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Stability estimates for a class of neutral type systems with distributed time-varying delay components

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Abstract

This paper investigates new stability estimates for a class of neutral type systems with distributed time-varying delay components. Initially, an appropriate Lyapunov–Krasovskii functional (LKF) is constructed to generate some solution estimates for the considered system. Then, using the estimates obtained in the employed method framework, it is determined whether the solutions are stable or not. The stabilization rates of the solutions at infinity are discussed in the context of asymptotic and exponential stability (ES). Finally, two numerical examples with simulations are presented to demonstrate the applicability of the proposed method on the current results.

Mathematics Subject Classification: 34K20; 34K40

Keywords: Time-varying delay components; Neutral type systems; LKF; Estimates for solutions; Stability

1 Introduction

Delay is a ubiquitous phenomenon we constantly encounter in our daily lives. In all physical systems, there is a momentary time delay, however short, between the occurrence of a stimulus and the corresponding response. Time delays are a general occurrence that frequently arises in practical application areas such as industrial production, grid transport, circuitry, and signaling. Since time delays can be a source of instability and poor performance, and are commonly found in various engineering, biological, and economic systems, they have been extensively investigated in recent years in the context of delayed differential equations or systems. The use of delayed differential equation systems and their qualitative behaviors are very actively studied in the field of dynamical systems. In recent years, there have been many intriguing findings about the qualitative properties of solutions, including stability, instability, exponential stability (ES), and asymptotic stability, among others. For a comprehensive overview of the qualitative properties of neutral type systems, we refer readers to the work and references in [1–38]. It is important to note that the presence of time-varying delays can significantly affect the performance of a system. Therefore, the stability analysis of time-varying delayed systems has become a major research topic in the last few decades [37, 38].

When examining the relevant literature, the commonly used method for analyzing the stability of time-delayed differential systems is Lyapunov stability theory. This theory is primarily considered from two key aspects. The first is to choose a suitable LKF, the second is to reduce the expansion when estimating its derivative. LKFs allow us to examine the qualitative behavior of the differential equations under consideration without explicitly determining their roots. However, using these functionals, it is not always enable us to obtain some estimates of the decay rate of solutions at infinity. For this purpose, different modifications of LKFs have been proposed, tailored to the specific differential equation system being studied. These modifications include discretized LKFs [18], augmented LKFs [21], delayed partitioning LKFs [35], etc. Moreover, in the case of constant [31] and periodic coefficients [7] in the linear part, a modified LKF was suggested and used in [11, 12] to obtain the estimates of exponential decay at the infinity of the solutions to systems of linear and quasi linear time-delay differential equations. Additionally, some approaches incorporate binary and triple integral terms in the LKF to further reduce conservatism [32]. Therefore, it is crucial to select the appropriate functional according to the form of the system in order to obtain a less conservative stability criterion [26].

Motivated by the ideas discussed above, in this research we consider neutral systems, which represent a more general class than delayed-type systems. From this perspective, the main goal and contribution of this work can be summarized as follows.

Recently, many approaches have been proposed to analyze the stability, asymptotic stability, ES and exponential decay of solutions of neutral delayed differential systems with constant, periodic and variable coefficients. For example, [7, 13, 14, 28, 31, 33] presented some new estimates characterizing the exponential decay rate at infinity of solutions of neutral type linear differential equation systems by using the LKFs. In [9, 12, 15, 16, 27, 29], improved results on exponential decay and ES of solutions of nonlinear neutral type systems based on LKF are obtained. In [30], some estimates were obtained for solutions to nonlinear systems in a class of non-autonomous systems with time-varying concentrated and distributed delays. In [3, 4], criteria for stability of fractional-order uncertain neutral systems with time-varying delay were constructed. In [36], stability analysis of time-varying neutral-type stochastic systems with both discrete and distributed delays was examined. Thus, this research on the stability of neutral systems is still under exploitation and development. This motivates the existing research. For further refinement to the work discussed above, we consider a new nonlinear neutral system with time-varying delays. We used the Lyapunov method, which is a convenient method in terms of the neutral system we considered in the study. To reduce the conservatism, we have produced some solution estimates for the system in question by constructing an appropriate LKF and using some fundamental inequalities. Then, two examples are given with numerical simulations to show the effectiveness of the proposed method on the theoretical results obtained. In this study, when we look at the similar studies in the literature in terms of both the neutral differential system we discussed and the method applied, it is clear that the qualitative behaviors of periodic or constant coefficient systems are generally examined [9, 12–16, 27–29, 31, 33]. However, this study examined a class of nonlinear neutral type variable coefficient systems. Compared to the existing results concerning linear neutral systems [6, 11, 28] and nonlinear neutral systems [7, 8, 29, 30] with variable delays, the results of this paper are more general and less conservative. In [6], the author investigates asymptotic stability estimates of linear neutral-type systems with time-varying delay components and variable matrix coefficients. In [11], the authors study the systems of quasilinear delay differential equations with periodic coefficients of linear terms. In [28], the author investigates the exponential stability of solutions of linear periodic systems of neutral type with time-varying delays. However, this study investigates new stability estimates for a class of nonlinear neutral-type systems with variable matrix-coefficient distributed time-varying delay components.

In [7, 8] and [29] the authors obtained some estimates on the exponential stability of solutions of nonlinear neutral-type systems with periodic coefficients. In [30], the author has obtained some estimates for solutions to a class of nonautonomous systems of neutral type with unbounded delay. The system considered in this study has variable matrix coefficients, not periodic coefficients, and is more general and comprehensive than the systems studied in [7, 8, 29] and [30]. Also, numerical examples with simulations are presented to show the applicability of the obtained theoretical results, unlike the above-mentioned studies, for the system considered in this study.

In this sense, we think that this study, whose theoretical results are exemplified by numerical simulations, may be useful to researchers working on the qualitative behavior of solutions of nonlinear neutral type systems.

The rest of this research is organized as follows: In Sect. 2, we introduce the problem statement and the necessary notations. The main results and their proofs are given in Sect. 3. To show the effectiveness of the proposed method on the current results, two numerical examples are shown in Sect. 4, and the research is finalized with Sect. 5.

