}. and the total annual precession, arising from the united forces of both, will be 50''. 00'''. 12^{iv}. the quantity of which motion agrees with the phænomena. For the precession of the equinoxes, by astronomical observations, is about 50''. yearly.

If the height of the Earth at the equator exceeds its height at the poles by more than 17$\frac{1}{6}$ miles, the matter thereof will be more rare near the surface, than at the center; and the precession of the equinoxes will be augmented by the excess of height, and diminished by the greater rarity.

And now we have deſcribed the ſyſtem of the Sun, the Earth, Moon and Planets, it remains that we add ſomething about the Comets.

L E M M A IV.

That the Comets are higher than the Moon, and in the regions of the Planets.

As the Comets were placed by aſtronomers above the Moon becauſe they were found to have no diurnal parallax; ſo their annual parallax is a convincing proof of their deſcending into the regions of the Planets. For all the Comets which move in a direct courſe according to the order of the ſigns, about the end of their appearance become more than ordinarily ſlow or retrograde, if the Earth is between them and the Sun: and more than ordinarily ſwift, if the Earth is approaching to a heliocentric oppoſition with them. Whereas, on the other hand, thoſe which move againſt the order of the ſigns, towards the end of their appearance, appear ſwifter than they ought to be, if the Earth is between them and the Sun; and ſlower, and perhaps retrograde, if the Earth is in the other ſide of its orbit. And theſe appearances proceed chiefly from the diverſe ſituations which the Earth acquires in the courſe of its motion, after the ſame manner as it happens to the Planets, which appear ſometimes retrograde, ſometimes more ſlowly, and ſometimes more ſwiftly, progreſſive, according as the motion of the Earth falls in with that of the Planet, or is directed the contrary way. If the Earth move the ſame way with the Comet, but, by an angular motion about the Sun, ſo much ſwifter that right lines drawn from the Earth to the Comet converge towards the parts beyond the Comet; the Comet ſeen from the Earth becauſe of its ſlower motion will appear retrograde; and even if the Earth is slower than the Comet, the motion of the Earth being subducted, the motion of the Comet will at least appear retarded. But if the Earth tends the contrary way to that of the Comet, the motion of the Comet will from thence appear accelerated. And from this apparent acceleration, or retardation, or regressive motion, the distance of the Comet may be inferr'd in this manner. Let ♈ *QA*, ♈ *QB*, ♈ *QC* (*Pl.* 15. *Fig.* 1.) be three observed longitudes of the Comet about the time of its first appearing, and ♈ *QF* its last observed longitude before its disappearing. Draw the right line *ABC*, whose parts *AB*, *BC*, intercepted between the right lines *QA* and *QB*, *QB* and *QC*, may be one to the other, as the two times between the three first observations. Produce *AC* to *G*, so as *AG* may be to *AB* as the time between the first and last observation to the time between the first and second; and join *QG*. Now if the Comet did move uniformly in a right line, and the Earth either stood still, or was likewise carried forwards in a right line by an uniform motion: the angle ♈ *QG* would be the longitude of the Comet at the time of the last observation. The angle therefore *FQG*, which is the difference of the longitude, proceeds from the inequality of the motions of the Comet and the Earth. And this angle, if the Earth and Comet move contrary ways, is added to the angle ♈ *QG*, and accelerates the apparent motion of the Comet. But if the Comet move the same way with the Earth, it is subtracted, and either retards the motion of the Comet, or perhaps renders it retrograde, as we have but now explained. This angle therefore, proceeding chiefly from the motion of the Earth, is justly to be esteem'd the parallax of the Comet; neglecting, to wit, some little increment or decrement that may arise from the unequal motion of the Comet in its orbit. And from this parallax we thus deduce the distance of the Comet.

Let

Let *S*, (Pl. 15. Fig. 2.) represent the Sun, *a c T* the *orbis magnus*, *a* the Earth's place in the first observation, *c* the place of the Earth in the third observation, *T* the place of the Earth in the last observation, and *T* ♈ a right line drawn to the beginning of Aries. Set off the angle ♈ *T V*, equal to the angle ♈ *Q F*, that is, equal to the longitude of the Comet at the time when the Earth is in *T*; join *a c*, and produce it to *g*, so as *a g* may be to *a c*, as *AG* to *AC*; and *g* will be the place at which the Earth would have arrived in the time of the last observation, if it had continued to move uniformly in the right line *a c*. Wherefore if we draw *g* ♈, parallel to *T* ♈, and make the angle ♈ *g V*, equal to the angle ♈ *Q G*, this angle ♈ *g V* will be equal to the longitude of the Comet seen from the place *g*, and the angle *T V g* will be the parallax which arises from the Earth's being transferr'd from the place *g* into the place *T*; and therefore *V* will be the place of the Comet in the plane of the ecliptic. And this place *V* is commonly lower than the orb of Jupiter.

The same thing may be deduced from the incurvation of the way of the Comets. For these bodies move almost in great circles, while their velocity is great, but about the end of their course, when that part of their apparent motion which arises from the parallax bears a greater proportion to their whole apparent motion, they commonly deviate from those circles, and when the Earth goes to one side, they deviate to the other. And this deflexion, because of its corresponding with the motion of the earth, must arise chiefly from the parallax. And the quantity thereof is so considerable, as, by my computation, to place the disappearing Comets a good deal lower than Jupiter. Whence it follows that when they approach nearer to us in their perigees and perihelions, they often descend below the orbs of Mars and the inferior Planets.

The near approach of the Comets is further confirmed from the light of their heads. For the light of a celestial body, illuminated by the Sun and receding to remote parts, is diminished in the quadruplicate proportion of the distance; to wit, in one duplicate proportion, on account of the increase of the distance from the Sun, and in another duplicate proportion, on account of the decrease of the apparent diameter. Wherefore if both the quantity of light and the apparent diameter of a Comet are given, its distance will be also given, by taking the distance of the Comet to the distance of a Planet, in the direct proportion of their diameters and the reciprocal subduplicate proportion of their lights. Thus in the Comet of the year 1682, Mr. *Flamstead* observed with a telescope of 16 feet, and measured with a micrometer, the least diameter of its head, 2′. 00. But the nucleus, or star in the middle of the head, scarcely amounted to the tenth part of this measure; and therefore its diameter was only 11″ or 12″. But in the light and splendor of its head, it surpass'd that of the Comet in the year 1680. and might be compared with the Stars of the first or second magnitude. Let us suppose that Saturn with its ring was about four times more lucid; and because the light of the ring was almost equal to the light of the globe within, and the apparent diameter of the globe is about 21″. and therefore the united light of both globe and ring would be equal to the light of a globe whose diameter is 30″. it follows that the distance of the Comet was to the distance of Saturn, as 1 to $\surd 4$ inversly and 12″ to 30 directly; that is, as 24 to 30, or 4 to 5. Again the Comet in the month of *April* 1665, as *Hevelius* informs us, excelled almost all the fixt Stars in splendor, and even Saturn it self, as being of a much more vivid colour. For this Comet was more lucid than that other which had appeared about the end of the preceding year and had been compared to the Stars of the first magnitude.

The

The diameter of its head was about 6′. but the nucleus, compared with the Planets by means of a telescope, was plainly less than Jupiter; and sometimes judged less, sometimes judged equal to the globe of Saturn within the ring. Since then the diameters of the heads of the Comets seldom exceed 8′ or 12′. and the diameter of the nucleus or central star is but about a tenth or perhaps fifteenth part of the diameter of the head; it appears that these stars are generally of about the same apparent magnitude with the Planets. But in regard their light may be often compared with the light of Saturn, yea and sometimes exceeds it; it is evident, that all Comets in their perihelions, must either be placed below, or not far above Saturn. And they are much mistaken, who remove them almost as far as the fixt Stars. For if it was so, the Comets could receive no more light from our Sun, than our Planets do from the fixt Stars.

So far we have gone, without considering the obscuration which Comets suffer from that plenty of thick smoak, which encompasseth their heads, and through which the heads always shew dull, as through a cloud. For by how much the more a body is obscured by this smoak, by so much the more near it must be allowed to come to the Sun, that it may vye with the Planets in the quantity of light which it reflects. Whence it is probable that the Comets descend far below the orb of Saturn, as we proved before from their parallax. But above all the thing is evinced from their tails, which must be owing either to the Sun's light reflected by a smoke arising from them, and dispersing it self through the æther, or to the light of their own heads. In the former case, we must shorten the distance of the Comets, lest we be obliged to allow that the smoak arising from their heads, is propagated through such a vast extent of space and with such a velocity and expansion, as will seem altogether incredible.

dible. In the latter caſe, the whole light of both head and tail is to be aſcribed to the central nucleus. But then if we ſuppoſe all this light to be united and condens'd within the diſc of the nucleus, certainly the nucleus will by far exceed Jupiter it ſelf in ſplendor, eſpecially when it emits a very large and lucid tail. If therefore, under a leſs apparent diameter, it reflects more light, it muſt be much more illuminated by the Sun, and therefore much nearer to it. And the ſame argument will bring down the heads of Comets ſometimes within the orb of Venus, *viz.* when being hid under the Sun's rays, they emit ſuch huge and ſplendid tails, like beams of fire, as ſometimes they do. For if all that light was ſuppoſed to be gathered together into one Star, it would ſometimes exceed not one Venus only, but a great many ſuch united into one.

Laſtly, the ſame thing is infer'd from the light of the heads, which increaſes in the receſs of the Comets from the Earth towards the Sun; and decreaſes in their return from the Sun towards the Earth. For ſo the Comet of the year 1665 (by the obſervations of *Hevelius*) from the time that it was firſt ſeen, was always loſing of its apparent motion, and therefore had already paſſed its perigee; but yet the ſplendor of its head was daily increaſing, till being hid under the Sun's rays, the Comet ceas'd to appear. The Comet of the year 1683 (by the obſervations of the ſame *Hevelius*) about the end of *July*, when it firſt appeared, moved at a very ſlow rate, advancing only about 40 or 45 minutes in its orb in a day's time. But from that time its diurnal motion was continually upon the increaſe, till *September* 4, when it aroſe to about 5 degrees. And therefore in all this interval of time, the Comet was approaching to the Earth. Which is likewiſe proved from the diameter of its head, meaſured with a micrometer. For *Auguſt* 6. *Hevelius* found it only 6'. 05'' including the *coma*, which *Sept.* 2. he obſerved to be

9'.07''.

9′.07″. and therefore its head appeared far less about the beginning, than towards the end of the motion: tho' about the beginning, because nearer to the Sun, it appeared far more lucid than towards the end, as the same *Hevelius* declares. Wherefore in all this interval of time, on account of its recess from the Sun, it decreas'd in splendor, notwithstanding its access towards the Earth. The Comet of the year 1618 about the middle of *December*, and that of the year 1680, about the end of the same month, did both move with their greatest velocity, and were therefore then in their perigees. But the greatest splendor of their heads was seen two weeks before, when they had just got clear of the Sun's rays; and the greatest splendor of their tails, a little more early, when yet nearer to the Sun. The head of the former Comet (according to the observations of *Cysatus*) *December* 1. appeared greater than the Stars of the first magnitude, and *December* 16. (then in the perigee) it was but little diminished in magnitude, but in the splendor and brightness of its light, a great deal. *January* 7, *Kepler* being uncertain about the head left off observing. *December* 12. the head of the latter Comet was seen and observ'd by Mr. *Flamstead*, when but 9 degrees distant from the Sun; which is scarcely to be done in a Star of the third magnitude. *December* 15 and 17. it appeared as a Star of the third magnitude, its lustre being diminished by the brightness of the clouds near the setting Sun. *December* 26. when it mov'd with the greatest velocity, being almost in its perigee, it was less than the mouth of *Pegasus*, a Star of the third magnitude. *Jan.* 3. it appeared as a Star of the fourth. *Jan.* 9. as one of the fifth. *Jan.* 13. it was hid by the splendor of the Moon then in her increase. *January* 25. it was scarcely equal to the Stars of the seventh magnitude. If we compare equal intervals of time, on one side and on the other, from the perigee, we shall find that the head of the Comet, which at both

both intervals of time, was far, but yet equally, remov'd from the Earth, and should have therefore shone with equal splendor, appear'd brightest on the side of the perigee towards the Sun; and disappeared on the other. Therefore from the great difference of light in the one situation and in the other, we conclude the great vicinity of the Sun and Comet in the former. For the light of Comets uses to be regular, and to appear greatest when the heads move fastest, and are therefore in their perigees; excepting in so far as it is increased by their nearness to the Sun.

COR. 1. Therefore the Comets shine by the Sun's light, which they reflect.

COR. 2. From what has been said, we may likewise understand, why Comets are so frequently seen in that hemisphere in which the Sun is, and so seldom in the other. If they were visible in the regions far above Saturn, they would appear more frequently in the parts opposite to the Sun. For such as were in those parts would be nearer to the Earth, whereas the presence of the Sun must obscure and hide those that appear in the hemisphere in which he is. Yet looking over the history of Comets, I find that four or five times more have been seen in the hemisphere towards the Sun, than in the opposite hemisphere; besides, without doubt, not a few, which have been hid by the light of the Sun. For Comets descending into our parts neither emit tails nor are so well illuminated by the Sun as to discover themselves to our naked eyes, until they are come nearer to us than Jupiter. But the far greater part of that spherical space, which is describ'd about the Sun with so small an interval, lies on that side of the Earth which regards the Sun; and the Comets in that greater part are commonly more strongly illuminated, as being for the most part nearer to the Sun.

COR. 3. Hence alſo it is evident, that the celeſtial ſpaces are void of reſiſtance. For though the Comets are carried in oblique paths, and ſometimes contrary to the course of the Planets, yet they move every way with the greateſt freedom, and preſerve their motions for an exceeding long time, even where contrary to the courſe of the Planets. I am out in my judgment, if they are not a ſort of Planets, revolving in orbits returning into themſelves with a perpetual motion. For as to what ſome writers contend, that they are no other than meteors, led into this opinion by the perpetual changes that happen to their heads, it ſeems to have no foundation. For the heads of Comets are encompaſſed with huge atmoſpheres, and the lowermoſt parts of theſe atmoſpheres muſt be the denſeſt. And therefore it is in the clouds only, not in the bodies of the Comets themſelves, that theſe changes are ſeen. Thus the Earth, if it was view'd from the Planets, would, without all doubt, ſhine by the light of its clouds, and the ſolid body would ſcarcely appear through the ſurrounding clouds. Thus alſo the belts of Jupiter are form'd in the clouds of that Planet, for they change their poſition one to another, and the ſolid body of Jupiter is hardly to be ſeen through them. And much more muſt the bodies of Comets be hid under their atmoſpheres, which are both deeper and thicker.

Proposition XL. Theorem XX.

That the Comets move in some of the conic sections, having their foci in the center of the Sun; and by radij drawn to the Sun describe areas proportional to the times.

This proposition appears from cor. 1. prop. 13. book 1. compared with prop. 8. 12. and 13. book 3.

Cor. 1. Hence if Comets are revolv'd in orbits returning into themselves, those orbits will be ellipses; and their periodic times be to the periodic times of the Planets in the sesquiplicate proportion of their principal axes. And therefore the Comets, which for the most part of their course are higher than the Planets, and upon that account describe orbits with greater axes, will require a longer time to finish their revolutions. Thus if the axe of a Comet's orbit was four times greater than the axe of the orbit of Saturn, the time of the revolution of the Comet would be to the time of the revolution of Saturn, that is, to 30 years, as $4\sqrt{4}$ (or 8) to 1, and would therefore be 240 years.

Cor. 2. But their orbits will be so near to parabolas, that parabolas may be us'd for them without sensible error.

Cor. 3. And therefore by cor. 7. prop. 16. book 1. the velocity of every Comet will always be to the velocity of any Planet, suppos'd to be revolv'd at the same distance in a circle about the Sun, nearly in the subduplicate proportion of double the distance of the Planet from the centre of the Sun, to the distance of the Comet from the Sun's centre very nearly. Let us suppose the radius of the *orbis magnus*, or the greatest semidiameter of the ellipse which the Earth describes, to to consist of 100000000 parts; and then the Earth by its mean diurnal motion will describe 1720212 of those parts, and $71675\frac{1}{2}$ by its horary motion. And therefore the Comet, at the same mean distance of the Earth from the Sun, with a velocity which is to the velocity of the Earth as $\sqrt{2}$ to 1, would by its diurnal motion describe 2432747 parts, and $101364\frac{1}{2}$ parts by its horary motion. But at greater or less distances both the diurnal and horary motion will be to this diurnal and horary motion in the reciprocal subduplicate proportion of the distances, and is therefore given.

Cor. 4. Wherefore, if the *latus rectum* of the parabola is quadruple of the radius of the *orbis magnus*, and the square of that radius is suppos'd to consist of 100000000 parts: the area which the Comet will daily describe by a radius drawn to the Sun will be $1216373\frac{1}{2}$ parts; and the horary area will be $50682\frac{1}{4}$ parts. But if the *latus rectum* is greater or less in any proportion, the diurnal and horary area will be less or greater, in the subduplicate of the same proportion reciprocally.

Lemma V.

To find a curve line of the parabolic kind, which shall pass through any given number of points. Pl. 15. Fig. 3.

Let those points be *A, B, C, D, E, F,* &c. and from the same to any right line *HN,* given in position, let fall as many perpendiculars *AH, BI, CK, DL, EM, FN,* &c.

Case 1. If *HI, IK, KL,* &c. the intervals of the points *H, I, K, L, M, N,* &c. are equal, take *b,* 2 *b,* 3 *b,* 4 *b,* 4 b, 5 b, &c. the first differences of the perpendiculars AH, BI, CK, &c. their second differences c, $2c$, $3c$, $4c$, &c. their third, d, $2d$, $3d$, &c. that is to say, so as $AH-BI$ may be $=b$, $BI-CK=2b$, $CK-DL=3b$, $DL+EM=4b$, $-EM+FN=5b$, &c. then $b-2b=c$, &c. and so on to the last difference, which is here f. Then erecting any perpendicular RS, which may be considered as an ordinate of the curve required; in order to find the length of this ordinate, suppose the intervals HI, IK, KL, LM, &c. to be units, and let $AH=a$, $-HS=p$, $\frac{1}{2}p$ into $-IS=q$, $\frac{1}{3}q$ into $+SK=r$, $\frac{1}{4}r$ into $+SL=s$, $\frac{1}{5}s$ into $+SM=t$; proceeding, to wit, to ME, the last perpendicular but one, and prefixing negative signs before the terms HS, IS, &c. which lye from S towards A; and affirmative signs before the terms SK, SL, &c. which lie on the other side of the point S. And observing well the signs, RS will be $=a+bp+cq+dr+es+ft,+$ &c.

Case 2. But if HI, IK, *&c.* the intervals of the points H, I, K, L, &c. are unequal, take b, $2b$, $3b$, $4b$, $5b$, &c. the first differences of the perpendiculars AH, BI, CK, &c. divided by the intervals between those perpendiculars; c, $2c$, $3c$, $4c$, &c. their second differences divided by the intervals between every two; d, $2d$, $3d$, &c. their third differences, divided by the interval between every three; e, $2e$, &c. their fourth differences, divided by the intervals between every four; and so forth; that is, in such manner, that b may be $=$

$$\frac{AH-BI}{HI},\ 2b=\frac{BI-CK}{IK},\ 3b=\frac{CK-DL}{KL},\ \&c.$$

then $c=\dfrac{b-2b}{HK}$, $2c=\dfrac{2b-3b}{IL}$, $3c=\dfrac{3b-4b}{KM}$, &c.

then $d=\dfrac{c-2c}{HL}$, $2d=\dfrac{2c-3c}{IM}$, &c. And those differences being found, let AH be $=a$, $-HS=p$,

p into

p into $-IS=q$, q into $+SK=r$, r into $+SL=s$, s into $+SM=t$; proceeding, to wit, to ME, the last perpendicular but one; and the ordinate RS will be $=a+bp+cq+dr+es+ft$, $+$ &c.

COR. Hence the areas of all curves may be nearly found. For if some number of points of the curve to be squar'd are found, and a parabola be suppos'd to be drawn through those points; the area of this parabola will be nearly the same with the area of the curvilinear figure propos'd to be squar'd. But the parabola can be always squar'd geometrically by methods vulgarly known.

LEMMA VI.

Certain observed places of a Comet being given, to find the place of the same to any intermediate given time.

Let HI, IK, KL, LM (in the preceding Fig.) represent the times between the observations; HA, IB, KC, LD, ME, five observ'd longitudes of the Comet, and HS the given time between the first observation and the longitude required. Then if a regular curve $ABCDE$ is suppos'd to be drawn through the points A, B, C, D, E, and the ordinate RS is found out by the preceding lemma, RS will be the longitude required.

After the same method, from five observ'd latitudes we may find the latitude to a given time.

If the differences of the observed longitudes are small, suppose of 4 or 5 degrees, three or four observations will be sufficient to find a new longitude and latitude. But if the differences are greater, as of 10 or 20 degrees, five observations ought to be used.

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LEMMA VII.

Through a given point P, (Pl. 15. Fig. 4.) *to draw a right line* BC, *whose parts* PB, PC, *cut off by two right lines* AB, AC, *given in position, may be, one to the other, in a given proportion.*

From the given point *P*, suppose any right line *PD* to be drawn to either of the right lines given as *AB*, and produce the same towards *AC* the other given right line, as far as *E*, so as *PE* may be to *PD* in the given proportion. Let *EC* be parallel to *AD*. Draw *CPB*, and *PC* will be to *PB*, as *PE* to *PD*. *Q. E. F.*

LEMMA VIII.

Let ABC (Pl. 16. Fig. 1.) *be a parabola, having its focus in* S. *By the chord* AC *bisected in* I *cut off the segment* ABCI, *whose diameter is* $I\mu$, *and vertex* μ. *In* $I\mu$ *produced take* μO *equal to one half of* $I\mu$. *Join* OS, *and produce it to* ξ, *so as* Sξ *may be equal to* 2SO. *Now, supposing a Comet to revolve in the arc* CBA, *draw* ξB, *cutting* AC *in* E; *I say, the point* E *will cut off from the chord* AC *the segment* AE, *nearly proportional to the time.*

For, if we join *EO*, cutting the parabolic arc *ABC* in *Y*, and draw μX touching the same arc in the vertex μ, and meeting *EO* in *X*, the curvilinear area

AEXμA

$AEX\mu A$ will be to the curvilinear area $ACY\mu A$, as AE to AC. And therefore since the triangle ASE is to the triangle ASC in the same proportion, the whole area $ASEX\mu A$ will be to the whole area $ASCY\mu A$, as AE to AC. But because ξO is to SO as 3 to 1, and EO to XO in the same proportion, SX will be parallel to EB: and therefore joining BX, the triangle SEB will be equal to the triangle XEB. Wherefore if to the area $ASEX\mu A$ we add the triangle EXB, and from the sum subduct the triangle SEB, there will remain the area $ASBX\mu A$ equal to the area $ASEX\mu A$, and therefore in proportion to the area $ASCY\mu A$ as AE to AC. But the area $ASBY\mu A$ is nearly equal to the area $ASBX\mu A$, and this area $ASBY\mu A$ is to the area $ASCY\mu A$, as the time of description of the arc AB to the time of description of the whole arc AC. And therefore AE is to AC nearly in the proportion of the times. *Q. E. D.*

COR. When the point B falls upon the vertex μ of the parabola, AE is to AC accurately in the proportion of the times.

SCHOLIUM.

If we join $\mu\xi$ cutting AC in δ, and in it take ξn in proportion to μB, as $27MI$ to $16M\mu$, and draw Bn: this Bn will cut the chord AC in the proportion of the times, more accurately than before. But the point n is to be taken beyond, or on this side the point ξ, according as the point B is more or less distant from the principal vertex of the parabola than the point μ.

LEMMA IX.

The right lines $I\mu$ and μM and the length $\frac{AIC}{4S\mu}$ *are equal among themselves.*

For $4S\mu$ is the latus rectum of the parabola belonging to the vertex μ.

LEMMA X.

Produce $S\mu$ *to* N *and* P, (Pl. 16. Fig. 1.) *so as* μN *may be one third of* μI, *and* SP *may be to* SN *as* SN *to* $S\mu$: *and in the time that a Comet would describe the arc* $A\mu C$, *if it was suppos'd to move always forwards with the velocity which it hath in a height equal to* SP, *it would describe a length equal to the chord* AC.

For if the Comet with the velocity, which it hath in μ, was in the said time suppos'd to move uniformly forwards in the right line which touches the parabola in μ; the area which it would describe by a radius drawn to the point *S*, would be equal to the parabolic area $ACS\mu A$. And therefore the space contain'd under the length describ'd in the tangent and the length $S\mu$, would be to the space contain'd under the lengths *AC* and *SM*, as the area $ASC\mu A$ to the triangle *ASC*, that is, as *SN* to *SM*. Wherefore *AC* is to the length describ'd in the tangent, as $S\mu$ to *SN*. But since the velocity of the Comet in the height *SP* (by cor. 6. prop. 16. book 1.) is to the velocity of the same

same in the height $S\mu$, in the reciprocal subduplicate proportion of SP to $S\mu$, that is, in the proportion of $S\mu$ to SN; the length describ'd with this velocity will be to the length in the same time describ'd in the tangent, as $S\mu$ to SN. Wherefore since AC, and the length describ'd with this new velocity, are in the same proportion to the length describ'd in the tangent, they must be equal betwixt themselves. *Q. E. D.*

COR. Therefore a Comet, with that velocity which it hath in the height $S\mu + \frac{2}{3} I\mu$, would, in the same time, describe the chord AC nearly.

LEMMA XI.

If a Comet void of all motion was let fall from the height SN, *or* $S\mu + \frac{1}{3} I\mu$, *towards the Sun; and was still impell'd to the Sun by the same force, uniformly continued, by which it was impell'd at first; the same in one half of that time in which it might describe the arc* AC *in its own orbit, would in descending, describe a space equal to the length* $I\mu$.

For in the same time that the Comet would require to describe the parabolic arc AC, it would (by the last lemma) with that velocity which it hath in the height SP, describe the chord AC; and therefore (by cor. 7. prop. 16. book 1.) if it was in the same time suppos'd to revolve by the force of its own gravity, in a circle whose semidiameter was SP, it would describe an arc of that circle, the length of which would be to the chord of the parabolic arc AC, in the subduplicate proportion of 1 to 2. Wherefore if with that weight, which in the height SP it hath towards the Sun, it should fall from that height towards the Sun, it would (by cor. 9. prop. 4. book 1.) in half the ſaid time deſcribe a ſpace equal to the ſquare of half the ſaid chord apply'd to quadruple the height SP, that is, it would deſcribe the ſpace $\frac{AI^2}{4SP}$. But ſince the weight of the Comet towards the Sun in the height SN, is to the weight of the ſame towards the Sun in the height SP, as SP to $S\mu$: the Comet, by the weight which it hath in the height SN, in falling from that height towards the Sun, would in the ſame time deſcribe the ſpace $\frac{AI^2}{4S\mu}$, that is, a ſpace equal to the length $I\mu$ or μM. *Q.E.D.*

PROPOSITION XLI. PROBLEM XXI.

From three obſervations given to determine the orbit of a Comet moving in a parabola.

This being a problem of very great difficulty, I try'd many methods of reſolving it; and ſeveral of thoſe problems, the compoſition whereof I have giv'n in the firſt book, tended to this purpoſe. But afterwards I contrived the following ſolution, which is ſomething more ſimple.

Select three obſervations diſtant one from another by intervals of time nearly equal. But let that interval of time in which the Comet moves more ſlowly, be ſomewhat greater than the other; ſo, to wit, that the difference of the times may be to the ſum of the times, as the ſum of the times to about 600 days; or that the point E (*Pl.* 16 *Fig.* 1.) may fall upon M nearly, and may err therefrom, rather towards I than towards A. If ſuch direct obſervations are not at hand, a new place of the Comet muſt be found by lem. 6.

Let

Let *S* (*Pl.* 16. *Fig.* 2.) represent the Sun; *T*, *t*, τ, three places of the Earth in the *orbis magnus*; *TA*, *tB*, τC, three observ'd longitudes of the Comet; *V* the time between the first observation and the second; *W* the time between the second and the third; *X* the length, which, in the whole time, $V + W$, the Comet might describe with that velocity which it hath in the mean distance of the Earth from the Sun: which length is to be found by cor. 3. prop. 40. book 3. and *tV* a perpendicular upon the chord $T\tau$. In the mean observed longitude *tB*, take at pleasure the point *B*, for the place of the Comet in the plane of the ecliptic; and from thence towards the Sun *S*, draw the line *BE*, which may be to the perpendicular *tV*, as the content under *SB* and St^2 to the cube of the hypotenuse of the right angl'd triangle, whose sides are *SB* and the tangent of the latitude of the Comet in the second observation to the radius *tB*. And through the point *E*, (by lemma 7.) draw the right line *AEC*, whose parts *AE* and *EC*, terminating in the right lines *TA* and τC, may be, one to the other, as the times *V* and *W*: then *A* and *C* will be nearly the places of the Comet in the plane of the ecliptic in the first and third observations, if *B* was its place rightly assum'd in the second.

