Advances in Continuous and Discrete Models

RESEARCH Open Access

Computational thinking in complex problem solving based on data analysis: exploring the role of problem representation using the Tower of Hanoi

Jinlong Fei[1], Shuyuan Dong[2] and Dingzhou Fei[2*]

*Correspondence: feeding_psy@whu.edu.cn
[2]Speech Analysis and Artificial Intelligence Lab, Wuhan University, Wuhan, China
Full list of author information is available at the end of the article

Abstract

This article investigates the role of problem representation in complex problem-solving based on quasi-experimental data analysis, specifically within the context of computational thinking. Using the Tower of Hanoi task, researchers compared the performance of computer science students (with algorithmic background) and non-computer science students (novices). The data research results revealed that participants with stronger problem representation skills, i.e., computer science students, demonstrated superior problem-solving abilities, particularly as task difficulty increased. Additionally, practice was found to significantly improve problem-solving efficiency for novices, suggesting that enhancing problem representation skills through experience can effectively boost complex problem-solving abilities. The data research highlights the critical role of problem representation in computational thinking and provides valuable insights for programming practices, emphasizing the importance of developing problem representation skills to foster effective problem-solving in complex scenarios.

Keywords: Complex problem; The Tower of Hanoi; Problem representation; Algorithm and procedures; Computational thinking; Big data

1 Introduction

The ability to solve complex problems in artificial intelligence has become pivotal for both individual and societal development and has attracted more and more researchers' attention in recent years. Computational thinking, as a systematic approach to problem-solving, underscores the central role of problem representation in addressing such challenges [1]. Encompassing dimensions such as problem decomposition, pattern recognition and knowledge discovery, abstraction, and algorithm design, computational thinking trains individuals to effectively analyze and resolve complex issues by breaking them down into smaller, more manageable sub-problems and devising appropriate algorithms to tackle them [2]. Moreover, it aids in identifying patterns and regularities among problems, thereby accelerating the discovery of solutions [3].

While previous research has elucidated the connection between cognitive factors like reasoning ability and working memory and problem-solving proficiency [4], recent literature has used different modes of presentations, i.e., computer, mental, or physical, in the experiments and found that the mode of presentation affected problem-solving performance, with the computer-based representation potentially reducing working memory load. However, the conclusions of these studies seem to contradict our practices, especially for complex problems. The reason for this is that these studies have overlooked the role of computational thinking in problem-solving. Complex problems often have underlying complex algorithmic mechanisms, and the understanding of these mechanisms differs between experts and novices.

As a result, the findings that there were no differences in the number of moves required to complete the puzzle across experience levels or presentation modes may not be applicable to more complex problems [5], where the representation modes are not reflected by the solver's understanding of the underlying algorithmic mechanisms, which truly play a crucial role in problem-solving performance [6]. The study in [7] pointed out that incorrect problem representation cannot lead to the correct resolution of problems. The recent research has also confirmed that enhancing problem representation skills can effectively improve students' abilities to solve mathematical and physical problems [8]. This study, employing the Tower of Hanoi as a tool, aims to investigate how problem representation impacts the problem-solving capabilities of college students. Consequently, prior research in complex problems solving appeared to overlook the importance and generality of computational thinking, the ability to identify relevant information and make appropriate inferences and solving plans, and emphasize abstract reasoning and mental model construction, which are crucial aspects of complex problem-solving. Like almost working mathematicians, we believe that complex problem solving essentially is an objective and logical reasoning process, although it is affected by a lot of psychological factors. Our philosophy for the study of complex problem solving is toward an objective and logical view, contrary to current subjective view that complex problem solving is a psychological process, controlled by various psychological factors, such as cognitive states.

The recursive algorithm underlying the Tower of Hanoi problem demands a certain degree of logical and advanced thinking. By comparing the performance of computer science students (with relevant algorithmic background) and non-computer science students (novices), this research seeks to reveal the key role of problem representation in complex problem-solving, hypothesizing that participants with stronger problem representation skills will demonstrate superior problem-solving abilities.

Furthermore, this study also explores how practice can improve problem representation skills. Through a carefully designed experiment, we observed changes in participants' problem-solving strategies and efficiency after a period of practice. These findings not only provide a new perspective for understanding the cognitive mechanisms of complex problem solving, but also offer valuable insights for educational practice and vocational training. In the context of computational thinking, the ability to solve complex problems is regarded as a crucial skill. Computational thinking is not merely a skill specific to programming or computer science; it is a way of thinking that helps us better understand and address complex issues [9].

While previous research on complex problem-solving has primarily focused on cognitive factors such as reasoning and working memory, it has often neglected the intricate cognitive mechanisms underlying problem-solving strategies. This gap in understanding stems from several limitations: behavioralist approaches prioritizing observable behaviors over internal processes; cognitive psychology's emphasis on macro-level cognitive structures and physical appearances rather than specific logical and computational mechanisms; and the lack of direct analysis framework for capturing the gist of thinking processes. Computational thinking can fit this purpose for offering a explicit framework, as which this study will show.

Despite these limitations, some potential features of thinking mechanisms have been identified, including dynamism, hierarchical structure, and individual differences. To advance our understanding, future research should focus on developing effective measurement tools, employing diverse methodologies, and constructing interdisciplinary theoretical frameworks. This will enable a more comprehensive exploration of the cognitive mechanisms underlying problem-solving strategies, leading to a deeper understanding of complex problem solving.

So, from the perspective of computational thinking, what is the key influencing factor in the ability to solve complex problems? Generally speaking, individuals must go through two cognitive stages when solving a problem: establishing problem representation and finding problem solutions. This suggests that establishing correct problem representation is the foundation of solving complex problems. Based on this, the present paper focuses on the ability to solve complex problems, using college students as the subject group, to explore the factors affecting this ability, discovering the critical role of problem representation in complex problem-solving, and further finding that practice can effectively enhance problem representation skills, thereby improving the ability to solve complex problems.

The ability to solve complex problems simply refers to the interaction ability between problem solver and dynamic task environment [10]. It is of great significance to explore the influencing factors of the ability to solve complex problems. Most of the previous studies focused on the education of primary and secondary school students, but there was no relevant research based on good measurement tools. As a classic measurement tool in the field of problem solving, the Tower of Hanoi has specific task rules, and due to the recursive algorithm behind it, it requires a certain degree of logical ability and advanced computational thinking process, which is more targeted and professional than previous studies. Therefore, based on the perspective of problem representation, this paper finds that problem representation plays a key role in solving complex problems, and practice can effectively improve the ability of problem representation, thus improving the ability to solve complex problems.

1.1 Complex problem

Problem solving refers to the behavior of transforming the current state into the goal state by means of a series of goal-oriented cognitive operations when there is no clear solution. According to the complexity of the problem situation, problems can be divided into simple problems and complex problems. Simple problems are usually well structured, that is, they provide all the relevant information to solve the problem, have a clear goal and a limited number of solutions, such as solving an equation; Complex problems are poorly structured and often lack information and unclear goals in the initial state of the problem. It is necessary to gather information in the process of interaction with the task to find solutions to the problem, such as writing a business plan.