2 Problem statement and preliminary

Consider a class of nonlinear neutral type systems with distributed time-varying delay components:

$$\begin{aligned}\frac{d}{dt}u(t) &= A(t)u(t) + B(t)u(t-\xi(t)) + C(t)\frac{d}{dt}u(t-\xi(t)) + D(t)\int_{t-\xi(t)}^{t} u(s)ds \\ &\quad + F\Big(t, u(t), u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t)), \int_{t-\xi(t)}^{t} u(s)ds\Big), \quad t \in [0,\infty),\end{aligned} \tag{2.1}$$

where $A(t)$, $B(t)$, $C(t)$ and $D(t)$ are $n \times n$ matrices with continuous real-valued entries, that is,

$$a_{ij}(t),\ b_{ij}(t),\ c_{ij}(t),\ d_{ij}(t) \in C(\bar{R}_+), \quad i,j = 1,2,\dots,n,$$

and $\xi(t) \in C^1(0,\infty)$,

$$0 < \xi_1 \le \xi(t) \le \xi_2 < \infty, \tag{2.2}$$

and

$$\xi_3 \le \xi'(t) \le \xi_4 < 1. \tag{2.3}$$

In addition, the continuous real value $F(t,u,v,w,\omega)$ function satisfies the Lipschitz condition respect to u on every compact set $G \subset [0,\infty) \times R^n \times R^n \times R^n \times R^n$ such that

$$\begin{aligned} &\|F(t,u,v,w,\omega)\| \leq q_1 \|u\| + q_2 \|v\| + q_3 \|w\| + q_4 \|\omega\|, \\ &t \in [0,\infty), \quad u,\ v,\ w,\ \omega \in R^n, \end{aligned} \tag{2.4}$$

for some constants $q_j \geq 0$, $(j = 1,\dots,4)$.

For the system (2.1), we deal with the following initial value problem

$$\begin{aligned} &\frac{d}{dt}u(t) = A(t)u(t) + B(t)u(t-\xi(t)) + C(t)\frac{d}{dt}u(t-\xi(t)) + D(t)\int_{t-\xi(t)}^{t} u(s)ds \\ &\qquad + F(t,u(t),u(t-\xi(t)),\frac{d}{dt}u(t-\xi(t)),\int_{t-\xi(t)}^{t} u(s)ds), \quad t \in [0,\infty), \\ &u(t) = \vartheta(t) \quad , \quad t \in [-\xi_2, 0], \\ &u(0^+) = \vartheta(0), \end{aligned} \tag{2.5}$$

where $\vartheta(t) \in C^1([-\xi_2,0])$ is the initial datum.

To state results, we introduce some notations.

There exist matrices $K(t,s)$ and $L(t,s)$ in $C^1(\bar{R}_+^2)$, such that

$$K(t,s) = K^*(t,s) \geq 0, \quad \frac{\partial}{\partial s}K(t,s) \leq 0, \ \ (t,s) \in \bar{R}_+^2, \tag{2.6}$$

and

$$L(t,s) = L^*(t,s) \geq 0, \quad \frac{\partial}{\partial s}L(t,s) \leq 0, \ \ (t,s) \in \bar{R}_+^2. \tag{2.7}$$

Hereafter we use the following dot product and vector norm, respectively:

$$\langle x,y \rangle = \sum_{j=1}^{n} x_j \bar{y}_j, \quad \|x\| = \sqrt{\langle x,x \rangle}.$$

We now state the definitions of the stability.

Definition 2.1 (([31])) The zero solution of initial value problem (2.5) is Lyapunov stable if for any $\varepsilon > 0$ there exists $\delta > 0$ such that $\max_{t\in[-\xi_2,0]} \|\vartheta(t)\| < \delta$ implies $\|u(t)\| < \varepsilon$ for all $t > 0$.

Definition 2.2 (([31])) The zero solution of initial value problem (2.5) is asymptotically stable if the zero solution of initial value problem (2.5) is Lyapunov stable, and also there exists $\delta_0 > 0$ such that $\max_{t\in[-\xi_2,0]} \|\vartheta(t)\| < \delta_0$ implies $\|u(t)\| \to 0$ as $t \to \infty$.

Definition 2.3 (([25])) The zero solution of initial value problem (2.5) is ES if there exist $K > 0$ and $\lambda > 0$ such that $\|u(t)\| \leq Ke^{-\lambda t} \max_{t\in[-\xi_2,0]} \|\vartheta(t)\|$ for all $t > 0$.

Remark 2.1 A zero solution would mean that $u = 0$ satisfies the equation for all t in the domain.

3 Main results

We first present a result for the ES of the trivial solution of system (2.1) with $F(t,u,v,w,\omega)\equiv 0$ as follows:

$$\frac{d}{dt}u(t)=A(t)u(t)+B(t)u(t-\xi(t))+C(t)\frac{d}{dt}u(t-\xi(t))+D(t)\int_{t-\xi(t)}^{t}u(s)ds,\quad t\in(0,\infty). \tag{3.1}$$

Theorem 3.1 *Suppose that there exist matrices $K(t,s)$ and $L(t,s)$ satisfying (2.6) and (2.7) in $C^1(\bar{R}_+^2)$, and $H(t)\in C^1(\bar{R}_+)$,*

$$H(t)=H^*(t)>0,\quad t\in[0,\infty), \tag{3.2}$$

such that the minimal eigenvalue $h(t)$ of $H(t)$ satisfies the inequality

$$h(t)\geq h_0>0, \tag{3.3}$$

also the matrix

$$\Pi(t,s)=\begin{pmatrix}\Pi_{11}(t,s) & \Pi_{12}(t,s) & \Pi_{13}(t,s) & \Pi_{14}(t,s)\\ \Pi_{12}^*(t,s) & \Pi_{22}(t,s) & \Pi_{23}(t,s) & \Pi_{24}(t,s)\\ \Pi_{13}^*(t,s) & \Pi_{23}^*(t,s) & \Pi_{33}(t,s) & \Pi_{34}(t,s)\\ \Pi_{14}^*(t,s) & \Pi_{24}^*(t,s) & \Pi_{34}^*(t,s) & \Pi_{44}(t,s)\end{pmatrix}, \tag{3.4}$$

with entries

$$\Pi_{11}(t,s)=-\frac{d}{dt}H(t)-H(t)A(t)-A^*(t)H(t)-A^*(t)L(t,0)A(t)-K(t,0),$$

$$\Pi_{12}(t,s)=-H(t)B(t)-B^*(t)L(t,0)A(t),\Pi_{13}(t,s)=-H(t)C(t)-C^*(t)L(t,0)A(t),$$

$$\Pi_{14}(t,s)=-H(t)D(t)-D^*(t)L(t,0)A(t),\Pi_{22}(t,s)=-B^*(t)L(t,0)B(t)+(1-\xi_4)K(t,\xi_2),$$

$$\Pi_{23}(t,s)=-C^*(t)L(t,0)B(t),\Pi_{24}(t,s)=-D^*(t)L(t,0)B(t),$$

$$\Pi_{33}(t,s)=-C^*(t)L(t,0)C(t)+(1-\xi_3)^{-1}L(t,\xi_2),\Pi_{34}(t,s)=-D^*L(t,0)C(t),$$

$$\Pi_{44}(t,s)=-\xi^2(t)D^*(t)L(t,0)D(t)-\xi(t)\left(\frac{\partial}{\partial t}K(t,s)+\frac{\partial}{\partial s}K(t,s)\right),$$

is positive definite. Then the zero solution to (3.1) is ES.

