

Software Implementation of the Optimal Temporal Intuitionistic Fuzzy Algorithm for Franchisee Selection

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Abstract—The selection of the most suitable franchisee applicant in an uncertain environment in a particular moment of time is a key decision for a franchisor and the success of a franchising business. In this work, for the first time, we describe a problem for choosing the optimal candidate for the franchise chain and algorithm for a solution in terms of temporal intuitionistic fuzzy pairs and index matrices as a means for data analysis in uncertain conditions over time. We also use our software utility to demonstrate the proposed algorithm and to apply the decision support approach to a franchisee selection for the largest fast food restaurant chain in Bulgaria.

I. INTRODUCTION

Franchising is an effective business strategy for entering new markets. The franchisor grants the right to its franchisees to use the brand, the business concept, and the products or services within a specific time frame [1]. The concept of fuzzy [10] and intuitionistic fuzzy logic [6], provides such a tool for creating an optimal algorithm for choosing a franchisee in conditions of ambiguity. The studies [12], [13] present fuzzy franchisee selection models using an Analytic Hierarchy Process (AHP) and neural networks.

In [16], we presented an optimal interval-valued intuitionistic fuzzy multicriteria decision-making problem in outsourcing and a software utility for its solution. We have also introduced in [19] an intuitionistic fuzzy approach (IFIMFr) to select the most suitable candidates for franchising in a patisserie using the theory of index matrices (IMs, [5]). The aim of the paper is to expand the IFIMFr approach so that it can be applied to temporal intuitionistic fuzzy data [3]. The work uses our custom programs to implement the proposed algorithm and to apply it to the largest fast food restaurant chain in Bulgaria. The remainder of our study includes 4 sections: Section II describes some definitions and properties of temporal intuitionistic fuzzy IMs and pairs. Section III describes a problem for choosing the optimal franchise candidate and algorithm

for solution in terms of temporal IFPs an IMs as a means for uncertain data analysis over time and the basic characteristics of our software utility. Section IV sets out the conclusions and aspects for future research.

II. DEFINITION AND PROPERTIES OF TEMPORAL INTUITIONISTIC FUZZY IMs AND PAIRS

Let us briefly give the definitions of temporal intuitionistic fuzzy IMs and TIFPs and some of their properties [3].

2.1. Temporal Intuitionistic Fuzzy Pair (TIFP)

Let $T = \{t_1, \dots, t_g, \dots, t_f\}$ be a fixed time-scale. A TIFP is in the form of $\langle \mu(t), \nu(t) \rangle$, where $\mu(t)$ and $\nu(t)$ are interpreted as degrees of membership and non-membership, $\mu, \nu : T \rightarrow [0, 1]$ and $\mu(t) + \nu(t) \leq 1$ for $t \in T$. Let us have two TIFPs $x = \langle \mu(t), \nu(t) \rangle$ and $y = \langle \rho(t), \sigma(t) \rangle$. Then, we recall some basic operations [3] with two TIFPs.

$$\begin{aligned} x(t) \wedge_1 y(t) &= \langle \min(\mu(t), \rho(t)), \max(\nu(t), \sigma(t)) \rangle \\ x(t) \vee_1 y(t) &= \langle \max(\mu(t), \rho(t)), \min(\nu(t), \sigma(t)) \rangle; \\ x(t) \wedge_2 y(t) &= x(t) + y(t) = \langle \mu(t) + \rho(t) - \mu(t) \cdot \rho(t), \nu(t) \cdot \sigma(t) \rangle \end{aligned} \quad (1)$$

Let $R_{\langle a(t), b(t) \rangle} = 0.5 \cdot (2 - a(t) - b(t)) \cdot (1 - a(t))$ following [4]. Then, as per [6], [20]:

$$x(t) \geq_R y(t) \text{ iff } R_{\langle a(t), b(t) \rangle} \leq R_{\langle c(t), d(t) \rangle}. \quad (2)$$

