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BHABHA SCATTERING MODEL FOR MULTI-TURN TRACKING SIMULATIONS AT THE FCC-ee

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Abstract

The measurement of Bhabha scattered leptons enables a direct estimate of luminosity in lepton colliders. Currently existing Monte Carlo event generators for this process are optimized for high precision detector background simulations. From a beam dynamics point of view, emitted photons will modify the bunch distribution and lead to beam losses due to the limited momentum acceptance of the machine. Hence the interest in building an event generator which is optimized for beam dynamics studies requiring efficient multi-turn tracking simulations. We discuss the implementation of such a model in the newly developed `Xsuite` simulation framework as well as its benchmarking and performance.

INTRODUCTION

In quantum electrodynamics (QED), the Coulomb attraction of two opposite charges (e.g. an electron and a positron) is called Bhabha scattering [1]. Depending on the scattering angle, small and high angle variants of the process can be distinguished, whose dynamics become qualitatively different at high center of mass energies ($\sqrt{s} \gtrsim m_Z \approx 91$ GeV/c^2, m_Z being the rest mass of the Z boson) [2]. The observation of the scattered primaries from small angle Bhabha scattering events (typically defined with a scattering angle below 6°) is a standard way to measure the integrated luminosity in high energy electron-positron colliders, such as LEP or the proposed FCC-ee [3, 4]. This type of process is dominated by the t-channel interaction, for which the Feynman diagram is shown in Fig. 1. In the QED description, the scattering particles exchange a virtual photon and the process can occasionally result in the emission of extra real photons, in which case it is called radiative Bhabha scattering, an analog of bremsstrahlung [5, 6].

The mathematical treatment of Bhabha scattering can be done using the method of equivalent photons [7, 8]. The essence of this approach lies in the fact that the electromagnetic field of a relativistic charged particle, say the positron from Fig. 1, is almost transversal and can therefore accurately be substituted by an appropriately chosen equivalent radiation field of photons. Thus, the cross section for the scattering of an electron with this positron (Bhabha scattering) can be approximated by that of the electron and a photon (Compton scattering). In this case, the equivalent photon corresponds to the exchanged virtual photon between the scattering primaries. The subsequent emission of bremsstrahlung photons can be treated in a numerical simulation as an inverse Compton scattering process [9]. In this, the virtual photons emitted by the positron will collide with the electron. Due to the relativistic dynamics of the participating leptons, the virtual photons have an energy which is often negligible compared to that of the leptons, thus we can treat them as real. The process is called inverse since here the electron will lose energy while the photons will gain energy, contrary to standard Compton scattering. The scattered photons are real and typically end up with an energy E'_γ comparable to the initial lepton energy E_e [10]. According to the law of energy conservation, this causes the scattered primaries to lose a large fraction of their initial energy and be lost in the accelerator shortly after the interaction point. In the FCC-ee, this process is one of the main limitations (the other being beamstrahlung) of the estimated beam lifetime [4].

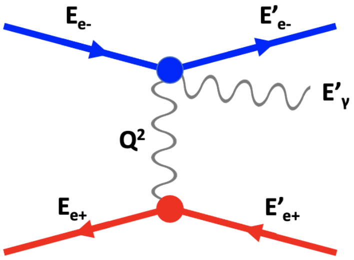

Figure 1: Feynman diagram of the t-channel Bhabha scattering process with the subsequent emission of a real photon from the scattered electron. The particle energies and the virtuality of the exchanged photon are indicated.

MODELING IN XSUITE

The generation of photons from radiative Bhabha scattering in `Xsuite` [11] can be divided into 3 steps. As a first step, we compute the charge density of the opposite bunch slice at the location of the macroparticle in the soft-Gaussian approximation [12]. From this we estimate the integrated luminosity of the collision of the macroparticle with the virtual photons represented by the slice, integrated over the time of passing through the slice. Then, we generate a set of virtual photons corresponding to the total energy of the opposite slice. Finally, we iterate over these photons and simulate the bremsstrahlung process as a series of inverse Compton scattering events between the lepton and virtual photon. In the following, we describe each of these steps in more details.