Upon *AC*, bisected in *I*, erect the perpendicular *Ii*. Through *B* draw the obscure line *Bi* parallel to *AC*. Join the obscure line *Si*, cutting *AC* in λ, and compleat the parallelogram $iI\lambda\mu$. Take $I\sigma$ equal to $3I\lambda$, and through the Sun *S*, draw the obscure line $\sigma\xi$ equal to $3S\sigma + 3i\lambda$. Then, cancelling the letters *A*, *E*, *C*, *I*, from the point *B* towards the point ξ, draw the new obscure line *BE*, which may be to the former *BE* in the duplicate proportion of the distance *BS* to the quantity $S\mu + \frac{1}{3}i\lambda$. And through the point *E*, draw again the right line *AEC* by the same rule as before, that is, so as its parts *AE* and *EC* may be one to the other as the times *V* and *W*, between the

the obſervations. Thus *A* and *C* will be the places of the Comet more accurately.

Upon *AC*, biſected in *I*, erect the perpendiculars *AM*, *CN*, *IO*, of which *AM* and *CN* may be the tangents of the latitudes in the firſt and third obſervations, to the radij *TA* and τC. Join *MN*, cutting *IO* in *O*. Draw the rectangular parallelogram $iI\lambda\mu$, as before. In *IA* produc'd, take *ID* equal to $S\mu + \frac{2}{3}\, i\lambda$. Then in *MN*, towards *N*, take *MP*, which may be to the above found length *X*, in the ſubduplicate proportion of the mean diſtance of the Earth from the Sun (or of the ſemidiameter of the *orbis magnus*) to the diſtance *OD*. If the point *P* fall upon the point *N*; *A*, *B*, and *C* will be three places of the Comet, through which its orbit is to be deſcrib'd in the plane of the ecliptic. But if the point *P* falls not upon the point *N*; in the right line *AC* take *CG* equal to *NP*, ſo as the points *G* and *P* may lie on the ſame ſide of the line *NC*.

By the ſame method, as the points *E*, *A*, *C*, *G*, were found from the aſſum'd point *B*, from other points *b* and β aſſum'd at pleaſure, find out the new points *e*, *a*, *c*, *g*; and ε, α, κ, γ. Then through *G*, *g*, and γ, draw the circumference of a circle $Gg\gamma$, cutting the right line τC in *Z*: and *Z* will be one place of the Comet in the plane of the ecliptic. And in *AC*, *ac*, $\alpha\kappa$, taking *AF*, *af*, $\alpha\phi$ equal reſpectively to *CG*, *cg*, $\kappa\gamma$; through the points *F*, *f*, and ϕ, draw the circumference of a circle $Ff\phi$, cutting the right line *AT* in *X*; and the point *X* will be another place of the Comet in the plane of the ecliptic. And at the points *X* and *Z*, erecting the tangents of the latitudes of the Comet to the radij *TX*, and τZ, two places of the Comet in its own orbit will be determin'd. Laſtly, if (by prop. 19. book 1.) to the focus *S*, a parabola is deſcrib'd paſſing through thoſe two places, this parabola will be the orbit of the Comet. *Q. E. I.*

The

The demonstration of this construction follows from the preceding lemmas: because the right line *AC* is cut in *E* in the proportion of the times by lem. 7, as it ought to be by lem. 8: and *BE*, by lem. 11, is a portion of the right line *BS* or *Bξ* in the plane of the ecliptic, intercepted between the arc *ABC* and the chord *AEC*; and *MP*, (by cor. lem. 10.) is the length of the chord of that arc, which the Comet should describe in its proper orbit between the first and third observation, and therefore is equal to *MN*, providing *B* is a true place of the Comet in the plane of the ecliptic.

But it will be convenient to assume the points *B, b, β*, not at random, but nearly true. If the angle *AQt*, at which the projection of the orbit in the plane of the ecliptic cuts the right line *tB*, is rudely known; at that angle with *Bt* draw the obscure line *AC*, which may be to $\frac{4}{3}T\tau$ in the subduplicate proportion of *SQ* to *St*. And drawing the right line *SEB*, so as its part *EB* may be equal to the length *Vt*, the point *B* will be determin'd which we are to use for the first time. Then cancelling the right line *AC*, and drawing a new *AC* according to the preceding construction, and moreover, finding the length *MP*; in *tB* take the point *b*, by this rule, that if *TA*, and *τC* intersect each other in *Y*, the distance *Yb* may be to the distance *YB* in a proportion compounded of the proportion of *MP* to *MN* and the subduplicate proportion of *SB* to *Sb*. And by the same method you may find the third point *β*, if you please to repeat the operation the third time. But if this method is follow'd, two operations generally will be sufficient. For if the distance *Bb* happens to be very small; after the points *F, f*, and *G, g*, are found, draw the right lines *Ff* and *Gg*, and they will cut *TA* and *τC* in the points requir'd *X* and *Z*.

Example.

Let the Comet of the year 1680 be propos'd. The following table shews the motion thereof, as observ'd by *Flamstead*, and calculated afterwards by him from his observations, and corrected by Dr. *Halley* from the same observations.

	Time Appar.	Time True.	Sun's Longitude.	Comet's Longitude.	Comet's Lat. N.
	h. ′ ″	h. ′ ″	° ′ ″	° ′ ″	° ′ ″
1680 *Dec.* 12	4.46	4.46. 0	♑ 1.51.23	♑ 6.32.30	8.28. 0
21	6.32½	6.36.59	11.06.44	♒ 5.08.12	21.42.13
24	6.12	6.17.52	14.09.26	18.49.23	25.23. 5
26	5.14	5.20.44	16.09.22	28.24.13	27.00.52
29	7.55	8.03.02	19.19.43	♓ 13.10.41	28.09.58
30	8.02	8.10.26	20.21.09	17.38.20	28.11.53
1681 *Jan.* 5	5.51	6.01.38	26.22.18	♈ 8.48.53	26.15. 7
9	6.49	7.00.53	♒ 0.29.02	18.44.04	24.11.56
10	5.54	6.06.10	1.27.43	20.40.50	23.43.52
13	6.56	7.08.55	4.33.20	25.59.48	22.17.28
25	7.44	7.58.42	16.45.36	♉ 9.35. 0	17.56.30
30	8.07	8.21.53	21.49.58	13.19.51	16.42.18
Feb. 2	6.20	6.34.51	24.46.59	15.13.53	16.04. 1
5	6.50	7.04.41	27.49.51	16.59.06	15.27. 3

To these you may add some observations of mine.

	Ap. Time.	Comet's Longitude.	Comet's Lat. North.
	h. ′	° ′ ″	° ′ ″
1681 *Feb.* 25	8.30	♉ 26.18.35	12.46.46
27	8.15	27.04.20	22.36.12
Mar. 1	11. 0	27.52.42	12.23.40
2	8. 0	28.12.48	12.19.38
5	11.30	29.18. 0	12.03.16
7	9.30	♊ 0. 4. 0	11.57. 0
9	8.30	0.43. 4	11.45.52

These observations were made by a telescope of 7 feet, with a micrometer and threads plac'd in the focus of

Fig. I. p. 336.

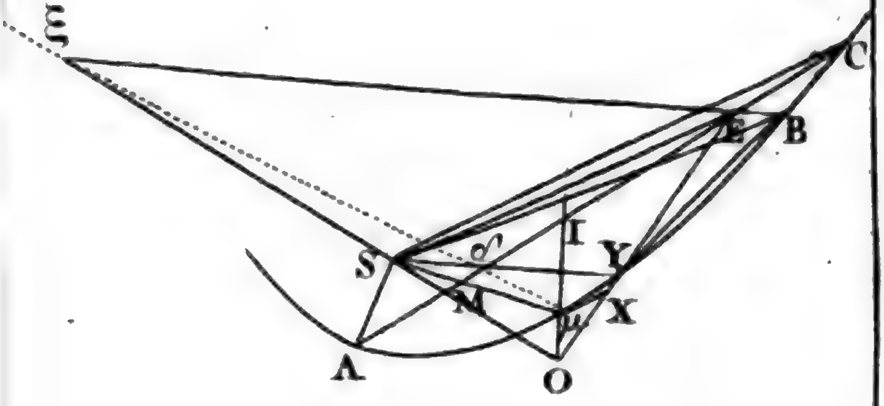


Fig. 2. p. 338.

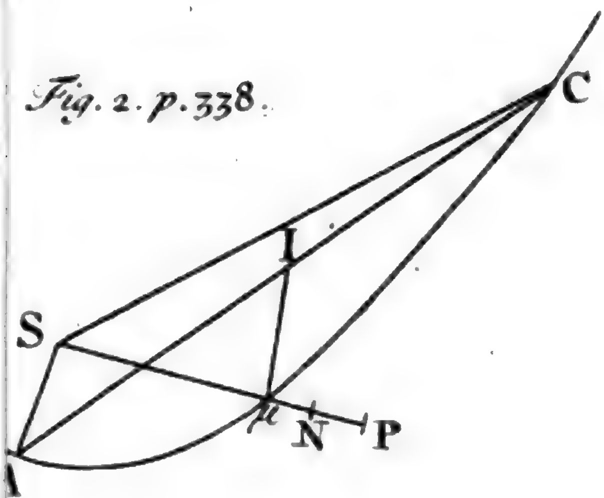


Fig. 3. p. 341.

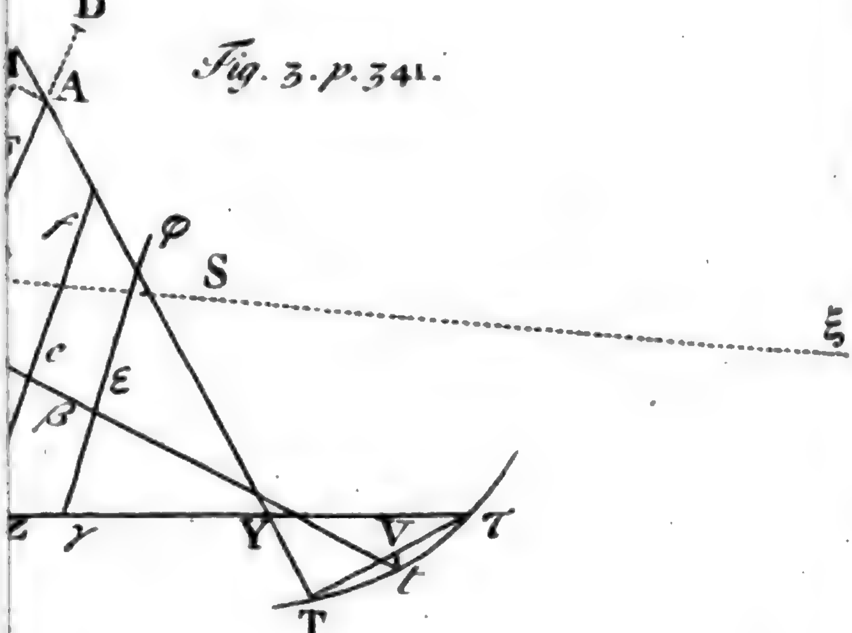
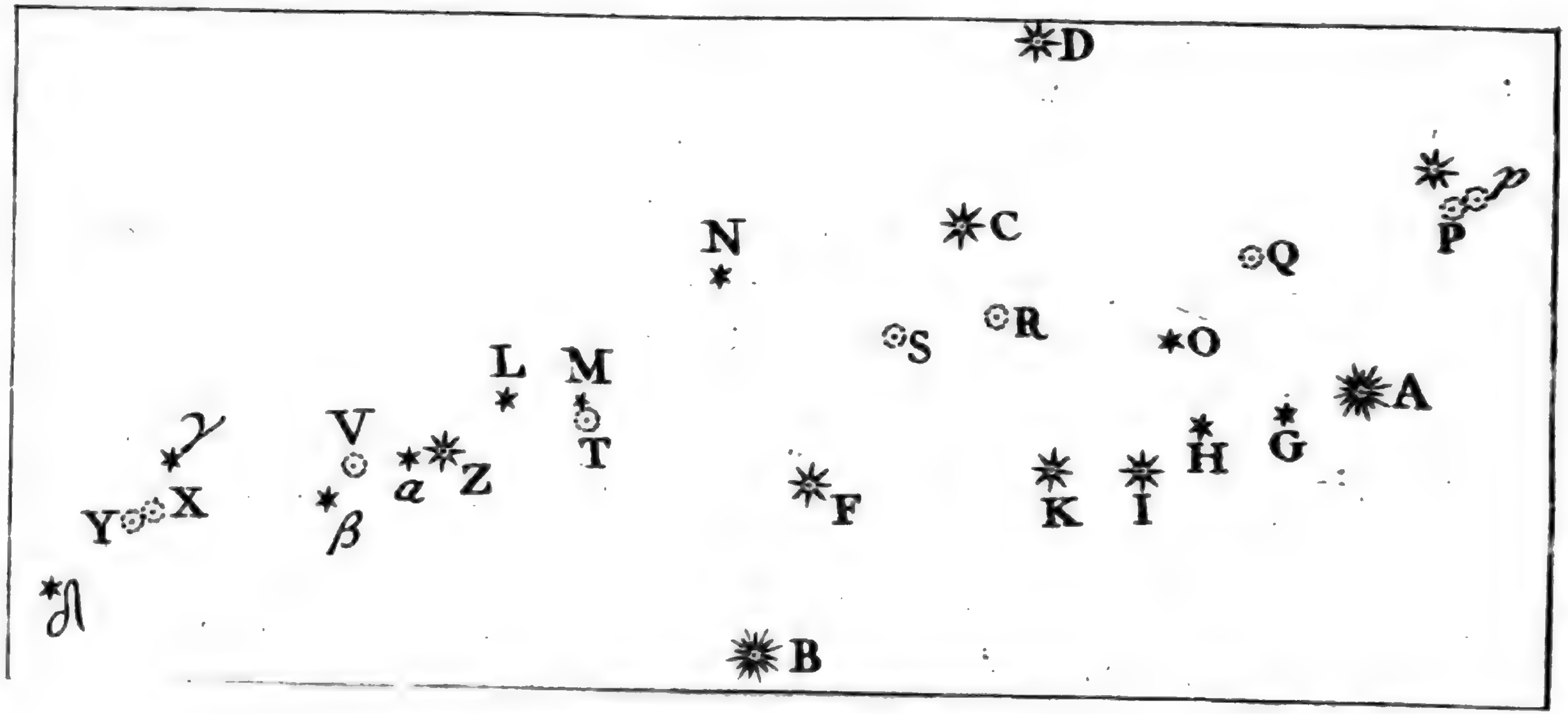


Plate XVII. Vol. II. Pag. 345.



of the telescope ; by which instruments we determin'd the positions both of the fixt Stars among themselves and of the Comet in respect of the fixt Stars. Let *A* (*Pl.* 17.) represent the Star of the fourth magnitude in the left heel of *Perseus*, (*Bayer's o*) *B* the following Star of the third magnitude in the left foot (*Bayers ζ*) *C* a Star of the sixth magnitude (*Bayer's n*) in the heel of the same foot, and *D, E, F, G, H, I, K, L, M, N, O, Z*, α, β, γ, δ, other smaller Stars in the same foot. And let *p, P, Q, R, S, T, V*, X, represent the places of the Comet in the observations above set down ; and reckoning the distance *AB* of $80\frac{7}{12}$ parts, *AC* was $52\frac{1}{4}$ of those parts, *BC*, $58\frac{5}{6}$; *AD*, $57\frac{5}{12}$; *BD*, $82\frac{6}{11}$; *CD*, $23\frac{2}{3}$; *AE*, $29\frac{4}{7}$; *CE*, $57\frac{1}{2}$; *DE*, $49\frac{11}{12}$; *AI*, $27\frac{7}{12}$; *BI*, $52\frac{1}{6}$; *CI*, $36\frac{7}{12}$; *DI*, $53\frac{5}{11}$; *AK*, $38\frac{2}{3}$; *BK*, 43 ; *CK*, $31\frac{5}{9}$; *FK*, 29 ; *FB*, 23 , *FC*, $36\frac{1}{4}$; *AH*, $18\frac{6}{7}$; *DH*, $50\frac{7}{8}$; *BN*, $46\frac{5}{12}$; *CN*, $31\frac{1}{3}$; *BL*, $45\frac{5}{12}$; *NL*, $31\frac{5}{7}$. *HO* was to *HI* as 7 to 6, and produc'd did pass between the Stars *D* and *E*, so as the distance of the Star *D* from this right line was $\frac{1}{6}$ *CD*. *LM* was to *LN* as 2 to 9, and produc'd did pass through the Star *H*. Thus were the positions of the fixt Stars determin'd in respect of one another.

Mr. *Pound* has since observed a second time the positions of these fixed Stars amongst themselves, and collected their longitudes and latitudes according to the following table.

The

The fixed Stars.	Their Longitudes.	Latitude North.	The fixed Stars.	Their Longitudes.	Latitude North.
	° ′ ″	° ′ ″		° ′ ″	° ′ ″
A	♉ 26.41.50	12. 8.36	L	♉ 29.33.34	12. 7.48
B	28.40.23	11.17.54	M	29.18.54	12. 7.20
C	27.58.30	12.40.25	N	28.48.29	12.31. 9
E	26.27.17	12.52. 7	Z	29.44.48	11.57.13
F	28.28.37	11.52.22	α	29.52. 3	11.55.48
G	26.56. 8	12. 4.58	β	♊ 0. 8.23	11.48.56
H	27.11.45	12. 2. 1	γ	0.40.10	11.55.18
I	27.25. 2	11.53.11	δ	1. 3.20	11.30.42
K	27.42. 7	11.53.26			

The positions of the Comet to these fix'd Stars were observ'd to be as follows.

Friday, *Feb.* 25. O. S. at $8\frac{1}{2}^h$, P. M. the distance of the Comet in *p* from the Star *E*, was less than $\frac{3}{13}AE$, and greater than $\frac{1}{5}AE$, and therefore nearly equal to $\frac{3}{14}AE$; and the angle *ApE* was a little obtuse, but almost right. For from *A*, letting fall a perpendicular on *pE*, the distance of the Comet from that perpendicular was $\frac{1}{5}pE$.

The same night at $9\frac{1}{2}^h$, the distance of the Comet in *P* from the Star *E*, was greater than $\frac{1}{4\frac{1}{2}}AE$, and less than $\frac{1}{5\frac{1}{4}}AE$, and therefore nearly equal to $\frac{10}{4\frac{7}{8}}$ of *AE*, or $\frac{8}{39}AE$. But the distance of the Comet from the perpendicular let fall from the Star *A* upon the right line *PE*, was $\frac{4}{5}PE$.

Sunday, *Feb.* 27 $8\frac{1}{4}^h$ P. M. the distance of the Comet in *Q*, from the Star *O*, was equal to the distance of the Stars *O* and *H*; and the right line *QO* produc'd pass'd between the Stars *K* and *B*. I could not, by reason of intervening clouds, determine the position of the Star to greater accuracy.

Tuesday, *March* 1. 11^h. P. M. the Comet in *R*, lay exactly in a line between the Stars *K* and *C*, so as the

the part CR of the right line CRK, was a little greater than $\frac{1}{3}CK$ and a little leſs than $\frac{1}{3}CK + \frac{1}{8}CR$, and therefore $= \frac{1}{3}CK + \frac{1}{16}CR$, or $\frac{16}{45}CK$.

Wedneſday, *March* 2. 8^h. P. M. the diſtance of the Comet in S from the Star C, was nearly $\frac{4}{9}FC$; the diſtance of the Star F from the right line CS produc'd was $\frac{1}{24}FC$; and the diſtance of the Star B from the ſame right line was five times greater than the diſtance of the Star F. And the right line NS produc'd paſs'd between the Stars H and I, five or ſix times nearer to the Star H than to the Star I.

Saturday, *March* 5. $11\frac{1}{2}^h$ P. M. when the Comet was in T, the right-line MT was equal to $\frac{1}{2}ML$, and the right-line LT produc'd paſs'd between B and F, four or five times nearer to F than to B, cutting off from BF a fifth or ſixth part thereof towards F: and MT produc'd paſs'd on the out-ſide of the ſpace BF, towards the Star B, four times nearer to the Star B than to the Star F. M was a very ſmall Star ſcarcely to be ſeen by the teleſcope, but the Star L was greater, and of about the eighth magnitude.

Monday, *March* 7. $9\frac{1}{2}$ P. M. The Comet being in V, the right line $V\alpha$ produced did paſs between B and F, cutting off, from BF towards F, $\frac{1}{10}$ of BF, and was to the right line $V\beta$ at 5 to 4. And the diſtance of the Comet from the right line $\alpha\beta$ was $\frac{1}{2}V\beta$.

Wedneſday, *March* 9. $8\frac{1}{2}^h$ P. M. the Comet being in X, the right line γX was equal to $\frac{1}{4}\gamma\delta$, and the perpendicular let fall from the Star δ upon the right γX was $\frac{2}{5}$ of $\gamma\delta$.

The ſame night at 12^h, the Comet being in Y, the right line γY was equal to $\frac{1}{3}$ of $\gamma\delta$, or a little leſs, as perhaps $\frac{1}{16}$ of $\gamma\delta$, and a perpendicular let fall from the Star δ on the right line γY was equal to about $\frac{1}{6}$ or $\frac{1}{7}\gamma\delta$. But the Comet being then extremely near the horizon was ſcarcely diſcernable, and therefore its place could not be determined with that certainty as in the foregoing observations.

From these observations, by constructions of figures and calculations, I deduced the longitudes and latitudes of the Comet: and Mr. *Pound* by correcting the places of the fixed Stars hath determined more correctly the places of the Comet, which correct places are set down above. Though my micrometer was none of the best, yet the errors in longitude and latitude (as derived from my observations) scarcely exceed one minute. The Comet (according to my observations) about the end of its motion, began to decline sensibly towards the north, from the parallel which it describ'd about the end of *February*.

Now in order to determine the orbit of the Comet out of the observations above describ'd; I selected those three which *Flamstead* made, *Dec.* 21. *Jan.* 5. and *Jan.* 25. From which I found St of 9842,1 parts, and Vt of 455, such as the semidiameter of the *orbis magnus* contains 10000. Then for the first observation, assuming tB of 5657 of those parts, I found SB 9747, BE for the first time 412, $S\mu$ 9503, $i\lambda$ 413, BE for the second time 421, OD 10186, X 8528,4; PM 8450, MN 8475, NP 25. From whence, by the second operation, I collected the distance tb 5640. And by this operation, I at last deduced the distances TX 4775 and τZ 11322. From which limiting the orbit, I found its descending node in ♋ and ascending node in ♑ 1° 53′; the inclination of its plane to the plane of the eclyptick 61°. 20′⅓; the vertex thereof (or the perihelion of the Comet) distant from the Node 8°. 38′, and in ♐ 27°. 43′, with latitude 7°. 34′ south; its *latus rectum* 236,8; and the diurnal area describ'd by a radius drawn to the Sun 93585, supposing the square of the semidiameter of the *orbis magnus*, 100000000; that the Comet in this orbit mov'd directly according to the order of

3 the

the signs, and on *Dec.* $8^d. 00^h. 04'$ P. M. was in the vertex or perihelion of its orbit. All which I determin'd by scale and compass, and the chords of angles, taken from the table of natural sines, in a pretty large figure, in which, to wit, the radius of the *orbis magnus* (consisting of 10000 parts,) was equal to $16\frac{1}{3}$ inches of an *English* Foot.

Lastly, in order to discover whether the Comet did truly move in the orbit so determin'd, I investigated its places in this orbit partly by arithmetical operations, and partly by scale and compass, to the times of some of the observations, as may be seen in the following table.

The Comet's							
	Dist. from Sun.	Longitude computed.	Latitud. computed.	Longitud. observ'd.	Latitude observ'd.	Dif. Lo.	Dif. Lat.
Dec. 12	2792	♑ 6°. 32′	8°.18½	♑ 6°.31⅓	8°.26	+1	− 7½
29	8403	♓ 13 . 13⅔	28. 00	♓ 13. 11¾	28 .10$\frac{1}{12}$	+2	−10$\frac{1}{12}$
Febr. 5	16669	♉ 17 . 00	15. 29⅔	♉ 16. 59⅞	15 .27⅖	+0	+ 2¼
Mar. 5	21737	29 . 19¾	12. 4	29. 20$\frac{6}{7}$	12 . 3½	−1	+ ½

But afterwards Dr. *Halley* did determine the orbit to a greater accuracy by an arithmetical calculus, than could be done by linear descriptions; and retaining the place of the nodes in ♋ and ♑ 1° 53′, and the inclination of the plane of the orbit to the ecliptic $61° 20\frac{1}{3}'$, as well as the time of the Comets being in perihelio, *Dec.* $8^d. 00. 04'$: he found the distance of the perihelion from the ascending node measur'd in the Comet's orbit 9°. 20′, and the *latus rectum* of the parabola 2430 parts, supposing the mean distance of the Sun from the Earth to be 100000 parts. And from these *data*, by an accurate arithmetical calculus, he computed the places of the Comet to the times of the observations as follows.

True

True Time.		Dist. from the Sun.	The Comet's Longitude Computed.	The Comet's Latitude Computed.	Errors in Long.	Errors in Lat.
	d. h. ′ ″		° ′ ″	° ′ ″	′ ″	′ ″
Dec.	12 . 4 . 46 .	28028	♑ 6 . 29 . 25	8 . 26 . 0 Bor.	− 3 . 5	− 2 . 0
	21 . 6 . 37 .	61076	♒ 5 . 6 . 30	21 . 43 . 20	− 1 . 42	+ 1 . 7
	24 . 6 . 18 .	70008	18 . 48 . 20	25 . 22 . 40	− 1 . 3	− 0 . 25
	26 . 5 . 20 .	75576	28 . 22 . 45	27 . 1 . 36	− 1 . 28	+ 0 . 44
	29 . 8 . 3 .	84021	♓ 13 . 12 . 40	28 . 10 . 10	+ 1 . 59	+ 0 . 12
	30 . 8 . 10 .	86661	17 . 40 . 5	28 . 11 . 20	+ 1 . 45	− 0 . 33
Jan.	5 . 6 . 1 . ½	101440	♈ 8 . 49 . 49	26 . 15 . 15	+ 0 . 56	+ 0 . 8
	9 . 7 . 0 .	110959	18 . 44 . 36	24 . 12 . 54	+ 0 . 32	+ 0 . 58
	10 . 6 . 6 .	113162	20 . 41 . 0	23 . 44 . 10	+ 0 . 10	+ 0 . 18
	13 . 7 . 9 .	120000	26 . 0 . 21	22 . 17 . 30	+ 0 . 33	+ 0 . 2
	25 . 7 . 59 .	145370	♉ 9 . 33 . 40	17 . 57 . 55	− 1 . 20	+ 1 . 25
	30 . 8 . 22 .	155303	13 . 17 . 41	16 . 42 . 7	− 2 . 10	− 0 .
Febr.	2 . 6 . 35 .	160951	15 . 11 . 11	16 . 4 . 15	− 2 . 42	+ 0 . 11
	5 . 7 . 4 . ½	166686	16 . 58 . 55	15 . 29 . 13	− 0 . 41	+ 2 . 14
	25 . 8 . 41 .	202570	26 . 15 . 46	12 . 48 . 0	− 2 . 49	+ 1 . 10
Mar.	5 . 11 . 39 .	216205	29 . 18 . 35	12 . 5 . 40	+ 0 . 35	+ 2 . 14

This Comet alſo appeared in the *November* before, and at *Coburg* in *Saxony* was obſerved by Mr. *Gottfried Kirch* on the *4th* of that Month, on the *6th* and *11th O. S*; from its poſitions to the neareſt fixed Stars obſerved with ſufficient accuracy, ſometimes with a two foot, and ſometimes with a ten foot teleſcope; from

from the difference of longitudes of *Coburg* and *London*, 11°, and from the places of the fixed Stars observed by Mr. *Pound*, Dr. *Halley* has determined the places of the Comet as follows.

Nov. 3*d.* 17^{h}. 2′, apparent time at *London*, the Comet was in ♌ 29 deg. 51′, with 1 deg. 17′. 45″. latitude north.

November 5. 15^{h}. 58′ the Comet was in ♍ 3°. 23′, with 1°. 6′. north lat.

November 10. 16^{h}. 31′, the Comet was equally distant from two Stars in ♌ which are σ and τ in *Bayer*; but it had not quite touched the right line that joins them, but was very little distant from it. In *Flamstead's* catalogue this Star σ was then in ♍ 14°. 15′, with 1 deg. 41′. lat. north nearly, and τ in ♍ 17°. $3'\frac{1}{2}$ with 0. deg. 34′. lat. south. And the middle point between those Stars was ♍ 15°. $39'\frac{1}{4}$, with 0°. $33'\frac{1}{2}$ lat. north. Let the distance of the Comet from that right line be about 10′ or 12′; and the difference of the longitude of the Comet and that middle point will be 7′; and the difference of the latitude nearly, $7'\frac{1}{2}$. And thence it follows, that the Comet was in ♍ 15°. 32′, with about 26′ lat. north.

The first observation from the position of the Comet with respect to certain small fixed Stars had all the exactness that could be desired. The second also was accurate enough. In the third observation, which was the least accurate, there might be an error of 6 or 7 minutes, but hardly greater. The longitude of the Comet, as found in the first and most accurate observation, being computed in the aforesaid parabolic orbit, comes out ♌ 29°. 30′. 22″, its latitude north 1°. 25′. 7″, and its distance from the Sun 115546.