Assuming the above requirements are satisfied, we establish some requirements for the estimate of solutions to the nonlinear system (2.1). Using this matrix function $H(t)$ together with the matrix functions $K(t,s)$ and $L(t,s)$,we introduce some notations below to state our results:

$$V(0,\vartheta)=\langle H(0)\vartheta(0),\vartheta(0)\rangle+\int_{-\xi(0)}^{0}\langle K(0,-s)\vartheta(s),\vartheta(s)\rangle\,ds+\int_{-\xi(0)}^{0}\left\langle L(0,-s)\frac{d}{ds}\vartheta(s),\frac{d}{ds}\vartheta(s)\right\rangle ds, \tag{3.5}$$

$$\begin{aligned}
\beta_1(t) &= 2\|H(t)\| + (2\|A(t)\| + q_1)\|L(t,0)\|, \\
\beta_2(t) &= (2\|B(t)\| + q_2)\|L(t,0)\|, \\
\beta_3(t) &= (2\|C(t)\| + q_3)\|L(t,0)\|, \\
\beta_4(t) &= (2\|D(t)\| + q_4)\|L(t,0)\|,
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
\alpha_1(t) &= q_1\beta_1(t) + \frac{q_1\beta_2(t) + q_2\beta_1(t) + q_1\beta_3(t) + q_3\beta_1(t) + q_1\beta_4(t) + q_4\beta_1(t)}{2}, \\
\alpha_2(t) &= q_2\beta_2(t) + \frac{q_1\beta_2(t) + q_2\beta_1(t) + q_2\beta_3(t) + q_3\beta_2(t) + q_2\beta_4(t) + q_4\beta_2(t)}{2}, \\
\alpha_3(t) &= q_3\beta_3(t) + \frac{q_1\beta_3(t) + q_3\beta_1(t) + q_2\beta_3(t) + q_3\beta_2(t) + q_3\beta_4(t) + q_4\beta_3(t)}{2}, \\
\alpha_4(t) &= q_4\beta_4(t) + \frac{q_1\beta_4(t) + q_4\beta_1(t) + q_2\beta_4(t) + q_4\beta_2(t) + q_3\beta_4(t) + q_4\beta_3(t)}{2},
\end{aligned} \tag{3.7}$$

and the matrix

$$\Pi^\alpha(t,s) = \Pi(t,s) - \begin{pmatrix} \alpha_1(t)I & 0 & 0 & 0 \\ 0 & \alpha_2(t)I & 0 & 0 \\ 0 & 0 & \alpha_3(t)I & 0 \\ 0 & 0 & 0 & \alpha_4(t)\xi(t)I \end{pmatrix}, \tag{3.8}$$

where I is the unit matrix.

Theorem 3.2 *Let the requirements of Theorem 3.1 be satisfied. The zero solution to system (2.1) is ES, if there are parameters $q_j, (j = 1, \dots, 4)$ and positive definite matrix $\Pi^\alpha(t,s)$, for all $t \in (0,\infty)$.*

Theorem 3.3 *Let the requirements of Theorem 3.2 be satisfied. Suppose that there exist matrices $K(t,s)$ and $L(t,s)$ satisfying (2.6) and (2.7) in $C^1(\bar{R}_+^2)$, and $H(t) \in C^1(\bar{R}_+)$ such that*

$$\left\langle \Pi^\alpha(t,s)\begin{pmatrix} u \\ v \\ w \\ \omega \end{pmatrix}, \begin{pmatrix} u \\ v \\ w \\ \omega \end{pmatrix} \right\rangle \geq p(t)\langle H(t)u, u\rangle + k(t)\xi(t)\langle K(t,s)\omega, \omega\rangle, \tag{3.9}$$
$$u, v, w, \omega \in R^n, (t,s) \in \bar{R}_+^2,$$

where $p(t), k(t) \in C(\bar{R}_+)$. If

$$\frac{d}{dt}L(t,s) + lL(t,s) \leq 0, \quad (t,s) \in \bar{R}_+^2, \tag{3.10}$$

for some $l > 0$, then the following estimate holds for the zero solution of $u(t)$ to (2.5),

$$\|u(t)\| \leq \sqrt{\frac{V(0,\vartheta)}{h(t)}} \exp\left(-\frac{1}{2}\int_0^t \lambda(\delta)d\delta\right), \qquad t \in (0,\infty), \tag{3.11}$$

where $V(0,\vartheta)$ is described in (3.5) and

$$\lambda(t) = \min\{p(t),\ k(t),\ l\}. \tag{3.12}$$

Remark 3.1 Inequality (3.11) allows us to generate estimate for solutions to the initial value problem (2.5) that characterize the exponential decay rate at infinity. The decay rate in (3.11) depends on $\lambda(t)$. Obviously, the statement of Theorem 3.1 and Theorem 3.2 follows immediately from (3.11) and so we need only to prove Theorem 3.3.

As is well known, there exists a unique solution to the initial value problem (2.5). The well-posedness of the above initial value problem (2.5) has been discussed in [20, pp. 36-56] and [24, pp. 1-43].