There are five characteristics of complex problems: 1) complexity, that is, the number of variables associated with the problem is large; 2) Correlation, that is, the relationship between various variables is complex and intensive; 3) Dynamic, that is, as the problem solver takes action, the problem situation itself changes, and in some cases, the problem situation even changes spontaneously over time; 4) Fuzziness, that is, the clarity of task characteristics, such as variable relations, task objectives and operations that can be taken, is low. Individuals must explore for themselves and constantly integrate information in the process of exploration to obtain necessary information; 5) Multiple goals. Complex problems usually have more than one goal, and there are often conflicts between different goals. Therefore, problem solvers need to actively identify different goals and classify their priority levels. Not all complex problems meet the above five characteristics at the same time, but the more conditions a particular problem meets, the problem is considered to be a complex problem, and dynamics is the most core characteristic of the complex problem.

The ability to solve complex problems refers to the collection of a series of self-regulating psychological processes that are necessary in the face of complex problems with ill-structured structure in the dynamic changing environment. This type of problem solving cannot be achieved through conventional behavior, but requires the creative combination of knowledge and advanced cognitive processes [11].

Studies have proved that reasoning ability is significantly correlated with the ability to solve complex problems, and participants with higher reasoning ability usually have higher ability to solve complex problem [12]. Other research results supported the important role of working memory in complex problem solving based on the cognitive load model, for instance, that the participants with higher working memory level show higher ability to solve complex problem [13].

It is obvious that solving problems cannot do without knowledge. Some studies have proved that it is an effective way to improve the ability to solve complex problems for students to master general problem-solving knowledge [14]. There are even studies that suggest motivation and emotion affect people's ability to solve complex problem. [15] found that individual intrinsic motivation was positively correlated with the performance of complex problem solving, and the stronger the individual intrinsic motivation was, the better the problem-solving result would be. Lin et al [16] found that positive emotions can improve individuals' performance in complex problem solving, possibly because positive emotions have a positive effect on cognitive flexibility.

But these factors are not the core elements that affect the ability to solve complex problems. The thinking process of complex problem solving is more complex and comprehensive, which requires not only knowledge but also knowledge integration and the application of advanced cognitive processing ability, not simple reasoning ability or working memory ability. In a study, Güss and Badibanga [17] compared the performance of business managers and students on complex tasks involving running a chocolate company and found that business managers performed better than students because they spent more time exploring complex situations before making decisions and had the flexibility to adjust their solutions to suit the situation. This means that compared with novices, expert problem solvers have better ability to extract effective information from problems due to their experience, so that they can quickly find solutions to problems. In other words, they have better problem representation ability, so they have higher ability to solve complex problems.

The study of [18] shows that individuals need to go through two cognitive stages to solve a problem, namely, constructing problem representation and finding a problem solution. The establishment of problem representation is the basis of solving complex problems, that is, only the establishment of correct problem representation can solve problems more quickly and accurately. At present, it has been proved that the improvement of problem representation ability can effectively improve students' ability to solve mathematical and physical problems [19]. However, many wrong problem representations could not lead to a correct solution of the problem.

From the above discussion, we conclude that the key to the improvement of the ability to solve complex problems is not simply the increase of knowledge, nor the improvement of reasoning ability and working memory level, but the improvement of the ability to establish correct problem representation. The establishment of correct problem representation is convenient for us to extract the required information from the problem, so as to integrate the information, find the breakthrough to solve the problem, and then solve the problem.

The prospects may involve the transferability of problem representation across domains: to explore whether computational thinking in the Hanoi Tower task can be transferred to other types of complex problem solving, such as programming, data analysis, mathematical modeling or engineering design, in particular, the chain of thought in large language models.

Therefore, the following hypothesis is proposed.

Hypothesis 1 The key factor in the ability to solve complex problems is problem representation, that is, individuals with higher problem representation ability will have a higher ability to solve complex problems.

1.2 Problem representation

Problem representation mainly refers to the process in which problem solvers interpret the perceived information of known conditions according to their own knowledge and experience to discover the structure of the problem, construct the problem space, and transform the external stimuli into internal psychological symbols, which is to construct the cognitive structure of the problem and form the problem schema in their minds [20]. Problem representation plays an important role in solving the problem smoothly.

The research [21] has shown that constructing problem representation is a process of continuous abstraction from surface to deep, which includes three stages: 1) Search and extract problem information. This stage mainly involves perceptual processes and requires the support of specialized knowledge, verbal and comprehension skills, and experience in problem solving. 2) Understanding and internalization of problem information. This stage involves the deep processing of perceived information, that is, the understanding, generalization and internalization of information, which needs the support of knowledge base, thinking ability and problem-solving skills. 3) Developing metaphorical constraints and consciousness. In order to solve the problem, it is necessary to infer the implied information not directly expressed and find the metaphorical constraints actively.

The paper [22] highlights that problem solvers' specialized knowledge, experience, and successful problem-solving history are key factors influencing problem representation ability. The stability, clarity and availability of these experiences directly affect the perceptual system's selection of information in the situation of the problem and its interpretation of the perceived information, and even can become an analogy example of problem representation.

Based on the above discussion, we infer that individuals with knowledge in the same professional field as the problem may possess better problem representation ability compared to those with knowledge in other fields. At the same time, Patrick, John, Ahmed, Afia [23] showed that practices can contribute to the improvement of the level of representation.

The following hypothesis is proposed:

Hypothesis 2 Practices can improve problem representation ability and thus improve the ability to solve complex problems.

1.3 Theoretic analysis framework

The complex problem solving in for the Tower of Hanoi required an evaluation calibrating for the participants' performances, The derived number of moves is exactly $2^n - 1$. This is the minimum number of moves required to solve the Tower of Hanoi puzzle with n disks. There is no way to solve it with fewer moves, while adhering to the rules.

The actual number of moves will always be greater than or equal to the derived number. Any solution that solves the puzzle requires at least $2^n - 1$ moves. If a solution uses more than $2^n - 1$ moves, it simply means it is not an optimal solution. The difference between the actual and derived number represents the inefficiency of the solution strategy. In the real contexts, people will stop trying to make more moves once they have achieved the goal of the task, so, we can assume that:

Hypothesis 3 The number of $2^n - 1$ moves is exactly the correct number for the Tower of Hanoi.

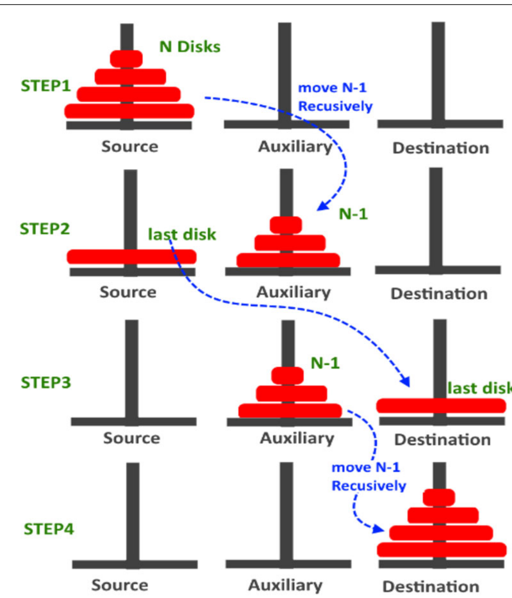

Figure 1 N-disks Hanoi Tower task can be done exactly with the recursion

Now derive the exact move number of the Tower of Hanoi using the above Fig. 1, which is a mathematical puzzle where the objective is to move a stack of disks of different sizes from one rod to another, obeying the following rules:

– Only one disk can be moved at a time.
– Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
– No larger disk may be placed on top of a smaller disk.