Moreover, Dr. *Halley* observing that a remarkable Comet had appeared four times at equal intervals of 575 years, that is, in the Month of *September* after *Julius Cæsar* was killed, *An. Chr.* 531 in the consulate of

Lam-

Lampadius and *Orestes*, *An. Chr.* 1106 in the Month of *February*, and at the end of the year 1680; and that with a long and remarkable tail (except when it was seen after *Cæsar*'s death, at which time, by reason of the inconvenient situation of the Earth, the tail was not so conspicuous:) set himself to find out an elliptic orbit whose greater axis should be 1382957 parts, the mean distance of the Earth from the Sun containing 10000 such; in which orbit a Comet might revolve in 575 years. And placing the ascending node in ♋ 2°, 2′; the inclination of the plane of the orbit to the plane of the ecliptic in an angle of 61°. 6′. 48″; the perihelion of the Comet in this plane in ♐ 22°. 44′. 25″; the equal time of the perihelion *December* 7^d^. 23^h^. 9′; the distance of the perihelion from the ascending node in the plane of the ecliptic 9°. 17′. 35″; and its conjugate axis 18481, 2; he computed the motions of the Comet in this ecliptic orbit. The places of the Comet, as deduced from the observations and as arising from computation made in this orbit, may be seen in the following table.

True

True time		Long. obs.	Lat. Nor. obs.	Long. comp.	Lat.
	d . h . '	° . ' . "	° . ' . "	° . ' . "	° . '
Nov.	3 . 16 . 47	♌ 29 . 51 . 0	1 . 17 . 45	♌ 29 . 51 . 22	1 . 17
	5 . 15 . 37	♍ 3 . 23 . 0	1 . 6 . 0	♍ 3 . 24 . 32	1 .
	10 . 16 . 18	15 . 32 . 0	0 . 27 . 0	15 . 33 . 2	0 . 2
	16 . 17 . 00	- - - - - - - - -	- - - - - - -	♎ 8 . 16 . 45	0 . 5
	18 . 21 . 34	- - - - - -	- - - - - - -	- 18 . 52 . 15	1 . 2
	20 . 17 . 0	- - - - - - - - -	- - - - - - -	- 28 . 10 . 36	1 . 5
	23 . 17 . 5	- - - - - - - - -	- - - - - - -	♏ 13 . 22 . 42	2 . 2
Dec.	12 . 4 . 46	♑ 6 . 32 . 30	8 . 28 . 0	♑ 6 . 31 . 20	8 . 2
	21 . 6 . 37	♒ 5 . 8 . 12	21 . 42 . 13	♒ 5 . 6 . 14	21 . 4
	24 . 6 . 18	18 . 49 . 23	25 . 23 . 5	18 . 47 . 30	25 . 2
	26 . 5 . 21	28 . 24 . 13	27 . 0 . 52	28 . 21 . 42	27 .
	29 . 8 . 3	♓ 13 . 10 . 41	28 . 9 . 58	♓ 13 . 11 . 14	28 . 1
	30 . 8 . 10	17 . 38 . 0	28 . 11 . 53	17 . 38 . 27	28 . 1
Jan.	5 . 6 . 1½	♈ 8 . 48 . 53	26 . 15 . 7	♈ 8 . 48 . 51	26 . 1
	9 . 7 . 1	18 . 44 . 4	24 . 11 . 56	18 . 43 . 51	24 . 1
	10 . 6 . 6	20 . 40 . 50	23 . 43 . 32	20 . 40 . 23	23 . 4
	13 . 7 . 9	25 . 59 . 48	22 . 17 . 28	26 . 0 . 8	22 . 1
	25 . 7 . 59	♉ 9 . 35 . 0	17 . 56 . 30	♉ 9 . 34 . 11	17 . 5
	30 . 8 . 22	13 . 19 . 51	16 . 42 . 18	13 . 18 . 28	16 . 4
Feb.	2 . 6 . 35	15 . 13 . 53	16 . 4 . 1	15 . 11 . 59	16 .
	5 . 7 . 4½	16 . 59 . 6	15 . 27 . 3	16 . 59 . 17	15 . 2
	25 . 8 . 41	26 . 18 . 35	12 . 46 . 46	26 . 16 . 59	12 . 4
Mar.	1 . 11 . 10	27 . 52 . 42	12 . 23 . 40	27 . 51 . 47	12 . 2
	5 . 11 . 39	29 . 18 . 0	12 . 3 . 16	29 . 20 . 11	12 .
	9 . 8 . 38	♊ 0 . 43 . 4	11 . 45 . 52	♊ 0 . 42 . 43	11 . 4

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omp.	Errors in Long.	Errors in Lat.
"	′ ″	′ ″
[illegible].32 N	+0.22	−0.13
6. 9	+1.32	+0. 9
[illegible]. 7	+1. 2	−1.53
[illegible]. 7 S	- - - - -	- - - -
5.54	- - - - -	- - - - -
3.35	- - - - -	- - - - -
9. 0	- - - - -	- - - - -
9. 6 N	−1.10	+1. 6
4.42	−1.58	+2.29
3.35	−1.53	+0.30
2. 1	−2.31	+1. 9
0.38	+0.33	+0.40
1.37	+0. 7	−0.16
4.57	−0. 2	−0.10
2.17	−0.13	+0.21
3.25	−0.27	−0. 7
6.32	+0.20	−0.56
6. 6	−0.49	−0.24
0. 5	−1.23	−2.13
2. 7	−1.54	−1.54
7. 0	+0.11	−0. 3
5.22	−1.36	−1.24
2.28	−0.55	−1.12
2.50	+2.11	−0.26
5.35	−0.21	−0.17

The observations of this Comet from the beginning to the end agree as perfectly with the motion of the Comet in the orbit just now described, as the motions of the Planets do with the theories from whence they are calculated, and by this agreement plainly evince that it was one and the same Comet that appeared all that time; and also that the orbit of that Comet is here rightly defined.

In the foregoing table we have omitted the observations of *Nov.* 16, 18, 20 and 23 as not sufficiently accurate. For at those times several persons had observed the Comet. *Nov.* 17. *O. S. Ponthæus* and his Companions at 6^h in the morning at *Rome* (that is $5^h. 10'$ at *London*) by threads directed to the fixt Stars, observ'd the Comet in ♎ $8^{\circ}. 30'$. with latitude, $0^{\circ}. 40'$. south. Their observations may be seen in a treatise, which *Ponthæus* publish'd concerning this Comet. *Cellius* who was present, and communicated his observations in a Letter to *Cassini*, saw the Comet at the same hour in ♎ $8^{\circ}. 30'$. with latitude $0^{\circ}. 30'$ south. It was likewise seen by *Galletius* at the same hour at *Avignon* (that is at $5^h. 42'$. morning at *London*) in ♎ 8°. without latitude. But by the theory the Comet was at that time in ♎ $8^{\circ}. 16'. 45''$. and its latitude was $0^{\circ}. 53'. 7''$. south.

Nov. 18. at $6^h. 30'$ in the morning at *Rome* (that is, at $5^h. 40'$. at *London*) *Ponthæus* observ'd the Comet in ♎ $13^{\circ}. 30'$. with latitude $1^{\circ}. 20'$. south; and *Cellius* in ♎ $13^{\circ}. 30'$. with latitude $1^{\circ}. 00'$. south. But at $5^h\ 30'$. in the morning at *Avignon* *Galletius* saw it in ♎ $13^{\circ}. 00'$. with latitude $1^{\circ}. 00'$ south. In the university of *La Fleche* in *France*, at 5^h in the morning (that is at $5^h. 9'$. at *London*) it was seen by *P. Ango*, in the middle between two small Stars, one of which is the middle of the three which lye in a right-line in the southern hand of Virgo, *Bayers* ψ, and the other is the outmost of the wing, *Bayers* θ. Whence the

Comet was then in ♎ 12°. 46′. with latitude 50′ south. And I was informed by Dr. *Halley* that on the same day, at *Boston* in *New-England*, in the latitude of 42 ½ deg. at 5^h in the morning, (that is, at 9^h. 44′ in the morning at *London*,) the Comet was seen near ♎ 14°, with latitude 1°. 30′ south.

Nov. 19. at 4^h ½ at *Cambridge*, the Comet (by the observation of a young man) was distant from *Spica* ♍ about 2° towards the north-west. Now the spike was at that time in ♎ 19°. 23′. 47″. with latitude 2°. 1′. 59″. south. The same day at 5^h in the morning at *Boston* in *New-England*, the Comet was distant from *Spica* ♍ 1° with the difference of 40′ in latitude. The same day in the island of *Jamaica*, it was about 1° distant from *Spica* ♍. The same day Mr. *Arthur Storer* at the river *Patuxent* near *Hunting Creek* in *Maryland* in the confines of *Virginia* in lat. 38½° at 5 in the morning (that is at 10^h at *London*) saw the Comet above *Spica* ♍, and very nearly join'd with it, the distance between them being about ¾ of one deg. And from these observations compar'd I conclude, that at 9^h 44′ at *London*, the Comet was in ♎ 18°. 50′ with about 1°. 25′ latitude south. Now by the theory the Comet was at that time in ♎ 18°. 52′. 15″. with 1°. 26′. 54″. lat. south.

Nov. 20. *Montenari* professor of astronomy at *Padua*, at 6^h in the morning at *Venice* (that is 5^h. 10′ at *London*) saw the Comet in ♎ 23°. with latitude 1°. 30′ south. The same day at *Boston*, it was distant from *Spica* ♍ by about 4° of longitude east, and therefore was in ♎ 23°. 24′ nearly.

Nov. 21. *Ponthæus* and his companions at $7\frac{1}{4}^h$ in the morning, observ'd the Comet in ♎ 27°. 50′ with latitude 1°. 16′. south. *Cellius* in ♎ 28°. *P. Ango* at 5^h in the morning, in ♎ 27°. 45′. *Montenari* in ♎ 27°. 51′. The same day in the island of *Jamaica*, it was seen near the beginning of ♏ and of about the same latitude

titude with *Spica* ♍, that is, 2°. 2′. The ſame day at 5ʰ morning at *Ballaſore* in the *Eaſt-Indies* (that is at 11ʰ, 20′ of the night preceding at *London*) the diſtance of the Comet from *Spica* ♍ was taken 7°. 35′. to the eaſt. It was in a right line between the ſpike and the ballance, and therefore was then in ♎ 26°. 58′. with about 1°. 11′. lat. ſouth; and after 5ʰ. 40′. (that is at 5ʰ morning at *London*) it was in ♎ 28°. 12′. with 1°. 16′. lat. ſouth. Now by the theory the Comet was then in ♎ 28°. 10′. 36″ with 1°. 53′. 35″ lat. ſouth.

Nov. 22. The Comet was ſeen by *Montenari* in ♏ 2°. 33′. But at *Boſton* in *New-England*, it was found in about ♏ 3°, and with almoſt the ſame latitude as before, that is, 1°. 30′. The ſame day at 5ʰ morning at *Ballaſore* the Comet was obſerv'd in ♏ 1°. 50′; and therefore at 5ʰ morning at *London* the Comet was in ♏ 3°. 5′ nearly. The ſame day at $6\frac{1}{2}$ʰ in the morning at *London*, Dr. *Hook* obſerv'd it in about ♏ 3°. 30′; and that in the right line which paſſeth through *Spica* ♍ and *Cor Leonis*; not indeed exactly, but deviating a little from that line towards the north. *Montenari* likewiſe obſerv'd, that this day and ſome days after, a right line drawn from the Comet through *Spica*, paſs'd by the ſouth ſide of *Cor Leonis*, at a very ſmall diſtance therefrom. The right line through *Cor Leonis* and *Spica* ♍ did cut the ecliptic in ♍ 3°. 46′ at an angle of 2°. 51′. And if the Comet had been in this line and in ♏ 3°. its latitude would have been 2°. 26′. But ſince *Hook* and *Montenari* agree, that the Comet was at ſome ſmall diſtance from this line towards the north, its latitude muſt have been ſomething leſs. On the 20th, by the obſervation of *Montenari*, its latitude was almoſt the ſame with that of *Spica*, that is about 1°. 30′. But by the agreement of *Hook*, *Montenari* and *Ango*, the latitude was continually increaſing and therefore muſt now on the 22d, be ſenſibly greater than

1°. 30′. And taking a mean between the extreme limits but now stated 2°. 26′ and 1°. 30′, the latitude will be about 1°. 58′. *Hook* and *Montenari* agree that the tail of the Comet was directed towards *Spica* ♍, declining a little from that Star towards the south according to *Hook*, but towards the north, according to *Montenari*. And therefore that declination was scarcely sensible; and the tail lying nearly parallel to the equator, deviated a little from the opposition of the Sun, towards the north.

Nov. 23. *O. S.* At 5^h morning at *Nuremberg* (that is at $4^h\frac{1}{2}$ at *London*) Mr. *Zimmerman* saw the Comet in ♏ 8°. 8′ with 2°. 31′ south lat. its place being collected by taking its distances from fixed Stars.

Nov. 24. Before Sun-rising the Comet was seen by *Montenari* in ♏ 12°. 52′ on the north side of the right line through *Cor Leonis* and *Spica* ♍, and therefore its latitude was something less than 2°. 38′. And since the latitude, as we said, by the concurring observations of *Montenari*, *Ango*, and *Hook*, was continually increasing; therefore it was now on the 24th something greater than 1°. 58′; and, taking the mean quantity, may be reckon'd 2°. 18′, without any considerable error. *Ponthæus* and *Galletius* will have it that the latitude was now decreasing; and *Cellius* and the observer in *New-England*, that it continued the same, *viz.* of about 1°, or $1\frac{1}{2}$°. The observations of *Ponthæus* and *Cellius* are more rude, especially those which were made by taking the azimuths and altitudes; as are also the observations of *Galletius*. Those are better which were made by taking the position of the Comet to the fixt Stars by *Montenari*, *Hook*, *Ango*, and the observer in *New-England*, and sometimes by *Ponthæus* and *Cellius*. The same day, at 5^h morning at *Ballasore* the Comet was observed in ♏ 11°. 45′; and therefore at 5^h morning at *London* was in ♏ 13° nearly. And by by the theory, the Comet was at that time in ♏ $13^{\circ}.\ 22'.\ 42''$.

Nov. 25. Before Sun-rise *Montenari* obſerv'd the Comet in ♏ $17^{\circ}\frac{3}{4}$ nearly; and *Cellius* obſerv'd at the ſame time that the Comet was in a right line between the bright Star in the right thigh of Virgo and the ſouthern ſcale of Libra; and this right line cuts the Comet's way in ♏ $18^{\circ}.\ 36'$. And by the theory the Comet was in ♏ $18^{\circ}\frac{1}{3}$ nearly.

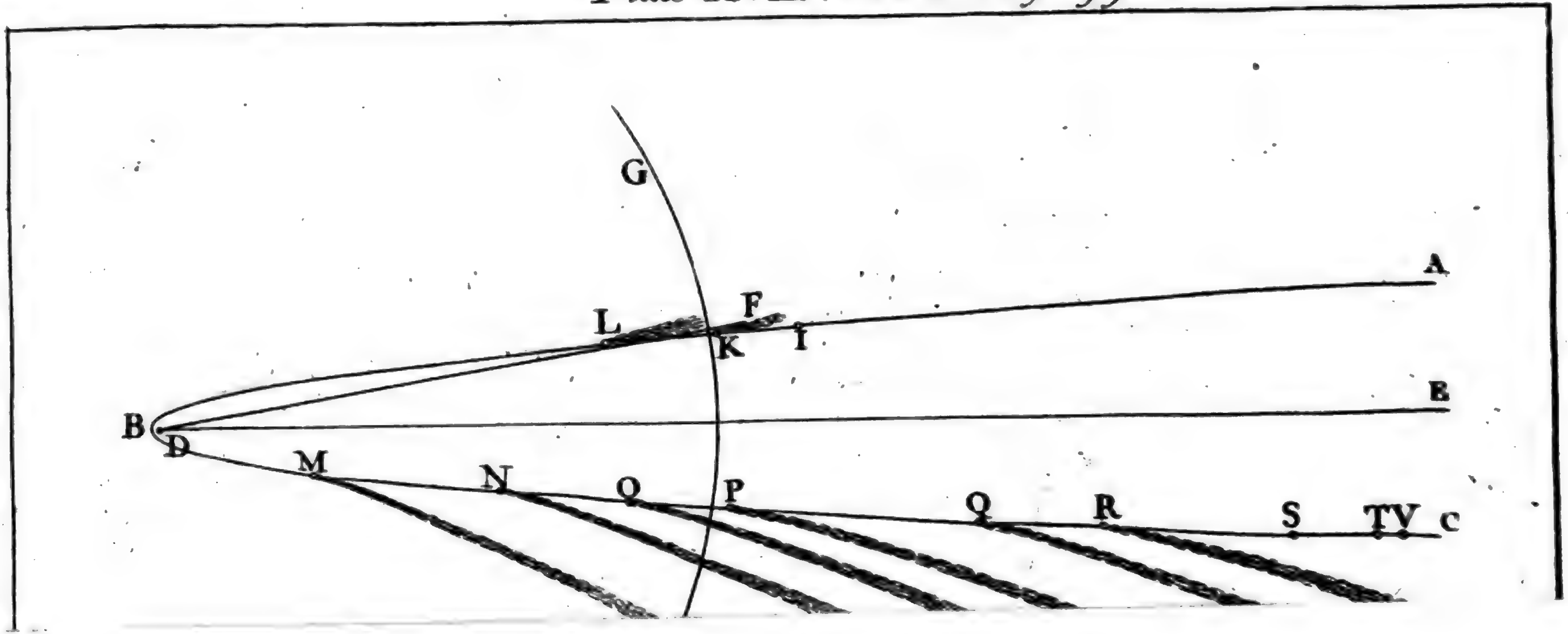
From all this it is plain that theſe obſervations agree with the theory, ſo far as they agree with one another, and by this agreement it is made clear that it was one and the ſame Comet that appeared all the time from *Nov.* 4. to *Mar.* 9. The path of this Comet did twice cut the plane of the ecliptic, and therefore was not a right line. It did cut the ecliptic, not in oppoſite parts of the heavens, but in the end of Virgo and beginning of Capricorn, including an arc of about 98°. And therefore the way of the Comet did very much deviate from the path of a great circle. For in the month of *Nov.* it declined at leaſt 3° from the ecliptic towards the ſouth; and in the month of *Dec.* following it declined 29° from the ecliptic towards the north; the two parts of the orbit in which the Comet deſcended towards the Sun, and aſcended again from the Sun, declining one from the other by an apparent angle of above 30°, as obſerv'd by *Montenari.* This Comet travel'd over 9 ſigns, to wit, from the laſt deg. of ♌ to the beginning of ♊, beſide the ſign of ♌, thro' which it paſs'd before it began to be ſeen. And there is no other theory by which a Comet can go over ſo great a part of the heavens with a regular motion. The motion of this Comet was very unequable. For about the 20th of *Nov.* it deſcrib'd about 5° a day. Then its motion being retarded, between *Nov.* 26. and *Dec.* 12. to wit, in the ſpace of $15\frac{1}{2}$ days, it deſcrib'd only 40°. But the motion thereof being afterwards accelerated, it describ'd near 5° a day, till its motion began to be again retarded. And the theory which justly corresponds with a motion so unequable, and through so great a part of the heavens, which observes the same laws with the theory of the Planets, and which accurately agrees with accurate astronomical observations, cannot be otherwise than true.

And thinking it would not be improper, I have giv'n (*Pl.* 18.) a true representation of the orbit which this Comet describ'd, and of the tail which it emitted in several places, in the annexed figure; protracted in the plane of the trajectory. In this scheme *ABC* represents the trajectory of the Comet, *D* the Sun, *DE* the axis of the trajectory, *DF* the line of the nodes, *GH* the intersection of the sphere of the *orbis magnus* with the plane of the trajectory, *I* the place of the Comet *Nov.* 4. *Ann.* 1680, *K* the place of the same *Nov.* 11, *L* the place of the same *Nov.* 19. *M* its place *Dec.* 12. *N* its place *Dec.* 21. *O* its place *Dec.* 29. *P* its place *Jan.* 5. following, *Q* its place *Jan.* 25. *R* its place *Feb.* 5. *S* its place *Feb.* 25. *T* its place *March* 5. and *V* its place *March* 9. In determining the length of the tail I made the following observations.

Nov. 4. and 6. the tail did not appear; *Nov.* 11. the tail just begun to shew itself, but did not appear above ½ deg. long through a 10 foot telescope; *Nov.* 17. the tail was seen by *Ponthaeus* more than 15° long; *Nov.* 18. in *New-England* the tail appear'd 30° long, and directly opposite to the Sun, extending itself to the planet Mars, which was then in ♍ 9°. 54'; *Nov.* 19. in *Mary-Land*, the tail was found 15° or 20° long. *Dec.* 10. (by the observation of Mr. *Flamstead*) the tail pass'd through the middle of the distance intercepted between the tail of the Serpent of *Ophiuchus* and the Star δ in the south wing of *Aquila*, and did terminate near the Stars *A*, ω, *b*, in *Bayer*'s tables. Therefore the end of the tail was in ♑ 19½°, with latitude about

34¼°

Plate XVIII. Vol. II. Pag. 358.



$34\frac{1}{4}°$ north; *Dec.* 11. it ascended to the head of *Sagitta* (*Bayer*'s α, β) terminating in ♑ 26°. 43′, with latitude 38°. 34′ north; *Dec.* 12. it pass'd through the middle of *Sagitta*, nor did it reach much farther; terminating in ♒ 4°, with latitude $42\frac{1}{2}°$ north nearly. But these things are to be understood of the length of the brighter part of the tail. For with a more faint light, observ'd too perhaps in a serener sky, at *Rome*, *Dec.* 12. 5h. 40′. by the observation of *Ponthæus*, the tail arose to 10° above the rump of the swan, and the side thereof towards the west and towards the north was 45′ distant from this star. But about that time the tail was 3° broad towards the upper end; and therefore the middle thereof was 2°. 15′ distant from that star towards the south, and the upper end was ♓ in 22° with latitude 61° north. And thence the tail was about 70° long. *Dec* 21. it extended almost to *Cassiopeia*'s chair, equally distant from β and from *Schedir*, so as its distance from either of the two was equal to the distance of the one from the other, and therefore did terminate in ♈ 24° with latitude $47\frac{1}{2}°$. *Dec.* 29. it reach'd to a contact with *Scheat* on its left, and exactly fill'd up the space between the two stars in the northern foot of *Andromeda*, being 54° in length; and therefore terminated in ♉ 19° with 35° of latitude. *Jan.* 5. it touch'd the Star π in the breast of *Andromeda* on its right side, and the Star μ of the girdle on its left; and according to our observations, was 40° long; but it was curved, and the convex side thereof lay to the south. And near the head of the Comet, it made an angle of 4° with the circle which pass'd through the Sun and the Comet's head. But towards the other end, it was inclin'd to that circle in an angle of about 10° or 11°. And the chord of the tail contain'd with that circle an angle of 8°. *Jan.* 13. the tail terminated between *Alamech* and *Algol*, with a light that was sensible enough; but with a faint light it ended over against the Star κ in

Perseus's side. The distance of the end of the tail from the circle passing through the Sun and the Comet, was 3°. 50′. And the inclination of the chord of the tail to that circle was 8½°. *Jan.* 25. and 26. it shone with a faint light to the length of 6° or 7°. And for a night or two after when there was a very clear sky, it extended to the length of 12°, or something more, with a light that was very faint and very hardly to be seen. But the axe thereof was exactly directed to the bright Star in the eastern shoulder of *Auriga*, and therefore deviated from the opposition of the Sun towards the north, by an angle of 10°. Lastly, *Feb.* 10. with a telescope I observ'd the tail 2° long. For that fainter light which I spoke of, did not appear through the glasses. But *Ponthaeus* writes that on *Feb.* 7. he saw the tail 12° long. *Feb.* 25. the Comet was without a tail, and so continued till it disappeared.

Now if one reflects upon the orbit describ'd, and duly considers the other appearances of this Comet, he will be easily satisfy'd that the bodies of Comets are solid, compact, fixt and durable, like the bodies of the Planets. For if they were nothing else but the vapours or exhalations of the Earth, of the Sun, and other Planets, this Comet in its passage by the neighbourhood of the Sun, would have been immediately dissipated. For the heat of the Sun is as the density of its rays, that is, reciprocally as the square of the distance of the places from the Sun. Therefore, since on *Dec.* 8. when the Comet was in its perihelion, the distance thereof from the centre of the Sun was to the distance of the Earth from the same as about 6 to 1000, the Sun's heat on the Comet was at that time to the heat of the Summer-Sun with us, as 1000000 to 36, or as 28000 to 1. But the heat of boiling water is about 3 times greater than the heat which dry earth acquires from the Summer-Sun, as I have try'd; and the heat of red-hot iron (if my conjecture is right) is about

about three or four times greater than the heat of boiling water. And therefore the heat, which dry earth on the Comet, while in its perihelion, might have conceived from the rays of the Sun, was about 2000 times greater than the heat of red-hot iron. But by so fierce a heat, vapours and exhalations, and every volatile matter must have been immediately consum'd and dissipated.

This Comet therefore must have conceiv'd an immense heat from the Sun, and retain that heat for an exceeding long time. For a globe of iron of an inch in diameter, expos'd red-hot to to the open air, will scarcely lose all its heat in an hour's time; but a greater globe would retain its heat longer in the proportion of its diameter, because the surface (in proportion to which it is cool'd by the contact of the ambient air) is in that proportion less in respect of the quantity of the included hot matter. And therefore a globe of red-hot iron, equal to our Earth, that is, about 40000000 feet in diameter, would scarcely cool in an equal number of days, or in above 50000 years. But I suspect that the duration of heat may, on account of some latent causes, increase in a yet less proportion than that of the diameter; and I should be glad that the true proportion was investigated by experiments.

It is further to be observ'd, that the Comet in the month of *December*, just after it had been heated by the Sun, did emit a much longer tail, and much more splendid, than in the month of *November* before, when it had not yet arriv'd at its perihelion. And universally, the greatest and most fulgent tails always arise from Comets, immediately after their passing by the neighbourhood of the Sun. Therefore the heat received by the Comet conduces to the greatness of the tail. From whence I think I may infer, that the tail is nothing else but a very fine vapour, which the head or nucleus of the Comet emits by its heat.

But

But we have had three several opinions about the tails of Comets. For some will have it, that they are nothing else but the beams of the Sun's light transmitted through the Comet's heads, which they suppose to be transparent; others that they proceed from the refraction which light suffers in passing from the Comet's head to the Earth: and lastly others, that they are a sort of clouds or vapour constantly rising from the Comet's heads, and tending towards the parts opposite to the Sun. The first is the opinion of such, as are yet unacquainted with optics. For the beams of the Sun are seen in a darkned room only in consequence of the light that is reflected from them by the little particles of dust and smoak which are always flying about in the air. And for that reason in air impregnated with thick smoak, those beams appear with great brightness, and move the sense vigorously; in a yet finer air they appear more faint, and are less easily discerned; but in the heavens, where there is no matter to reflect the light, they can never be seen at all. Light is not seen as it is in the beam, but as it is thence reflected to our eyes. For vision can be no otherwise produced than by rays falling upon the eyes. And therefore there must be some reflecting matter in those parts where the tails of the Comets are seen: for otherwise, since all the celestial spaces are equally illuminated by the Sun's light, no part of the heavens could appear with more splendor than another. The second opinion is liable to many difficulties. The tails of Comets are never seen variegated with those colours which commonly are inseparable from refraction. And the distinct transmission of the light of the fixt Stars and Planets to us, is a demonstration that the æther or celestial medium is not endow'd with any refractive power. For as to what is alledg'd that the fixt Stars have been sometimes seen by the *Egyptians*, environ'd with a *Coma*, or *Capillitium*, because that has but rarely happen'd, it is is rather to be ascrib'd to a casual refraction of clouds; and so the radiation and scintillation of the fixt Stars, to the refractions both of the eyes and air. For upon laying a telescope to the eye those radiations and scintillations immediately disappear. By the tremulous agitation of the air and ascending vapours, it happens that the rays of light are alternately turn'd aside from the narrow space of the pupil of the eye; but no such thing can have place in the much wider aperture of the object-glass of a telescope. And hence it is, that a scintillation is occasion'd in the former case, which ceases in the latter. And this cessation in the latter case is a demonstration of the regular transmission of light through the heavens, without any sensible refraction. But to obviate an objection that may be made from the appearing of no tail, in such Comets as shine but with a faint light; as if the secondary rays were then too weak to affect the eyes, and for that reason it is that the tails of the fixt Stars do not appear; we are to consider, that by the means of telescopes the light of the fixt Stars may be augmented above an hundred fold, and yet no tails are seen; that the light of the Planets is yet more copious without any tail; but that Comets are seen sometimes with huge tails, when the light of their heads is but faint and dull. For so it happen'd in the Comet of the year 1680, when in the month of *Dec.* it was scarcely equal in light to the Stars of the second magnitude, and yet emitted a notable tail, extending to the length of 40°, 50°, 60° or 70°, and upwards; and afterwards on the 27 and 28 of *January* when the head appear'd but as a Star of the 7th magnitude, yet the tail (as was said above) with a light that was sensible enough, though faint, was stretcht out to 6 or 7 degrees in length, and with a languishing light that was more difficultly seen, ev'n to 12°. and upwards. But on the 9 and 10 of *February*, when to the naked eye the head appear'd no more, through a telea telescope I view'd the tail of 2° in length. But farther, if the tail was owing to the refraction of the celestial matter, and did deviate from the opposition of the Sun, according to the Figure of the heavens; that deviation in the same places of the heavens should be always directed towards the same parts. But the Comet of the year 1680 *December* 28^{d}, $8\frac{1}{2}^{h}$. *P. M.* at *London* was seen in ♓ 8°. 41′. with latitude north 28°. 6′; while the Sun was in ♑ 18°. 26′. And the Comet of the year 1577 *Dec.* 29^{d}. was in ♓ 8°. 41′, with latitude north 28°. 40′, and the Sun as before in about ♑ 18°. 26′. In both cases the situation of the Earth was the same, and the Comet appear'd in the same place of the heavens: Yet in the former case the tail of the Comet (as well by my observations as by the observations of others) deviated from the opposition of the Sun towards the north, by an angle of $4\frac{1}{2}$ degrees, whereas in the latter, there was (according to the observations of *Tycho*) a deviation of 21 degrees towards the south. The refraction therefore of the heavens being thus disprov'd, it remains that the *phænomena* of the tails of Comets must be deriv'd from some reflecting matter.