2.2. Three-Dimensional Temporal Intuitionistic Fuzzy Index Matrices (3-D TIFIM)

A 3-D TIFIM [5] $A(T) = [K, L, T, \{\langle \mu_{k_i, l_j, t_g}, \nu_{k_i, l_j, t_g} \rangle\}]$

$$= \begin{array}{c|ccc} t_g \in T & l_1 & \dots & l_n \\ \hline k_1 & \langle \mu_{k_1, l_1, t_g}, \nu_{k_1, l_1, t_g} \rangle & \dots & \langle \mu_{k_1, l_n, t_g}, \nu_{k_1, l_n, t_g} \rangle \\ \vdots & \vdots & \dots & \vdots \\ k_m & \langle \mu_{k_m, l_1, t_g}, \nu_{k_m, l_1, t_g} \rangle & \dots & \langle \mu_{k_m, l_n, t_g}, \nu_{k_m, l_n, t_g} \rangle \end{array}, \quad (3)$$

where \mathcal{S} be a fixed set of indices, $(K, L, T \subset \mathcal{S})$, and its elements $\langle \mu_{k_i, l_j, t_g}, \nu_{k_i, l_j, t_g} \rangle$ are TIFPs. T is a fixed temporal scale and its elements $t_g (g = 1, \dots, g, \dots, f)$ are time moments.

In [5], [17], [18], operations with 3-D TIFIMs, analogous to those with the classical matrices were introduced, but there are also specific ones such as projection, substitution, aggregation operations, internal subtraction of IMs' components, term-wise multiplication and subtraction. Let us recollect some operations with an application in temporal IFIMFr.

Aggregation operation by one dimension [17]: Let us have

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two TIFPs $x = \langle a, b \rangle$ and $y = \langle c, d \rangle$ and $(1 \leq q \leq 3)$. An aggregation operation by one dimension is

$$\alpha_{K, \#_q}(A(T), k_0) = \begin{array}{c|ccc} t_g & l_1 & \dots & l_n \\ \hline k_0 & \#_q^m \langle \mu_{k_i, l_1, t_g}, \nu_{k_i, l_1, t_g} \rangle & \dots & \#_q^m \langle \mu_{k_i, l_n, t_g}, \nu_{k_i, l_n, t_g} \rangle \end{array} \quad (4)$$

If we use $\#_1^* = \langle \min(a(t), c(t)), \max(b(t), d(t)) \rangle$ we perform a super pessimistic aggregation operation, with $\#_2^* = \langle \text{average}(a(t), c(t)), \text{average}(b(t), d(t)) \rangle$ we have an averaging aggregation operation, and with $\#_3^* = \langle \max(a(t), c(t)), \min(b(t), d(t)) \rangle$ we perform a super optimistic aggregation operation.

Projection: Let $W \subseteq K$, $V \subseteq L$ and $U \subseteq H$. Then, $pr_{W, V, U}A(T) = [W, V, U, \{\langle R_{pr, q_s, e_d}, S_{pr, q_s, e_d} \rangle\}]$, where for each $k_i \in W, l_j \in V$ and $t_g \in U$, $\langle R_{pr, q_s, e_d}, S_{pr, q_s, e_d} \rangle = \langle \mu_{k_i, l_j, t_g}, \nu_{k_i, l_j, t_g} \rangle$.

A Level Operator for Decreasing the Number of Elements of TIFIM: Let $\langle \alpha(t), \beta(t) \rangle$ is an TIFP, then according to [9] $N_{\alpha(t), \beta(t)}^>(A(T)) = [K, L, T, \{\langle \rho_{k_i, l_j, t_g}, \sigma_{k_i, l_j, t_g} \rangle\}]$, where

$$= \begin{cases} \langle \mu_{k_i, l_j, t_g}, \nu_{k_i, l_j, t_g} \rangle & \text{if } \langle \rho_{k_i, l_j, t_g}, \sigma_{k_i, l_j, t_g} \rangle > \langle \alpha(t), \beta(t) \rangle \\ (0, 1) & \text{otherwise} \end{cases} \quad (5)$$

III. AN OPTIMAL TEMPORAL INTUITIONISTIC FUZZY ALGORITHM FOR SELECTION OF THE MOST ELIGIBLE FRANCHISEE (OTIFAFr)

This section proposes an OTIFAFr, used the concepts of IMs and TIFPs. Let us formulate the optimal problem as follows:

A large franchise has decided to turn to experts to select the best franchisee candidate for expanding its brand. The franchise candidates for studied brand v_e need to be evaluated by the experts. The experts assess the IF priorities pk_{c_j, v_e, t_f} of the criteria c_j in the evaluation system of the franchise chain v_e at a particular moment t_f . The IF ratings of the experts are defined on the basis of their participation in previous franchise evaluation procedures and given to the experts at a time t_f . All candidates for a franchisee have been evaluated by the experts at a particular moment t_f and their evaluations ev_{k_i, c_j, d_s, t_f} at a current moment t_f are temporal intuitionistic fuzzy data. The estimations of the same applicants from previous evaluation procedures are given as elements of TIFIM at time points $t_1, \dots, t_g, \dots, t_{f-1}$. The aim of the problem is to select the most eligible franchisee for this brand.