Let T_n be the minimum number of moves required to solve the Tower of Hanoi puzzle with n disks.

- Base Case: For $n = 1$ disk, only one move is required. Therefore, $T_1 = 1$.
- Recursive Step: To move n disks from the source rod to the destination rod, we can break the problem into three steps:
  i. Move the top $n - 1$ disks from the source rod to the auxiliary rod. This requires T_{n-1} moves.
  ii. Move the largest disk (the nth disk) from the source rod to the destination rod. This requires 1 move.
  iii. Move the $n - 1$ disks from the auxiliary rod to the destination rod. This requires another T_{n-1} moves.

Therefore, the total number of moves required is:

$$T_n = T_{n-1} + 1 + T_{n-1} = 2T_{n-1} + 1 \ (T_1 = 1).$$

This is a recurrence relation. We can solve it iteratively. Its proof is below: let $a_n = T_n + 1$, then this recurrence is rewritten as $a_n - 1 = 2(a_{n-1} - 1)$, so $a_n - 1 = 2^n(a_1 = 2)$, $T_n = 2^n - 1(T_1 = 1)$.

2 Method

Students from a college were selected as participants in the experiment. Since the students majoring in computing have studied the algorithms associated with the Tower of Hanoi, understand the arithmetic logic behind it, have a background in the same area of expertise as it, and conform to the conditions capable of good problem representation constructed, therefore, it is assumed that the students majoring in computing have better problem representation ability than the students not majoring in computing. So, students majoring in computing are the professional participants, while students not majoring in computing are the newer participants.

It is warned that solving complex problems requires high reasoning abilities and specialized knowledge. Therefore, we chose college students as our subjects, the college students generally possess high cognitive and reasoning abilities, making them an ideal group for studying complex and hard problems.

The measuring tool is the Tower of Hanoi. It is the classic measurement tool in the field of problem solving, satisfying the five characteristics of complex problems [24]. At the same time, it is difficult and requires a certain logical ability, which can well meet the task standard for the evaluation of the ability to solve complex problems of college students. And the Tower of Hanoi has a clear and accurate quantitative index, so it is selected as the measurement tool of this experiment. The whole study flowchart is followed by Fig. 2.

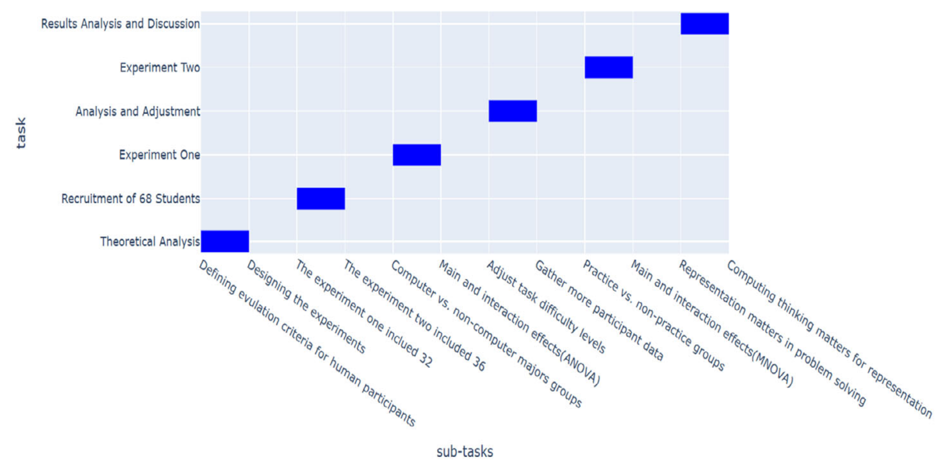

Figure 2 The overview of study procedure

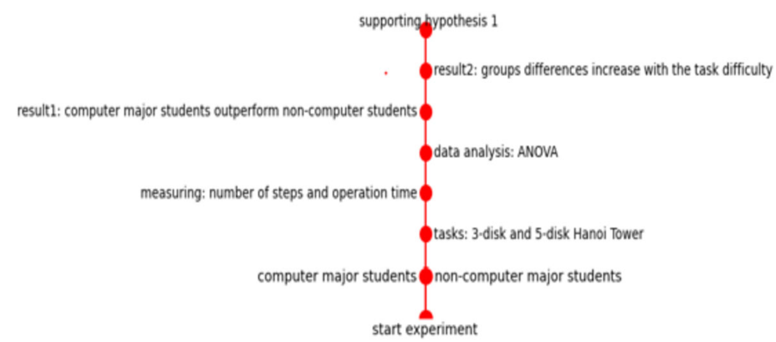

Figure 3 The flowchart for the Experiment 1

2.1 Experiment 1

The flowchart for Experiment 1 is given in Fig. 3.

2.1.1 Participants

The grouping of subjects into computer science majors and non-computer science majors is more reasonable than the other groupings, mainly because computer science majors have systematic knowledge of algorithms and data structures, as well as extensive programming experience, which gives them a significant advantage in solving the Tower of Hanoi problem. Non-computing majors, on the other hand, due to the lack of these systematic knowledge and experience, although they may have occasional inspirations in some cases, it is actually difficult for them to solve the Tower of Hanoi problem effectively without the corresponding knowledge and experience. Therefore, this grouping better reflects the effect of different professional backgrounds on problem solving ability.

Our study has a basic premise based on the item response theory (IRT) that an individual's performance on a test depends primarily on his or her latent abilities (e.g., knowledge, skills, etc.) rather than other irrelevant factors (e.g., age, cognitive skills, and cognitive state). In the Tower of Hanoi task, an individual's performance depends primarily on his or her knowledge of algorithms and data structures rather than age, cognitive skills, and cognitive state. Therefore, grouping subjects with computer science majors and non-computer science majors is more reasonable than other groupings and better reflects the effects of different professional backgrounds on problem-solving ability.

A total of 32 college students were randomly selected from a university, ranging in age from 20 to 24 years old, including 12 boys. The professional: 15 students majoring in computing, 7 boys. The newer: 17 students not majoring in computing, 5 boys. All participants were right-handed, had normal visual acuity or corrected visual acuity, and were proficient in computer operation.

2.1.2 The experiment design

The experiment was a two-factor mixed experimental design of 2 (participant type) $\times$ 2 (task difficulty). Participant type was a variable between participants, which was divided into the newer and the professional; Task difficulty was a variable within participants, which was divided into 3 disks and 5 disks. The dependent variables were the number of moving steps and operation time of the participants to complete the Tower of Hanoi.