Proof of Theorem 3.3 Let $u(t)$ be a solution of system (2.5). Using the matrix functions $H(t)$, $K(t,s)$ and $L(t,s)$, consider the following LKF:

$$V(t,u) = \langle H(t)u(t), u(t)\rangle + \int_{t-\xi(t)}^{t} \langle K(t,t-s)u(s), u(s)\rangle \, ds + \int_{t-\xi(t)}^{t} \left\langle L(t,t-s)\frac{d}{ds}u(s), \frac{d}{ds}u(s)\right\rangle ds. \tag{3.13}$$

It is obvious that the functional $V(t,u)$ is positive. Differentiating of this functional along solutions of system (2.5) with respect to t, we find

$$\begin{aligned}
\frac{d}{dt}V(t,u) &= \left\langle \frac{d}{dt}H(t)u(t), u(t)\right\rangle + \left\langle H(t)\frac{d}{dt}u(t), u(t)\right\rangle + \left\langle H(t)u(t), \frac{d}{dt}u(t)\right\rangle \\
&+ \langle K(t,0)u(t), u(t)\rangle - (1-\xi'(t))\,\langle K(t,\xi(t))u(t-\xi(t)), u(t-\xi(t))\rangle \\
&+ \int_{t-\xi(t)}^{t} \left\langle \frac{d}{dt}K(t,t-s)u(s), u(s)\right\rangle ds + \left\langle L(t,0)\frac{d}{dt}u(t), \frac{d}{dt}u(t)\right\rangle \\
&- (1-\xi'(t))^{-1}\left\langle L(t,\xi(t))\frac{d}{dt}u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t))\right\rangle \\
&+ \int_{t-\xi(t)}^{t} \left\langle \frac{d}{dt}L(t,t-s)\frac{d}{ds}u(s), \frac{d}{ds}u(s)\right\rangle ds.
\end{aligned}$$

By (2.2)–(2.3) and using the conditions (2.6)–(2.7), we easily obtain the following expressions

$$\begin{aligned}
&(1-\xi'(t))\,\langle K(t,\xi(t))u(t-\xi(t)), u(t-\xi(t))\rangle \\
&\geq (1-\xi_4)\,\langle K(t,\xi_2)u(t-\xi(t)), u(t-\xi(t))\rangle,
\end{aligned} \tag{3.14}$$

and

$$\begin{aligned}
&(1-\xi'(t))^{-1}\left\langle L(t,\xi(t))\frac{d}{dt}u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t))\right\rangle \\
&\geq (1-\xi_3)^{-1}\left\langle L(t,\xi_2)\frac{d}{dt}u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t))\right\rangle.
\end{aligned} \tag{3.15}$$

We introduce the notation as

$$z(t) = F(t, u(t), u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t)), \int_{t-\xi(t)}^{t} K(t,t-s)u(s)ds).$$

By (3.14) and (3.15), considering $u(t)$ satisfying the system (2.5), we get

$$
\begin{aligned}
\frac{d}{dt}V(t,u) = {} & \left\langle \left[\frac{d}{dt}H(t) + H(t)A(t) + A^{*}(t)H(t) + K(t,0) + A(t)L(t,0)A^{*}(t) \right] u(t), u(t) \right\rangle \\
& + \langle H(t)B(t)u(t-\xi(t)), u(t)\rangle + \langle B^{*}(t)H(t)u(t), u(t-\xi(t))\rangle \\
& + \langle B^{*}(t)L(t,0)A(t)u(t), u(t-\xi(t))\rangle + \langle A^{*}(t)L(t,0)B(t)u(t-\xi(t)), u(t)\rangle \\
& + \left\langle H(t)C(t)\frac{d}{dt}u(t-\xi(t)), u(t) \right\rangle + \left\langle C^{*}(t)H(t)u(t), \frac{d}{dt}u(t-\xi(t)) \right\rangle \\
& + \left\langle C^{*}(t)L(t,0)A(t)u(t), \frac{d}{dt}u(t-\xi(t)) \right\rangle \\
& + \left\langle A^{*}(t)L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), u(t) \right\rangle \\
& + \left\langle H(t)D(t)\int_{t-\xi(t)}^{t} u(s)ds, u(t) \right\rangle + \left\langle D^{*}(t)H(t)u(t), \int_{t-\xi(t)}^{t} u(s)ds \right\rangle \\
& + \left\langle A^{*}(t)L(t,0)D(t)\int_{t-\xi(t)}^{t} u(s)ds, u(t) \right\rangle \\
& + \left\langle D^{*}(t)L(t,0)A(t)u(t), \int_{t-\xi(t)}^{t} u(s)ds \right\rangle \\
& + \langle B^{*}(t)L(t,0)B(t)u(t-\xi(t)), u(t-\xi(t))\rangle \\
& - (1-\xi_4)\,\langle K(t,\xi_2)u(t-\xi(t)), u(t-\xi(t))\rangle \\
& + \left\langle B^{*}(t)L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), u(t-\xi(t)) \right\rangle \\
& + \left\langle C^{*}(t)L(t,0)B(t)u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t)) \right\rangle \\
& + \left\langle B^{*}(t)L(t,0)D(t)\int_{t-\xi(t)}^{t} u(s)ds, u(t-\xi(t)) \right\rangle \\
& + \left\langle D^{*}(t)L(t,0)B(t)u(t-\xi(t)), \int_{t-\xi(t)}^{t} u(s)ds \right\rangle \\
& + \left\langle C^{*}(t)L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t)) \right\rangle \\
& - (1-\xi_3)^{-1}\left\langle L(t,\xi_2)\frac{d}{dt}u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t)) \right\rangle \\
& + \left\langle D^{*}(t)L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), \int_{t-\xi(t)}^{t} u(s)ds \right\rangle \\
& + \left\langle C^{*}(t)L(t,0)D(t)\int_{t-\xi(t)}^{t} u(s)ds, \frac{d}{dt}u(t-\xi(t)) \right\rangle \\
& + \left\langle D(t)L(t,0)D^{*}(t)\int_{t-\xi(t)}^{t} u(s)ds, \int_{t-\xi(t)}^{t} u(s)ds \right\rangle \\
& + \int_{t-\xi(t)}^{t} \left\langle \frac{d}{dt}K(t,t-s)u(s), u(s) \right\rangle ds \\
& + \langle H(t)z(t), u(t)\rangle + \langle H(t)u(t), z(t)\rangle + \langle L(t,0)A(t)u(t), z(t)\rangle
\end{aligned}
$$

$$
\begin{aligned}
&+ \langle L(t,0)B(t)u(t-\xi(t)), z(t)\rangle + \left\langle L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), z(t)\right\rangle \\
&+ \left\langle L(t,0)D(t)\int_{t-\xi(t)}^{t} u(s)ds, z(t)\right\rangle + \langle A^*(t)L(t,0)z(t), u(t)\rangle \\
&+ \langle B^*(t)L(t,0)z(t), u(t-\xi(t))\rangle + \left\langle C^*(t)L(t,0)z(t), \frac{d}{dt}u(t-\xi(t))\right\rangle \\
&+ \left\langle D^*L(t,0)z(t), \int_{t-\xi(t)}^{t} u(s)ds\right\rangle + \langle L(t,0)z(t), z(t)\rangle \\
&+ \int_{t-\xi(t)}^{t} \left\langle \frac{d}{dt}L(t,t-s)\frac{d}{ds}u(s), \frac{d}{ds}u(s)\right\rangle ds.
\end{aligned}
$$