A. An Optimal Temporal Intuitionistic Fuzzy Algorithm for Assignment of a Franchisee

The procedure of OTIFAFr includes the following steps:

Step 1. The team of experts needs to evaluate the candidates for the brand v_e according to the approved criteria in the company at a particular moment t_f . The estimations of the $d_s (1 \leq s \leq D)$ expert are described by the TIFP $ev_{k_i, c_j, d_s, t_f} = \langle \mu_{k_i, c_j, d_s, t_f}, \nu_{k_i, c_j, d_s, t_f} \rangle$ by criterion $c_j (1 \leq j \leq n)$ for the the k_i -th $(1 \leq i \leq m)$ candidate at a particular moment t_f . Expert assessments are uncertain due to galloping inflation and the existing pandemic. The data values are transformed into TIFPs as demonstrated in [3], [20]. The TIFP $\{ev_{k_i, c_j, d_s, t_f}\}$ presents the degrees of perception (the positive evaluation of the d_s -th expert for the k_i -th candidate by the c_j -th criterion divided by the (maximum-minimum) evaluation) and non-perception (the

negative evaluation of the d_s -th expert for the k_i -th candidate by the c_j -th criterion divided by the (maximum-minimum) evaluation) of the d_s -th expert for the k_i -th candidate by the c_j -th criterion at a particular moment t_f . The hesitation degree $\mu_{k_i, c_j, d_s, t_f} = 1 - \mu_{k_i, c_j, d_s, t_f} - \nu_{k_i, c_j, d_s, t_f}$ corresponds to the uncertain evaluation of the d_s -th expert for the k_i -th candidate by the c_j -th criterion at a particular moment t_f .

The experts have the opportunity to include assessments for the same candidates from the previous evaluation procedures at time points $t_1, \dots, t_g, \dots, t_{f-1}$. A TIFIM $EV_s[K, C, T, \{ev_{k_i, c_j, d_s, t_g}\}]$ is built with the dimensions $K = \{k_1, k_2, \dots, k_m\}$, $C = \{c_1, c_2, \dots, c_n\}$ and $T = \{t_1, t_2, \dots, t_f\}$ for each expert $d_s (s = 1, \dots, D)$:

$$EV_s[K, C, T, \{ev_{k_i, c_j, d_s, t_g}\}] = \begin{array}{c|ccc} t_g \in T & c_1 & \dots & c_n \\ \hline k_1 & \langle \mu_{k_1, c_1, d_s, t_g}, \nu_{k_1, c_1, d_s, t_g} \rangle & \dots & \langle \mu_{k_1, c_n, d_s, t_g}, \nu_{k_1, c_n, d_s, t_g} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ k_m & \langle \mu_{k_m, c_1, d_s, t_g}, \nu_{k_m, c_1, d_s, t_g} \rangle & \dots & \langle \mu_{k_m, c_n, d_s, t_g}, \nu_{k_m, c_n, d_s, t_g} \rangle \end{array},$$

where $K = \{k_1, k_2, \dots, k_m\}$, $C = \{c_1, c_2, \dots, c_n\}$, $T = \{t_1, t_2, \dots, t_f\}$ and the element $\{ev_{k_i, c_j, d_s, t_g}\}$ is the estimate of the d_s -th expert for the k_i -th candidate by the c_j -th criterion at a particular moment t_g . Let us apply the α_T -th aggregation operation (4) to find the aggregated evaluation of the d_s -th expert $(s = 1, \dots, D)$ for the k_i -th candidate for the period T .