2.1.3 Instruments and materials

(1) Experimental instruments: the Tower of Hanoi software (automatically record the number of moving steps and operation time), computer.
(2) Experimental materials for the Tower of Hanoi: 3 cylinders with trays and several disks; The three cylinders are exactly the same; The disks are identical except in size; The difference between the diameters of any two disks exceeds the difference threshold and is easy to distinguish.
(3) Operation rules of the Tower of Hanoi: in the initial state, several disks are stacked on the left cylinder in ascending order, and the task objective is to move all disks to the right cylinder with the help of the middle cylinder, and also put them in ascending order. Only one disk can be moved at a time. A large disk cannot be placed on a small disk; Any disk that is not moving must be placed on the cylinder.

2.1.4 Experimental procedure

(1) Inform the participants of the Tower of Hanoi rules and matters needing attention to ensure that each participant clearly defined the rules.
(2) The participants completed the Tower of Hanoi on the computer, and the software automatically recorded the number of moving steps and operation time. All participants were required to complete the tasks of 3 disks and 5 disks, and there was no limit on the time to complete the tasks.
(3) Participants were given 2-5 minutes to rest between tasks.
(4) After the task is completed, the participants are asked whether there is any strategy during the task. If so, please specify.

2.1.5 Data analysis

SPSS was used for repeated measures ANOVA.

2.1.6 Results

(1) Dependent variable is the number of moving steps

Table 1 Descriptive statistical results of moving steps as dependent variable

Task difficulty	Participant type	M	SD	*n*
3 disks	the professional	7.79	1.19	14
	the newer	14.06	6.89	16
5 disks	the professional	54.57	27.4	14
	the newer	83.75	33.43	16

Table 2 ANOVA results of moving steps as dependent variable

Variables	*p*	Effect size
task difficulty	<0.001	0.8
participant type	0.005	0.25
task difficulty*participant type	0.05	0.13

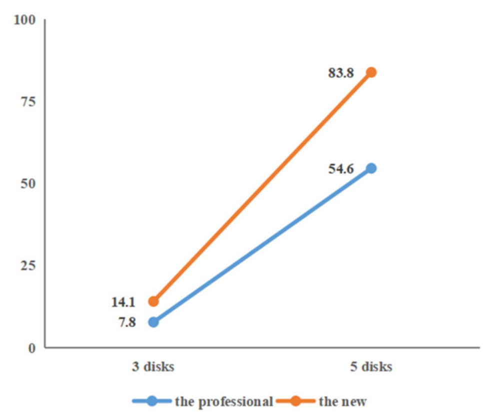

Figure 4 Interaction between task difficulty, participant type and the number of moving steps

Participant type was the variable between participants, task difficulty was the variable within participants and the number of moving steps was a dependent variable. Repeated measures ANOVA was performed.

1) Descriptive statistical results

After removing one extreme value data from the newer group and the professional group, the statistical results were obtained as shown in Table 1.

2) ANOVA

As can be seen from Table 2, the main effect of task difficulty is significant, indicating that the number of moving steps of participants under different task difficulty has significant difference; The main effect of the participant type was significant, indicating that there was a significant difference in the number of moving steps between the newer and the professional, and the speed of the professional was significantly higher than that of the newer; The interaction between task difficulty and participant type was marginal significant, indicating that with the change of task difficulty, the difference degree between the newer and the professional also changed.

3) Interaction

Figure 4 shows that under different task difficulties, there are differences in the number of moving steps of the newer and the professional. When the task difficulty was 3 disks, there was little difference between the newer and the professional. When the task difficulty

Table 3 Simple effect analysis results of moving steps as dependent variable

Variables	Llevel	*p*
task difficulty	3 disks	0.002
	5 disks	0.015
participant type	the professional	<0.001
	the newer	<0.001

Table 4 Descriptive statistical results of operation time as dependent variable

Task difficulty	Participant type	M	SD	*n*
3 disks	the professional	19.43	10.8	14
	the newer	54.31	32.06	16
5 disks	the professional	110.71	74.7	14
	the newer	269.88	128.93	16

Table 5 ANOVA results of operation time as dependent variable

Variables	*p*	Effect size
task difficulty	<0.001	0.7
participant type	<0.001	0.43
task difficulty*participant type	0.003	0.27

was 5 disks, there was a large difference between the newer and the professional, so it can be inferred that the greater the task difficulty, the greater the difference between the newer and the professional.

4) Simple effect analysis

It can be seen from the above results that there is an interaction between the participant type and the task difficulty, so the simple effect is analyzed. The statistical results are shown in Table 3. As can be seen from Table 3, there is a significant difference in the number of moving steps between the newer and the professional, regardless of 3 disks or 5 disks. There was a significant difference in the number of moving steps at different task difficulty for both the newer and the professional.

(2) Dependent variable is operation time

Participant type was the variable between participants, task difficulty was the variable within participants and operation time was a dependent variable. Repeated measures ANOVA was performed.

1) Descriptive statistical results

After removing one extreme value data from the newer group and the professional group, the statistical results were obtained as shown in Table 4.

2) ANOVA

As can be seen from Table 5, the main effect of task difficulty is significant, indicating that the operation time of participants under different task difficulty has significant difference; The main effect of the participant type was significant, indicating that there was a significant difference in the operation time between the newer and the professional, and the speed of the professional was significantly higher than that of the newer; The interaction between task difficulty and participant type was significant, indicating that with the change of task difficulty, the difference degree between the newer and the professional also changed.

3) Interaction

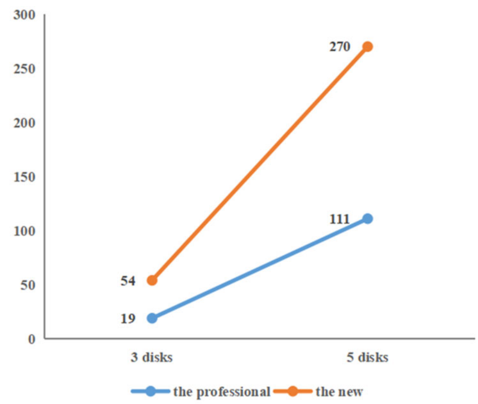

Figure 5 Interaction between task difficulty, participant type and the operation time

Table 6 Simple effect analysis results of the operation time as dependent variable

Variables	Level	*p*
task difficulty	3 disks	0.001
	5 disks	<0.001
participant type	the professional	0.003
	the newer	<0.001

Figure 5 shows that under different task difficulties, there are differences in the operation time of the newer and the professional. When the task difficulty was 3 disks, there was little difference between the newer and the professional. When the task difficulty was 5 disks, there was a large difference between the newer and the professional, so it can be inferred that the greater the task difficulty, the greater the difference between the newer and the professional.

4) Simple effect analysis

It can be seen from the above results that there is an interaction between the participant type and the task difficulty, so the simple effect is analyzed. The statistical results are shown in Table 6. As can be seen from Table 6, there is a significant difference in the operation time between the newer and the professional, regardless of 3 or 5 disks. There was a significant difference in the operation time at different task difficulty for both the newer and the professional.