And that the tails of Comets do arise from their heads, and tend towards the parts opposite to the Sun, is further confirm'd from the laws which the tails observe. As that lying in the planes of the Comet's orbits which pass through the Sun, they constantly deviate from the opposition of the Sun towards the parts which the Comet's heads in their progress along these orbits have left. That to a spectator, plac'd in those planes, they appear in the parts directly opposite to the Sun; but as the spectator recedes from those planes, their deviation begins to appear, and daily becomes greater. That the deviation, *cæteris paribus*, appears less, when the tail is more oblique to the orbit of the Comet, as well as when the head of the Comet approaches nearer to the Sun, eſpecially if the angle of deviation is eſtimated near the head of the Comet. That the tails which have no deviation appear ſtraight, but the tails which deviate are likewiſe bended into a certain curvature. That this curvature is greater when the deviation is greater; and is more ſenſible, when the tail, *cæteris paribus,* is longer: for in the ſhorter tails the curvature is hardly to be perceiv'd. That the angle of deviation is leſs near the Comet's head, but greater towards the other end of the tail; and that becauſe the convex ſide of the tail regards the parts, from which the deviation is made, and which lye in a right line drawn out infinitely from the Sun through the Comet's head. And that the tails that are long and broad, and ſhine with a ſtronger light, appear more reſplendent and more exactly defin'd on the convex than on the concave ſide. Upon which accounts, it is plain that the *phænomena* of the tails of Comets, depend upon the motions of their heads, and by no means upon the places of the heavens in which their heads are ſeen, and that therefore the tails of Comets do not proceed from the refraction of the heavens, but from their own heads, which furniſh the matter that forms the tail. For, as in our air, the ſmoak of a heated body aſcends, either perpendicularly if the body is at reſt, or obliquely, if the body is mov'd obliquely; ſo in the heavens, where all bodies gravitate towards the Sun, ſmoak and vapour muſt (as we have already ſaid) aſcend from the Sun, and either riſe perpendicularly, if the ſmoaking body is at reſt; or obliquely, if the body, in all the progreſs of its motion, is always leaving thoſe places from which the upper or higher parts of the vapour had riſen before. And that obliquity will be leaſt, where the vapour aſcends with moſt velocity, (to wit) near the ſmoaking body, when that is near the Sun. But becauſe the obliquity varies, the column of vapour will be incurvated; and becauſe the vapour in the preceding ſide is

ſome-

ſomething more recent, that is, has aſcended ſomething more late from the body, it will therefore be ſomething more denſe on that ſide, and muſt on that account reflect more light, as well as be better defin'd. I add nothing concerning the ſudden uncertain agitation of the tails of Comets, and their irregular figures, which Authors ſometimes deſcribe, becauſe they may ariſe from the mutations of our air, and the motions of our clouds, in part obſcuring thoſe tails; or perhaps from parts of the *Via Lactea*, which might have been confounded with and miſtaken for parts of the tails of the Comets as they paſſed by.

But that the atmoſpheres of Comets may furniſh a ſupply of vapour, great enough to fill ſo immenſe ſpaces, we may eaſily underſtand from the rarity of our own air. For the air near the ſurface of our Earth, poſſeſſes a ſpace 850 times greater than water of the ſame weight. And therefore a cylinder of air 850 feet high, is of equal weight with a cylinder of water, of the ſame breadth and but one foot high. But a cylinder of air, reaching to the top of the atmoſphere, is of equal weight with a cylinder of water, about 33 feet high: and therefore, if from the whole cylinder of air, the lower part of 850 feet high is taken away, the remaining upper part will be of equal weight with a cylinder of water 32 feet high. And from thence (and by the hypotheſis, confirm'd by many experiments, that the compreſſion of air is as the weight of the incumbent atmoſphere, and that the force of gravity is reciprocally as the ſquare of the diſtance from the center of the Earth) raiſing a calculus, by cor. prop. 22. book 2. I found, that at the heighth of one ſemidiameter of the Earth, reckon'd from the Earth's ſurface, the air is more rare than with us, in a far greater proportion than of the whole ſpace within the orb of Saturn to a ſpherical ſpace of one inch in diameter. And therefore if a ſphere of our air, of but one inch in thickneſs

ness, was equally rarify'd with the air at the heighth of one semi-diameter of the Earth from the Earth's surface, it would fill all the regions of the Planets to the orb of Saturn and far beyond it. Wherefore since the air at greater distances is immensely rarify'd, and the *coma* or atmosphere of Comets is ordinarily about ten times higher, reckoning from their centers, than the surface of the nucleus, and the tails rise yet higher, they must therefore be exceedingly rare. And tho' on account of the much thicker atmospheres of Comets and the great gravitation of their bodies towards the Sun, as well as of the particles of their air and vapours mutually one towards another, it may happen that the air in the celestial spaces and in the tails of Comets, is not so vastly rarify'd; yet from this computation it is plain, that a very small quantity of air and vapour is abundantly sufficient to produce all the appearances of the tails of Comets. For that they are indeed of a very notable rarity appears from the shining of the Stars through them. The atmosphere of the Earth, illuminated by the Suns light, tho' but of a few miles in thickness, quite obscures and extinguishes the light not only of all the Stars, but ev'n of the Moon itself: whereas the smallest Stars are seen to shine through the immense thickness of the tails of Comets, likewise illuminated by the Sun, without the least diminution of their splendor. Nor is the brightness of the tails of most Comets ordinarily greater than that of our air an inch or two in thickness, reflecting in a darken'd room the light of the Sun beams let in by an hole of the window-shut.

And we may pretty nearly determine the time spent during the ascent of the vapour from the Comet's head to the extremity of the tail, by drawing a right line from the extremity of the tail to the Sun, and marking the place where that right line intersects the Comet's orbit. For the vapour that is now in the extremity of the

tail,

tail, if it has ascended in a right line from the Sun, must have begun to rise from the head, at the time when the head was in the point of intersection. It is true, the vapour does not rise in a right line from the Sun, but retaining the motion which it had from the Comet before its ascent, and compounding that motion with its motion of ascent, arises obliquely. And therefore, the solution of the problem will be more exact, if we draw the line which intersects the orbit parallel to the length of the tail; or rather (because of the curvilinear motion of the Comet,) diverging a little from the line or length of the tail. And by means of this principle I found, that the vapour which *Jan.* 25. was in the extremity of the tail, had begun to rise from the head before *Dec.* 11. and therefore had spent in its whole ascent 45 days; but that the whole tail which appear'd on *Dec.* 10. had finish'd its ascent in the space of the two days then elaps'd from the time of the Comet's being in its perihelion. The vapour therefore, about the beginning and in the neighbourhood of the Sun, rose with the greatest velocity, and afterwards continu'd to ascend with a motion constantly retarded by its own gravity; and the higher it ascended, the more it added to the length of the tail. And while the tail continu'd to be seen, it was made up of almost all that vapour, which had risen since the time of the Comet's being in its perihelion; nor did that part of the vapour which had risen first, and which form'd the extremity of the tail, cease to appear, till its too great distance, as well from the Sun from which it receiv'd its light, as from our eyes, render'd it invisible. Whence also it is, that the tails of other Comets which are short, do not rise from their heads with a swift and continual motion, and soon after disappear; but are permanent and lasting columns of vapours and exhalations; which ascending from the heads with a slow motion of many days, and partaking of the motion of the

the heads which they had from the beginning, continue to go along together with them through the heavens. From whence again we have another argument proving the celestial spaces to be free and without resistance, since in them not only the solid bodies of the Planets and Comets, but also the extremely rare vapours of Comets tails, maintain their rapid motions with great freedom, and for an exceeding long time.

Kepler ascribes the ascent of the tails of the Comets to the atmospheres of their heads; and their direction towards the parts opposite to the Sun, to the action of the rays of light carrying along with them the matter of the Comet's tails. And without any great incongruity we may suppose, that in so free spaces, so fine a matter as that of the æther may yield to the action of the rays of the Sun's light, though those rays are not able sensibly to move the gross substances in our parts, which are clogg'd with so palpable a resistance. Another author thinks, that there may be a sort of particles of matter endow'd with a principle of levity, as well as others are with a power of gravity; that the matter of the tails of Comets may be of the former sort, and that its ascent from the Sun, may be owing to its levity. But considering that the gravity of terrestrial bodies is as the matter of the bodies, and therefore can be neither more nor less in the same quantity of matter, I am inclin'd to believe that this ascent may rather proceed from the rarefaction of the matter of the Comet's tails. The ascent of smoak in a chimney is owing to the impulse of the air, with which it is entangled. The air rarefy'd by heat ascends, because its specific gravity is diminish'd, and in its ascent carries along with it the smoak, with which it is engag'd. And why may not the tail of a Comet rise from the Sun after the same manner? For the Sun's rays do not act upon the mediums which they pervade otherwise than by reflection and refraction. And thoſe reflecting particles heated by this action, heat the matter of the æther which is involv'd with them. That matter is rarefied by the heat which it acquires; and becauſe by this rarefaction the ſpecific gravity with which it tended towards the Sun before is diminiſh'd, it will aſcend therefrom, and carry along with it the reflecting particles, of which the tail of the Comet is compos'd. But the aſcent of the vapours is further promoted by their circumgyration about the Sun, in conſequence whereof they endeavour to recede from the Sun, while the Sun's atmoſphere and the other matter of the heavens are either altogether quieſcent, or are only mov'd with a ſlower circumgyration deriv'd from the rotation of the Sun. And theſe are the cauſes of the aſcent of the tails of the Comets in the neighbourhood of the Sun, where their orbits are bent into a greater curvature, and the Comets themſelves are plung'd into the denſer, and therefore heavier parts of the Sun's atmoſphere; upon which account they do then emit tails of an huge length. For the tails which then ariſe, retaining their own proper motion, and in the mean time gravitating towards the Sun, muſt be revolv'd in ellipſes about the Sun in like manner as the heads are, and by that motion muſt always accompany the heads, and freely adhere to them. For the gravitation of the vapours towards the Sun can no more force the tails to abandon the heads, and deſcend to the Sun, than the gravitation of the heads can oblige them to fall from the tails. They muſt by their common gravity, either fall together towards the Sun, or be retarded together in their common aſcent therefrom. And therefore, (whether from the cauſes already deſcrib'd, or from any others) the tails and heads of Comets may eaſily acquire, and freely retain any poſition one to the other, without diſturbance or impediment from that common gravitation.

The tails therefore that rise in the perihelion positions of the Comets will go along with their heads into far remote parts, and together with the heads will either return again from thence to us, after a long course of years; or rather, will be there rarefied, and by degrees quite vanish away. For afterwards in the descent of the heads towards the Sun, new short tails will be emitted from the heads with a slow motion; and those tails by degrees will be augmented immensly, especially in such Comets as in their perihelion distances descend as low as the Sun's atmosphere. For all vapour in those free spaces is in a perpetual state of rarefaction and dilatation. And from hence it is, that the tails of all Comets are broader at their upper extremity, than near their heads. And it is not unlikely, but that the vapour, thus perpetually rarefy'd and dilated, may be at last dissipated, and scatter'd through the whole heavens, and by little and little be attracted towards the Planets by its gravity, and mixed with their atmosphere. For as the seas are absolutely necessary to the constitution of our Earth, that from them, the Sun, by its heat, may exhale a sufficient quantity of vapours, which being gather'd together into clouds, may drop down in rain, for watering of the earth, and for the production and nourishment of vegetables; or being condens'd with cold on the tops of mountains, (as some philosophers with reason judge) may run down in springs and rivers; so for the conservation of the seas, and fluids of the Planets, Comets seem to be requir'd, that from their exhalations and vapours condens'd, the wastes of the Planetary fluids, spent upon vegetation and putrefaction, and converted into dry earth, may be continually supplied and made up. For all vegetables entirely derive their growths from fluids, and afterwards in great measure are turn'd into dry earth by putrefaction; and a sort of slime is always found to settle at the bottom of putrified fluids. And hence it is, that the bulk of the solid earth is continually increaſed, and the fluids, if they are not ſupplied from without, muſt be in a continual decreaſe, and quite fail at laſt. I ſuſpect moreover, that 'tis chiefly from the Comets that ſpirit comes, which is indeed the ſmalleſt, but the moſt ſubtle and uſeful part of our air, and ſo much required to ſuſtain the life of all things with us.

The atmoſpheres of Comets, in their deſcent towards the Sun, by running out into the tails are ſpent and diminiſh'd, and become narrower, at leaſt on that ſide which regards the Sun; and in receding from the Sun, when they leſs run out into the tails, they are again enlarg'd, if *Hevelius* has juſtly mark'd their appearances. But they are ſeen leaſt of all juſt after they have been moſt heated by the Sun, and on that account then emit the longeſt and moſt reſplendent tails; and perhaps at the ſame time the nuclei are environ'd with a denſer and blacker ſmoak, in the lowermoſt parts of their atmoſphere. For ſmoak that is rais'd by a great and intenſe heat, is commonly the denſer and blacker. Thus the head of that Comet which we have been deſcribing, at equal diſtances both from the Sun and from the Earth, appear'd darker after it had paſs'd by its perihelion, than it did before. For in the month of *December* it was commonly compar'd with the Stars of the third magnitude, but in *November*, with thoſe of the firſt or ſecond. And ſuch as ſaw both appearances, have deſcrib'd the firſt, as of another and greater Comet than the ſecond. For *November* 19. this Comet appear'd to a young man at *Cambridge*, though with a pale and dull light, yet equal to *Spica Virginis*; and at that time it ſhone with greater brightneſs than it did afterwards. And *Montenari*, *Nov.* 20. ſt. vet. obſerved it larger than the Stars of the firſt magnitude, its tail being then 2 deg. long. And Mr. *Storer*, (by letters which have come into my hands) writes, that in the month of *Dec.* when the tail appear'd of the greateſt bulk

and

and ſplendor, the head was but ſmall, and far leſs than that which was ſeen in the month of *November* before Sun-riſing; and conjecturing at the cauſe of the appearance, he judg'd it to proceed from there being a greater quantity of matter in the head at firſt, which was afterwards gradually ſpent.

And, which further makes for the ſame purpoſe, I find, that the heads of other Comets, which did put forth tails of the greateſt bulk and ſplendor, have appeared but obſcure and ſmall. For in *Braſile, March* 5. 1668. 7^h *P. M. St. N. P. Valentinus Eſtancius* ſaw a Comet near the horizon, and towards the ſouth weſt, with a head ſo ſmall as ſcarcely to be diſcern'd, but with a tail above meaſure ſplendid, ſo that the reflection thereof from the ſea was eaſily ſeen by thoſe who ſtood upon the ſhoar. And it look'd like a fiery beam extended 23° in length from weſt to ſouth, almoſt parallel to the horizon. But this exceſſive ſplendor continu'd only three days, decreaſing apace afterwards; and while the ſplendor was decreaſing, the bulk of the tail increas'd. Whence in *Portugal*, it is ſaid to have taken up one quarter of the heavens, that is, 45 degrees, extending from weſt to eaſt with a very notable ſplendor, though the whole tail was not ſeen in thoſe parts, becauſe the head was always hid under the horizon. And from the increaſe of the bulk, and decreaſe of the ſplendor of the tail, it appears that the head was then in its receſs from the Sun, and had been very near to it in its perihelion, as the Comet of 1680 was. And we read, in the *Saxon* chronicle, of a like Comet appearing in the year 1106, *the Star whereof was ſmall and obſcure*, (as that of 1680.) *but the ſplendour of its tail was very bright, and like a huge fiery beam ſtretch'd out in a direction between the eaſt and north*, as *Hevelius* has it alſo from *Simeon* the monk of *Durham*. This Comet appear'd in the beginning of *February*, about the evening, and towards the ſouth weſt part of heaven:

From whence, and from the position of the tail, we infer, that the head was near the Sun. *Matthew Paris* says, *It was distant from the Sun by about a cubit, from three of the clock* (rather six) *till nine, putting forth a long tail.* Such also was that most resplendent Comet, described by *Aristotle*, lib. 1. *Meteor.* 6. *The head whereof could not be seen, because it had set before the Sun, or at least was hid under the Sun's rays; but next day it was seen as well as might be. For having left the Sun but a very little way, it set immediately after it. And the scatter'd light of the head, obscur'd by the too great splendor* (of the tail) *did not yet appear. But afterwards* (as *Aristotle* says) *when the splendor* (of the tail) *was now diminish'd* (the head of) *the Comet recover'd its native brightness; and the splendour* (of its tail) *reach'd now to a third part of the heavens* (that is, to 60°.) *This appearance was in the winter season,* (an. 4. olymp. 101.) *and rising to* Orion's *girdle, it there vanish'd away.* It is true that the Comet of 1618, which came out directly from under the Sun's rays, with a very large tail, seem'd to equal, if not to exceed, the Stars of the first magnitude. But then abundance of other Comets have appear'd yet greater than this, that put forth shorter tails; some of which are said to have appear'd as big as Jupiter; others as big as Venus, or even as the Moon.

We have said, that Comets are a sort of Planets, revolv'd in very eccentric orbits about the Sun. And as in the Planets which are without tails, those are commonly less, which are revolv'd in lesser orbits, and nearer to the Sun; so in Comets it is probable, that those which in their perihelion approach nearer to the Sun, are generally of less magnitude, that they may not agitate the Sun too much by their attractions. But as to the transverse diameters of their orbits, and the periodic times of their revolutions, I leave them to be determin'd by comparing Comets together which after long intervals of time return again in the same or-

bit,

bit. In the mean time, the following proposition may give some light in that enquiry.

Proposition XLII. Problem XXII.

To correct a Comet's trajectory found as above.

Operation 1. Assume that position of the plane of the trajectory which was determin'd according to the preceding proposition. And select three places of the Comet, deduc'd from very accurate observations, and at great distances one from the other. Then suppose A to represent the time between the first observation and the second; and B the time between the second and the third. But it will be convenient that in one of those times the Comet be in its perigeon, or at least not far from it. From those apparent places find by trigonometric operations the three true places of the Comet in that assum'd plane of the trajectory; then through the places found, and about the center of the Sun as the focus, describe a conic section by arithmetical operations, according to prop. 21. book 1. Let the areas of this figure which are terminated by radij drawn from the Sun to the places found, be D and E, to wit, D the area between the first observation and the second, and E the area between the second and third. And let T represent the whole time, in which the whole area D + E should be described with the velocity of the Comet found by prop. 16. book 1.

Oper. 2. Retaining the inclination of the plane of the trajectory to the plane of the ecliptic, let the longitude of the nodes of the plane of the trajectory be increas'd by the addition of 20 or 30 minutes, which call P. Then from the foresaid three observ'd places of the Comet, let the three true places be found (as before) in this new plane, as also the orbit passing through those places, and the two areas of the same describ'd between the two observations, which call d and e; and let t be the whole time in which the whole area $d+e$ should be describ'd.

Oper. 3. Retaining the longitude of the nodes in the first operation, let the inclination of the plane of the trajectory to the plane of the ecliptic be increas'd by adding thereto 20′ or 30′, which call Q. Then from the foresaid three observ'd apparent places of the Comet, let the three true places be found in this new plane, as well as the orbit passing through them, and the two areas of the same describ'd between the observation, which call δ and ε, and let τ be the whole time in which the whole area $\delta+\varepsilon$ should be describ'd.

Then taking C to 1, as A to B; and G to 1, as D to E; and g to 1, as d to e; and γ to 1, as δ to ε; let S be the true time between the first observation and the third; and observing well the signs $+$ and $-$, let such numbers m and n be found out as will make $2G-2C, = mG-mg+nG-n\gamma$; and $2T-2S = mT-mt+nT-n\tau$. And, if in the first operation I represents the inclination of the plane of the trajectory to the plane of the ecliptic, and K the longitude of either node, then $I+nQ$ will be the true inclination of the plane of the trajectory to the plane of the ecliptic; and $K+mP$ the true longitude of the node. And lastly, if in the first, second, and third operations, the quantities R, r, and ϱ, represent the parameters of the trajectory, and the quantities $\frac{1}{L}$, $\frac{1}{l}$, $\frac{1}{\lambda}$, the transverse diameters of the same; then $R+mr-mR+n\varrho-nR$ will be the true parameter, and $\frac{1}{L+ml-mL+n\lambda-nL}$ will be the true transverse diameter of the trajectory which agree with the observations, will appe[illegible] ... nexed table, calculated by Dr. *Halley*.

...om	The obſerv'd Places		The places computed in the orb.
′. 20″	*Long.* ♎	7ᵈ. 01′. 00″	♎ 7°. 01′. 29″
2 . 10	*Lat. S.*	21 . 39 . 00	21 . 38 . 50
2 . 45	*Long.* ♎	6 . 15 . 00	♎ 6 . 16 . 05
2 . 40	*Lat. S.*	22 . 24 . 00	22 . 24 . 00
8 . 00	*Long.* ♎	3 . 06 . 00	♎ 3 . 07 . 33
6 . 40	*Lat. S.*	25 . 22 . 00	25 . 21 . 40
. 15	*Long.* ♌	2 . 56 . 00	♌ 2 . 56 . 00
. 30	*Lat. S.*	49 . 25 . 00	49 . 25 . 00
. 50	*Long.* ♊	28 . 40 . 30	♊ 28 . 43 . 00
. 00	*Lat. S.*	45 . 48 . 00	45 . 46 . 00
. 00	*Long.* ♊	13 . 03 . 00	♊ 13 . 05 . 00
. 00	*Lat. S.*	39 . 54 . 00	39 . 53 . 00
. 25	*Long.* ♊	2 . 16 . 00	♊ 2 . 18 . 30
. 00	*Lat. S.*	33 . 41 . 00	33 . 39 . 40
. 00	*Long.* ♉	24 . 24 . 00	♉ 24 . 27 . 00
. 30	*Lat. S.*	27 . 45 . 00	27 . 46 . 00
. 00	*Long.* ♉	9 . 00 . 00	♉ 9 . 02 . 28
. 00	*Lat. S.*	12 . 36 . 00	12 . 34 . 13
. 00	*Long.* ♉	7 . 05 . 40	♉ 7 . 08 . 45
. 00	*Lat. S.*	10 . 23 . 00	10 . 22 . 13

which

which the Comet deſcribes. And from the tranſverſe diameter given the periodic time of the Comet is alſo given. *Q. E. I.* But the periodic times of the revolutions of Comets, and the tranſverſe diameters of their orbits, cannot be accurately enough determin'd, but by comparing Comets together which appear at different times. If after equal intervals of time, ſeveral Comets are found to have deſcrib'd the ſame orbit, we may thence conclude, that they are all but one and the ſame Comet revolv'd in the ſame orbit. And then from the times of their revolutions, the tranſverſe diameters of their orbits will be given; and from thoſe diameters the elliptic orbits themſelves will be determin'd.

To this purpoſe, the trajectories of many Comets ought to be computed, ſuppoſing thoſe trajectories to be parabolic. For ſuch trajectories will always nearly agree with the *phænomena*, as appears not only from the parabolic trajectory of the Comet of the year 1680, which I compar'd above with the obſervations, but likewiſe from that of the notable Comet, which appear'd in the years 1664, and 1665, and was obſerv'd by *Hevelius*; who, from his own obſervations, calculated the longitudes and latitudes thereof, though with little accuracy. But from the ſame obſervations Dr. *Halley* did again compute its places; and from thoſe new places determin'd its trajectory; finding its aſcending node in ♊ 21°. 13'. 55''; the inclination of the orbit to the plane of the ecliptic 21°. 18'. 40''; the diſtance of its perihelion from the node, eſtimated in the Comet's orbit 49°. 27'. 30''. its perihelion in ♌ 8°. 40'. 30''; with heliocentric latitude ſouth, 16°. 01'. 45''; the Comet to have been in its perihelion *Nov.* 24^{d}. 11^{h}. 52'; *P. M.* equal time at *London*, or 13^{h}. 8', at *Dantzick*, *O. S.* and that the *latus rectum* of the parabola was 410286 ſuch parts as the Sun's mean diſtance from the Earth is ſuppos'd to contain 100000. And how nearly the places of the Comet computed in this orbit agree with the obſervations, will appear from the annexed table, calculated by Dr. *Halley*.

In *February*, the beginning of the year 1665. the 1ſt Star of Aries, which I ſhall hereafter call γ, was in ♈ 28°. 30′. 15″, with 7°. 8′. 58″. north lat. The 2d Star of Aries was in ♈ 29°. 17′. 18″, with 8°. 28′. 16″. north lat. And another Star of the ſeventh magnitude which I call A, was in ♈ 28°. 24′. 45″, with 8°. 28′. 33″, north lat. The Comet *Feb.* 7^d. 7^h. 30′, at *Paris* (that is *Feb.* 7^d. 8^h. 37′, at *Dantzick*) O. S. made a triangle with thoſe Stars γ and A, which was right-angled in γ. And the diſtance of the Comet from the Star γ was equal to the diſtance of the Stars γ and A, that is 1°. 19′. 46″, of a great circle; and therefore in the parallel of the latitude of the Star γ it was 1°. 20′. 26″. Therefore if from the longitude of the Star γ there be ſubducted the longitude 1°. 20′. 26″, there will remain the longitude of the Comet ♈ 27°. 9′. 49″. M. *Auzout*, from this obſervation of his, placed the Comet in ♈ 27°. 0′, nearly. And by the ſcheme in which Dr. *Hooke* delineated its motion, it was then in ♈ 26°. 59′. 24″. I place it in ♈ 27°. 4′. 46″, taking the middle between the two extremes.

From the ſame obſervation, M. *Auzout* made the latitude of the Comet at that time, 7° and 4′ or 5′ to the north. But he had done better to have made it 7°. 3′. 29″, the difference of the latitudes of the Comet and the Star γ being equal to the difference of the longitude of the Stars γ and A.

Feb. 22^d. 7^h. 30′, at *London*, that is, *Feb.* 22^d. 8^h. 46′, at *Dantzick*, the diſtance of the Comet from the Star A, according to Dr. *Hooke*'s obſervation, as was delineated by himſelf in a ſcheme, and alſo by the obſervations of M. *Auzout*, delineated in like manner by M. *Petit*, was a 5th part of the diſtance between the Star A and the firſt Star of Aries, or 15′. 57″; and the diſtance of the Comet from a right line joining the Star A and the firſt of Aries, was a fourth part of the ſame 5th part, that is 4′. And therefore the Comet was in ♈ 28°. 29′. 46″, with 8°. 12′. 36″, north lat.

Mar.

Mar. 1. 7^h. 0′, at *London*, that is, *Mar.* 1. 8^h. 16′, at *Dantzick*, the Comet was obſerv'd near the 2d Star in Aries, the diſtance between them being to the diſtance between the firſt and ſecond Stars in Aries, that is, to 1^o. 33′, as 4 to 45 according to Dr. *Hooke*, or as 2 to 23 according to M. *Gottignies*. And therefore the diſtance of the Comet from the 2d Star in Aries was 8′. 16″, according to Dr. *Hooke*, or 8′. 5″, according to M. *Gottignies*; or taking a mean between both 8′. 10″. But according to M. *Gottignies*, the Comet had gone beyond the 2d Star of Aries, about a 4th or a 5th part of the ſpace, that it commonly went over in a day, to wit, about 1′. 35″; (in which he agrees very well with M. *Auzout*) or according to Dr. *Hooke*, not quite ſo much, as perhaps only 1′. Wherefore if to the longitude of the 1ſt Star in Aries, we add 1′, and 8′. 10″, to its latitude, we ſhall have the longitude of the Comet ♈ 29^o. 18′, with 8^o. 36′. 26″, north lat.