By the integration inequality described in [19, Lemma 1, pp. 2806], the inequality $\left\langle \int_{t-\xi(t)}^{t} u(s)ds, \int_{t-\xi(t)}^{t} u(s)ds\right\rangle \leq \xi(t)\int_{t-\xi(t)}^{t}\langle u(s), u(s)\rangle\, ds$ is easily achieved. Then, we can write the following expression

$$
\begin{aligned}
\frac{d}{dt}V(t,u) \leq &-\frac{1}{\xi(t)}\int_{t-\xi(t)}^{t} \left\langle \Pi(t,t-s)\begin{pmatrix} u(t) \\ u(t-\xi(t)) \\ \frac{d}{dt}u(t-\xi(t)) \\ u(s)\end{pmatrix}, \begin{pmatrix} u(t) \\ u(t-\xi(t)) \\ \frac{d}{dt}u(t-\xi(t)) \\ u(s)\end{pmatrix}\right\rangle ds \\
&+ \langle H(t)z(t), u(t)\rangle + \langle H(t)u(t), z(t)\rangle + \langle L(t,0)A(t)u(t), z(t)\rangle \\
&+ \langle L(t,0)B(t)u(t-\xi(t)), z(t)\rangle + \left\langle L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), z(t)\right\rangle \\
&+ \left\langle L(t,0)D(t)\int_{t-\xi(t)}^{t} u(s)ds, z(t)\right\rangle + \langle A^*(t)L(t,0)z(t), u(t)\rangle \\
&+ \langle B^*(t)L(t,0)z(t), u(t-\xi(t))\rangle + \left\langle C^*(t)L(t,0)z(t), \frac{d}{dt}u(t-\xi(t))\right\rangle \\
&+ \left\langle D^*L(t,0)z(t), \int_{t-\xi(t)}^{t} u(s)ds\right\rangle + \langle L(t,0)z(t), z(t)\rangle \\
&+ \int_{t-\xi(t)}^{t} \left\langle \frac{d}{dt}L(t,t-s)\frac{d}{ds}u(s), \frac{d}{ds}u(s)\right\rangle ds
\end{aligned} \tag{3.16}
$$

where the matrix $\Pi(t,s)$ described in (3.4).

The above inequality (3.16) can be summarized as follows:

$$
\begin{aligned}
\frac{d}{dt}V(t,u) \leq &-\frac{1}{\xi(t)}\int_{t-\xi(t)}^{t} \left\langle \Pi(t,t-s)\begin{pmatrix} u(t) \\ u(t-\xi(t)) \\ \frac{d}{dt}u(t-\xi(t)) \\ u(s)\end{pmatrix}, \begin{pmatrix} u(t) \\ u(t-\xi(t)) \\ \frac{d}{dt}u(t-\xi(t)) \\ u(s)\end{pmatrix}\right\rangle ds + W(t) \\
&+ \int_{t-\xi(t)}^{t} \left\langle \frac{d}{dt}L(t,t-s)\frac{d}{ds}u(s), \frac{d}{ds}u(s)\right\rangle ds.
\end{aligned} \tag{3.17}
$$

Obviously,

$$
\begin{aligned}
W(t) = &\langle H(t)z(t), u(t)\rangle + \langle H(t)u(t), z(t)\rangle + \langle L(t,0)A(t)u(t), z(t)\rangle \\
&+ \langle L(t,0)B(t)u(t-\xi(t)), z(t)\rangle + \left\langle L(t,0)C(t)\frac{d}{dt}u(t-\xi(t)), z(t)\right\rangle \\
&+ \left\langle L(t,0)D(t)\int_{t-\xi(t)}^{t} u(s)ds, z(t)\right\rangle + \langle A^*(t)L(t,0)z(t), u(t)\rangle
\end{aligned}
$$

$$
\begin{aligned}
&+\left\langle B^*(t)L(t,0)z(t),u(t-\xi(t))\right\rangle+\left\langle C^*(t)L(t,0)z(t),\frac{d}{dt}u(t-\xi(t))\right\rangle\\
&+\left\langle D^*L(t,0)z(t),\int_{t-\xi(t)}^{t}u(s)ds\right\rangle+\langle L(t,0)z(t),z(t)\rangle\\
\le{}&(2\,\|H(t)\|\,\|u(t)\|+2\,\|L(t,0)\|\,\|A(t)u(t)+B(t)u(t-\xi(t))+C(t)\frac{d}{dt}u(t-\xi(t))\\
&+D(t)\int_{t-\xi(t)}^{t}u(s)\,ds\|+\|L(t,0)\|\,\|z(t)\|)\,\|z(t)\|\,.
\end{aligned}
$$

Applying (2.4), we have

$$
\begin{aligned}
W(t)\le{}&(\beta_1(t)\,\|u(t)\|+\beta_2(t)\,\|u(t-\xi(t))\|\\
&+\beta_3(t)\left\|\frac{d}{dt}u(t-\xi(t))\right\|+\beta_4(t)\left\|\int_{t-\xi(t)}^{t}u(s)ds\right\|)\\
&\times(q_1\,\|u(t)\|+q_2\,\|u(t-\xi(t))\|+q_3\left\|\frac{d}{dt}u(t-\xi(t))\right\|+q_4\left\|\int_{t-\xi(t)}^{t}u(s)ds\right\|)
\end{aligned}
$$

where $\beta_j(t),(j=1,\dots,4)$ are described in (3.6). Clearly,

$$
\begin{aligned}
W(t)\le{}&\alpha_1(t)\,\|u(t)\|^2+\alpha_2(t)\,\|u(t-\xi(t))\|^2+\alpha_3(t)\left\|\frac{d}{dt}u(t-\xi(t))\right\|^2\\
&+\alpha_4(t)\left\|\int_{t-\xi(t)}^{t}u(s)ds\right\|^2,
\end{aligned}
\tag{3.18}
$$

where $\alpha_j(t)$, $(j=1,\dots,4)$ are described in (3.7). Thus, combining inequalities (3.17) and (3.18), we can derive the following estimation

$$
\begin{aligned}
\frac{d}{dt}V(t,u)\le{}&-\frac{1}{\xi(t)}\int_{t-\xi(t)}^{t}\left\langle\Pi^{\alpha}(t,t-s)\begin{pmatrix}u(t)\\u(t-\xi(t))\\\frac{d}{dt}u(t-\xi(t))\\u(s)\end{pmatrix},\begin{pmatrix}u(t)\\u(t-\xi(t))\\\frac{d}{dt}u(t-\xi(t))\\u(s)\end{pmatrix}\right\rangle ds\\
&+\int_{t-\xi(t)}^{t}\left\langle\frac{d}{dt}L(t,t-s)\frac{d}{ds}u(s),\frac{d}{ds}u(s)\right\rangle ds.
\end{aligned}
$$

where the matrix $\Pi^{\alpha}(t)$ is described in (3.8).