2.1.7 Discussion

The experimental results showed that there were significant differences in the number of moving steps and operation time between the professional and the newer in both high and low difficulty Tower of Hanoi, and the number of moving steps and operation time of the professional was significantly lower than that of the newer, that is, the speed of the professional solving the problem was significantly faster than that of the newer, supporting Hypothesis 1. It was also found that the difference between the professional and the newer became more significant as the task difficulty increased. Tables 1, 2, 3 and Fig. 4 collectively support the conclusion that there is a significant interaction effect between participant type and task difficulty in the experimental results. Tables 4, 5 and 6 are consistent with the results for the operation time in Fig. 5.

It can be seen from Experiment 1 that the efficiency of problem solving is greatly affected by the problem representation ability, so whether the newer who do not have good problem representation ability can improve the speed of problem solving through practice, and then improve the problem representation to improve the solving efficiency, Experiment 2 was designed to answer this issue.

2.2 Experiment 2

The flowchart for Experiment 2 is given in Fig. 6.

2.2.1 Participants

In selecting participants for the experiment, we were still guided by the knowledge and skills required for solving the complex problem and IRT theory, as the same as the Experiment 1.

A total of 36 college students were randomly selected from a university, ranging in age from 20 to 24 years old, including 14 boys. The professional: 17 students majoring in computer science, 7 boys. randomly assigned to the control group and the experimental group, of which 8 were in the control group; The newer: 19 students not majoring in computer science, 7 boys. Randomly assigned to the control group and the experimental group, of

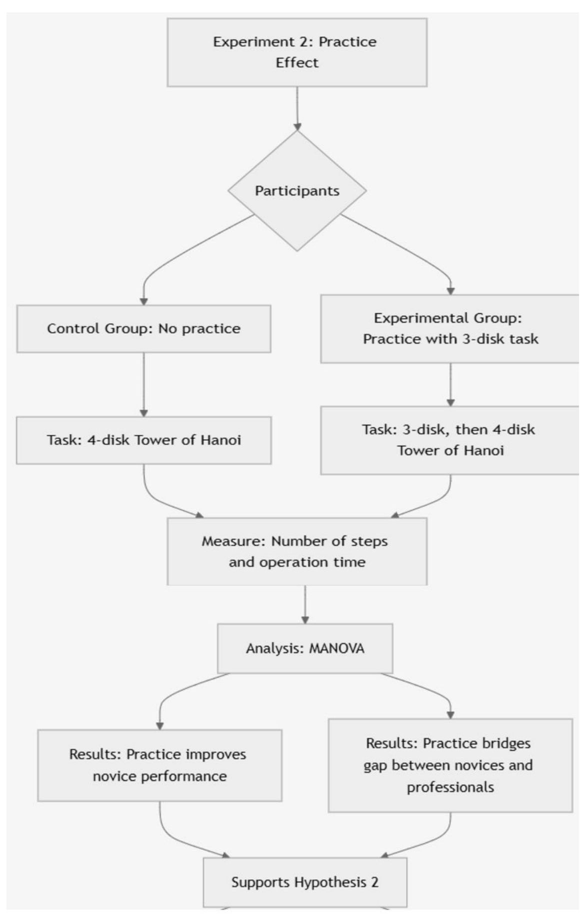

Figure 6 The flowchart for Experiment 2

which 9 were in the control group. All participants were right-handed, had normal visual acuity or corrected visual acuity, and were proficient in computer operation.

2.2.2 The experiment design

The experiment was a two-factor experiment design between participants of 2 (participant type) $\times$ 2 (group: the experimental group, the control group). Participant type was divided into the newer and the professional; Group was divided into the experimental group and the control group. The experimental group completed 3 disk tasks first, then 4 disk tasks; The control group completed 4 disk tasks directly. The dependent variables were the number of moving steps and operation time of the participants to complete the 4-disk Tower of Hanoi. The flowchart is given as below:

2.2.3 Instruments and materials

The experimental instruments and materials are the same as in Experiment 1.

2.2.4 Experimental procedure

(1) Inform the participants of the Tower of Hanoi rules and matters needing attention to ensure that each participant clearly defined the rules.
(2) The participants completed the Tower of Hanoi on the computer, and the software automatically recorded the number of moving steps and operation time. The experimental group completed the task with 3 disks and 4 disks, while the control group only completed the task with 4 disks. There was no limit on the time to complete the tasks.
(3) Participants were given 2-5 minutes to rest between tasks.
(4) After the task is completed, the participants are asked whether there was any strategy during the task. If so, please specify.

2.2.5 Data analysis

SPSS was used for MANOVA.

2.2.6 Results

(1) Dependent variable is the number of moving steps

The participant type and group were the independent variables and the number of moving steps was a dependent variable. MANOVA was performed.

1) Descriptive statistical results

See Table 7.

2) MANOVA

As can be seen from Table 8, the main effect of group is significant, indicating that the number of moving steps of the control group and the experimental group has significant

Table 7 Descriptive statistical results of moving steps as dependent variable

Group	Participant type	M	SD	*n*
control group	the professional	24.13	7.4	8
experimental group	the professional	15.89	2	9
control group	the newer	44	14.64	9
experimental group	the newer	23	8.54	10

Table 8 MANOVA results of moving steps as dependent variable

Variables	*p*	Effect size
group	<0.001	0.41
participant type	<0.001	0.37
group*participant type	0.049	0.12

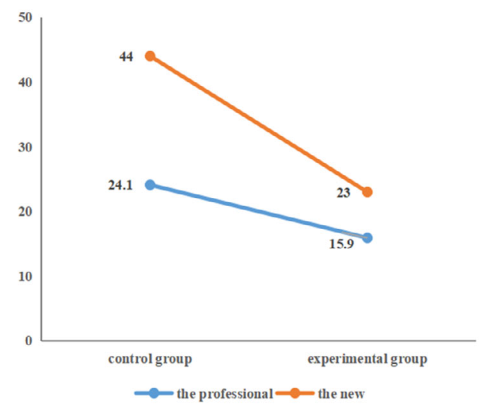

Figure 7 Interaction between group participant type and the number of moving steps

Table 9 Simple effect analysis results of moving steps as dependent variable

Variables	Group	*p*
group	control group	<0.001
	experimental group	0.107
participant type	the professional	0.079
	the newer	<0.001

difference; The main effect of the participant type was significant, indicating that there was a significant difference in the number of moving steps between the newer and the professional, and the speed of the professional was significantly higher than that of the newer; The interaction between group and participant type was marginal significant, indicating that the difference degree between the newer and the professional is also different in different groups.

3) Interaction

Figure 7 shows that both the control group and the experimental group have differences in the number of moving steps of the newer and the professional. There was a great difference between the newer and the professional in the control group. After practice, there was little difference between the newer and the professional in the experimental group. It follows that, to some extent, practice can bridge the gap between the new and the professional. Figure 7 further illustrates the interaction effect between experimental group and participant type, with the results aligned with the analysis outcomes from Tables 7, 8 and 9.