Mar. 7. 7^h. 30′, at *Paris* (that is, *Mar.* 7. 8^h. 37′, at *Dantzick*) from the obſervations of M. *Auzout*, the diſtance of the Comet from the 2d Star in Aries, was equal to the diſtance of that Star from the Star A, that is, 52′. 29″; and the difference of the longitude of the Comet and the 2d Star in Aries was 45′, or 46′, or taking a mean quantity 45′. 30″. And therefore the Comet was in ♉ 0^o. 2′. 48″. From the ſcheme of the obſervations of M. *Auzout*, conſtructed by M. *Petit*, *Hevelius* collected the latitude of the Comet 8^o. 54′. But the engraver did not rightly trace the curvature of the Comet's way toward the end of the motion: and *Hevelius* in the ſcheme of M. *Auzout*'s obſervations which he conſtructed himſelf, corrected this irregular curvature, and ſo made the latitude of the Comet 8^o. 55′. 30″. And by farther correcting this irregularity the latitude may become 8^o. 56′, or 8^o. 57′.

This Comet was also seen *Mar.* 9, and at that time its place must have been in ♉ 0°. 18′ with 9°. 3′ ½ north lat. nearly.

This Comet appeared three months together, in which space of time it travell'd over almost six signs, and in one of the days thereof describ'd almost 20 deg. Its course did very much deviate from a great circle, bending towards the north, and its motion towards the end from retrograde became direct. And notwithstanding its course was so uncommon, yet by the table it appears that the theory, from beginning to end, agrees with the observations no less accurately than the theories of the Planets usually do with the observations of them. But we are to subduct about 2′. when the Comet was swiftest, which we may effect by taking off 12″ from the angle between the ascending node and the perihelion, or by making that angle 49°. 27′. 18″. The annual parallax of both these Comets (this and the preceding) was very conspicuous, and by its quantity demonstrates the annual motion of the Earth in the *orbis magnus.*

This theory is likewise confirm'd by the motion of that Comet, which in the year 1683 appear'd retrograde, in an orbit whose plane contain'd almost a right angle with the plane of the ecliptic, and whose ascending node (by the computation of Dr. *Halley*) was in ♍ 23°. 23′; the inclination of its orbit to the ecliptic 83°. 11′; its perihelion in ♊ 25°. 29′. 30″; its perihelion distance from the Sun 56020 of such parts as the radius of the *orbis magnus* contains 100000; and the time of its perihelion *July* 2d. 3h. 50′. And the places thereof computed by Dr. *Halley* in this orbit, are compar'd with the places of the same observ'd by Mr. *Flamsteed*, in the following table.

1683

1683 Eq. Time	Sun's Place.	Comet's Long. comp.	Lat. Nor. comput.	Comet's Long. obser.	Lat. Nor. observ'd	Diff. Long.	Diff. Lat.
d. h. ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Jul. 13.12.55	♌ 1.02.30	♋ 13.05.42	29.28.13	♋ 13.06.42	29.28.20	+ 1.00	+ 0.07
15.11.15	2.53.12	11.37.48	29.34.00	11.39.43	29.34.50	+ 1.55	+ 0.50
17.10.20	4.45.45	10.07.06	29.33.30	10.08.40	29.34.00	+ 1.34	+ 0.30
23.13.40	10.38.21	5.10.27	28.51.42	5.11.30	28.50.28	+ 1.03	− 1.14
25.14. 5	12.35.28	3.27.53	24.24.47	3.27.00	28.23.40	− 0.53	− 1.07
31. 9.42	18.09.22	♊ 27.55.03	26.22.52	♊ 27.54.24	26.22.25	− 0.39	− 0.27
31.14.55	18.21.53	27.41.07	26.16.57	27.41.08	26.14.50	+ 0.01	− 2.07
Aug. 2.14.56	20.17.16	25.29.32	25.16.19	25.28.46	25.17.28	− 0.46	+ 1.09
4.10.49	22.02.50	23.18.20	24.10.49	23.16.55	24.12.19	− 1.25	+ 1.30
6.10. 9	23.56.45	20.42.23	22.47.05	20.40.32	22.49.05	− 1.51	+ 2.00
9.10.26	26.50.52	16.07.57	20.06.37	16.05.55	20.06.10	− 2.02	− 0.27
15.14. 1	♍ 2.47.13	3.30.48	11.37.33	3.26.18	11.32.01	− 4.30	− 5.32
16.15.10	3.48.02	0.43.07	9.34.16	0.41.55	9.34.13	− 1.12	− 0.03
18.15.44	5.45.33	♉ 24.52.53	5.11.15	♉ 24.49.05	5.09.11	− 3.48	− 2.04
			South		South		
22.14.44	9.35.49	11.07.14	5.16.58	11.07.12	5.16.58	− 0.02	− 0.03
23.15.52	10.36.48	7.02.18	8.17.09	7.01.17	8.16.41	− 1.01	− 0.28
26.16. 2	13.31.10	♈ 24.45.31	16.38.00	♈ 24.44.00	16.38.20	− 1.31	+ 0.20

This theory is yet further confirm'd by the motion of that retrograde Comet, which appear'd in the year 1682. The ascending node of this (by Dr. *Halley*'s computation) was in ♉ 21°. 16′. 30″; the inclination of its

its orbit to the plane of the ecliptic 17°. 56′. 00″; its perihelion in ♒ 2°. 52′. 50″; its perihelion distance from the Sun 58328 parts, of which the radius of the *orbis magnus* contains 100000; the equal time of the Comet's being in its perihelion *Sept.* 4^d. 7^h. 39′. And its places, collected from Mr. *Flamsteed*'s observations, are compar'd with its places computed from our theory, in the following table.

1682 App. Time	Sun's Place.	Comet's Lon. comp.	Lat. Nor. comp.	Com. Long. observ'd.	Lat. Nor. observ'd.	Diff. Longit.	Diff. Latitude
d h ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	′ ″	′ ″
Aug. 19.16.38	♍ 7.00.07	♌ 18.14.28	25.50.07	♌ 18.14.40	25.49.55	− 0.12	+ 0.12
20.15.38	7.55.52	24.46.23	26.14.42	24.46.22	26.12.52	+ 0.01	+ 1.50
21.08.21	8.36.14	29.37.15	26.20.03	29.38.02	26.17.37	− 0.47	+ 2.26
22.08.08	9.33.55	♍ 6.29.53	26.08.42	♍ 6.30.03	26.07.12	− 0.10	+ 1.30
29.08.20	16.22.40	♎ 12.37.54	18.37.47	♎ 12.37.49	18.34.05	+ 0.05	+ 3.42
30.07.45	17.19.41	15.36.01	17.26.43	15.35.18	17.27.17	+ 0.43	− 0.34
Sept. 1.07.33	19.16.09	20.30.53	15.13.00	20.27.04	15.09.49	+ 3.49	+ 3.11
4.07.22	22.11.28	25.42.00	12.23.48	25.40.58	12.22.00	+ 1.02	+ 1.48
5.07.32	23.10.29	27.00.46	11.33.08	26.59.24	11.33.51	+ 1.22	− 0.43
8.07.16	26.05.58	29.58.44	9.26.46	29.58.45	9.26.43	− 0.01	+ 0.03
9.07.26	27.05.09	♏ 0.44.10	8.49.10	♏ 0.44.04	8.48.25	+ 0.06	+ 0.45

This

This theory is alſo confirmed by the retrograde motion of the Comet that appeared in the year 1723. The aſcending node of this Comet (according to the computation of Mr. *Bradley*, Savilian Profeſſor of Aſtronomy at *Oxford*) was in ♈ 14°. 16′. The inclination of the orbit to the plane of the ecliptic 49°. 59′. Its perihelion was in ♉ 12°. 15′. 20″. Its perihelion diſtance from the Sun 998651 parts, of which the radius of the *orbis magnus* contains 1000000, and the equal time of its perihelion *September* 16ᵈ. 16ʰ. 10′. The places of this Comet computed in this orbit by Mr. *Bradley*, and compared with the places obſerved by himſelf, his uncle Mr. *Pound*, and Dr. *Halley*, may be ſeen in the following table.

1723. Eq. Time.	Comet's Long. obſ.	Lat. Nor. obſ.	Comet's Lon. com.	Lat. Nor. comp.	Diff. Lon.	Diff. Lat.
d h ′	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	″
Oct. 9.8. 5	♒7.22.15	5. 2. 0	♒7.21.26	5. 2.47	+49	−47
10.6.21	6.41.12	7.44.13	6.41.42	7.43.18	−50	+55
12.7.22	5.39.58	11.55. 0	5.40.19	11.54.55	−21	+ 5
14.8.57	4.59.49	14.43.50	5. 0.37	14.44. 1	−48	−11
15.6.35	4.47.41	15.40.51	4.47.45	15.40.55	− 4	− 4
21.6.22	4. 2.32	19.41.49	4. 2.21	19.42. 3	+11	−14
22.6.24	3.59. 2	20. 8.12	3.59.10	20. 8.17	− 8	− 5
24.8. 2	3.55.29	20.55.18	3.55.11	20.55. 9	+18	+ 9
29.8.56	3.56.17	22.20.27	3.56.42	22.20.10	−25	+17
30.6.20	3.58. 9	22.32.28	3.58.17	22.32.12	− 8	+16
Nov. 5.5.53	4.16.30	23.38.33	4.16.23	23.38. 7	+ 7	+26
8.7. 6	4.29.36	24. 4.30	4.29.54	24. 4.40	−18	−10
14.6.20	5. 2.16	24.48.46	5. 2.51	24.48.16	−35	+30
20.7.45	5.42.20	25.24.45	5.43.13	25.25.17	−53	−32
Dec. 7.6.45	8. 4.13	26.54.18	8. 3.55	26.53.42	+18	+36

From theſe examples it is abundantly evident, that the motions of Comets are no leſs accurately repreſented by our theory, than the motions of the Planets commonly are by the theories of them. And therefore, by means of this theory, we may enumerate the orbits of Comets, and ſo diſcover the periodic time of a Comet's revolution in any orbit; whence at laſt we

ſhall

ſhall have the tranſverſe diameters of their elliptic orbits and their aphelion diſtances.

That retrograde Comet which appear'd in the year 1607, deſcrib'd an orbit whoſe aſcending node (according to Dr. *Halley's* computation) was in ♉ 20°. 21′; and the inclination of the plane of the orbit to the plane of the ecliptic 17°. 2′; whoſe perihelion was in ♒ 2°. 16′; and its perihelion diſtance from the Sun 58680 of ſuch parts as the radius of the *orbis magnus* contains 100000. And the Comet was in its perihelion *October* 16d. 3h. 50′. Which orbit agrees very nearly with the orbit of the Comet which was ſeen in 1682. If theſe were not two different Comets, but one and the ſame, that Comet will finiſh one revolution in the ſpace of 75 years. And the greater axe of its orbit will be to the greater axe of the *orbis magnus*, as $\sqrt[3]{:75 \times 75}$ to 1, or as 1778 to 100, nearly. And the aphelion diſtance of this Comet from the Sun will be to the mean diſtance of the Earth from the Sun as about 35 to 1. From which data it will be no hard matter to determine the elliptic orbit of this Comet. But theſe things are to be ſuppoſed, on condition, that after the ſpace of 75 years the ſame Comet ſhall return again in the ſame orbit. The other Comets ſeem to aſcend to greater heights, and to require a longer time to perform their revolutions.

But becauſe of the great number of Comets, of the great diſtance of their aphelions from the Sun, and of the ſlowneſs of their motions in the aphelions, they will, by their mutual gravitations, diſturb each other: ſo that their eccentricities and the times of their revolutions will be ſometimes a little increaſed, and ſometimes diminiſhed. Therefore we are not to expect that the ſame Comet will return exactly in the ſame orbit, and in the ſame periodic times. It will be ſufficient if we find the changes no greater, than may ariſe from the cauſes juſt ſpoken of.

And

And hence a reason may be assign'd why Comets are not comprehended within the limits of a zodiac as the Planets are; but, being confin'd to no bounds, are with various motions dispers'd all over the heavens; namely, to this purpose, that in their aphelions, where their motions are exceeding slow, receding to greater distances one from another they may suffer less disturbance from their mutual gravitations. And hence it is, that the Comets which descend the lowest, and therefore move the slowest in their aphelions, ought also to ascend the highest.

The Comet which appear'd in the year 1680. was in its perihelion less distant from the Sun than by a sixth part of the Sun's diameter: and because of its extreme velocity in that proximity to the Sun, and some density of the Sun's atmosphere, it must have suffer'd some resistance and retardation; and therefore, being attracted something nearer to the Sun in every revolution will at last fall down upon the body of the Sun. Nay in its aphelion, where it moves the slowest, it may sometimes happen to be yet farther retarded by the attractions of other Comets, and in consequence of this retardation descend to the Sun. So fixed Stars that have been gradually wasted by the light and vapours emitted from them for a long time, may be recruited by Comets that fall upon them; and from this fresh supply of new fewel, those old Stars, acquiring new splendor, may pass for new Stars. Of this kind are such fixed Stars as appear on a sudden and shine with a wonderful brightness at first, and afterwards vanish by little and little. Such was that Star which appeared in *Cassiopeias* chair; which *Cornelius Gemma* did not see upon the 8th of *November* 1572, though he was observing that part of the heavens upon that very night, and the skie was perfectly serene; but the next night (*Nov.* 9.) he saw it shining much brighter than any of the fixed Stars, and ſcarcely inferiour to *Venus* in ſplendor. *Tycho Brahe* ſaw it upon the 11th of the ſame month when it ſhone with the greateſt luſtre; and from that time he obſerv'd it to decay by little and little; and in 16 months time it entirely diſappear'd. In the month of *November*, when it firſt appeared, its light was equal to that of *Venus*. In the month of *December* its light was a little diminiſhed, and was now become equal to that of *Jupiter*. In *January* 1573. it was leſs than *Jupiter* and greater than *Sirius*, and about the end of *February* and the beginning of *March* became equal to that Star. In the months of *April* and *May* it was equal to a Star of the 2d magnitude. In *June*, *July* and *Auguſt* to a Star of the 3d magnitude. In *September*, *October* and *November* to thoſe of the 4th magnitude, in *December* and *January* 1574. to thoſe of the 5th, in *February* to thoſe of the 6th magnitude, and in *March* it entirely vaniſhed. Its colour at the beginning was clear, bright and inclining to white, afterwards it turned a little yellow, and in *March* 1573. it became ruddy like *Mars* or *Aldebaran*; in *May* it turned to a kind of dusky whiteneſs like that we obſerve in *Saturn*, and that colour it retained ever after, but growing always more and more obſcure. Such alſo was the Star in the right foot of *Serpentarius*, which *Kepler*'s ſcholars firſt obſerved *September* 30. *O. S.* 1604, with a light exceeding that of *Jupiter*, tho' the night before it was not to be ſeen. And from that time it decreas'd by little and little, and in 15 or 16 months entirely diſappeared. Such a new Star, appearing with an unuſual ſplendor, is ſaid to have moved *Hipparchus* to obſerve, and make a catalogue of, the fixed Stars. As to thoſe fixed Stars that appear and diſappear by turns, and encreaſe ſlowly and by degrees, and ſcarce ever exceed the Stars of the 3d magnitude, they ſeem to be of another kind, which revolve about their axes, and having a light and a dark ſide, ſhew thoſe two

different

different ſides by turns. The vapours which ariſe from the Sun, the fixed Stars, and the tails of the Comets, may meet at laſt with, and fall into, the atmoſpheres of the Planets by their gravity; and there be condenſed and turned into water and humid ſpirits, and from thence by a ſlow heat paſs gradually into the form of ſalts, and ſulphurs, and tinctures, and mud, and clay, and ſand, and ſtones, and coral, and other terreſtial ſubſtances.

General Scholium.

The hypotheſis of Vortices is preſs'd with many difficulties. That every Planet by a radius drawn to the Sun may deſcribe areas proportional to the times of deſcription, the periodic times of the ſeveral parts of the Vortices ſhould obſerve the duplicate proportion of their diſtances from the Sun. But that the periodic times of the Planets may obtain the ſeſquiplicate proportion of their diſtances from the Sun, the periodic times of the parts of the Vortex ought to be in the ſeſquiplicate proportion of their diſtances. That the ſmaller Vortices may maintain their leſſer revolutions about *Saturn*, *Jupiter*, and other Planets, and ſwim quietly and undiſturb'd in the greater Vortex of the Sun, the periodic times of the parts of the Sun's Vortex ſhould be equal. But the rotation of the Sun and Planets about their axes, which ought to correſpond with the motions of their Vortices, recede far from all theſe proportions. The motions of the Comets are exceeding regular, are govern'd by the ſame laws with the motions of the Planets, and can by no means be accounted for by the hypotheſis of Vortices. For Comets are carry'd with very eccentric motions through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a Vortex.

Bodies, projected in our air, suffer no resistance but from the air. Withdraw the air, as is done in Mr. *Boyle*'s vacuum, and the resistance ceases. For in this void a bit of fine down and a piece of solid gold descend with equal velocity. And the parity of reason must take place in the celestial spaces above the Earth's atmosphere; in which spaces, where there is no air to resist their motions, all bodies will move with the greatest freedom; and the Planets and Comets will constantly pursue their revolutions in orbits given in kind and position, according to the laws above explain'd. But though these bodies may indeed persevere in their orbits by the mere laws of gravity, yet they could by no means have at first deriv'd the regular position of the orbits themselves from those laws.

The six primary Planets are revolv'd about the Sun, in circles concentric with the Sun, and with motions directed towards the same parts and almost in the same plane. Ten Moons are revolv'd about the Earth, Jupiter and Saturn, in circles concentric with them, with the same direction of motion, and nearly in the planes of the orbits of those Planets. But it is not to be conceived that mere mechanical causes could give birth to so many regular motions: since the Comets range over all parts of the heavens, in very eccentric orbits. For by that kind of motion they pass easily through the orbs of the Planets, and with great rapidity; and in their aphelions, where they move the slowest, and are detain'd the longest, they recede to the greatest distances from each other, and thence suffer the least disturbance from their mutual attractions. This most beautiful System of the Sun, Planets and Comets, could only proceed from the counsel and dominion of an intelligent and powerful being. And if the fixed Stars are the centers of other like systems, these being form'd by the like wise counsel, must be all subject to the dominion of One; especially, since the light of the fixed Stars is of the same nature with the light of the Sun, and from every system light passes into all the other systems. And lest the systems of the fixed Stars should, by their gravity, fall on each other mutually, he hath placed those Systems at immense distances one from another.

This Being governs all things, not as the soul of the world, but as Lord over all: And on account of his dominion he is wont to be called *Lord God* παντοκράτωρ, or *Universal Ruler*. For *God* is a relative word, and has a respect to servants; and *Deity* is the dominion of God, not over his own body, as those imagine who fancy God to be the soul of the world, but over servants. The supreme God is a Being eternal, infinite, absolutely perfect; but a being, however perfect, without dominion, cannot be said to be Lord God; for we say, my God, your God, the God of *Israel*, the God of Gods, and Lord of Lords; but we do not say, my Eternal, your Eternal, the Eternal of *Israel*, the Eternal of Gods; we do not say, my Infinite, or my Perfect: These are titles which have no respect to servants. The word *God* usually [a] signifies *Lord*; but every lord is not a God. It is the dominion of a spiritual being which constitutes a God; a true, supreme or imaginary dominion makes a true, supreme or imaginary God. And from his true dominion it follows, that the true God is a Living, Intelligent and Powerful Being; and from his other perfections, that he is Supreme or most Perfect. He is Eternal and Infinite, Omnipotent and Omniscient; that is, his duration reaches from Eternity to Eternity; his

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[a] Dr. *Pocock* derives the Latin word *Deus* from the *Arabic du*, (in the oblique case *di*,) which signifies *Lord*. And in this sense Princes are called *Gods*, *Psal.* lxxxii. ver. 6. and *John* x. ver. 35. And *Moses* is called a *God* to his brother *Aaron*, and a *God* to *Pharaoh* (*Exod.* iv. ver. 16. and vii. ver. 8. And in the same sense the souls of dead Princes were formerly, by the Heathens, called *gods*, but falsly, because of their want of dominion.

presence from Infinity to Infinity; he governs all things, and knows all things that are or can be done. He is not Eternity or Infinity, but Eternal and Infinite; he is not Duration or Space, but he endures and is present. He endures for ever, and is every where present; and by existing always and every where, he constitutes Duration and Space. Since every particle of Space is *always*, and every indivisible moment of Duration is *every where*, certainly the Maker and Lord of all things cannot be *never* and *no where*. Every soul that has perception is, though in different times and in different organs of sense and motion, still the same indivisible person. There are given successive parts in duration, co-existent parts in space, but neither the one nor the other in the person of a man, or his thinking principle; and much less can they be found in the thinking substance of God. Every man, so far as he is a thing that has perception, is one and the same man during his whole life, in all and each of his organs of sense. God is the same God, always and every where. He is omnipresent, not *virtually* only, but also *substantially*; for virtue cannot subsist without substance. In him [b] are all things contained and moved; yet neither affects the other: God suffers nothing from the motion of bodies; bodies find no resistance from the omnipresence of God. 'Tis allowed by all that the supreme God exists necessarily; and by the same necessity he exists

[b] This was the opinion of the Ancients. So *Pythagoras* in *Cicer. de Nat. Deor.* lib. i. *Thales*, *Anaxagoras*, *Virgil*, Georg. lib. iv. ver. 220. and Æneid. lib. vi. ver. 721. *Philo Allegor.* at the beginning of lib. i. *Aratus* in his Phænom. at the beginning. So also the sacred Writers, as St. *Paul*, *Acts* xvii. ver. 27, 28. St. *John's* Gosp. chap. xiv. ver. 2. *Moses* in *Deut.* iv. ver. 39. and x. ver. 14. *David*, *Psal.* cxxxix. ver. 7, 8, 9. *Solomon*, 1 *Kings* viii. ver. 27. *Job* xxii. ver. 12, 13, 14. *Jeremiah* xxiii. ver. 23, 24. The Idolaters supposed the Sun, Moon and Stars, the Souls of Men, and other parts of the world, to be parts of the supreme God, and therefore to be worshipped: but erroneously.

always and *every where.* Whence also he is all similar, all eye, all ear, all brain, all arm, all power to perceive, to understand, and to act; but in a manner not at all human, in a manner not at all corporeal, in a manner utterly unknown to us. As a blind man has no idea of colours, so have we no idea of the manner by which the all-wise God perceives and understands all things. He is utterly void of all body and bodily figure, and can therefore neither be seen, nor heard, nor touched; nor ought he to be worshipped under the representation of any corporeal thing. We have ideas of his attributes, but what the real substance of any thing is, we know not. In bodies we see only their figures and colours, we hear only the sounds, we touch only their outward surfaces, we smell only the smells, and taste the savours; but their inward substances are not to be known, either by our senses, or by any reflex act of our minds; much less then have we any idea of the substance of God. We know him only by his most wise and excellent contrivances of things, and final causes; we admire him for his perfections; but we reverence and adore him on account of his dominion. For we adore him as his servants; and a God without dominion, providence, and final causes, is nothing else but Fate and Nature. Blind metaphysical necessity, which is certainly the same always and every where, could produce no variety of things. All that diversity of natural things which we find, suited to different times and places, could arise from nothing but the ideas and will of a Being necessarily existing. But by way of allegory, God is said to see, to speak, to laugh, to love, to hate, to desire, to give, to receive, to rejoice, to be angry, to fight, to frame, to work, to build. For all our notions of God are taken from the ways of mankind, by a certain similitude which, though not perfect, has some likeness however. And thus much concerning God; to discourse of whom from the appearances of things, does certainly belong to Natural Philosophy.

Hitherto we have explain'd the phænomena of the heavens and of our sea, by the power of Gravity, but have not yet assign'd the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centers of the Sun and Planets, without suffering the least diminution of its force; that operates, not according to the quantity of the surfaces of the particles upon which it acts, (as mechanical causes use to do,) but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides, to immense distances, decreasing always in the duplicate proportion of the distances. Gravitation towards the Sun, is made up out of the gravitations towards the several particles of which the body of the Sun is compos'd; and in receding from the Sun, decreases accurately in the duplicate proportion of the distances, as far as the orb of Saturn, as evidently appears from the quiescence of the aphelions of the Planets; nay, and even to the remotest aphelions of the Comets, if those aphelions are also quiescent. But hitherto I have not been able to discover the cause of those properties of gravity from phænomena, and I frame no hypotheses. For whatever is not deduc'd from the phænomena, is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferr'd from the phænomena, and afterwards render'd general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered. And to us it is enough, that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

And now we might add something concerning a certain most subtle Spirit, which pervades and lies hid in all gross bodies; by the force and action of which Spirit, the particles of bodies mutually attract one another at near distances, and cohere, if contiguous; and electric bodies operate to greater distances, as well repelling as attracting the neighbouring corpuscles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this Spirit, mutually propagated along the solid filaments of the nerves, from the outward organs of sense to the brain, and from the brain into the muscles. But these are things that cannot be explain'd in few words, nor are we furnish'd with that sufficiency of experiments which is required to an accurate determination and demonstration of the laws by which this electric and elastic spirit operates.

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APPENDIX.

Among the Explications, (given by a Friend,) of some Propositions in this Book, not demonstrated by the Author, the Editor finding these following, has thought it proper to annex them. Thus,

To Cor. 2. *Prop.* 91. *Book* 1. *Pag* 303.

1. TO find the force whereby a sphere ($AdBg$,) on the diameter AB, attracts the body P. (*Pl.* 19. *Fig.* 1.)

Let $SA = SB = r$, $PS = d$, $PE = x$, $PB = a = d + r$, $PA = \alpha = d - r$; Theref. $a\alpha = dd - rr$; also $a + \alpha = 2d$, $a - \alpha = 2r$; Therefore $aa - \alpha\alpha = 4dr$: And $SE = d - x$, $AE = x - \alpha$, $BE = a - x$.

Now the force whereby the circle, whose radius is Ed, attracts the body P, is as $1 - \frac{PE}{Pd}$ (by Cor. 1. Prop. 90.)

 And

And $\overline{Ed}^2 = (AE \times EB = \overline{SA}^2 - \overline{SE}^2 = rr - dd + 2dx - xx =) - a\alpha + 2dx - xx$. Also $\overline{Pd}^2 = (\overline{Ed}^2 + \overline{EP}^2 = 2dx - a\alpha - xx + xx =)\ 2dx - a\alpha$: Th. $\frac{PE}{Pd} = \frac{x}{\sqrt{-a\alpha + 2dx}}$. Therefore

$1 - \frac{x}{\sqrt{-a\alpha + 2dx}}$ or $\dot{x} - \frac{x\dot{x}}{\sqrt{-a\alpha + 2dx}}$ is the fluxion of the attractive force of the sphere on the body P, or the ordinate of a curve whose area represents that force.

But the fluent of $\dot{x}$ is x; and the fluent of $\frac{\dot{x}x}{\sqrt{-a\alpha + 2dx}}$ is $\frac{a\alpha + dx}{3dd}\sqrt{-a\alpha + 2dx}$ (by *Tab.* 1. *Form* 4. *Cas.* 2. *Quadr. of Curv.*)

Therefore $x - \frac{a\alpha + dx}{3dd}\sqrt{-a\alpha + 2dx}$ is the general expression of the area of the curve.

Now let $x = a$, then area $= (a - \frac{a\alpha + da}{3dd}\sqrt{-a\alpha + 2da} =) \frac{d^3 + r^3}{3dd} = A$.

Also let $x = \alpha$, then area $= (\alpha - \frac{a\alpha + d\alpha}{3dd}\sqrt{-a\alpha + 2d\alpha} =) \frac{d^3 - r^3}{3dd} = B$.

And the force whereby the sphere attracts the body P is as $(A - B$ or as $\frac{2r^3}{3d^2} =) \frac{2\overline{SA}^3}{3\overline{PS}^2}$.

 2. The

2. The force whereby the spheroid $ADBG$, attracts the body P, may, in the same manner, be found thus. Let $SC = c$,

The force of a circle whose radius is ED, to attract P, is as $1 - \frac{PE}{PD}$, (by Cor. 1. Prop. 90.) Now $\overline{ED}^2 = \frac{\overline{SC}^2}{\overline{SA}^2} \times AEB = \frac{cc}{rr} \times \overline{-aa + 2dx - xx}$ (by the Conics;) and $\overline{PD}^2 = (\overline{ER}^2 = \overline{ED}^2 + \overline{EP}^2 = \frac{-aacc + 2dccx - ccxx}{rr} + xx =)$ $\frac{-aacc + 2dccx + \overline{rr - cc}\, xx}{rr}$. Therefore $(1 - \frac{PE}{PD} = 1 - \frac{x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr-cc}{rr}xx}}$, or) $\dot{x} - \frac{\dot{x}x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr-cc}{rr}xx}}$ is the fluxion of the attractive force of the spheroid on the body P, or the ordinate of a curve whose area is the measure of that force.