The result IM $\alpha_{EV_s(T), \#_q}$ has the form

$$\alpha_{EV_s(T), \#_q} = \begin{array}{c|ccc} d_s & \dots & c_n \\ \hline k_1 & \dots & \#_q^f \langle \mu_{k_1, c_n, d_s, t_g}, \nu_{k_1, c_n, d_s, t_g} \rangle_{g=1} & \dots \\ \vdots & \ddots & \vdots & \ddots \\ k_m & \dots & \#_q^f \langle \mu_{k_m, c_n, d_s, t_g}, \nu_{k_m, c_n, d_s, t_g} \rangle_{g=1} & \dots \end{array},$$

where $1 \leq q \leq 3$ depending on whether the pessimistic, averaging or optimistic scenarios are accepted in the process of decision making in the franchise chain. Then we create aggregated TIFIM $EV[K, C, E, \{ev_{k_i, c_j, d_s, t_f}\}]$ with the evaluations of all experts for all candidates by all criteria:

$$EV(t_f) = \alpha_{EV_1, \#_q}(T, d_1) \oplus_{(max, min)} \dots \oplus_{(max, min)} \alpha_{EV_D, \#_q}(T, d_D) \quad (6)$$

Then we go to Step 2.

Step 2. Let the present score coefficient $r_s(t_f)$ of each expert $(s \in E)$ is defined by an TIFP $\langle \delta_s(t_f), \varepsilon_s(t_f) \rangle$, which elements can be interpreted respectively as his degree of competence and of incompetence at a particular moment t_f . Then is created the IM $V(t_f)[K, C, E, \{v_{k_i, c_j, d_s, t_f}\}]$

$$= r_1 pr_{K, C, d_1, t_f} EV \oplus_{(max, min)} r_2 pr_{K, C, d_2, t_f} EV \dots \oplus_{(max, min)} r_D pr_{K, C, d_D, t_f} EV.$$

The IM $EV(t_f) := V(ev_{k_i, l_j, d_s, t_f} = v_{k_i, l_j, d_s, t_f}, \forall k_i \in K, \forall l_j \in L, \forall d_s \in E)$ contains the final score of each franchise candidate at a particular moment t_f .

The total assessment of the k_i -th candidate on the c_j -th criterion at a particular moment $t_f \notin E$ is calculated by an application of the α_E -th aggregation operation as follows

$$R(h_0) = \alpha_{E, \#_q}(EV(t_f), h_0) = \left\{ \begin{array}{c|ccc} c_j & h_0 & & \\ \hline k_1 & \#_q^D \langle \mu_{k_1, c_j, d_s, t_f}, \nu_{k_1, c_j, d_s, t_f} \rangle & & \\ \vdots & \vdots & & \\ k_m & \#_q^D \langle \mu_{k_m, c_j, d_s, t_f}, \nu_{k_m, c_j, d_s, t_f} \rangle & & \end{array} \right\} \Big| c_j \in C, \quad (7)$$

where $(1 \leq q \leq 3)$.

If we use $\#_1^* = \langle \min, \max \rangle$, then we accept super pessimistic aggregation operation, with $\#_2^* = \langle \text{average}, \text{average} \rangle$ we assume averaging aggregation operation and with $\#_3^* = \langle \max, \min \rangle$ we accept super optimistic aggregation operation for the assessment of the applicant.

If the franchise chain has a requirement for the candidates, so that their total score is not less than a predetermined TIFP $\langle \alpha(t_f), \beta(t_f) \rangle$, then in this case it is necessary to apply the level operator (5) to TIFIM B to remove from the ranking candidates who do not meet this requirement. Go to Step 3.

Step 3. This step creates a TIFIM $PK(h_0)$ with the coefficients determining the importance of the evaluation criteria for the franchisor v_e at a particular moment t_f by :

$$PK(h_0)[C, v_e, h_0, \{pk_{c_j, v_e, h_0}\}] = \begin{array}{c|c} h_0 & v_e \\ \hline c_1 & pk_{c_1, v_e, h_0} \\ \vdots & \vdots \\ c_j & pk_{c_j, v_e, h_0} \\ \vdots & \vdots \\ c_n & pk_{c_n, v_e, h_0} \end{array},$$

Then we calculate the evaluation TIFIM

$$B(h_0)[K, v_e, h_0, \{b_{k_i, v_e, h_0}\}] = R(h_0) \odot_{(\circ, *)} PK(h_0),$$

containing the total estimates of the k_i -th candidate (for $1 \leq i \leq m$) at a particular moment t_f for the brand v_e , where $\langle \circ, * \rangle$ is an operation from (4). Go to Step 4.