4) Simple effect analysis

It can be seen from the above results that there is an interaction between the participant type and the group, so the simple effect is analyzed. The statistical results are shown in Table 9. As can be seen from Table 9, there were significant differences in the number of moving steps between the newer and the professional in the control group, and there

Table 10 Descriptive statistical results of operation time as dependent variable

Group	Participant type	M	SD	*n*
control group	the professional	53.25	32.78	8
experimental group	the professional	22.56	9.28	9
control group	the newer	187.11	152.43	9
experimental group	the newer	55.6	24.15	10

Table 11 MANOVA results of operation time as dependent variable

Variables	*p*	Effect size
group	0.004	0.23
participant type	0.003	0.24
group $*$ participant type	0.065	0.1

was no significant difference between the newer and the professional in the experimental group, indicating that after practice, the difference between the newer and the professional became smaller. In the newer group, there was a significant difference in the number of moving steps between the experimental group and the control group, indicating that after practice, the newer solving speed was effectively improved; In the professional group, there was no significant difference in the number of moving steps between the experimental group and the control group, indicating that practice could not lead to a significant improvement in the professional's solving speed.

(2) Dependent variable is operation time

Participant type and group were the independent variables and operation time was a dependent variable. MANOVA was performed.

1) Descriptive statistical results

See Table 10.

2) MANOVA

As can be seen from Table 11, the main effect of group is significant, indicating that the operation time of the control group and the experimental group has significant difference; The main effect of the participant type was significant, indicating that there was a significant difference in the operation time between the newer and the professional, and the speed of the professional was significantly higher than that of the newer; The interaction between group and participant type was marginal significant, indicating that the difference degree between the newer and the professional is also different in different groups.

3) Interaction

Figure 8 shows that both the control group and the experimental group have differences in the operation time of the newer and the professional. There was a great difference between the new and the professional in the control group. After practice, there was little difference between the new and the professional in the experimental group. It follows that, to some extent, practice can bridge the gap between the new and the professional.

4) Simple effect analysis

It can be seen from the above results that there is an interaction between the participant type and the group, so the simple effect is analyzed. The statistical results are shown in Table 12. As can be seen from Table 12, there were significant differences in the operation time between the newer and the professional in the control group and there was no significant difference between the newer and the professional in the experimental group, indicating that after practice, the difference between the newer and the professional be-

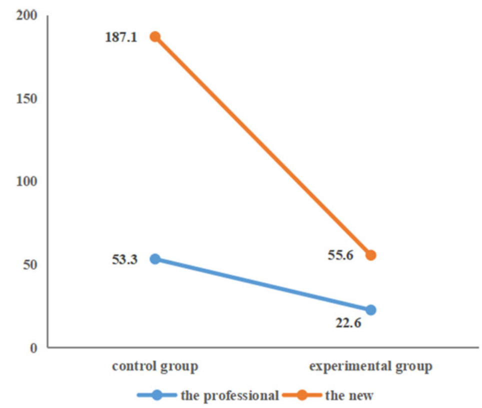

Figure 8 Interaction between group, participant type and the operation time

Table 12 Simple effect analysis results of operation time as dependent variable

Variables	Group	*p*
group	control group	0.001
	experimental group	0.369
participant type	the professional	0.429
	the newer	0.001

came smaller. In the newer group, there was a significant difference in the operation time between the experimental group and the control group, indicating that after practice, the novices' solving speed was effectively improved; In the professional group, there was no significant difference in the operation time between the experimental group and the control group, indicating that practice could not lead to a significant improvement in the professional's solving speed. Figure 8 further illustrates the interaction effect between experimental group and participant type, with the results consistent with the analysis outcomes from Tables 10, 11 and 12.

2.2.7 Discussion

Experiments showed that there was a significant difference between the experimental and control groups of the newer group, and there was no significant difference between the newer and the professional in the experimental group, indicating that practice can indeed accelerate the speed of the newer to solve problems and bridge the gap between the newer and the professional, verifying Hypothesis 2. Further analysis found that there was no significant difference between the control group and the experimental group in the professional group, indicating that practice could not lead to a significant increase in the speed of problem solving of the professional, which may be because the professional had a relatively complete problem representation, so the ability to solve complex problems could not be significantly improved through practice.

2.2.8 The supportive qualitive evidence from the interviews with participants

We collected some of the participants' comments on doing the Hanoi Tower puzzle after completing the tasks. The participants were from a variety of majors and not disciplinary. They were randomly selected for interviews. We recorded the talks with emphasis on

Table 13 The interviews for the participants in the two experiments

Theme	Theme Summary	Sample Comments
Problem Understanding	Participants were able to grasp the basic rules of the Tower of Hanoi, but struggled to comprehend its underlying structure.	Participant A (Mathematics) stated that he initially believed the problem was simple, assuming he could find the optimal solution through trial and error. However, as the number of disks increased, he found the problem becoming more complex. He attempted to use mathematical induction to derive the optimal solution but was unsuccessful. Eventually, he tried an iterative approach and felt more confident in his approach.
Strategy Selection	Participants employed various strategies to solve the problem, such as recursion, iteration, and observational learning.	Participant B (Psychology) mentioned that he improved his own methods by observing others, highlighting the effectiveness of observational learning in enhancing problem-solving skills.
Practice Effectiveness	Practice effectively improved participants' problem-solving abilities, enabling them to find optimal solutions more quickly.	Participant C (Psychology) indicated that through repeated practice, he gradually mastered the strategies for solving 3, 4, and 5 disks of the Tower of Hanoi. However, he remained unsure about how to generalize these strategies to other situations.
Challenges and Difficulties	Participants encountered challenges and difficulties during the problem-solving process, such as forgetting rules and being easily distracted.	Participant D (History) admitted to occasionally forgetting completed steps in the Tower of Hanoi, leading to difficulties in completing the task.
Knowledge Transfer	Participants were able to apply existing knowledge or experience to new problem-solving scenarios.	Participant F (Computer Science) explained that his experience with writing C language code, particularly with recursive and iterative algorithms, facilitated his success in solving the Tower of Hanoi. Although he was unsure of how to derive the optimal solution, the algorithmic structure provided him with the necessary steps to solve the problem efficiently.

the strategies and understanding of problem solving, not psychological questionaries. We used these comments in order to facilitate the understanding for the two main hypotheses in this research: (1) whether the strategies are successful in solving complex problem depends on the understanding of the hidden structures in complex problems; (2) these hidden structures are explicated in problem representations; (3) the problem representation is abstract relations(not only in spatial or temporal forms) and organizes clues and information in the solving problems in order to search for right solutions for the complex problem; (4) the computational thinkings are the core elements for successfully solving complex problems.

See Table 13 for the interviews.

3 Final discussion

The prior literature on complex problem-solving research has tended to focus on cognitive factors, such as reasoning skills, working memory, etc., while ignoring the critical role of problem representation skills in the process. These studies tend to view problem solving as a purely mental process, influenced by various psychological factors, such as cognitive state. However, we believe that complex problem solving is by its nature an objective logical reasoning process, although it can indeed be influenced by psychological factors. In contrast to this subjective view, we advocate studying complex problem solving from an objective and logical perspective, emphasizing the role of objective knowledge hidden in complex problems as a guide to problem solving strategies. This knowledge can be viewed as a concrete expression of computational thinking, with the appropriate problem representation being the driving force behind the associated solution strategy.