Now the fluent of $\dot{x}$ is x; and (by *Cas.* 2. *Form* 8. *Tab.* 2. *Quad. Cur.*) the fluent of $\frac{\dot{x}x}{\sqrt{-\frac{aacc}{rr} + \frac{2dcc}{rr}x + \frac{rr-cc}{rr}xx}}$ is $\left(\frac{-\frac{8dcc}{rr}s + \frac{4dcc}{rr}xv - \frac{4aacc}{rr}v}{-\frac{4aacc}{rr} \times \frac{rr-cc}{rr} - \frac{4ddc^4}{r^4}} = \frac{-2drrs + drrxv - aarrv}{aa \times \overline{cc - rr} - ddcc} = \frac{-2ds + dxv - axv}{-cc - dd + rr} = \right)$

$=) \frac{2ds - dxv + a\alpha v}{cc + dd - rr}$. Therefore $x + \frac{dxv - a\alpha v - 2ds}{cc + dd - rr}$ is the general expression for the area of the curve.

But $v = PD = ER = \sqrt{-\frac{a\alpha cc}{rr} + \frac{2dcc}{rr}x - \frac{cc - rr}{rr}xx}$ is an ordinate to a conic section, whose abscissa is x; and s, σ, the areas NMB, NKA, adjacent to the ordinates BM, AK: Put $D = s - \sigma$.

Let $x = a$, or $PE = PB = BM$; then $v = a$, or $PD = PB = BM$, and the area $= a + \frac{daa - \alpha aa - 2ds}{cc + dd - rr} = A$. And let $x = \alpha$, or $PE = PA = AK$; then $v = \alpha$, or $PD = PA = AK$, and the area $= \alpha + \frac{d\alpha\alpha - a\alpha\alpha - 2d\sigma}{cc + dd - rr} = B$.

And the attractive force of the spheroid on P, is as $(A - B = a - \alpha + \frac{d \times \overline{aa - \alpha\alpha} - a\alpha \times \overline{a - \alpha} - 2d \times \overline{s - \sigma}}{cc + dd - rr} = 2r + \frac{2ddr + 2r^3 - 2dD}{cc + dd - rr} =) \frac{2rcc + 2d \times \overline{2dr - D}}{cc + dd - rr}$.

But $2d = (a + \alpha =)\ BM + AK$, therefore $2dr =$ trapezium $ABMK$; and $D = (s - \sigma =)$ area $AKRMB$; therefore $D - 2dr =$ mixtilinear area $KRMLK = C$; consequently $2dr - D = -C$; therefore $2d \times \overline{2dr - D} = -2dC$; therefore the attractive force of the spheroid on P, is as $\frac{2rcc - 2dC}{cc + dd - rr} = \frac{2AS \times \overline{SC}^2 - 2PS \times KRMK}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2}$. Consequently, the attractive force of the spheroid upon the body P will be to the attractive force of a sphere, whose diameter is AB, upon the same body P, as $\frac{rcc - dC}{cc + dd - rr}$ to $\frac{r^3}{3dd}$, or as $\frac{AS \times \overline{SC}^2 - PS \times KRMK}{\overline{SC}^2 + \overline{PS}^2 - \overline{AS}^2}$ to $\frac{\overline{AS}^3}{3\overline{PS}^2}$.

To

To Schol. Prop. 34. Book 2. p. 119. l. 20.

For let it be proposed to find the vertex of the cone, a frustum of which has the describ'd property.

Let *CFGB* be the frustum, and *S* the vertex required. (*Pl.* 19. *Fig.* 2.)

Now conceive the medium to consist of particles which strike the surface of a body (moving in it) in a direction opposite to that of the motion; then the resistance will be the force which is made up of the efficacy of the forces of all the strokes.

In any line *Pp*, parallel to the axis of the cone, and meeting its surface in *p*, take *pm* of a given length, for the space describ'd by each point of the cone in a given time: Draw *mq* perpendicular to the side (*CF*) of the cone, and *qn* perpendicular to *pm*.

Therefore the line *pm* will represent the velocity, or force, with which a particle of the medium strikes the surface of the cone obliquely in *p*.

But the force *mp* is equivalent to two forces, the one (*mq*) perpendicular, the other (*pq*) parallel to the side of the cone; which last is therefore of no effect.

And the perpendicular force *mq* is equivalent to two forces, the one (*mn*) parallel to the axis of the cone, the other (*qn*) perpendicular to it; which also is destroy'd by the contrary action of another particle on the opposite side of the cone.

There remains only the force *mn*, which has any effect in resisting or moving the cone in the direction of its axis.

Therefore the whole force of a single particle, or the effect of the perpendicular stroke of a particle, upon the base of a circumscribing cylinder, is to the effect of the oblique stroke upon the surface of the cone (in *p*) as *mp* to *mn*, or as $\overline{mp}^2$ to ($mp \times mn =$) $\overline{mq}^2$, or as $\overline{CF}^2$ to $\overline{CH}^2$.

 Now

Now the number of particles ſtriking in a parallel direction on any ſurface, is as the area of a plane figure perpendicular to that direction, and that would juſt receive thoſe ſtrokes.

Therefore, the number of particles ſtriking againſt the fruſtum, that is, againſt the ſurfaces deſcrib'd by the rotation of FD, and CF, each particle with the forces mp, and mn reſpectively, is as the circle deſcrib'd by (FD or) OH, and the annulus deſcribed by CH, that is, as $\overline{OH}^2$ to $\overline{CO}^2 - \overline{OH}^2$.

But the whole force of the medium in reſiſting, is the ſum of the forces of the ſeveral particles.

Therefore, the reſiſtance of the medium, or the whole efficacy of the force of all the ſtrokes againſt the end FG of the fruſtum, is to the reſiſtance againſt the convex ſurface thereof, as ($mp \times \overline{OH}^2$ to $mn \times \overline{\overline{CO}^2 - \overline{OH}^2}$ or as $\overline{CF}^2 \times \overline{OH}^2$ to $\overline{CH}^2 \times \overline{\overline{CO}^2 - \overline{OH}^2}$ or as) $\overline{OH}^2$ to $\dfrac{\overline{CH}^2 \times \overline{\overline{CO}^2 - \overline{OH}^2}}{\overline{CF}^2}$.

Theref. the whole reſiſtance of the medium againſt the fruſtum may be repreſented by ($\overline{OH}^2 + \dfrac{\overline{CH}^2 \times \overline{\overline{CO}^2 - \overline{OH}^2}}{\overline{CF}^2}$

$$= \frac{\overline{CF}^2 \times \overline{OH}^2 - \overline{CH}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}$$

$= $) $\dfrac{\overline{HF}^2 \times \overline{OH}^2 + \overline{CH}^2 \times \overline{OC}^2}{\overline{CF}^2}$, which call z;

that is, (putting $OC = r$, $OD = 2a$, $OS = y$, then $CH = (\dfrac{OC \times FH}{OS} =) \dfrac{2ar}{y}$, and $OH = \dfrac{ry - 2ar}{y}$,)

$z = \dfrac{r^4 + r^2y^2 - 4ar^2y + 4a^2r^2}{r^2 + y^2}$; therefore $r^4 + r^2y^2 - 4ar^2y + 4a^2r^2 = r^2z + y^2z$: Conſequently

$2r^2$

$2r^2 y\dot{y} - 4ar^2\dot{y} = 2yz\dot{y} + y^2\dot{z} + r^2\dot{z}$; But z is a minimum; therefore $rry - 2arr = zy$; consequently $(z =) \frac{r^2 y - 2ar^2}{y} = \frac{r^4 + r^2 y^2 - 4ar^2 y + 4a^2 r^2}{rr + yy}$.

Hence $yy - 2ay = rr$; and making $OQ = QD = a$; then $(y - a =) QS = (\sqrt{rr + aa} =) QC$.

To the same Schol. p. 120. l. 10.

On the right-line BC, (*Pl.* 19. *Fig* 3.) suppose the parallelograms $BG\gamma b$, $MN\nu m$, of the least breadth, to be erected, whose hights BG, MN, their distance Mb, and half the sum of their bases $\frac{1}{2}Mm + \frac{1}{2}Bb = a$, are given: Let half the difference of the bases $\frac{1}{2}Mm - \frac{1}{2}Bb$ be called x: Let G and N be points in the curve GND; and producing $b\gamma$, and $m\nu$ to g and n, (so that $\gamma g = \nu n = b$,) the points g and n may also be in the same curve.

Now if the figure $CDNGB$, revolving about the axis BC, generates a solid, and that solid moves forwards in a rare and elastic medium from C towards B, (the position of the right-line BC remaining the same;) then will the sum of the resistances against the surfaces generated by the lineolæ Gg, Nn, be the least possible, when $\overline{Gg}^4$ is to $\overline{Nn}^4$ as $BG \times Bb$ to $MN \times Mm$.

For the force of a particle on Gg and Nn, to move them in the direction BC, is as $\frac{1}{\overline{Gg}^2}$ and $\frac{1}{\overline{Nn}^2}$; and the number of particles that strike in the same time on the surfaces generated by Gg and Nn, are as (the annuli describ'd by $g\gamma$ and $n\nu$, that is, as $BG \times g\gamma$ and $MN \times n\nu$, or as) BG and MN; therefore the resistances against those surfaces are as $\frac{BG}{\overline{Gg}^2}$ to $\frac{MN}{\overline{Nn}^2}$, that is (putting y for $\overline{Gg}^2$, and z for $\overline{Nn}^2$,) as $\frac{BG}{y}$ to $\frac{MN}{z}$.

But the sum of these resistances ($\frac{BG}{y} + \frac{MN}{z}$) is a minimum. Therefore $-BG \times \frac{\dot{y}}{yy} - MN \times \frac{\dot{z}}{zz} = 0$, or $MN \times \frac{\dot{z}}{zz} = -BG \times \frac{\dot{y}}{yy}$: But $y = (\overline{Gg}^2 = \overline{Bb}^2 + \overline{\gamma g}^2 =)\ aa - 2ax + xx + bb$; and $z = (\overline{Nn}^2 = \overline{Mm}^2 + \overline{vn}^2 =)\ aa + 2ax + xx + bb$; therefore $\dot{y} = 2x\dot{x} - 2a\dot{x}$, and $\dot{z} = 2a\dot{x} + 2x\dot{x}$: consequently $\frac{MN}{zz} \times 2\dot{x} \times \overline{a+x} = \frac{BG}{yy} \times 2\dot{x} \times \overline{a-x}$; or $(\frac{MN}{zz} \times \overline{a+x} =)\ \frac{MN}{zz} \times Mm = (\frac{BG}{yy} \times \overline{a-x} =)\ \frac{BG}{yy} \times Bb$. Therefore $(yy)\ \overline{Gg}^4 : (zz)\ \overline{Nn}^4 :: BG \times Bb : MN \times Mm$.

Consequently, that the sum of the resistances against the surfaces generated by the lineolæ Gg and Nn, may be the least possible, $\overline{Gg}^4$ must be to $\overline{Nn}^4$ as GBb to NMm.

Wherefore, if γg be made equal to γG, so that the angle γGg may be 45°, and the angle BGg 135°; also $\overline{Gg}^2 = 2\,\overline{\gamma g}^2$, and $\overline{Gg}^4 = 4\,\overline{\gamma g}^4$; then $4\,\overline{\gamma g}^4 : \overline{Nn}^4 :: GBb : NMm$; and since GR is parallel to Nn, and BG, BR parallel to nv, Nv; also $nv = g\gamma = \gamma G$; it follows that $(nv = \gamma G =)\ Bb : (Nv =)\ Mm :: BG : BR$; therefore $Bb = \frac{BG \times Mm}{BR}$; also $(nv =)\ \gamma G : Nn :: BG : GR$. Consequently $\frac{4\overline{\gamma g}^4}{\overline{Nn}^4} =)\ \frac{4\overline{BG}^4}{\overline{GR}^4} = (\frac{GBb}{NMm} =)\ \frac{\overline{BG}^2}{MN \times BR}$. Therefore $4\overline{BG}^2 \times BR$ is to $\overline{GR}^3$ as GR to MN.

FINIS.

THE
LAWS
OF THE
MOON's
MOTION
According to
GRAVITY.

The Laws of the

MOON's

MOTION.

IN justice to the editor of this translation of Sir *Isaac Newton*'s *Principia*, it is proper to acquaint the reader, that it was with my consent, he published an advertisement, at the end of a volume of miscellanies, concerning a small tract which I intended to add to his book by way of appendix; my design in which was to deliver some general elementary propositions, serving, as I thought, to explain and demonstrate the truth of the rules in Sir *Isaac Newton*'s Theory of the Moon.

THE occaſion of the undertaking was merely accidental; for he ſhewing me a paper which I communicated to the author, in the year 1717, relating to the motion of the nodes of the Moon's orbit; I recollected, that the method made uſe of in ſettling the Equation for that motion, was equally applicable to any other motion of revolution. And therefore I thought that it would not be unacceptable to a reader of the *Principia*, to ſee the uſes of the ſaid method explained in the other Equations of the Moon's motion: Eſpecially ſince the greateſt part of the Theory of the Moon is laid down without any proof; and ſince thoſe propoſitions relating to the Moon's motion, which are demonſtrated in the *Principia*, do generally depend upon calculations very intricate and abſtruſe, the truth of which is not eaſily examined, even by thoſe that are moſt skilful; and which however might be eaſily deduced from other principles.

But in my progreſs in this deſign, happening to find ſeveral general propoſitions relating to the Moon's motions, which ſerve to determine many things, which have hitherto been taken from the obſervations of Aſtronomers: And having

having reaſon to think, that the Theory of the Moon might by theſe means, be made more perfect and compleat than it is at preſent; I retarded the publication of the book, 'till I could procure due ſatisfaction by examining obſervations on places of the Moon. But finding this to be a work requiring a conſiderable time, not only in procuring ſuch places as are proper, but alſo in performing calculations, upon a new method, not yet accommodated to practiſe by convenient rules, or aſſiſted by tables; I thought it therefore more convenient for the Bookſeller, not to ſtop the publication of his impreſſion any longer upon this account. But that I may in ſome meaſure, ſatisfy thoſe who are well converſant in Sir *Iſaac Newton's Principia*, (and I could wiſh that none but ſuch would look over theſe papers,) that the ſaid advertiſement was not without ſome foundation; and that I may remove any ſuſpicion that the deſign is entirely laid aſide, I have put together, altho' in no order, as being done upon a ſudden reſolution, ſome of the Propoſitions, among many others, that I have by me, which ſeem chiefly to be wanting in a Theory of the Moon, as it is a ſpeculation founded on a phyſical cauſe; and thoſe are what relate to the ſtating of the mean motions. For altho' it be of little or no uſe in Aſtronomy to know the rules for aſcertaining the mean motions of the Node or Apogee, ſince the fact is all that is wanting, and that is otherwiſe known by comparing the obſervations of former ages with thoſe of the preſent; yet in matter of ſpeculation, this is the chief and moſt neceſſary thing required: ſince there is no other way to know that the cauſe is rightly aſſigned, but by ſhewing that the motions are ſo much and no more than what they ought to be.

But that it may not be altogether without its uſe, I have added all the rules for the equation of the Moon's motion, except two; one of which is a monthly equation of the variation depending on the Moon's anomaly; and the other an equation ariſing from the Earth's being not in the focus of the Moon's orbit, as it has been ſuppoſed to be, in all the modern theories ſince *Horrox*.

For not having had time to examine over the obſervations which are neceſſary, but being oblig'd inſtead thereof, to take Sir *Iſaac Newton*'s theory for my chief guide and direction, I cannot venture

venture to depart from it too far, in eſtabliſhing equations entirely new; ſince I am well aſſured, upon the beſt authority, that it is never found to err more than ſeven or eight minutes.

And therefore, hoping that the reader, who conſiders the ſudden occaſion and neceſſity of my publiſhing theſe Propoſitions at this time, will make due allowance for the want of order and method, and look upon them only as ſo many diſtinct Rules and Propoſitions not connected: I ſhall begin, without any other preface, with ſhewing the origine of that inequality, which is called the Variation or Reflection of the Moon.

The Variation of the Moon.

THE variation or reflection is that monthly inequality in the Moon's motion, wherein it more manifeſtly differs from the laws of the motion of a planet in an elliptic orbit. *Tycho Brahe* makes this inequality to ariſe from a kind of libratory motion backwards and forwards, whereby the Moon is accelerated and retarded by turns, moving ſwifter in the firſt and third quarter, and ſlower in the ſecond and fourth, which inequality is principally obſerved in the octants.

Sir *Iſaac Newton* accounts for the

variation from the different force of gravity of the Moon and Earth to the Sun, arising from the different distances of the Moon in its several aspects.

The mean gravity of the Moon to the Sun, he supposes, is satisfied by the annual motion of the Moon round the Sun; the gravity of the Moon to the Earth, he supposes, is satisfied by a revolution of the Moon about the Earth. But the difference of the Moon's gravity to the Sun more or less than the Earth's gravity, he supposes, produces two effects; for as this difference of force may be resolved into two forces, one acting in the way, or contrary to the way, of the Moon about the Earth, and the other acting in the line to or from the Earth: the first causes the Moon to describe a larger or smaller area in the same time about the Earth, according as it tends to accelerate or retard it; the other changes the form of the lunar orbit from what it ought to be merely from the Moon's gravity to the Earth, and both together make up that inequality which is called the variation.

But since the real motion of the Moon, tho' a simple motion, caused by a continual deflection from a streight line, by the

the joint force of its gravity to the Sun and Earth, thereby deſcribing an orbit, which incloſes not the Earth but the Sun, is yet conſidered as a compound motion, made from two motions, one about the Sun, and the other about the Earth; becauſe two ſuch motions are requiſite to anſwer the two forces of its gravity, if ſeparately conſidered: For the very ſame reaſon, the Moon's motion ought to be reſolved into a third motion of revolution, ſince there remains a third force to be ſatisfied, and that is the force ariſing from the alteration of the Moon's gravity to the Sun. And this when conſidered, will require a motion in a ſmall ellipſis, in the manner here deſcribed.

THE circle *ADFH* repreſents the orbit of the Moon about the Earth in the center *T*, as it would be at a mean diſtance, ſuppoſing the Moon had no gravity to any other body but the Earth. The diameter *ATF* divides that part of the orbit which is towards the Sun, ſuppoſe *ADF*, from the part oppoſite to the Sun, ſuppoſe *AHF*. The diameter at right angles *HTD*, is the line of the Moon's conjunction with or oppoſition to the Sun. The figure *PQLK* is an Ellipſis, whoſe center Fig. 1.

ter

ter is carried round the Earth in the orbit *ABDEFH*, having its longer axis *PL* in length double of the ſhorter axis *QK*, and lying always parallel to *TD*, the line joining the centers of the Earth and Sun. Whilſt the ſaid figure is carried from *A* to *B*, the Moon revolves the contrary way from *Q* to *N*, ſo as to deſcribe equal areas in equal times about the centre of it; and to perform its revolution in the ſame time as the center of the ſaid Elliptic epicycle (if it may be ſo called,) performs its revolution; the Moon being always in the remoter extremity of its ſhorter axis in *Q* and *K* when it is in the quarters, and in the neareſt extremity of its longer axis at the time of the new and full Moon.

THE ſhorter ſemiaxis of this Ellipſis *AQ*, is to the diſtance of its center from the Earth *AT*, in the duplicate proportion of the Moon's periodical time about the Earth to the Sun's periodical time: Which proportion, if there be 2139 revolutions of the Moon to the Stars in 160 ſydereal years, is that of 47 to 8400.

THE figure which is deſcribed by this compound motion of the Moon in the Elliptic epicycle, whilſt the center of it is carried round the Earth, very nearly repreſents the form of the Lunar orbit; ſuppoſing it without eccentricity, and that the

the plane was coincident with the plane of the ecliptic, and that the Sun continu'd in the ſame place during the whole revolution of the Moon about the Earth.

FROM the above conſtruction it appears, that the proportion between the mean diſtance of the Moon and its greateſt or leaſt diſtances, is eaſily aſſigned; being ſomething larger than that which is aſſigned by Sir *Iſaac Newton* in the 8th propoſition of his third book. But as the computation there given, depends upon the ſolution of a biquadratic equation, affected with numeral coefficients; which renders it impoſſible to compare the proportions with each other, ſo as to ſee their agreement or diſagreement, except in a particular application to numbers; I ſhall therefore ſet down a rule, in general terms, derived from his method, which will be exact enough, unleſs the periods of the Sun and Moon ſhould be much nearer equal than they are. Let L be the periodical time of the Moon, S the period of the Sun, M the ſynodical period of the Moon to the Sun, and D be the difference of the periods of the Sun and Moon; then, according to Sir *Iſaac Newton*'s method, the difference of the two axes of the Moon's elliptic orbit, as it is contracted by the action

action of the Sun, is to the ſum of the ſaid axes as $3L \times \frac{M+L}{2}$ to $4DD - SS$. But according to the conſtruction before laid down, the ſaid proportion is as $3LL$ to $2SS - LL$.

By Sir *Iſaac Newton*'s rule, the difference will be to the ſum, nearly as 5 to 694; and conſequently the diameters will be nearly as 689 to 699, or 69 to 70: But by the latter rule, the difference will be to the ſum, nearly as 1 to 119; and the diameters or diſtances of the Moon, in its conjunction and quadrature with the Sun, will be as 59 to 60. Dr. *Halley*, (who in his remarks upon the Lunar theory, at the end of his catalogue of the Southern ſtars, firſt took notice of this contraction of the Lunar orbit in the Syzygies from the phenomena of the Moon's motion) makes the difference of the diameters to the ſum, as 1 to 90; and conſequently the greater axis to the leſſer, as $45\frac{1}{2}$ to $44\frac{1}{2}$.

But the difference, in theſe proportions of the extream diſtances, tho' it may appear conſiderable, is not, however, to be diſtinguiſh'd by the obſervations on the diameters of the Moon, whilſt the variations of the diameters, from

from this cause, are intermixt with the other much greater variations, arising from the eccentricity of the orbit.

THE angle of the Moon's elongation from the center, designed by *BTN*, is properly the variation or reflection of the Moon. The properties of which are evident from the description. Fig. 1.

FIRST, It is as the sine of the double distance of the Moon from the quadrature or conjunction with the Sun: For it is the difference of the two angles *BTA* and *NTA*, whose tangents, by the construction, are in a given proportion.

SECONDLY, The variation is, *cæteris paribus*, in the duplicate proportion of the synodical time of the Moon's revolution to the Sun. For the variation is in proportion to the mean diameter of the epicycle, and that is in the duplicate proportion of the synodical time of revolution.

THE greatest variation is an angle, whose sine is to the radius, as the difference of the greatest and least distances *TQ* and *TL*, that is 3*AQ*, to their sum. According to the proportion of the lines before described, this rule makes the elongation near 29 minutes; which would be be the variation, ſuppoſing the Moon perform'd its revolution to the Sun in the time of its revolution round the Earth. But if that elongation of 29 Minutes be increaſed in the duplicate proportion of the ſynodical time to the periodical time of revolution, it will produce near 34 minutes for the variation.

It is to be noted, that what is ſaid of the epicycle, is upon ſuppoſition, that the Earths orbit round the Sun is a circle; if the eccentricity of the annual orbit be conſidered, the mean diameter of the epicycle muſt increaſe or diminiſh reciprocally in the triplicate proportion of the Sun's diſtance.

The method of finding the inequalities in any Revolution.

The conſtruction which I communicated to Sir *Iſaac Newton*, for the annual motion of the nodes of the Moon's orbit, (which is printed in the ſcholium to the 33d propoſition of his 3d book) is a caſe of a general method, for ſhewing the inequality of any motion round a center, when the hourly motion or velocity of the object varies, according to any rule, depending on its aſpect to ſome other object. For in any revolution, the mean motion and inequality are to be aſſigned by means of a curvilinear figure

figure, wherein equal areas are deſcribed about the center in equal times; the property of which figure is, that the rays from the center, are always reciprocally in the ſubduplicate proportion of the hourly motion or velocity about the center.

Thus in the figure deſcribed in my conſtruction, where *T N* is the line of the nodes, *T A* the line drawn to the Sun, is ſuppoſed to revolve round the center *T*, with the velocity of the Sun's motion from the node; and the ray *T B*, which is taken always in the ſubduplicate proportion of that velocity, will deſcribe equal areas in equal times; ſo that the ſector *NTB* will be the mean motion of the Sun; the ſector *NTA* the motion of the Sun from the node; and conſequently the area *NAB* the motion of the node; which will be a retrograde motion if the area be within the circle, and direct if it falls without. From whence it follows, Fig. 2.

1. That the periodical time of the Sun's revolution to the node, will be to the periodical time of the Sun's revolution, as the area of the curvilinear figure, to the area of the circle.

2. That if a circle be deſcribed, whoſe area is equal to the area of the curvilinear

near figure, it will cut that figure in the place where the Sun has the mean motion from the node.

3. If an angle *NTF* be made, which ſhall comprehend an area in the ſaid circle, equal to the ſector *N T B* in the figure, that angle will be the mean motion of the Sun from the node. And conſequently,

4. The angle *FTB*, which is the difference between the Sun's true motion from the node, deſigned by *ATN*, and the Sun's mean motion from the node, deſigned by *FTN*, will be the equation for the Sun's motion from the node, when the Sun's poſition to the node is deſigned by the angle *ATN*.

From all which it appears, that what is ſaid of the Sun's motion from the node, will hold as to any other motion round a center; as of the Sun from the Moon, or the Moon from the node or apogee. In any ſuch revolution, a curvilinear figure may be deſcribed about the center, by the areas of which, the relation between the mean and true motion may be ſhewn; and conſequently the inequality or equation of the motion.

Thus

And as in every revolution there is a certain figure which is proper to ſhew this relation, ſuch a figure may be call'd an Equant for that motion or revolution.

And in every revolution where the Equant is a figure of the ſame property, the inequalities or equations will alter according to the ſame rule.

Thus, if the Equant be an ellipſis about the center, as in that for the motion of the Sun from the node,

Firſt, The mean motion in the whole revolution, will be a geometrical mean proportional, between the greateſt motion in the extremity of the leſſer axis, and the leaſt motion in the extremity of the longer axis: For the radius of the circle, which is equal to an ellipſis, is a mean proportional between the two ſemiaxes.

Secondly, The tangents of the angles of the mean and true motion, are in the given proportion of the two axes of the ellipſis. Thus the tangents of the angles of the true and mean motion of the Sun from the node, *viz.* the tangents of the angles *ATN* and *FTN*, are in proportion as the ordinates *BG* and *FG*, that is, as the ſemiaxes *TH* and *TN*. Fig. 2.

Thirdly, THE ſine of the angle of the greateſt inequality in the octants is

 to

to the radius, as half the ſum of the axes to half their difference.

It is to be noted, that the equant is an ellipſis about the center, in every motion, where the exceſs of the velocity about the center above the leaſt velocity, is always in the duplicate proportion of the ſine of the angle of the true motion, from the place where the velocity about the center is leaſt. From which remark, upon examination it will appear, that the following motions are to be reduced to an Elliptic equant deſcribed about the center.

The monthly motion of the Moon from the node.

The annual motion of the Sun from the node.

The motion of the Moon from the Sun, as it is accelerated or retarded, by the alteration of the area deſcrib'd about the Earth, according to Sir *Iſaac Newton*'s 26th prop. 3d book.

And the annual Motion of the Sun from the apogee. How theſe ſeveral equants are determin'd will appear by what follows.

The motion of the Nodes.

The node is in its ſwifteſt retrograde motion, when the Sun and Moon are in conjunction or oppo-

oppoſition, and in a quadrature with the line of the nodes. According to Sir *Iſaac Newton*'s method, (explain'd at the end of the thirtieth propoſition of the third book) the force of the Sun to produce a motion in the node, at this time, is equal to three times the mean Solar force; that is, by the conſtruction of the elliptic epicycle, equal to a force, which is to the force of gravity, as 3 *AQ* to *AT*, or three times the leſſer ſemiaxis of the ellipſis to the diſtance of its center from the center of the Earth. But if the Moon revolve in the elliptic epicycle as before deſcribed, the force to make a motion in the node at the time mention'd, will be to the force of gravity, as 3*DL* to *DT*, or three times the longer ſemiaxis to the diſtance of the center; which is the double of the former force. But then, according to Sir *Iſaac*'s method, the motion of the node at this time, is to the Moon's motion, as the ſolar force to create a motion in the node is to the force of gravity. But if the Moon be conceived as revolving in a circle, with the velocity of its motion from the node at this time, when the node moves ſwifteſt, and the plane of the ſaid circle be ſuppoſed to have a rotation Fig. 1.

upon an axis perpendicular to the plane of the ecliptic, and the contrary way to the motion of the Moon, ſo as to produce the motion of the node, and leave the Moon to move with its own motion about the Earth; the force to make a motion in the node ſeems to be the difference of the forces to retain it with the velocity of its motion in the moveable and immoveable planes: But the velocities of bodies revolving in circles are in the ſubduplicate proportion of the central forces. From whence it follows, that

The motion of the Moon from the node at this time, when the node moves ſwifteſt, is to the motion of the Moon, in the ſubduplicate proportion of the ſum of the forces to the force of gravity, or as the ſum of TD *and* 3DL *to* TD.

And this would be the greateſt motion of the node, upon ſuppoſition that the plane of the Moon's orbit was almoſt co-incident with the plane of the ecliptic; but if the inclination be conſidered, the motive force for the node muſt be diminiſhed, in the proportion of the ſine-complement of the inclination to the radius. How much this

this motion is, will appear by the following short calculation.