Step 4. At this step we choose the most optimal franchisee for v_e by using the aggregation operation by $K - \alpha_{K, \#_q}(B(h_0), k_0)$ using pessimistic, average or optimistic scenarios

$$\alpha_{K, \#_q}(B(h_0), k_0) = \begin{array}{c|c} & v_e \\ \hline k_0 & \begin{array}{c} m \\ \#_q \\ \langle \mu_{k_i, v_e, h_0}, \nu_{k_i, v_e, h_0} \rangle \\ i=1 \end{array} \end{array}, \quad (8)$$

where $k_0 \notin K, 1 \leq q \leq 3$. Go to Step 5.

Step 5. After finding the most effective franchisee, we will optimize the evaluation system for the next procedures using the intercriteria method (ICrA, [7], [8], [14]).

Let $\langle \alpha, \beta \rangle$ is an TIFP. The criteria C_k and C_l are in:

- $\langle \alpha(t_f), \beta(t_f) \rangle$ -positive consonance at a particular moment t_f , if $\mu_{C_k, C_l}(t_f) > \alpha(t_f)$ and $\nu_{C_k, C_l}(t_f) < \beta(t_f)$;
- $\langle \alpha(t_f), \beta(t_f) \rangle$ -negative consonance at a particular moment, if $\mu_{C_k, C_l}(t_f) < \beta(t_f)$ and $\nu_{C_k, C_l}(t_f) > \alpha(t_f)$;
- $\langle \alpha(t_f), \beta(t_f) \rangle$ -dissonance at a particular moment t_f , otherwise.

ICrA is applied over the matrix $R(h_0)$ to find the criteria, which are in a consonance. More complex criteria are reduced from the evaluation franchise system using the IM reduction operation over $R(h_0)$. Go to Step 6.

Step 6. This step obtains the new rank coefficients of the experts. Let the expert d_s ($s = 1, \dots, D$) has participated in γ_s evaluation procedures for the selection of a franchisee, on the basis of which his score $r_s(t_f) = \langle \delta_s(t_f), \varepsilon_s(t_f) \rangle$ is determined, then after his participation in the next procedure, his new score $\langle \delta_s^{(t_f+1)}, \varepsilon_s^{(t_f+1)} \rangle$ will be changed by [6]:

$$\begin{cases} \langle \frac{\delta(t_f)\gamma+1}{\gamma+1}, \frac{\varepsilon(t_f)\gamma}{\gamma+1} \rangle, & \text{if the expert has assessed correctly} \\ \langle \frac{\delta(t_f)\gamma}{\gamma+1}, \frac{\varepsilon(t_f)\gamma}{\gamma+1} \rangle, & \text{if the expert had not given any estimation} \\ \langle \frac{\delta(t_f)\gamma}{\gamma+1}, \frac{\varepsilon(t_f)\gamma+1}{\gamma+1} \rangle, & \text{if the expert has assessed incorrectly} \end{cases} \quad (9)$$

The complexity of OTIFAFr algorithm is $O(Dm^2n^2)$ [15]. For the application of the OTIFAFr algorithm, we will use an updated version of the C++ utility we previously developed for the IFIMOA and IVIFIMOA algorithms. As we outlined before [2], [16], it is based on a template class which allows us to replace its type with any C++ type or class that implements basic comparison and arithmetic operators. This has allowed us to use the same code for both IFPs (as we will do here) and IVIFPs (as we have done in [16], [21]). The program is command-line based. It expects the following input arguments: a 3-D TIFIM of the experts' evaluations, a matrix of the experts' rating coefficients and a matrix of the weight coefficients of each criterion for each service. The expert evaluations can be given either directly as an index matrix of IFPs, or as a matrix of mark intervals. For the latter case, the first argument of the program must be "interval" followed by the lowest and highest possible mark that an expert can give [21].