By using the requirement (3.9), we arrive at the inequality

$$
\begin{aligned}
\frac{d}{dt}V(t,u)\le{}&-p(t)\,\langle H(t)u(t),u(t)\rangle-k(t)\int_{t-\xi(t)}^{t}\langle K(t,t-s)u(s),u(s)\rangle\,ds\\
&-l\int_{t-\xi(t)}^{t}\left\langle L(t,t-s)\frac{d}{ds}u(s),\frac{d}{ds}u(s)\right\rangle ds.
\end{aligned}
$$

According to the LKF definition of (3.13), we find

$$
\frac{d}{dt}V(t,u)\le-\lambda(t)V(t,u),
$$

where $\lambda(t)$ is given in (3.12). This differential inequality gives the following estimate

$$V(t,u) \leq V(0,\vartheta)\exp\left(-\int_0^t \lambda(\delta)d\delta\right),$$

where $V(0,\vartheta)$ is described in (3.5). Obviously,

$$h(t)\|u(t)\|^2 \leq \langle H(t)u(t), u(t)\rangle \leq \|H(t)\|\,\|u(t)\|^2,$$

where $h(t)$ is given in (3.3). Then,

$$\|u(t)\|^2 \leq \frac{1}{h(t)}\langle H(t)u(t), u(t)\rangle \leq \frac{V(t,u)}{h(t)} \leq \frac{V(0,\vartheta)}{h(t)}\exp\left(-\int_0^t \lambda(\delta)d\delta\right),$$

whence (3.11) follows.

This finishes the proof. □

Repeating the steps in the proof of Theorem 3.3 for $F(t,u,v,w,\omega) = 0$,we reach easily to the statement of Theorem 3.1.

Remark 3.2 The theoretical results acquired above can be utilized to investigate the robust stability for (3.1). In fact, take into account uncertain systems of the form

$$\begin{aligned} \frac{d}{dt}u(t) &= A(t)u(t) + B(t)u(t-\xi(t)) + C(t)\frac{d}{dt}u(t-\xi(t)) + D(t)\int_{t-\xi(t)}^{t} u(s)ds \\ &\quad + \Delta A(t)u(t) + \Delta B(t)u(t-\xi(t)) \\ &\quad + \Delta C(t)\frac{d}{dt}u(t-\xi(t)) + \Delta D(t)\int_{t-\xi(t)}^{t} u(s)ds, \end{aligned} \tag{3.19}$$

where $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$ and $\Delta D(t)$ are unknown $n \times n$ matrices such that

$$\|\Delta A(t)\| \leq q_1, \quad \|\Delta B(t)\| \leq q_2, \quad \|\Delta C(t)\| \leq q_3, \quad \|\Delta D(t)\| \leq q_4.$$

Clearly, in this case the following vector-function

$$F(t,u,v,w,\omega) = \Delta A(t)u + \Delta B(t)v + \Delta C(t)w + \Delta D(t)\omega,$$

satisfies (2.4). Thus, Theorem 3.2 provides us the requirements of the robust ES for (3.1). By using Theorem 3.3, we obtain some estimates of the exponential decay of the solutions in (3.19).

Corollary 3.1 *Let the requirements of Theorem 3.3 be satisfied. The zero solution to (2.1) is stable, if*

$$\int_0^t \lambda(\delta)d\delta \geq 0,$$

for all $t \in [0,\infty)$*;moreover, for a solution* $u(t)$ *to* (2.5)*, the following estimate holds*

$$\|u(t)\| \le \sqrt{\frac{V(0,\vartheta)}{h_0}},$$

where h_0 *is described in* (3.3).

Corollary 3.2 *Let the requirements of Theorem* 3.3 *be satisfied. Then the zero solution to* (2.1) *is asymptotically stable, if*

$$\int_{t_0}^{t} \lambda(\delta)d\delta \to \infty, \quad t \to \infty,$$

for some $t_0 \in [0,\infty)$*; moreover, the stabilization rate is determined by the following function*

$$\exp\left(-\frac{1}{2}\int_{t_0}^{t} \lambda(\phi)d\phi\right).$$

Corollary 3.3 *Let the requirements of Theorem* 3.3 *be satisfied. The zero solution to* (2.1) *is ES, if*

$$\int_{0}^{t} \lambda(\delta)d\delta \ge \delta_1 t + \delta_2, \quad \delta_1 \in (0,\infty),$$

for all $t \in (0,\infty)$*;moreover, for a solution* $u(t)$ *to* (2.5)*, the following estimate holds*

$$\|u(t)\| \le \sqrt{\frac{V(0,\vartheta)}{h_0}} \exp\left(-\frac{\delta_1 t}{2} - \frac{\delta_2}{2}\right).$$

4 Numerical examples

This section provides two numerical examples to verify the effectiveness of the proposed method for the considered system on current results. Numerical results and simulations in the examples are easily obtained using Matlab-Simulink.

Example 4.1 As a special case of (2.1), for $F(t,u,v,w,\omega) = 0$ and $F(t,u,v,w,\omega) \neq 0$, $(u,v,w,\omega \in R)$,we consider the below neutral type systems with distributed time-varying delay components:

$$\begin{aligned} \frac{d}{dt}u(t) &= -2.4u(t) + 0.03u(t-\xi(t)) + 0.02\frac{d}{dt}u(t-\xi(t)) + 0.08\int_{t-\xi(t)}^{t} u(s)ds \\ &\quad + F(t,u(t),u(t-\xi(t)),\frac{d}{dt}u(t-\xi(t)),\int_{t-\xi(t)}^{t} u(s)ds), \end{aligned} \tag{4.1}$$

where

$$\xi_1 = 0.5 \le \xi(t) = 0.5\sin t + 1 \le 1.5 = \xi_2,$$

for all $t \in [0,\infty)$.