Computational thinking is essential to mathematics, such as Newton's iterative method for differential equations, the proof of Stokes' Theorem with its recursive approach, and problem-solving in mathematical modeling. These applications highlight the significance of iterative approximation, recursive problem-solving, and effective problem representation in advancing mathematical research. In artificial intelligence, the chain of thought (CoT) is crucial to improve the reasoning of large language models, and appropriate problems and task representations will booster the model's performances.

Recursion and iteration are the core elements in computational thinking, then are essential to theoretical computer science and discrete mathematics. On the one hand, problem representation determines algorithm selection: Problem representation involves understanding and analyzing the problem, which dictates how we break it down into smaller sub-problems and the relationships between them. Different problem representations lead to different algorithm choices. For instance, with the Tower of Hanoi, if we represent it as "moving all disks from one rod to another," we might opt for a recursive algorithm. Conversely, if we represent it as "moving a single disk," we might choose an iterative algorithm. On the other hand, algorithm selection affects problem representation: Algorithm choice can also influence how we represent the problem. For example, if we choose a recursive algorithm to solve the Tower of Hanoi, we need to represent the problem as "moving $n-1$ disks, moving the nth disk, and then moving $n-1$ disks again." This representation aligns with the structure of the recursive algorithm. The difference between the professional and the newer lies in their different representation of the same problem, and the difficulty of the problem is just an appearance. The key lies in their different analysis of the composition and structure of the problem. The representation of the newer is more superficial, while the professional master the internal deep structure of the problem. Therefore, the problem representation constructed by the newer is far worse than that of the professional, and the professional can grasp more effective problem-solving strategies, thus the ability to solve complex problems of the professional is far better than that of the newer.

Patsenko and Altmann's attentional selection model [25] based on the Tower of Hanoi showed that individuals with different concerns into the problem, namely selective attention, had different efficiency in solving the Tower of Hanoi; The research of Luigi [26] also shows that search and problem representation can affect the rate of problem solving. But neither study shows the key underlying factors that really affect the ability to solve complex problems. Some study verified that targeted training could improve the rate of individuals solving the Tower of Hanoi, but it does not reveal the mechanism through which the training improves the individual's ability to solve complex problem.

Based on the Tower of Hanoi, we found that problem representation ability is a key variable that affects the ability to solve complex problems. The number of moving steps and operation time of participants with good problem representation ability (the moderate expertise) were significantly less than those with poor problem representation ability (the novice), and with the increase of task difficulty, the difference between the professional and the newer became larger. This result is consistent with the results of previous studies. In addition, it was found that practice can effectively reduce the number of moving steps and operation time for the newer to complete the Tower of Hanoi. Although there is still a certain gap with the professional, compared with Experiment 1, the gap has been significantly reduced. Which shows that practice can indeed improve the problem representation ability of the newer, thus improving the ability to solve complex problems. This result also supports that practice can improve the ability of problem representation construction ability. In addition, it was found that the improvement of the problem representation construction ability of the professional was less than that of the newer. Although there was some improvement, it was significantly less than that of the newer. We conclude that this result may be due to the shortcomings of the problem representation established by the newer, and practice can improve the problem representation. However, since the problem representation established by the professional at the beginning tends to be perfected, practice cannot play a better role in improvement on this basis, but can only play a practice effect and simply improve their proficiency in problem solving. In conclusion, this study finds that problem representation plays a key role in the ability to solve complex problems, and practice can improve the problem representation construction ability of individuals with poor problem representation, thus improving the ability to solve complex problems [27].

To advance the study's support for computational thinking, it is essential to broaden the discussion to include applications in AI and machine learning. The strategies employed by participants, such as trial and error, recursive algorithms, and iterative approaches, align with principles utilized in AI algorithms like Monte Carlo Tree Search (MCTS) and Reinforcement Learning (RL). For instance, trial and error resembles the exploration phase of reinforcement learning, where the algorithm experiments with different actions to learn the optimal strategy. Recursive algorithms, on the other hand, are analogous to the recursive nature of MCTS, where the algorithm builds a search tree by recursively simulating possible moves. By integrating multidisciplinary references based on the intelligence on human inspiration, including AI and machine learning, the study can demonstrate the practical relevance of computational thinking beyond academic settings, further reinforcing its importance as a fundamental skill for solving complex problems.

4 Limitations and prospects

This paper found that the ability to solve complex problems is problem representation, and practice can improve the problem representation of novice students and thus improve their solving speed. But there are still several problems as follows.

(1) The definition of the newer and the expertise is vague and lacks specific professional definition. Although students majoring in computer science have learned relevant algorithms of the Tower of Hanoi, it can be regarded as their professional ability to solve the Tower of Hanoi to a large extent, but there is no clear evidence. Students not majoring in computer science do not exclude students who have studied the Tower of Hanoi, who may not belong to the newer group.

(2) The ability to solve complex problems is not directly measured, but inferred through other data. In the experiment, the number of moving steps and operation time of problem solving were used to determine the ability to solve complex problems of the participants. Although it has a certain predictive effect, it is not a direct and accurate measurement data of the ability to solve complex problems.

(3) The cognitive process of formation new representations, such as the sudden moments for discovery of new ideas for solution is not fully provided. The *Aha!*-insight problem solving in this study has come into the minds of participants

in the experiments a few times, and this occurrence needs to be further explored, and the methods to effectively improve the ability to solve complex problems need to be further explored.

The prospects may involve the transferability of problem representation across domains: to explore whether computational thinking in the Hanoi Tower task can be transferred to other types of complex problem solving, such as programming, data analysis, mathematical modeling or engineering design, in particular, the chain of thought in large language models.

Historically, George Polya, a renowned mathematician and problem-solving expert, explored themes such as mathematical discovery, plausible reasoning, and creative thinking in his works. Polya emphasizes the importance of problem representation, proposes a four-step problem-solving framework ("Understand the Problem, Devise a Plan, Carry Out the Plan, and Look Back"), and introduces strategies like induction, deduction, and analogy. His ideas have profoundly influenced the fields of mathematics, education and problem-solving. His work is much deeper than ones by cognitive sciences and psychology of reasoning. Moreover, his work was from examples covering many professional and mathematical fields, such as analysis and combinatorial mathematics, and shown that the highest creativity lies in the study in modern mathematics.

It is important to note that the study was on the Hanoi Tower task, whereas Polya's theory is for a broader range of problem-solving processes. Therefore, the findings of the paper do not fully validate Polya's theory, but they provide important empirical support for Polya's theory.

To enhance the study's impact and scope, future research should adopt a comprehensive roadmap. and holistic approach: including but not limited in analyzing demographic factors—age, gender, and cultural background—to explore their influence on problem-solving strategies and computational thinking effectiveness. Additionally, interdisciplinary applications in fields like mathematics, artificial intelligence, and engineering should be highlighted surprised relevance. Grasping computational thinking will be great helpful in improving educational practices and real-world problem solving.