The distance *TD* being as before equal to 8400, and 3*DL* being 282, the inclination of the plane in this position is 4°. 59′. 35″; the sine-complement of which is to the radius, as 525 to 527 nearly; therefore the force of gravity is to the motive force for the node thus diminished, in the compound proportion of 8400 to 282, and of 527 to 525, that is, in the proportion of 4216 to 141. So that the greatest motion of the Moon from the node is to the motion of the Moon, in the subduplicate proportion of 4357 to 4216, that is, in the proportion nearly of 613 to 603. According to which calculation, the greatest hourly motion of the node ought to be 32″. 47‴. By Sir *Isaac Newton*'s method, it amounts to 33″. 10‴½.

This is the swiftest retrograde motion of the node, when the line of the nodes is in a quadrature with the Sun, and the Moon is in its greatest latitude in conjunction or opposition to the Sun. But the equant for the motion of the Moon from the node in this month, when the line of the nodes is in quadrature with the Sun, is an ellipsis about the center; and therefore the mean motion in this month will be known by the following rule:

The mean motion of the Moon from the node, in that month when the line of the nodes is in a quadrature with the Sun, is a geometrical mean proportional, between the greatest motion of the Moon from the node and the motion of the Moon.

And therefore this mean motion, will be to the motion of the Moon, in the ſubduplicate proportion of 613 to 603, that is, nearly in the proportion of 1221 to 1211. So that the mean motion of the node in this month, will be to the motion of the Moon, as 10 to 1211, which makes the mean hourly motion 16″. 19‴ $\frac{1}{40}$. According to Sir *Iſaac Newton* it amounts to 16″. 35‴; but, by the corrections which he afterwards uſes, it is reduced to 16″. 16‴$\frac{2}{3}$.

But the equant for the annual motion of the Sun from the node being alſo an ellipſis, it follows, that

The mean motion of the Sun from the node, is a geometrical mean proportional, between the motion of the Sun and the mean motion of the Sun from the node, in the month when the line of the nodes is in quadrature with the Sun.

How

How near this rule agrees with the observations, will appear by this calculation.

Since the mean motion of the node in that month, when the line of nodes is in quadrature to the Sun, was before ſhewn to be to the Moon's mean motion, as 10 to 1211; and the motion of the Sun is to the motion of the Moon, as 160 to 2139: it follows, that the motion of the node and the motion of the Sun will be in the proportion of 154 and 1395; and therefore, by the rule, the Sun's mean motion from the node, is to the Sun's mean motion, in the ſubduplicate proportion of 1549 to 1395, that is, nearly as 98 to 93. Which correſponds with the obſervations; there being 98 revolutions of the Sun to the node in 93 revolutions of the Sun. The ſubduplicate proportion taken more nearly, is as 941 to 893, which will produce 19°. 21′. 3″, for the motion of the node from the fix'd Stars, in a ſydereal year. The motion (as obſerv'd) is 19°. 21′. 22″.

Had the calculation from the rule, been more exactly made in large numbers, the annual motion produced would be 19°. 21′. 07″½, which is 14″

 leſs

leſs than the motion, as obſerved by the Aſtronomers.

Which difference may very probably ariſe from the Sun's parallax; and if ſo, it may perhaps furniſh the beſt and moſt certain method of adjuſting and fixing the true diſtance of the Sun. For the Sun's force being ſomething more on that half of the orb which is towards the Sun, than what it is on the other half, the elliptic epicycle is accordingly larger in the firſt caſe, than in the latter. And by calculation, I find that the mean motion of the node, ariſing after conſideration is had of this difference, is more than the mean motion from the mean magnitude of the epicycle, by near 2″ in the year, for every minute in the parallactic angle of the orbit of the Moon, or for every ſecond of the Sun's parallax. And by the beſt computation I have yet made, this difference of 14″, in the annual motion of the node, will ariſe from about 8″ of parallax; which will make the Sun's diſtance above 25000 ſemi-diameters of the Earth.

In like manner as the equant for the motion of the node, in that month when the line of the nodes is in quadrature with the Sun, is an ellipſis; ſo in any other

other month it is alſo an ellipſis: the motion of the node being direct and retrograde by turns, in the Moon's paſſing from the quadrature to the Sun to the place of its node, and from the place of its node to the quadrature.

The Inclination of the Plane of the Moon's orbit to the Plane of the Ecliptic.

BUT theſe elliptic equants do not only ſerve to ſhew the inequality of the motion of the node, but alſo the inclination of the plane of the Moon's orbit to the plane of the ecliptic. Thus the rays in the elliptic equants, for the motion of the Moon from the node in each month, deſign the inclinations of the plane of its orbit to the plane of the ecliptic, in the ſeveral reſpective poſitions of the Moon to the line of the nodes. And the rays of the elliptic equant for the annual motion of the Sun from the node, in my Conſtruction, (in the ſchol. to prop. 33. book 3. of Sir *Iſaac Newton's Principia*) deſign the different mean inclinations of the ſaid plane, to the plane of the ecliptic in each month, when the Sun is in each reſpective aſpect to the line of the nodes.

THUS if *NT* (the ſemi-tranſverſe axis of the elliptic equant for the motion of the Sun from the node,) deſign the Fig. 2.

the mean inclination of the plane, or, which is the ſame thing, if it repreſent the mean diſtance between the pole of the ecliptic and the pole of the Moon's orbit, in that month when the Sun is in the line of the nodes; *TH*, the ſemiconjugate axis of the ſaid ellipſis, will deſign the mean inclination or mean diſtance of the poles in that month when the line of nodes is in quadrature to the Sun; and *TB*, any other ſemidiameter of the ſaid ellipſis, will repreſent the mean diſtance between the ſaid poles, when the Sun is in that aſpect to the line of the nodes, which is deſigned by the angle *NTA*. For example, if the leaſt inclination, deſigned by the ſhorter ſemiaxis *TH* be 5°. 00′. 00″; ſince *TH* is to *TK* as the motion of the Sun to the mean motion of the Sun from the node, by the property of this equant; and ſince there are 98 revolutions of the Sun to the node in 93 revolutions of the Sun; it follows, that *HK*, the difference between the greateſt and leaſt of the mean inclinations in the ſeveral months of the year, is to *TH* the leaſt, as 5 to 93; by which proportion, the ſaid difference will amount to 16′. 10″. According to Sir *Iſaac Newton*'s computation in the 35th prop. of the third book, it is

16.

16′. 23″½. But if the ſaid number be leſſen'd in the proportion of 69 to 70, according to the author's note at the end of the 34th prop. the ſaid difference will become 16′. 9″.

And in like manner, the inclinations of the plane of the Moon's orbit, in that month when the motion of the node is ſwifteſt, (being ſituated in the line of quadratures with the Sun,) are determined by the equant for the motion of the Moon from the node, in that month.

Thus, let *TH* be to *TN* in the ſubduplicate proportion of the Moon's motion, to its greateſt motion from the node, when the Moon is in the conjunction in *TH*; that is, (as was before determined) let *TH* be to *TN* in the proportion of 1211 to 1221; and the ellipſis deſcribed on the ſemiaxes *TH* and *TN*, will be the equant for the motion of the Moon from the node in that month. And the rays of the ſaid equant will deſign the inclinations of the plane in the ſeveral aſpects of the Moon to the line of the nodes. That is, if *TN* be the inclination of the plane, or the diſtance of the pole of the ecliptic from the pole of the Moon's orbit, when the Moon is Fig. 2.

is in *TN* the line of the nodes, the ray *TB* will repreſent the diſtance of the ſaid poles, or the inclination of the plane, in that aſpect which is deſigned by the angle *NTB*.

Which being laid down, it follows that the whole variation of the inclination, in the time the Moon moves from the line of the nodes to its quadrature in *THK*, is to the leaſt inclination, as *KH* to *TH*, that is, as 10 to 1211. Wherefore if the leaſt inclination be 4°. 59′. 35″, the whole variation will be 2′. 29″. This is upon ſuppoſition that the Sun continued in the ſame poſition to the line of the nodes, during the time that the Moon moves from the node to its quadrature. But the Sun's motion protracting the time of the Moon's period to the Sun, in the proportion of 13 to 12; the variation muſt be increaſed in the ſame proportion, and will therefore be 2′. 41″. According to Sir *Iſaac Newton*'s computation, as delivered in the corollaries to the 34th prop. of the 3d book, for ſtating this greateſt variation, (the intermediate variations in this or any other month not being computed or ſhewn by any method) it amounts to 2′. 43″. But if the ſaid quantity be diminiſh'd in the

the proportion of 70 to 69, according to his note at the end of the ſaid propoſition, it will become the ſame preciſely as it is here deriv'd from the equant.

The Variation of the Area deſcribed by the Moon about the Earth.

The motion of the Moon from the Sun, as it is accelerated or retarded by the increment of the area deſcribed about the Earth, (according to the 26th prop. of the 3d book) is alſo to be reduced to an elliptic equant; by taking the ſhorter axis to the longer axis, in the ſubquadruplicate proportion of the force of the Moon's gravity to the Earth, to the ſaid force added to three times the mean Solar force, that is, as *TA* to the firſt of three mean proportionals between *TA* and *TA*+3*AQ*. And in the ſame proportion is the area deſcribed by the Moon about the Earth, when in quadrature with the Sun, to the mean area, or as the mean area to the area deſcribed in the ſyzygies: So that the greateſt area in the ſyzygies is to the leaſt in the quadratures, in the ſubduplicate proportion of *TA*+3*AQ* to *TA*, or as $\sqrt{8541}$ to $\sqrt{8400}$. This is upon ſuppoſition, that the Moon revolves to the Sun in the ſame time as it revolves about the Earth; which will be found to agree Fig. 1.

gree very nearly with Sir *Isaac Newton*'s computation, in the before-cited proposition.

The Motion of the Apogee.

And after the same manner an elliptic equant might be constructed, which would very nearly shew the mean motion of the apogee, according to the rules deliver'd by Sir *Isaac Newton* (in the corollaries of the 45th prop. of the first book) for stating the motion of the apogee, namely, by taking the greatest retrograde motion of the apogee, from the force of the Sun upon the Moon in the quarters; and the greatest direct motion, from the force of the Sun upon the Moon when in the conjunction or opposition; each according to his rule, deliver'd in the second corollary to the said proposition. And if an ellipsis be made whose axes are in the subduplicate proportion of the Moon's motion from the apogee, when in the said swiftest direct and retrograde motions, the said ellipsis will be nearly the equant for the motion of the Moon from the apogee, and will be found to be nearly of the form of that above for the increment of the area.

But the motion of the apogee, according to this method, will be found to to be no more than 1°. 37′. 22″, in the revolution of the Moon from apogee to apogee, which (according to the obſervations) ought to be 3°. 4′. 7″½.

So that it ſeems there is more force neceſſary to account for the motion of the Moon's apogee, than what ariſes from the variation of the Moon's gravity to the Sun, in its revolution about the Earth.

But if the cauſe of this motion be ſuppoſed to ariſe from the variation of the Moon's gravity to the Earth, as it revolves round in the elliptic epicycle, this difference of force, which is near double the former, will be found to be ſufficient to account for the motion; but not with that exactneſs as ought to be expected. Neither is there any method that I have ever yet met with upon the commonly received principles, which is perfectly ſufficient to explain the motion of the Moon's apogee.

The rules which follow concerning the motion of the apogee, and the alteration of the eccentricity, are founded upon other principles, which I may have occaſion hereafter to explain, it being, as I apprehend, impoſſible to derive theſe, and many other ſuch propoſitions

positions from the laws of centripetal forces.

Fig. 1. LET *TC* (in the above construction of the Lunar orbit) be the mean distance of the Moon, or half the sum of its greatest and least distances, *viz.* *TQ* and *TL*; and let *CL* be the mean semidiameter of the elliptic epicycle, or half the sum of the semiaxes; and take a distance *LM*, on the other side towards the centre, equal to *CL*; then,

The mean motion of the Moon from its apogee, is to the mean motion of the Moon, in the subduplicate proportion of TM to TC.

FOR example, Half the shorter axis or *DC* is $23\frac{1}{2}$; therefore *TC* the mean distance is $8376\frac{1}{2}$; *CM* or 2 *CL*, the sum of the semiaxes, is 141; so that *TM* is $8235\frac{1}{2}$. Wherefore the motion of the Moon from the apogee is to the motion of the Moon, in the subduplicate proportion of $8235\frac{1}{2}$ to $8376\frac{1}{2}$, or of 16471 to 16753, that is, nearly as 117 to 118, or more nearly, as 352 to 355; or yet more nearly, as 1877 to 1893; so that there ought to be about 16 revolutions of the apogee in 1893 revolutions of the Moon; which agrees to great preciseness with the most modern numbers of Astronomy; according to which proportion,

portion, the mean motion of the apogee, in a ſydereal year, ought to be 40°. 40′. 40½″. But by the numbers in Sir *Iſaac Newton*'s theory of the Moon, the ſaid motion is 40°. 40′. 43″. According to the numbers of *Tycho Brahe*, it ought to be 40°. 40′. 47″.

THE mean motion of the apogee being ſtated, I find the following rule for the alteration of the eccentricity.

The Variation of the Eccentricity.

The leaſt eccentricity is to the mean eccentricity, in the duplicate proportion of the Sun's mean motion from the apogee of the Moon's orbit, to the Sun's mean motion. Or in the duplicate proportion of the periodical time of the Sun's revolution, to the mean periodical time of its revolution to the Moon's apogee.

BY the foregoing rule for the mean motion of the apogee, there are 16 revolutions of the apogee in 1893 revolutions of the Moon; but there being 254 revolutions of the Moon in 19 revolutions of the Sun; there muſt be about 7 revolutions of the apogee in about 62 revolutions of the Sun, or rather about 20 in 177. So that the periods of the Sun to the Stars, and of the Sun to the Moon's apogee, are in proportion nearly as the numbers 157 and 177. The duplicate of which proportion is that of 107 to 136; which, according to the rule, ought to be the proportion of the leaſt eccentricity to the mean eccentricity.

So that by this rule, the mean eccentricity, (or half the ſum of the greateſt and leaſt,) ought to be to the difference of the mean from the leaſt, (or half the difference of the greateſt and the leaſt,) as 136 to 29.

How near this agrees with the Obſervations, will appear from the numbers of Mr. *Horrox* or Mr. *Flamſted*, and of Sir *Iſaac Newton*.

The mean eccentricity according to Mr. *Flamſted* or Mr. *Horrox* is 0.055236, half the difference between the greateſt and leaſt is 0.011617; which numbers are in the proportion of $135\frac{1}{2}$ to 28, nearly.

According to Sir *Iſaac Newton*, the mean eccentricity is 0.05505, half the difference of the greateſt and leaſt is 0.01173; which numbers are in proportion nearly as $135\frac{47}{48}$ to $28\frac{47}{48}$, each of which proportions is very near that above aſſigned.

But it is to be noted, that the rule, which is here laid down, is true only upon ſuppoſition that the eccentricity is exceeding

exceeding ſmall. There is another rule derived from a different method, which preſuppoſes the knowledge of the quantity of the mean eccentricity; and which will not only determine the variation of the eccentricity according to the laws of gravity, with greater exactneſs, but ſerve alſo to correct an hypotheſis in the modern theories of the Moon, in which their greateſt error ſeems to conſiſt; and that is, in placing the earth in the focus of that ellipſis, which is deſcribed on the extreme diameters of the lunar orbit; whereas it ought to be in a certain point nearer the perigee, as I may have occaſion to explain more fully hereafter.

The Equation of the Apogee.

THE greateſt and leaſt eccentricity being determined; the equant for the motion of the Sun from the apogee is an ellipſis, whoſe greater and leſſer axes are the greateſt and leaſt eccentricities: and therefore, by the property of ſuch an equant as before laid down,

The ſine of the greateſt equation of the apogee will be to the radius, as the difference of the axes of the equant is to their ſum; that is, as the difference of the greateſt and leaſt eccentricities to their ſum.

For example, ſince the difference is to the ſum as 29 to 136, by what was determined in the foregoing article, the greateſt equation of the apogee will be about 12°. 18'. 40''. Sir *Iſaac Newton* has determined it from the obſervations to be 12°. 18'.

The greateſt and leaſt eccentricities being determined; the eccentricity and equation of the apogee, in any given aſpect of the Sun, are determined by the equant, in the following manner.

Fig 2. Let *TN* be the greateſt eccentricity, *TH* the leaſt, the ellipſis on the ſemi-axes *TN* and *TH*, the equant for the motion of the apogee.

Then if the angle *NTF*, be made equal to the mean diſtance or mean motion of the Sun from the apogee, the angle *NTB* will be the true diſtance or motion of the Sun from the apogee; the difference *BTF*, the equation of the apogee; and the ray *TB*, the eccentricity of the orbit, in that aſpect of the Sun to the apogee deſigned by the angle *NTB*. Hence ariſes this rule.

The tangent of the mean diſtance, viz. NTF, *is to the tangent of the true diſtance* NTB, *in the given proportion of the*

the greatest eccentricity TN *to the least* TH, *that is, as* 165 *to* 107.

FROM what has been laid down concerning the general property of an equant, that it is a curve line described about the center, whose rays are reciprocally in the subduplicate proportion of the velocity at the center, or the velocity of revolution, it will not be difficult to describe the proper curve for any motion that is proposed; and where the inequality of the motion throughout the revolution is but small, there is no need of any nice or scrupulous exactness in the quadrature of the curve for shewing what the equation is. Thus all the small annual equations of the Moon's motion arising from the different distances of the Sun, at different times of the year, may be reduced to one rule exact enough for the purpose.

FOR since the Sun's force to create these annual alterations, is reciprocally in the triplicate proportion of the distance; the rays of the equant for such a motion, will be in the sesquiplicate proportion of the distance. From whence it will not be difficult to prove, that if the revolution of the motion to be equated, were performed in the time of the Sun's revolution, the equation would be to the equation of the Sun's center, nearly as 3 to 2 : and ſo if the force decreaſed as any other power of the Sun's diſtance, ſuppoſe that whoſe index is *m*, the equation would be to that of the Sun's center as *m* to 2. But if the motion be performed in any other period, the equation will be more or leſs, in the proportion of the period of the revolution to the Sun, to the period of the revolution of the motion to be equated. Thus if it were the node or apogee of the Moon's orbit, the equation is to the former as the period of the Sun to the node or apogee, to the period of the node or apogee. Which rule makes the greateſt equation for the node about 8′. 56″, being a ſmall matter leſs than that in Sir *Iſaac Newton*'s theory; and the greateſt equation for the apogee about 21′. 57″, being ſomething larger than that in the ſame theory.

The like rule will ſerve for the annual equation of the Moon's mean motion. If inſtead of the equation for the Sun's center, another ſmall equation be taken in proportion to it as the force, by Sir *Iſaac Newton* called the mean ſolar force, to the force of the Moon's gravity, or as 47 to 8400; the ſaid equation increaſed in the proportion of the Sun's

Sun's period to the mean ſynodical period of the Moon to the Sun, or of 99 to 8, will be the annual equation of the Moon's mean motion. According to this, the equation, when greateſt, will be 12′. 5″.

WHAT is ſaid may be ſufficient for the preſent purpoſe, which is only to lay down the principal laws and rules of the ſeveral motions of the Moon, according to gravity. Some other propoſitions, which ſeem no leſs neceſſary than the former, for compleating the theory of the Moon's motion, as to its aſtronomical uſe, I reſerve to another time.

BUT to make ſome amends for the ſhortneſs and confuſedneſs of the preceeding propoſitions, I ſhall add one example to ſhew the uſe of the equant more at large, in what is commonly called the ſolution of the *Keplerian* problem; that being one of the things which I propoſed to explain, when the elements for the theory of the Moon were advertiſed.

An example of the uſe of the equant in finding the equation of the center.

Fig. 3. LET the figure *ADP* be the orbit in which a body revolves, deſcribing equal areas in equal times by lines drawn from a given point *S*; and let it be propos'd to find the equant for the apparent motion of the ſaid body, about any other place within the orbit, ſuppoſe *F*.

LET there be a line *FR* indefinitely produc'd, which revolves with the body as it moves through the arch *AR*; and in the ſaid line take a diſtance *Fp*, which ſhall be to *FR*, the diſtance of the body from the given point *F*, in the ſubduplicate proportion of the perpendicular let fall upon the tangent of the orbit at *R* from the point *S*, to the perpendicular on the ſaid tangent let fall from the given point *F*; and the curvilinear figure, deſcrib'd by the point *p*, ſo taken every where, will be the equant for the motion of the body about the point *F*.

FOR ſince the areas deſcribed at the diſtances *Fp* and *FR* are in the duplicate

cate proportion of thoſe lines, that is, by the conſtruction, in the proportion of the perpendiculars on the tangents let fall from *S* and *F*; the areas which the body deſcribes, in moving through the arch *AR* about the points *S* and *F*, are in the proportion of the ſame perpendiculars. And therefore the area deſcribed by the revolution of the line *Fp* in the figure, will be equal to that which is deſcribed by the revolution of the line *SR* in the orbit. So that the areas deſcribed in the figure will be equal in equal times, as they are in the orbit. And conſequently the rays *Fp* of the figure will conſtantly be in the ſubduplicate proportion of the velocity of the motion, as it appears at the center *F*, which is the property of the equant.

FROM which conſtruction, it will be eaſy to ſhew, that in the caſe where a body deſcribes equal areas in equal times about a fixed point, there may be a place found out within the orbit, about which the body will appear to revolve with a motion more uniform than about any other place.

THUS ſuppoſe the orbit *ADP* was a figure, wherein the remoteſt and neareſt apſis *A* and *P* were diametrically oppoſite, in a line paſſing through the point

S,

S, *viz.* the point about which the equal areas are defcribed; then if the point *F* be taken at the fame diftance from the remoteft apfis *A*, as the point *S* is from the neareft apfis *P*, the faid center *F* will be the place, about which the body will appear to have the moft uniform motion. For in this cafe the point *F* will be in the middle of the figure *LpDl*, which is the equant for the motion about that point. So that the body will appear to move about the center *F*, as fwift when it is in its floweft motion in the remoter apfis *A*, as it does when it is in its fwifteft motion in the neareft apfis *P*.

For by the conftruction, when the body is at *A*, the ray of the equant *FL* is a mean proportional between *AF* and *AS*; and when the body is at *P*, the ray of the equant *Fl* is a mean proportional between the two diftances *PS* and *PF*, which are refpectively equal to the former.

And in like manner in an orbit of any other given form, a place may be found about which the motion is moft regular.

If what has been faid be applied to the cafe of a body revolving in an elliptic orbit, and defcribing equal areas in in equal times about one of the foci, as is the caſe of a planet about the Sun, and a ſecondary planet about the primary one; it will ſerve to ſhew the foundation of the ſeveral hypotheſes and rules which have been invented by the modern Aſtronomers, for the equating of ſuch motions; and likewiſe ſhew how far each of them are deficient or imperfect.

FOR if the ellipſis *ADP* be the orbit of a planet deſcribing equal areas about the Sun in the focus *S*, the other focus, ſuppoſe *F*, will be the place about which the motion is moſt regular, from what has been already ſaid; that focus being at the ſame diſtance from the aphelion *A*, as the Sun at *S* is from the perihelion *P*. And by the conſtruction, each ray (*Fp*) of the equant will always be a mean proportional between *FR* and *RS*, the two diſtances of the planet from the two foci in that place where the ray *Fp* is taken. For the rays *SR* and *RF*, making equal angles with the tangent at *R*, by the property of the ellipſis, are in the proportion of the perpendiculars from *S* and *F*, let fall on thoſe tangents. And therefore *Fp* being to *FR* in the ſubduplicate proportion

portion of SR to FR, it will be a mean proportional between thoſe diſtances.

1. HENCE when the planet is in the aphelion A, or perihelion P, the rays of the equant FL and Fl are the ſhorteſt, each being equal to CD, the leſſer ſemi-axis of the orbit: For by the property of the ellipſis, the rectangle of the extream diſtances from the focus is equal to the ſquare of the leſſer ſemi-axis.

2. WHEN the planet is at its mean diſtance from the Sun in D or d, the extremities of the leſſer axis, the equant cuts the orbit in the ſame place; the rays of the equant being then the longeſt, being each equal to the greater ſemi-axis CA. For in thoſe points of the orbit, the diſtances from the foci and the mean proportional are the ſame.

FROM which form of the equant it appears,

1. THAT the velocity of the revolution about the focus F diminiſhes, in the motion of the planet from the aphelion or perihelion to the mean diſtance; and increaſes in paſſing from the mean diſtance to the perihelion or aphelion. For the rays of the equant increaſe in the firſt caſe, and diminiſh in the latter; and the velocity of revolution increaſes in

in the duplicate proportion, as the rays diminiſh.

2. In any place of the orbit, ſuppoſe *R*, the velocity of the revolution about the focus *F*, is in proportion to the mean velocity, as the rectangle of the ſemi-axes of the orbit *CD* and *CA*, to the rectangle of the focal diſtances *RF* and *RS*. For the equant and the orbit, being figures of the ſame area, are each equal to a circle, whoſe radius is a mean proportional between the two ſemi-axes *CD* and *CA*. But the mean motion about the focus *F*, is in thoſe places, where the ſaid circle cuts the equant; and in other places, the velocity of the revolution is reciprocally as the ſquare of the diſtance, that is, reciprocally as the rectangle of the focal diſtances *RF* and *RS*.

3. So that the planet is in its mean velocity of revolution about the focus *F*, in four places of the orbit, that is, where the rectangle of the focal diſtances is equal to the rectangle of the ſemi-axes; which places in orbits nearly circular, ſuch as thoſe of the planets, are about 45 degrees from the aphelion or perihelion; but may be aſſigned in general, if need be, by taking a point in the orbit, ſuppoſe *R*, whoſe neareſt diſtance from the leſſer axis of the orbit *CD* is to the longer

longer semi-axis *CA*, in the subduplicate proportion of the longer axis to the sum of the two axes; as may be easily proved.

WHAT has been said, may be enough to shew the form of the equant, and the manner of the motion about the upper focus in general. But the precise determination of the inequality of the motion, requires the knowledge of the quadrature of the several sectors of the equant, or at least, if any other method be taken, of that which is equivalent to such a quadrature.

There are divers methods for shewing the relation between the mean and true motion of a planet round the Sun, or round the other focus, some more exact than others. But the following seems the most proper for exhibiting in one view, all the several hypotheses, and rules, which are in common use in the modern Astronomy, whereby it may easily appear, how far they agree or differ from each other, and how much each of them errs from the precise determination of the motion, according to the true law of an equal description of areas about the Sun.

UPON the center *F* describe the ellipsis *LNl*, equal and similar to the elliptic orbit *ADP*; but having its axes

FN

FN and *FL* contrarily posited, that is, the shorter axis *LF* lying in the longer axis of the orbit *AP*, and the longer axis *FN* parallel to the shorter *CD*. Let the focus of the said ellipsis be in *f*. And suppose two other ellipsis *LBl* and *Lfl*, to be drawn upon the common axis *Ll*, one passing through the point *B*, where the perpendicular *FN* intersects the orbit, and the other through the focus *f*. Let the line *FR*, revolving with the planet in the orbit, be indefinitely produced, till it intersect the first ellipsis *LNl* (which was similar to the orbit) in *Q*, the equant in *p*, and the ellipsis *LBl* (drawn through the intersection *B*,) in *K*. From the point *K* let fall *KH* perpendicular to the line of apsides *AP*, and let it be produced till it intersect the first ellipsis *LNl* in *O*, and the ellipsis *Lfl* (passing through the focus *f*) in *E*. And lastly, in the ellipsis *LNl*, let *GM* be an ordinate equal and parallel to *EH*. In which construction it is to be noted, that the ellipsis *Lfl* and *LBl* are supposed as drawn only to divide the line *OKH* in given proportions, that *KH* may be to *OH*, as the latus rectum of the orbit to the transverse axis; and that *EH* or *GM*, the base of the elliptic segment *GLM*, may

may be to OH, as the diſtance of the foci to the tranſverſe axis.

WHICH being premiſed, it will be eaſy to prove, that the ſector pFL in the equant, or, which is the ſame thing, the ſector RSA in the orbit, is equal to the curvilinear area $OKFMG$, that is, equal to the elliptic ſector QFL, deducting the ſegment LMG, and adding or ſubducting the trilinear ſpace QKO, according as the angle RFA is leſs or greater than a right angle. Wherein it is to be noted, that theſe ſigns of addition and ſubduction are to be uſed in general, if the angle AFR is taken from the aphelion in the firſt ſemi-circle, but towards the aphelion in the latter ſemicircle. But if the angle AFR be taken the ſame way throughout the whole revolution, as is the method in Aſtronomical calculations, then the ſegment and the trilinear ſpace in the latter ſemi-circle muſt be taken with the contrary ſigns to what are laid down.

HENCE it appears, that the inequality in the motion of a planet about the upper focus F, conſiſts of three parts.