B. An Application of OTIFAFr to the Largest Fast Food Restaurant Chain in Bulgaria

In this section, the proposed OTIFAFr model from Sect. III-A is demonstrated with a real case study for choosing a franchisee for the largest fast food restaurant chain in Bulgaria. The optimal problem is defined below: The largest fast food restaurant chain in Bulgaria has given a decision to expand its business through the selection of a franchisee. The franchisor decides to invite a team of 3 experts to evaluate the 3 candidates at a particular moment t_f . The evaluation system consists of 4 groups of criteria: C_1 - owner profitability and business experience level; C_2 - brand marketing and franchise brand development concept; C_3 - opportunities to quickly start a franchise and actively participate in the management of the restaurant and C_4 - restaurant traffic management and parking options, strategic location of the restaurant and successful traffic management around it with provided parking opportunities. Each criteria has priority coefficient as TIFPs pk_{c_j, v_e, t_f} according to their importance from the franchisor's point of view at a current moment t_f . The experts' ratings are defined by TIFP $\{r_1(t_f), r_2(t_f), r_3(t_f)\}$ at a particular moment t_f . In the final ranking we admit only candidates with an overall score higher than $\langle 0.6, 0.01 \rangle$ Now we need to optimally rank the candidates and select the most eligible one.

Solution of the problem:

Step 1. At this step, we create the expert evaluation TIFIM $EV[K, C, E, \{es_{k_i, c_j, d_s}\}]$ with the estimates of the d_s -th expert for the k_i -th candidate by the c_j -th criterion (for $1 \leq i \leq 3, 1 \leq j \leq 4, 1 \leq s \leq 3$) and its form is:

$$\begin{array}{c|cccc} d_1 & c_1 & c_2 & c_3 & c_4 \\ \hline k_1 & \langle 0.4, 0.2 \rangle & \langle 0.3, 0.4 \rangle & \langle 0.7, 0.1 \rangle & \langle 0.3, 0.4 \rangle \\ k_2 & \langle 0.2, 0.7 \rangle & \langle 0.5, 0.3 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.5, 0.3 \rangle \\ k_3 & \langle 0.5, 0.1 \rangle & \langle 0.2, 0.6 \rangle & \langle 0.3, 0.3 \rangle & \langle 0.7, 0.1 \rangle \\ \hline d_2 & c_1 & c_2 & c_3 & c_4 \\ \hline k_1 & \langle 0.5, 0.3 \rangle & \langle 0.2, 0.6 \rangle & \langle 0.8, 0.0 \rangle & \langle 0.4, 0.4 \rangle \\ k_2 & \langle 0.3, 0.7 \rangle & \langle 0.4, 0.4 \rangle & \langle 0.7, 0.1 \rangle & \langle 0.7, 0.0 \rangle \\ k_3 & \langle 0.4, 0.3 \rangle & \langle 0.4, 0.5 \rangle & \langle 0.2, 0.6 \rangle & \langle 0.5, 0.3 \rangle \\ \hline d_3 & c_1 & c_2 & c_3 & c_4 \\ \hline k_1 & \langle 0.2, 0.6 \rangle & \langle 0.3, 0.6 \rangle & \langle 0.5, 0.3 \rangle & \langle 0.5, 0.3 \rangle \\ k_2 & \langle 0.2, 0.7 \rangle & \langle 0.4, 0.5 \rangle & \langle 0.3, 0.5 \rangle & \langle 0.6, 0.1 \rangle \\ k_3 & \langle 0.4, 0.4 \rangle & \langle 0.3, 0.6 \rangle & \langle 0.4, 0.5 \rangle & \langle 0.5, 0.4 \rangle \end{array}$$

Step 2. The rating coefficients of the experts at t_f are: $\{r_1(t_f), r_2(t_f), r_3(t_f)\} = \{(0.7, 0.05), (0.6, 0.05), (0.8, 0.05)\}$.