We start first by thinking about the linear case $F(t,u,v,w,\omega) = 0$, i.e. $q_j = 0$, $(j = 1,\dots,4)$. Choose the function $H(t)$ together with functions $K(t,s)$ and $L(t,s)$ as follows

$$H(t) = 2.3 - 0.4\sin t, \quad K(t,s) = 0.24e^{-0.06t}, \quad L(t,s) = 0.1e^{-0.2t-0.4s}.$$

Obviously, these functions satisfy the requirements in (2.2)–(2.3) and (3.2)–(3.3) with $l = 0.6$. Thus, $H(t)$ and its minimal eigenvalue $h(t)$ can be easily calculated as

$$h(t) \geq 1.9 = h_0 > 0 \quad , \quad \|H(t)\| \leq 2.7.$$

Then, it is simple to verify that the matrix $\Pi(t,s) > 0$ for the earlier specific choices. Thus, considering the assumptions of Theorem 3.1, we can say that the zero solution of (4.1) with $q_j = 0$, $(j = 1,\dots,4)$ is ES.

Let $p(t) = 0.72$ and $k(t) = 0.8$. Since $q_j = 0$, $(j = 1,\dots,4)$, it is clear that $\Pi(t,s) = \Pi^\alpha(t,s)$. $\lambda(t) = \min\{p(t),k(t),l\} = 0.6$. By (3.11), we establish the following estimate

$$\|u(t)\| \leq \gamma e^{-0.3t}, \quad \gamma > 0,$$

for the solutions to (4.1) with $q_j = 0$, $(j = 1,\dots,4)$.

Let us now examine the case of $F(t,u,v,w,\omega) \neq 0$, for the system (4.1). Considering the requirements in (2.4) and (3.2)–(3.3), we choose the functions $H(t)$, $K(t,s)$, $L(t,s)$ and constants $q_j \geq 0$, $(j = 1,\dots,4)$ as follows

$$H(t) = 2.3 - 0.4\sin t, \quad K(t,s) = 0.22e^{-0.4t-0.01s}, \quad L(t,s) = 0.2e^{-0.1t-0.4s},$$

and

$$q_1 = 0.002, \quad q_2 = q_4 = 0.001, \quad q_3 = 0.004. \tag{4.2}$$

Obviously, these functions satisfy the requirements in (2.2)–(2.4) and (3.2)–(3.3) with $l = 0.5$. Thus, $H(t)$ and its minimal eigenvalue $h(t)$ can be easily calculated as

$$h(t) \geq 1.9 = h_0 > 0 \quad , \quad \|H(t)\| \leq 2.7.$$

In this case, it is $\Pi(t,s) > 0$ for all $t \in [0,\infty)$. Then, by Theorem 3.1, the zero solution to (4.1) with $q_j \geq 0$, $(j = 1,\dots,4)$ is ES.

For $t \in [0,\infty)$,it is not hard to indicate that $\Pi^\alpha(t,s)$ described in (3.8) is positive definite. Let $p(t) = 0.62$ and $k(t) = 0.56$. In this case, $\lambda(t) = \min\{p(t),k(t),l\} = 0.5$.

By (3.11), we establish the following estimate

$$\|u(t)\| \leq \gamma e^{-0.25t} \quad , \quad \gamma > 0,$$

for the solutions to (4.1) with (4.2).

The following Fig. 1 and Fig. 2 show the behavior of the trajectories of the solutions of the considered system:

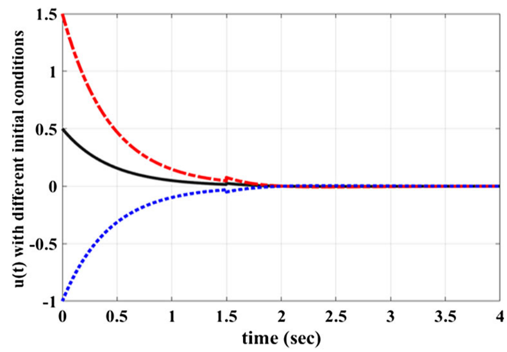

Figure 1 Trajectories of system (4.1) when $F(t,u,v,w,\omega) = 0$, for $\xi(t) = 0.5\sin t + 1$

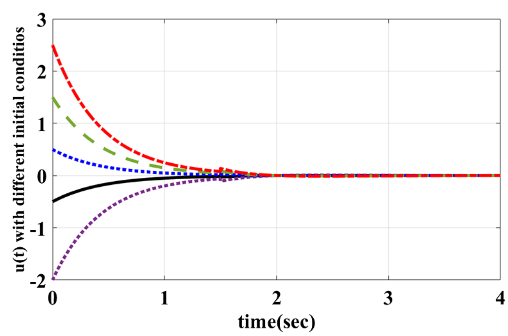

Figure 2 Trajectories of system (4.1) when $F(t,u,v,w,\omega) \neq 0$, for $\xi(t) = 0.5\sin t + 1$

Example 4.2 As a special case of (2.1), for $F(t,u,v,w,\omega) = 0$ and $F(t,u,v,w,\omega) \neq 0$, $(u,v,w,\omega \in R^2)$,we consider the below neutral type systems with distributed time-varying delay components:

$$\begin{aligned}\frac{d}{dt}u(t) &= \begin{pmatrix} -5.2 & 1-0.4\cos t \\ 1.2 & -4.6 \end{pmatrix} u(t) + \begin{pmatrix} 0.4\sin t & 0 \\ 0.02\sin t & 0.1\cos t \end{pmatrix} u(t-\xi(t)) \\ &\quad + \begin{pmatrix} 0.01 & 0.04 \\ 0.01 & 0.02 \end{pmatrix} \frac{d}{dt}u(t-\xi(t)) + \begin{pmatrix} 0.03 & 0 \\ 0 & 0.01 \end{pmatrix} \int_{t-\xi(t)}^{t} u(s)ds \\ &\quad + F\Big(t, u(t), u(t-\xi(t)), \frac{d}{dt}u(t-\xi(t)), \int_{t-\xi(t)}^{t} u(s)ds\Big),\end{aligned} \tag{4.3}$$

where

$$\xi_1 = 0.5 \le \xi(t) = 0.5\sin t + 1 \le 1.5 = \xi_2,$$

for all $t \in [0,\infty)$.