I. THE firſt and principal of which is the inequality in the alteration of the angle QFL, in making equal areas in the ellipſis

ellipſis *LNl*. For if a circle equal to the ellipſis be deſcribed upon the center *F*, ſince the radius (being a mean proportional between the two ſemi-axes) will fall without the ellipſis about the line of apſides, and within it about the middle diſtances, the angle *QFL*, which is proportional to the area deſcribed in the circle, will therefore increaſe faſter about the line of apſides, and ſlower about the middle diſtances, in deſcribing equal areas in the ellipſis, than it ought to do in the hypotheſis of Biſhop *Ward*, who makes the planet revolve uniformly about the focus. The equation to rectify this inequality is determined by the following rule. Fig. 3.

The tangent of the angle *QFL*, is to the tangent of the angle in the circle including the ſame area, as the longer axis of the ellipſis to the ſhorter axis; and the difference of the angles, whoſe tangents are in this proportion, is the equation; as is manifeſt from what was before ſaid on the properties of an elliptic equant. From the ſame it alſo follows, that

1. The greateſt equation is an angle, whoſe ſine is to the radius as the difference of the axes to their ſum, or, which is the ſame thing, as the ſquare of the diſtance

of the foci, to the ſquare of half the ſum of the axes. So that in ellipſis nearly circular, of different eccentricities, this greateſt equation will vary nearly in the duplicate proportion of the eccentricity.

2. IN ellipſis nearly circular, the equation at any given angle QFL, is to the greateſt equation, nearly as the ſine of the double of the given angle to the radius; which follows from hence, that the equation is the difference of two angles, whoſe tangents are in a given proportion, and nearly equal.

3. THIS equation adds to the mean motion in the firſt and third quadrant of mean anomaly, and ſubducts in the ſecond and fourth; as will eaſily appear from that the line QF, in deſcribing equal areas in the ellipſis, makes the angle to the line of the apſides, leſs acute than it would be in an uniform revolution.

THIS is the equation which is accounted for in the hypotheſis of *Bullialdus*. For he ſuppoſes the motion of the planet in its orbit to be ſo regulated about the upper focus, that the tangents of the angles, from the lines of apſides, ſhall always be to the tangents of the angles anſwering to the mean anomaly, in the proportion of the ordinates

in

in the ellipſis to the ordinates in the circle circumſcribed; which in effect is the ſame, as if he had made the true equant for its motion about the focus *F*, to be the ellipſis as above deſcribed.

THE ſame equation is alſo uſed by Sir *Iſaac Newton*, in his ſolution of the *Keplerian* problem, in the ſcholium to the 31ſt prop. of the 1ſt book, and is there deſigned by the letter *V*.

BUT ſince the true equant *L D l* coincides with the elliptic equant in the extremities of the ſhorter axis at *L* and *l*, and falls within the ſame at its interſection with the longer axis *F N*, it follows, that the motion of the planet in the ſemi-circle about the aphelion, is ſwifter than according to the hypotheſis of an equal deſcription of areas in the ellipſis *L N l*, and for the ſame reaſon ſlower in the other ſemi-circle about the perihelion; the velocity about the center *F* being always reciprocally in the duplicate proportion of the diſtance.

WHICH leads to the ſecond part of the inequality of the motion about the focus.

II. THE equation to rectify this inequality, is an angle anſwering to the ſegment *G L M*; which angle is to be added to the mean anomaly, to make the area of the elliptic ſector *Q F L*.

THIS angle or equation is determined by the following rule. Let *R* be an angle ſubtended by an arch equal in length to the radius of the circle, *viz.* 57,29578 degrees; and let *A* be an angle, whoſe ſine is to the radius as *GM*, the baſe of the ſegment, to *FN* the ſemi-tranſverſe axis; alſo let *B* be an arch in proportion to *R*, as the ſine of the double of the angle *A* to the radius: Then the equation for the ſegment will be equal to $A - \frac{1}{2}B$.

THIS equation is at its maximum, when the angle *LFQ* is a right angle; the baſe of the ſegment becoming equal to *Ff*, half the diſtance of the foci, and the angle *A*, being in this caſe half the angle *FDS* formed at the extremity of the leſſer axis, and ſubtended by *FS*, the diſtance of the foci; which is commonly called the greateſt equation of the center. And conſequently the arch *B*, in this caſe, is to *R*, as the ſine of the ſaid greateſt equation of the center, is to the radius. So that according to this rule, for the meaſure of the ſegment, it will follow, That

1. THIS greateſt equation is in proportion to the greateſt equation of *Bullialdus*, as found in the preceding article for the elliptic equant, nearly

nearly as three times the tranſverſe axis, to eight times the diſtance of the foci. Or, otherwiſe, the greateſt equation is to the angle deſigned by *R*, as twice the cube of the diſtance between the foci, to three times the cube of the tranſverſe axis. Either of which rules may be derived from the true angle, as before determined; or by taking $\frac{2}{3}$ of the rectangle of *GM* and *LM*, the baſe and height of the ſegment, for the meaſure of that ſegment.

So that in elliptic orbits nearly circular, this greateſt equation for the ſegment is in the triplicate proportion of the eccentricity.

2. This equation at any given angle *QFL*, is to the greateſt equation, in the triplicate proportion of the ordinate *OH* to the ſemi-tranſverſe; that is, nearly as the cube of the ſine of the mean anomaly joined to the double of *Bullialdus*'s equation to the cube of the radius. For the ſegment *GML*, which is proportional to the equation, is in the triplicate proportion of its baſe nearly; and the baſe is proportional to the ordinate *OH*, by the conſtruction.

But the ordinate *OH* (in a circle deſcribed upon the radius *FN*,) becomes the ſine of an angle, whoſe tangent

gent is to the tangent of the angle *QFL*, in the proportion of the tranſverſe axis to the conjugate; but the tangent of the ſame angle *QFL*, is to the tangent of the mean motion, anſwering to the area of the elliptic equant *QFL* in the ſame proportion. So that the ordinate *OH* is to the ſine of that angle of mean motion, in the duplicate of the ſaid proportion; and conſequently the ordinate *OH*, in the circle on the radius *FN*, is the ſine of an angle, nearly equal to the mean anomaly joined to the double of *Bullialdus*'s equation.

3. THIS equation adds to the mean motion in paſſing from the aphelion to the perihelion, and ſubducts in paſſing from the perihelion to the aphelion; as is evident from the tranſit of the point of interſection *E* round the periphery of the ellipſis *Lfl*.

IN Sir *Iſaac Newton*'s rule (in the before-cited ſcholium to the 31ſt prop. 1ſt book,) the angle *X* anſwers to this equation for the ſegment; excepting that it is there taken in the triplicate proportion of the ſine of the mean anomaly, inſtead of the triplicate proportion of the ordinate *OH*. The error of this rule makes

III. The third part of the inequality, anſwering to the trilinear ſpace OKQ, being the difference of the elliptic ſector OFQ and the triangle OFK.

The ſector OQF is proportional to an angle, which is the difference of two angles, whoſe tangents are in the given proportion of the ſemi-latus rectum FB and the ſemi-tranſverſe FN, or in the duplicate proportion of the leſſer axis to the axis of the orbit. So that this ſector, when at a maximum, is as an angle, whoſe ſine is to the radius, as the difference of the latus rectum and tranſverſe to their ſum; or as the difference of the ſquares of the ſemi-axes to their ſum.

The triangle OFK is proportional to the rectangle of the co-ordinates OH and HF; that is, as the rectangle of the ſine OH and its coſine, in the circle on the radius FN; or as the ſine of the double of that angle, whoſe ſine is OH; that is, the double of the angle, whoſe tangent is to the tangent of the angle QFL, in the given ratio of the greater to the leſſer axis; or whoſe tangent is the tangent of the angle of mean motion anſwering to the elliptic ſector QFL, in the duplicate of the ſaid ratio.

 ratio.

ratio. But this triangle *OFK*, when at a maximum, makes an angle of mean motion, which is to the angle called *R*, as *BN*, half the difference between the latus rectum and transverse axis, is to the double of the transverse axis.

So that the sector or triangle in orbits nearly circular, is always nearly equal to the double of *Bullialdus*'s equation.

The triangle and sector being thus determined, the equation for the trilinear space is accordingly determined. From what has been said, it appears, that

1. This equation for the trilinear space *OKQ*, is to that for the triangle *OKF*, in a ratio compounded of *BN*, the difference between the semi-transverse and semi-latus rectum to the semi-latus rectum, and of the duplicate proportion of the sine *OH* to the radius; or *OKQ* is to *OKF*, in a proportion compounded of the duplicate proportion of the distance of the foci to the square of the lesser axis, and the duplicate proportion of the sine *OH* to the radius. For the trilinear figure *OKQ* and the triangle *OKF*, are nearly as *OK* and *KH*, which are in that proportion; and consequently it holds in this proportion to the double of *Bullialdus*'s equation.

2. This

2. THIS equation, in different angles, is as the content under the ſine complement and the cube of the ſine. For the triangle *OKF*, is as the rectangle of the ſine and the ſine complement.

3. IT is at a maximum, at an angle whoſe ſine complement is to the radius, as the ſquare of the greater axis is to the ſum of the ſquares of the two axes; which in orbits nearly circular, is about 60 degrees of mean anomaly.

4. IN orbits of different eccentricities, it increaſes in the quadruplicate proportion of the eccentricity.

5. IT obſerves the contrary ſigns to that for the elliptic equant, called *Bullialdus*'s equation; ſubducting from the mean motion in the firſt and third quadrants, and adding in the ſecond and fourth, if the motion is reckoned from the aphelion.

THE uſe of theſe equations, in finding the place of a planet from the upper focus, will appear from the following rules, which are eaſily proved from what has been ſaid.

LET *t* be equal to *CA* the ſemi-tranſverſe, *c* equal to *FC* the diſtance of the center from the focus, *b* equal to *CD* the ſemi-conjugate, and *R* an angle ſubtended by an arch equal to the

the radius, *viz.* 57°. 17′. 44″. 48‴, or 57, 2957795 degrees. Take an angle $T = \frac{cc}{2tt} R$; $E = \frac{b}{2t} T$; $S = \frac{4c}{3b} T$.

The angle T be will the greatest equation for the triangle OFK; the angle S will be the greatest equation for the segment LMG; and the angle E will be the greatest equation for the area $OKFL$. Which greatest equations being found, the equations at any angle of mean anomaly, will be determined by the following rules.

Let M be the mean anomaly; and let τ be to T as the sine of the angle $2M$ to the radius: In which proportion, as also in the following, there is no need of any great exactness, it being sufficient to take the proportions in round numbers.

Take e to E as the sine of $2M \pm 2\tau$ to the radius; and s to S as the cube of the sine of $M \pm \tau$ to the cube of the radius.

Then the angle QFL is equal to $M+e+s$, in the first quadrant LN, or $M-e+s$, in the second quadrant Nl, or $M+e-s$ in the third quadrant, or $M-e-s$ in the fourth quadrant.

Note, That the small equation τ is always of the same sign with the equation e; and

and in the caſe of the planets, always near the double of that equation.

THE angle RFA at the upper focus F being known, the angle RSA at the Sun in the other focus, is found by the common rule of Biſhop *Ward*; *viz.* the tangent of half the angle RSA, is to be to the tangent of half the angle RFA, always in the given proportion of the perihelion diſtance SP to the aphelion diſtance SA. How theſe equations are in the ſeveral eccentricities of the Moon's orbit, will appear by the following Table.

Eccentr.	E.	S.
	′ ″	″
0.040	1.23	09
0.045	1.45	13
0.050	2.09	17
0.055	2.36	23
0.060	3.06	30
0.065	3.38	38
0.070	4.14	47

TO add one example; ſuppoſe the eccentricity 0.060, the mean anomaly 30°. The ſine of the double of the mean anomaly, that is, the ſine of 60 is to the radius, nearly as 87 to 100; whence, if the equation $E = 3'.06''$, be divided in that propor-

proportion, it will produce 2'.40" nearly, for the equation e: the ſine of M is, in this caſe, equal to $\frac{1}{2}$ the radius, the cube is $\frac{1}{8}$ of the cube of the radius; whence if the equation S=30" be divided in the ſame proportion, it will produce near 4" for the equation s. Therefore the angle RFA, which is $M+e+s$, will be 30°.2'.44"; and the half is 15°.1'.22"; wherefore if the tangent of this angle be diminiſhed, in the proportion of 1.06, the aphelion diſtance, to 94 the perihelion diſtance, it will produce the tangent of 13°.23'.13"; the double of which 26°.46'.26", is the true anomaly or angle at the Sun RSA. And conſequently, the equation of the center is 3°.13'.34" to be ſubducted, at 30 degrees mean anomaly.

WHEN the place of a planet is found by this, or any other method; the place may be corrected to any degree of exactneſs by the common property of the equant, *viz.* that the rays are reciprocally in the duplicate proportion of the velocity about the center. For in this caſe, if there be a difference between the mean motion belonging to the angle aſſumed at the upper focus, and the given mean motion, the error of the angle aſſumed is to the difference, as the rectangle of the ſemi-axes to the rect-

angle

angle of the diſtances from the foci. But in orbits like thoſe of the planets, the rules as they are delivered above are ſufficient of themſelves without further correction.

POSTSCRIPT.

UPON reviewing theſe few ſheets after they were printed off, which happened a little ſooner than I expected, I fear the apology I have offered for delivering the propoſitions relating to the Moon's motion, in this rude manner, without giving any proof of them, or ſo much as mentioning the fundamental principles of their demonſtration, will ſcarcely paſs as a ſatisfactory one; eſpecially ſince there are among theſe propoſitions, ſome which, I am apt to think, cannot eaſily be proved to be either true or falſe, by any methods which are now in common uſe.

WHEREFORE to render ſome ſatisfaction in this article, I ſhall add a few words concerning the principles from whence theſe propoſitions, and others of the like nature

nature are derived: and also take the opportunity to subjoin a few remarks, which ought to have been made in their proper places.

First, THERE is a law of motion, which holds in the case where a body is deflected by two forces, tending constantly to two fixed points.

WHICH is, *That the body, in such a case, will describe, by lines drawn from the two fixt points, equal solids in equal times, about the line joining the said fixt points.*

THE law of *Kepler*, that bodies describe equal areas in equal times, about the center of their revolution, is the only general principle, in the modern doctrine of centripetal forces.

BUT since this law, as Sir *Isaac Newton* has proved, cannot hold, whenever a body has a gravity or force to any other than one and the same point; there seems to be wanting some such law as I have here laid down, that may serve to explain the motions of the Moon and Satellites, which have a gravity towards two different centers.

IT follows as a corollary to the law here laid down, that if a body, gravitating towards two fixt centers, be supposed, for given small intervals of time, as moving in a plane paſſing through one of the fixt centers, the inclination of the ſaid plane, to the line joining the centers, will vary according to the area deſcribed; that is, if the area be greater, the inclination will be leſs; and if the area be leſs, the inclination will be greater, in order to make the ſolids equal.

THIS corollary, when rightly applied, will ſerve to explain the variation of the inclination of the plane of the Moon's orbit to the plane of the ecliptic.

AND how extremely difficult it is to compute the variation of the inclination in any particular caſe, without the knowledge of ſome ſuch principle as this is, will beſt appear, if any one conſider the intricacy of the calculations, uſed in the corollaries to the 34 prop. of the third book of the *Principia*, in order to ſtate the greateſt quantity of variation, in that month, when the line of the nodes is in quadrature with the Sun, and that only in particular Numbers, whereby it is determined to be 2'.43".

WHEREAS, there is a plain and general rule in this caſe, which follows from what is laid down, though not immediately; namely, that the greateſt variation in the ſaid poſition of the Moon's orbit, is

to

to the mean inclination of the plane as the difference of the greateſt and leaſt areas deſcribed in the ſame time by the Moon about the earth, when in the conjunction and in the quarters to the mean area.

WHEREFORE, if S be to L, as the Sun's period to the Moon's period: The greateſt area is to the leaſt, as $\sqrt{SS+3LL}$ to S, or as $S+\frac{3LL}{2S}$ to S nearly, by what is ſaid on this article in the 29th page. So that the difference of areas is to the mean area, as $\frac{3}{2}LL$ to $SS+\frac{3}{4}LL$; and in the ſame proportion is the greateſt variation of the inclination of the plane in this month to the mean inclination, which agrees nearly with Sir *Iſaac*'s computation.

Secondly, THERE is a general method for aſſigning the laws of the motion of a body to and from the center, abſtractly conſider'd, from its motion about the center.

THE motion to and from the center is called by *Kepler* a Libratory motion; the knowledge of which ſeems abſolutely requiſite, to define the laws of the revolution of a body, in reſpect of the apſides of its orbit.

FOR the revolution of a body, from apſis to apſis, is performed in the time of of the whole libratory motion; the apsides of the orbit being the extreme points, wherein the libratory motion ceases.

So that, according to this method, the motion of a body round the center, is not consider'd as a continued deflection from a streight line; but as a motion compounded of a circulatory motion round the center, and a rectilinear motion to or from the center.

Each of which motions require a proper *Equant*. Of the equant for the motion round the center, I have already given several examples. And in the case of all motions, which are governed by a gravity or force tending to a fixt point, the real orbit in which the body moves, is the equant for this motion. In all other cases it is a different figure.

The *Equant* for the libratory motion, is a curve line figure, the areas of which serve to shew the time wherein the several spaces of the libration are performed.

Which figure is to be determined, by knowing the law of the gravity to the center: For the libratory force, to accelerate or retard the motion to or from the center, is the difference between the gravity of the body to the center, and the centrifugal force arising from the circulatory motion. But the latter is always under one rule: For in all revolutions round a center, in any curve line, whether described by a centripetal force or not, the centrifugal force is directly in the duplicate proportion of the area described in a given small time, and reciprocally in the triplicate proportion of the distance; which is an immediate consequence of a known proposition of Mr. *Huygens*. The like proportion also holds as to the centripetal force in all circular motions, from a known proposition of Sir *Isaac Newton*. But what is true of the centripetal force in circles, is universally true of the other force in orbits of any form.

So that by knowing the gravity of the body, since the other force is always known, the difference, which is the absolute force to move the body to or from the center, will be known; and from thence the velocity of the motion, and the space described in any given time, may be found, and the equant described. These hints may be sufficient to shew what the method is.

To add an example. If the gravity be reciprocally as the square of the distance; the equant for the libratory motion, will be found to be an ellipsis

similar

ſimilar to the orbit, whoſe longer axis is the double of the eccentricity; the center of the libratory motion, that is the place where it is ſwifteſt, will be in the focus; the time of the libration, through the ſeveral ſpaces, is to be meaſured by ſectors of the ſaid ellipſis, ſimilar to thoſe deſcribed by the body round the focus of the orbit; and the period of the libratory motion will be the ſame with the period of the revolution.

In any other law of gravity, the equant for the libratory motion, will either be of a form different from the orbit, or if it be of the ſame form, it muſt not be ſimilarly divided.

I may juſt mention, that the equant for the libratory motion, in the caſe of the Moon, is a curve of the third kind, or whoſe equation is of four dimenſions; but is to be deſcribed by an ellipſis, the center of the libration not being in the focus.

From this method of reſolving the motion, it will not be difficult to ſhew the general cauſes of the alteration of the eccentricity and inequality in the motion of the apogee. For when the line of apſides is moving towards the Sun, it may be eaſily ſhewn, that ſince the external force in the apſides, is then centrifugal, it will contribute to lengthen the ſpace and time of the libration; by lengthening the ſpace, it increaſes the eccentricity; and by lengthening the time of the libration, it protracts the time of the revolution to the apſis, and cauſes what is improperly called a motion of the apſis forward. But when the line of apſides is moving to the quadratures, the external force in the apſides, is at that time centripetal; which will contribute to ſhorten the ſpace and time of libration; and by ſhortening the ſpace will thereby leſſen the eccentricity, and by ſhortening the time of libration, will thereby contract the time of the revolution to the apſis; and cauſe what is improperly called a retrograde motion of the apſis.

I ſhall only add a few remarks, which ought to have been made in their proper places.

As to the motion of the Moon in the elliptic epicycle (page 9.) it ſhould have been mentioned, that there is no need of any accurate and perfect deſcription of the curve called an ellipſis, it being only to ſhew the elongation of the Moon, from the center of the epicycle; which doth not require any ſuch accurate deſcription.

It

It should have been said, that when the Moon is in any place of its orbit, suppose somewhere at *N*, in that half of the orbit which is next the Sun, it then being nearer the Sun than the Earth, has thereby a greater gravity to the Sun than the Earth: which excess of gravity, according to Sir *Isaac Newton*'s method, consists of two parts; one acting in the line *NV*, parallel to that which joins the Earth and Sun; and the other acting in the line *VB* directed to the Earth; and these two forces, being compounded into one, make a force directed in the line *NB*; which is in proportion to the force of gravity, as that line *NB* is to *TB* nearly. Wherefore, as there is a force constantly impelling the Moon somewhere towards the point *B*, this force is supposed to inflect the motion of the Moon into a curve line about that point; for the same reason as the gravity of it to the Earth, is supposed to inflect its motion into a curve line about the Earth: not that the Moon can actually have so many distinct motions, but the one simple motion of the Moon round the Sun is supposed to arise from a composition of these several motions. Fig. 1.

In

In the laſt article on the ſmall annual equations, (page 38.) theſe rules ought to have been added.

Let *Æ* be the equation of the Sun's center; *P* the mean periodical time of the node or apogee; *S* the mean ſynodical time of the Sun's revolution to the node or apogee: Then will $\frac{3S}{2P}$ *Æ* be the annual equation of the node or apogee, according as *S* and *P* are expounded.

The like rule will ſerve for the annual equation of the Moon's mean motion. If *S* be put for the Sun's period; *P* for the mean ſynodical period of the Moon to the Sun; and *L* for the Moon's period to the Stars: The annual equation of the Moon's mean motion will be $\frac{3LL}{2PS}$ Æ.

According to theſe rules when expounded, the equation for the node will be found to be always in proportion to the equation of the Sun's center, nearly as 1 to 13.

The equation of the apogee to the equation of the Sun's center, as 10 to 53.

And the equation of the Moon's mean motion to the ſame, as 8 to 77.

It

Fig. 1

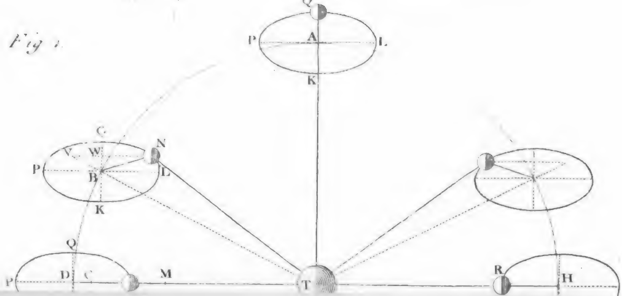
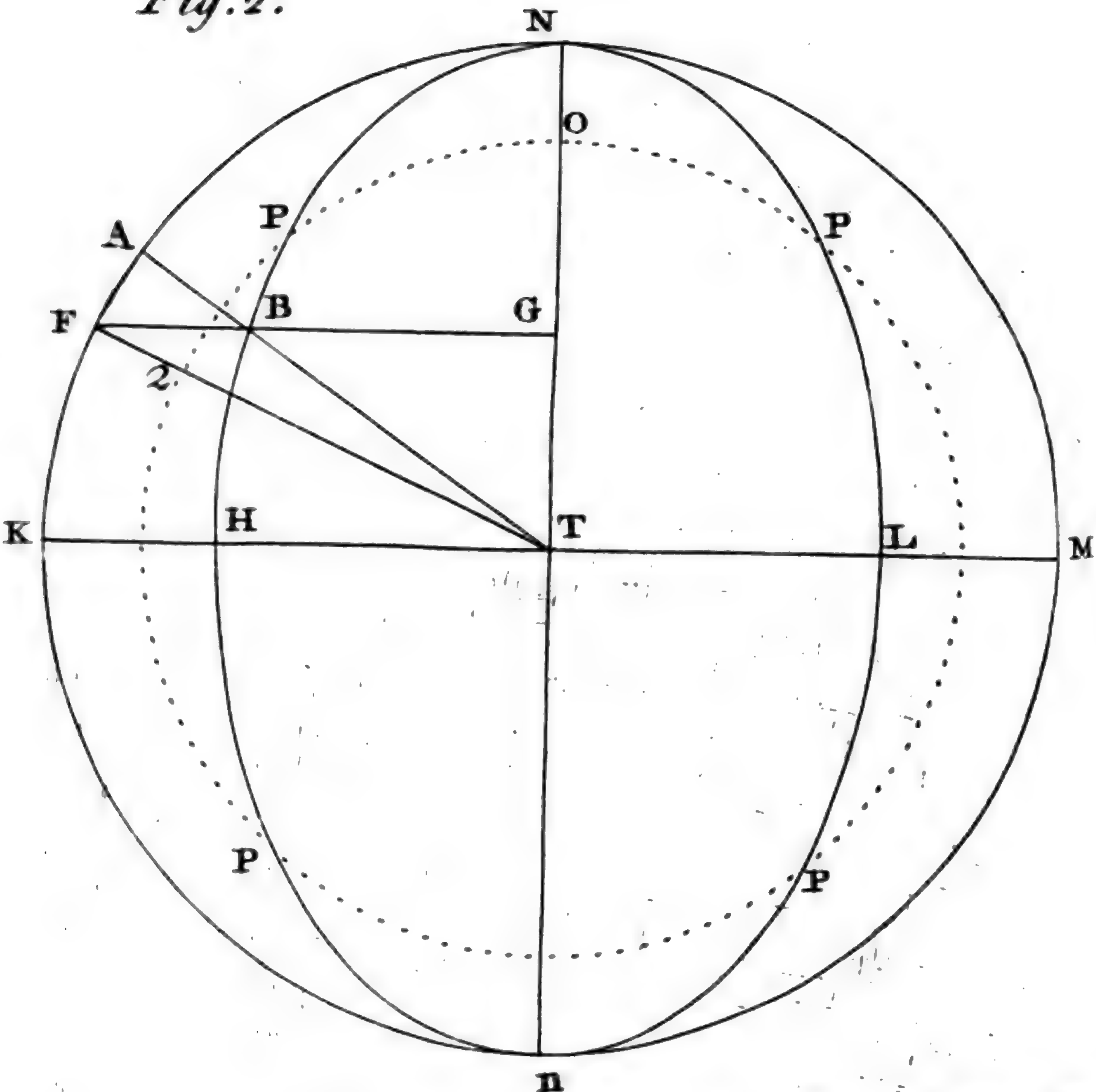


Fig. 2.



Place this at ye end of ye Book

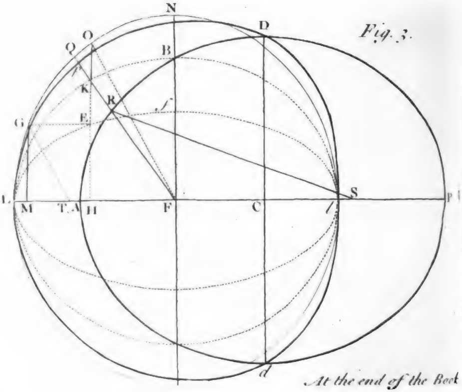


Fig. 3.

At the end of the Book

It may be throughout obſerved, that the propoſitions are in general terms, ſo as to ſerve, *mutatis mutandis*, for any other ſatellite, as well as the Moon.

There might have been ſeveral other obſervations and remarks made in many other places, had there been ſufficient time for it. But perhaps what I have already ſaid may be too much, conſidering the manner in which it is delivered.

ERRATUM.

Page 11. l. 11. for 8th, read 28th.

FINIS.

ERRATA.

VOLUME I.

PAGE 117. for *drawing*, read *draw.* p. 156. f. 2*AB*, r. $\frac{1}{2}$*AB*. p. 164. l. 7. 10, 20. p. 165. l. 9. p. 171. l. 27. f. *right line whose power is the area* &c. r. *right line whose square is equal to the area* &c. p. 166. l. 25, 29. f. *right line whose power is the rectangle* &c. r. *right line whose square is equal to the rectangle* &c. p. 192. l. 23, 24. r. $A^{\frac{1}{64}-3}$, or $A^{\frac{1}{16}-3}$, or $A^{\frac{1}{4}-3}$, or $A^{\frac{4}{9}-3}$, l. 29. r. $A^{\frac{nn}{mm}-3}$, p. 203. dele *if.* p. 229. l. penult. dele *is.* p. 240. l. 26. dele *near.* p. 243. l. 21. f. *when.* r. *because.* p. 272. l. 3. r. *is in the same ratio.*

VOLUME II.

PAGE 6. Line 21. for *its*, read *the*. p. 24 l. 21. dele *Fig.* 2. p. 50. l. 7. from the bottom. f. *Fig.* 5, 6, 7. r. *Fig.* 6, 7, 8. and so in page following. p. 95. l. 4. from the bottom, and p. 100. l. 5. f. *Averdupois*, r. *Troy*. p. 130. l. 28. r. *and the water*, &c. p. 140. l. 14. f. *and the*, r. *and whose.* p. 144. l. ult. f. *but*, r. *this.* p. 161. l. 2. f. *may*, r. *will.* p. 169. l. 6. f. *leave for some time*, r. *would otherwise leave.* l. 21. r. *receding from the parts* of the body *where it is pressed*, &c. p. 338. l. 7. f. *Fig.* 1. r. *Fig.* 2. p. 341. l. 1. f. *Fig.* 2. r. *Fig.* 3. p. 352. l. [illegible] *elliptic.*