5 Conclusions

This study provides valuable insights into the role of problem representation in complex problem-solving, particularly within the context of computational thinking. The findings demonstrate a clear link between stronger problem representation abilities and superior performance in solving the Tower of Hanoi task. This relationship becomes increasingly pronounced as task complexity increases, highlighting the crucial role of problem representation in addressing complex challenges.

The study's results contribute to existing literature by specifically focusing on the problem representation skills of college students and exploring the impact of practice on these skills. The findings indicate that novices can significantly improve their problem-solving abilities through practice, effectively narrowing the gap between novices and experts. This discovery has important implications for educational practices and vocational training, emphasizing the importance of incorporating problem representation skill development into curricula and professional development programs.

While the study's focus on the Tower of Hanoi provides a specific context for exploring problem representation, its findings have broader implications for understanding complex problem-solving in various domains. The study suggests that developing strong problem representation skills through practice and structured learning is essential for effectively addressing complex challenges in diverse areas, including academia, industry, and everyday life.

Future research could explore the transferability of problem representation skills across different problem domains and investigate the effectiveness of specific training interventions for improving problem representation abilities. Additionally, investigating the neural mechanisms underlying problem representation and its relationship with other cognitive processes, such as working memory and reasoning, would provide further insights into the complex nature of complex problem-solving.

Overall, this study provides a compelling case for the importance of problem representation in complex problem-solving and offers valuable insights for educators, researchers, and practitioners seeking to enhance problem-solving abilities.

Acknowledgements
We would like to express our gratitude to Sudhir Sharma for permission to use the picture.

Author contributions
JF conceptualized this study and completed a small pilot study. SD performed the experiments and analyzed the data. DF designed the experiments, made supervision, and wrote the paper. All authors reviewed and approved the final manuscript.

Funding
This work was not supported by any fund.

Data availability
The data that support the findings of this study are available on request from the corresponding author.

Declarations

Competing interests
The authors declare that they have no competing interests.

Author details
[1]Hubei University affiliated High School, Wuhan, China. [2]Speech Analysis and Artificial Intelligence Lab, Wuhan University, Wuhan, China.

Received: 27 September 2024 Accepted: 30 December 2024 Published online: 16 January 2025

References

1. Celik, I.: Exploring the determinants of artificial intelligence (AI) literacy: digital divide, computational thinking, cognitive absorption. Telemat. Inform. **83**, 10206, 1–11 (2023)
2. Fleischer, P., Hélie, S., Pizlo, Z.: The role of problem representation in producing near-optimal TSP tours. J. Probl. Solving (2018). https://doi.org/10.7771/1932-6246.1212
3. Hélie, S., Pizlo, Z.: When is psychology research useful in artificial intelligence? A case for reducing computational complexity in problem solving. Top. Cogn. Sci. (2021). https://doi.org/10.1111/tops.12572
4. Kipman, U., et al.: Personality traits and complex problem solving: personality disorders and their effects on complex problem-solving ability. Front. Psychol. **13**, 788402 (2022)
5. Patrick, J., Ahmed, A.: Facilitating representation change in insight problems through training. J. Exp. Psychol. Learn. Mem. Cogn. **40**(2), 532–543 (2014)
6. Cartwright, K.: Interpreting young children's multiplicative strategies through their drawn representations. Math. Educ. Res. J. **36**(2), 367–397 (2024)
7. Tareq, Z.-A., Raja Yusof, R.J.: Modeling a problem-solving approach through computational thinking for teaching programming. IEEE Trans. Ed. **67**(2), 282–291 (2024)
8. Lee, B., Lee, Y.: A study examining the effects of a training program focused on problem-solving skills for young adults. Think. Skills Creat. **37**, 100692 (2020)
9. Salmon-Mordekovich, N.: Success starts with the first step: insight problem-solving stages and their underlying cognitive strategies. J. Cogn. Psychol. **36**(4), 523–534 (2024)
10. Molnár, G., Greiff, S.: Understanding transitions in complex problem-solving: why we succeed and where we fail. Think. Skills Creat. **50**, 101408 (2023)
11. Nicolay, B., et al.: Unsuccessful and successful complex problem solvers—a log file analysis of complex problem-solving strategies across multiple tasks. Intelligence **101**, 101793 (2023)

12. Chua, B.L., Tan, O.S., Liu, W.C.: Journey into the problem-solving process: cognitive functions in a PBL environment. Innov. Educ. Teach. Int. **53**(2), 191–202 (2016)
13. Urhahne, D.: Motivation to learn and problem solving. Educ. Psychol. **41**(9), 1079–1081 (2021)
14. Lin, W.-L., Tsai, P.-H., Lin, H.-Y., Chen, H.-C.: How does emotion influence different creative performances? The mediating role of cognitive flexibility. Cogn. Emot. **28**(5), 834–844 (2014)
15. Güss, C.D., Badibanga, A.: Comparing business experts and novices in complex problem solving. J. Intell. **5**(2), 1–18 (2017)
16. Bocanegra, B.R., Poletiek, F.H., Ftitache, B., Clark, A.: Intelligent problem-solvers externalize cognitive operations. Nat. Hum. Behav. **3**(2), 136–142 (2019)
17. Fansher, M., Shah, P., Hélie, S.: The effect of mode of presentation on Tower of Hanoi problem solving. Cognition **224**, 105041 (2022)
18. Lu, X., Kaiser, G.: Creativity in students' modeling competencies: conceptualization and measurement. Educ. Stud. Math. **109**(2), 287–311 (2022)
19. Marengo, L.: Representation, search, and the evolution of routines in problem solving. Ind. Corp. Change **24**(5), 951–980 (2015)
20. Williams, D.J., Noyes, J.M.: Effect of experience and mode of presentation on problem solving. Comput. Hum. Behav. **23**(1), 258–274 (2007)
21. Daniels, J.S., Moreau, D., Macnamara, B.N.: Learning and transfer in problem-solving progressions. J. Intell. **10**(4), 85 (2022)
22. Dörner, D., Funke, J.: Complex problem solving: what it is and what it is not. Front. Psychol. **8**, 1153 (2017)
23. Patsenko, E.G., Altmann, E.M.: How planful is routine behavior? A selective-attention model of performance in the Tower of Hanoi. J. Exp. Psychol. Gen. **139**(1), 95–116 (2010)
24. Marengo, L.: Representation, search, and the evolution of routines in problem solving. Ind. Corp. Change **24**(5), 951–980 (2014)
25. Vakil, E., Heled, E.: The effect of constant versus varied training on transfer in a cognitive skill learning task: the case of the Tower of Hanoi puzzle. Learn. Individ. Differ. **47**, 207–214 (2016)
26. Wiley, J., Danek, A.H.: Restructuring processes and Aha! experiences in insight problem solving. Nat. Rev. Psychol. **3**(1), 42–55 (2024)
27. Polya, G.: Mathematics and Plausible Reasoning. Princeton University Press, Princeton (1954)

Publisher's Note