We start first by thinking about the linear case $F(t,u,v,w,\omega) = 0$, i.e. $q_j = 0$, $(j = 1,\dots,4)$. Choose the matrix function $H(t)$ together with matrix functions $K(t,s)$ and $L(t,s)$ as follows

$$H(t)=\begin{pmatrix}2-0.2\sin t & 1-0.4\sin t\\ 1-0.4\sin t & 4+1.2\sin t\end{pmatrix},\quad K(t,s)=0.3e^{-0.16t-0.12s}\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix},$$
$$L(t,s)=0.02e^{-0.14t-0.12s}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

Obviously, these functions satisfy the requirements in (2.2)–(2.3) and (3.2)–(3.3) with $l=0.26$. Thus, $H(t)$ and its minimal eigenvalue $h(t)$ can be easily calculated as

$$h_0(>0)=1.0682\leq h(t)\leq 1.6989,\quad \text{and}\quad 3.9318\leq \|H(t)\|\leq 5.3028.$$

Then, it is simple to verify that the matrix $\Pi(t,s)>0$ for the earlier specific choices. Thus, considering the assumptions of Theorem 3.1, we can say that the zero solution of (4.3) with $q_j=0$, $(j=1,\dots,4)$ is ES.

Let $p(t)=0.42$ and $k(t)=0.36$. Since $q_j=0$, $(j=1,\dots,4)$, it is clear that $\Pi(t,s)=\Pi^{\alpha}(t,s)$. $\lambda(t)=\min\{p(t),k(t),l\}=0.26$. By (3.11), we establish the following estimate

$$\|u(t)\|\leq \gamma e^{-0.13t},\quad \gamma>0,$$

for the solutions to (4.3) with $q_j=0$, $(j=1,\dots,4)$.

Let us now examine the case of $F(t,u,v,w,\omega)\neq 0$, for the system (4.3). Considering the requirements in (2.4) and (3.2)–(3.3), we choose the matrix functions $H(t)$, $K(t,s)$, $L(t,s)$ and constants $q_j\geq 0$, $(j=1,\dots,4)$ as follows

$$H(t)=\begin{pmatrix}2-0.2\sin t & 1-0.4\sin t\\ 1-0.4\sin t & 4+1.2\sin t\end{pmatrix},\quad K(t,s)=0.3e^{-0.16t-0.12s}\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix},$$
$$L(t,s)=0.02e^{-0.13t-0.12s}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

and

$$q_1=0.002,\quad q_2=0.003,\quad q_3=0.0002,\quad q_4=0.0001. \tag{4.4}$$

Obviously, these functions satisfy the requirements in (2.2)–(2.4) and (3.2)–(3.3) with $l=0.25$. Thus, $H(t)$ and its minimal eigenvalue $h(t)$ can be easily calculated as

$$h_0(>0)=1.0682\leq h(t)\leq 1.6989,\quad \text{and}\quad 3.9318\leq \|H(t)\|\leq 5.3028.$$

In this case, it is $\Pi(t,s)>0$ for all $t\in[0,\infty)$. Then, by Theorem 3.1, the zero solution to (4.3) with $q_j\geq 0$, $(j=1,\dots,4)$ is ES.

For $t\in[0,\infty)$,it is not hard to indicate that $\Pi^{\alpha}(t,s)$ described in (3.8) is positive definite. Let $p(t)=0.32$ and $k(t)=0.42$. In this case, $\lambda(t)=\min\{p(t),k(t),l\}=0.25$.

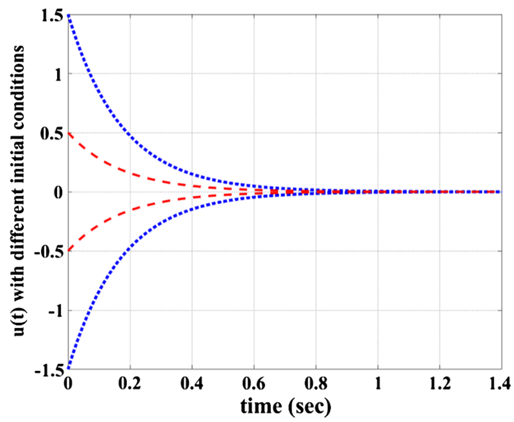

Figure 3 Trajectories of system (4.3) when $F(t,u,v,w,\omega) = 0$, for $\xi(t) = 0.5\sin t + 1$

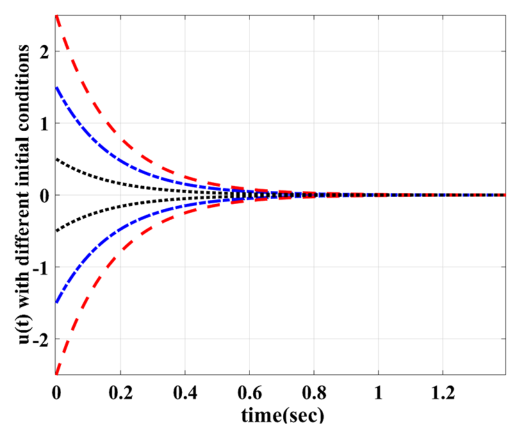

Figure 4 Trajectories of system (4.3) when $F(t,u,v,w,\omega) \neq 0$, for $\xi(t) = 0.5\sin t + 1$

By (3.11), we establish the following estimate

$$\|u(t)\| \leq \gamma e^{-0.125t}, \quad \gamma > 0,$$

for the solutions to (4.3) with (4.4).

The following Fig. 3 and Fig. 4 show the behavior of the trajectories of the solutions of the considered system:

5 Conclusions

This research presents some stability estimates for a class of neutral type differential systems with distributed time varying delay components. A set of sufficient conditions is obtained by constructing an appropriate Lyapunov-Krasovskii functional (LKF) to generate some solution estimates for the considered system. Using the estimations obtained in the framework of Lyapunov-Krasovskii theory, theoretical findings about the stability of the solutions are obtained. Two numerical examples with simulations are given to support the obtained theoretical results and to show the effectiveness and advantage of the criteria. The results obtained in the applied examples and the trajectories after a certain time interval under different initial conditions of the considered system reveal that the applied method is correct. As a result, it is seen that the obtained findings contribute to the results of the existing studies that have been examined previously on this subject in the literature.

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Competing interests
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