The TIFIM $V(t_f)[K, C, E, \{v_{k_i, c_j, d_s, t_f}\}]$, which contains the final score of each franchisee at a current moment t_f , is constructed by

$$V(t_f) = r_1 pr_{K,C,d_1,t_f} EV \oplus_{(max,min)} r_2 pr_{K,C,d_2,t_f} EV \oplus_{(max,min)} \oplus_{(max,min)} r_3 pr_{K,C,d_3,t_f} EV(t_f); EV(t_f) := V(t_f) \quad (10)$$

Then we apply the operation $\alpha_{E, \#_q}(EV, h_0) = R[K, C, h_0]$

h_0	c_1	c_2	c_3	c_4
k_1	$\langle 0.3, 0.24 \rangle$	$\langle 0.24, 0.3 \rangle$	$\langle 0.49, 0.05 \rangle$	$\langle 0.4, 0.34 \rangle$
k_2	$\langle 0.18, 0.72 \rangle$	$\langle 0.35, 0.34 \rangle$	$\langle 0.42, 0.15 \rangle$	$\langle 0.48, 0.05 \rangle$
k_3	$\langle 0.35, 0.15 \rangle$	$\langle 0.24, 0.53 \rangle$	$\langle 0.32, 0.34 \rangle$	$\langle 0.49, 0.15 \rangle$

to calculate the aggregated value of the k_i -th applicant about c_j -th criterion at a current moment $h_0 \notin D$, where $\#_q$ is equal to 1, 2 or 3 depending on whether the pessimistic, averaging or optimistic scenarios are chosen. The franchise chain has a requirement for the candidates, so that their total score is not less than a predetermined TIFP $\langle 0.6, 0.01 \rangle$. The level operator (5) is applied to the TIVIFIM B and it is established that all candidates meet this requirement.

Step 3. At this step, a TIFIM $PK(h_0)$ of the weight coefficients of the assessment criterion according to its antecedence is created from the franchisor v_e :

$$PK(h_0)[C, v_e, t_f, \{pk_{c_j, v_e, h_0}\}] = \begin{array}{c|c} h_0 & v_e \\ \hline c_1 & \langle 0.8, 0.1 \rangle \\ c_2 & \langle 0.7, 0.1 \rangle \\ c_3 & \langle 0.5, 0.2 \rangle \\ c_4 & \langle 0.7, 0.1 \rangle \end{array}$$

$$\text{and } B = R \odot_{(\circ,*)} PK = \begin{array}{c|c} h_0 & v_e \\ \hline k_1 & \langle 0.656, 0.0148 \rangle \\ k_2 & \langle 0.661, 0.0136 \rangle \\ k_3 & \langle 0.669, 0.0142 \rangle \end{array}$$

Step 4. The optimistic aggregation operation $\alpha_{K, \#_3}(B, k_0)$ finds that k_3 is the optimal franchisee for the franchise chain of fast food restaurants in Bulgaria v_e with the maximum degree of acceptance (d.a.) 0.669 and the minimum degree of rejection (d.r.) 0.0142 in an optimistic scenario, in a pessimistic scenario – k_1 with the minimum d.a. 0.656 and the maximum d.r. 0.0148. The closest to the average scenario is k_2 with the d.a. 0.661 and the d.r. 0.0136.

Step 5. At this step, we apply the ICRA with $\alpha = 0.8$ and $\beta = 0.10$ over $R(h_0)$. The conclusion is that the evaluation system in the chain is optimized.

The results, obtained from the ICRA application [11], are in the form of IM in $\mu - v$ view result matrix:

	c_1	c_2	c_3	c_4
c_1	\perp	$\langle 0.33, 0.48 \rangle$	$\langle 0.62, 0.29 \rangle$	$\langle 0.57, 0.29 \rangle$
c_1	$\langle 0.33, 0.48 \rangle$	\perp	$\langle 0.52, 0.29 \rangle$	$\langle 0.57, 0.19 \rangle$
c_1	$\langle 0.62, 0.29 \rangle$	$\langle 0.52, 0.29 \rangle$	\perp	$\langle 0.67, 0.19 \rangle$
c_1	$\langle 0.57, 0.29 \rangle$	$\langle 0.57, 0.19 \rangle$	$\langle 0.67, 0.19 \rangle$	\perp

Step 6. At last step, the experts' assessments are correct from the point of view of IF logic [6] and their new rating coefficients are equal to $\{(0.82, 0.05), (0.64, 0.05), (0.81, 0.05)\}$.

IV. CONCLUSION

In the study, we have defined the OTIFAFr procedure for selection of the most suitable franchisee over temporal IF evaluations. A software implementation of the proposed algorithm was presented and the decision making procedure

applied for a franchisee selection. In the future, the study will continue with the development of OTIFAFr approach, so that it can be applied over the data, saved in extended TIFIMs [5] and also with software for its implementation.

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