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Luminosity Computation

We model the charge density of a longitudinal slice of the opposing beam as a 2D Gaussian distribution $\rho(x, y)$, with its corresponding average transverse positions and sizes. Considering an infinitesimal area $\delta x \delta y$ around the transverse position x, y of a given macroparticle at the collision point with the slice, we can write the number of charges with which the macroparticle will interact:

$$N_e(x, y) = N_{b,s}\rho(x, y)\delta x \delta y, \tag{1}$$

with $N_{b,s}$ the number of charges in the opposing slice. To model the Bhabha scattering process, we first estimate the number of possible collisions between the macroparticle and the virtual photons generated by these charges. The number of events is proportional to the cross section σ of the process (inverse Compton scattering in our case) and L, the luminosity integrated over the passage of the particle through the slice. In our case, we consider the collision of $N_e(x, y)$ charges in the opposing slice with $N_{b,m}$ charges represented by each macroparticle, thus the integrated luminosity can be given, using Eq. (1), as:

$$L = \frac{N_{b,m} \cdot N_e(x, y)}{\delta x \delta y} = N_{b,m}N_{b,s}\rho(x, y). \tag{2}$$

Virtual Photon Generation

Equation (2) describes the integrated luminosity of lepton-lepton collisions. However, our aim is to simulate the collision of the leptons with virtual photons. Our basic assumption is therefore that the virtual photon distribution $N_\gamma(x, y)$ is proportional to that of the primary charges:

$$N_\gamma(x, y) = nN_e(x, y), \tag{3}$$

where n is a proportionality factor denoting the number of virtual photons corresponding to one elementary charge. The number density spectrum of virtual photons is given by:

$$\frac{dn}{dxdQ^2} = \frac{\alpha}{2\pi}\frac{1 + (1 - x)^2}{x}\frac{1}{Q^2}, \tag{4}$$

where $x = \frac{\hbar\omega}{E_e} = \frac{E_\gamma}{E_e}$ is the total energy of the virtual photon normalized to the primary energy and Q^2 is the squared virtuality of the virtual photon [13]. The virtual photon energies and virtualities can be drawn using the method of inverse CDF (Cumulative Distribution Function) sampling. The sampling algorithm in `Xsuite` has been adapted from `GUINEA-PIG` [14], a Particle In Cell (PIC) based single beam-beam collision simulation software. For each macroparticle in the beam, we first compute the total amount of equivalent photons using the energy of the opposite bunch slice. Subsequently, the energy and virtuality of each photon will be sampled. In the current implementation all virtual photons inherit the dynamical variables of the strong bunch slice centroid. Note that the virtual photons sampled this way will also be ”macroparticles” in the sense that they represent the dynamics of all virtual photons generated by all charges in a primary macroparticle.

An example energy spectrum of the virtual photons is shown in Fig. 2, produced with a simplified setup using the FCC-ee $t\bar{t}$ parameters with a beam energy E_e = 182.5 GeV [4]. The simulation has been simplified by setting the number of slices in the beam-beam model to 1, the crossing angle to 0 and the longitudinal coordinates of all macroparticles to 0. We have performed the same task with `Xsuite` and `GUINEA-PIG`. From the figure it can be seen that both the spectrum's shape and the number of sampled photons from both codes match well.

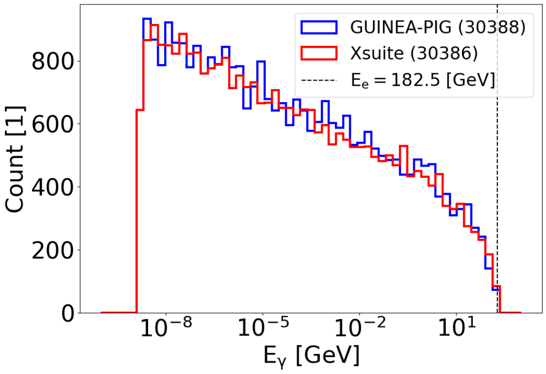

Figure 2: Comparison of virtual photon energy spectra. The numbers in the legend indicate the total number of generated photons. The black line denotes the nominal beam energy.

Inverse Compton Scattering of Virtual Photons

We account for the proportionality of the primary charge and virtual photon distributions described by Eq. (3) by resampling the photons for each macroparticle. With each photon, we simulate the bremsstrahlung process in the form of a set of inverse Compton scattering events. The number of Compton events can be described as:

$$R = \sigma_{C,tot}(s)L = \sigma_{C,tot}(s)N_{b,m}N_{b,s}\rho(x, y), \tag{5}$$

where $s \approx \frac{4E_\gamma E_e}{m_e^2c^4}$ is the center of mass energy squared of the photon-primary Compton interaction, normalized to the rest mass of the primary [15], and $\sigma_{C,tot}(s)$ denotes the total Compton scattering cross section, given by:

$$\begin{aligned}\sigma_{C,tot}(s) = \frac{2\pi r_e^2}{s}\Bigg[\ln(s + 1)\left(1 - \frac{4}{s} - \frac{8}{s^2}\right)\\ + \frac{1}{2} + \frac{8}{s} - \frac{1}{2(s + 1)^2}\Bigg],\end{aligned} \tag{6}$$

with r_e being the classical electron radius. For each event, we sample the scattered photon energy from the differential cross section:

$$\frac{d\sigma_C}{dy} = \frac{2\pi r_e^2}{s}\left[\frac{1}{1 - y} + 1 - y - \frac{4y}{s(1 - y)} + \frac{4y^2}{s^2(1 - y)^2}\right], \tag{7}$$

which describes the scattering of a beam of unpolarized photons on the primary charge [9]. Here $y = \frac{\hbar\omega'}{E_e} = \frac{E'_\gamma}{E_e}$ is the energy of the scattered photon in units of the total energy of the colliding primary. Given the energy E'_γ, we can compute the scattering angle of the primary and the photon as well as their momenta, using the constraints given by energy and momentum conservation. While the emitted photon spectrum corresponds to the sum of all charges represented by a macroparticle, a given macroparticle should represent the dynamics of a single primary charge. Thus, the dynamical variables of the macroparticles are updated according to energy and momentum conservation accounting for the emission of only a fraction of the photons. The latter are picked randomly based on a probability corresponding to the inverse of the number of charges per macroparticle.

Figure 3 shows an example energy spectrum of the emitted photons in a simplified simulation, identical to that for the production of Fig. 2, except for the bunch intensity which we have increased by a factor 10 compared to the nominal $t\bar{t}$ value in order to obtain better statistics on the histogram.

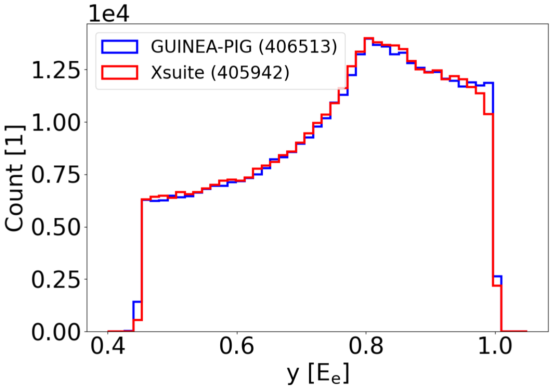

Figure 3: Comparison of the energy spectrum of the scattered Compton photons, normalized to the energy of the primary. The numbers in the legend indicate the total number of generated photons.

The number of photons and their energy distribution are similar with both codes, except a small deviation in the spectrum at the lowest and highest energies where `GUINEA-PIG` yields slightly more photons. This discrepancy is negligible and we attribute it to the different architecture of the two codes, namely the soft-Gaussian approximation in `Xsuite` and the PIC solver in `GUINEA-PIG`.

STUDIES IN REALISTIC SCENARIOS

We have tested the Bhabha event generation in the nominal FCC-ee $t\bar{t}$ configuration [4]. We have simulated 10 collisions (without tracking in the arc) with 100 slices and 10^4 macroparticles in both bunches. The energy spectrum of the emitted photons is shown on Fig. 4.

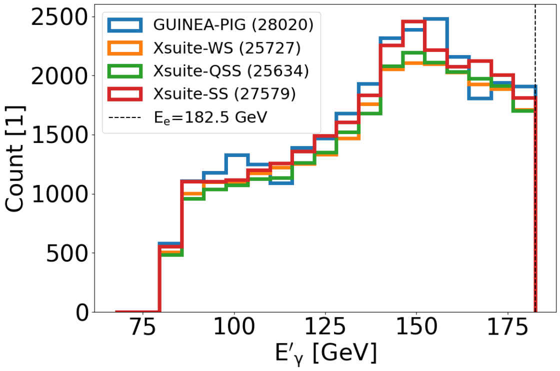

Figure 4: Spectrum of emitted Bhabha photons in the nominal FCC-ee $t\bar{t}$ configuration. The numbers in the legend indicate the total number of generated photons. The black line denotes the nominal beam energy.

We have simulated the spectrum in `Xsuite` with all beam-beam models: weak-strong, quasi-strong-strong and strong-strong, and by using our benchmark software `GUINEA-PIG`. We have observed an good overall agreement with a marginal difference between the models in `Xsuite` in terms of the number of emitted photons, with the strong-strong model being the most similar to `GUINEA-PIG`. The other models produce slightly less photons likely due to the lack of modeling of the distortion of the bunch during the pass. Note that most of the emitted photons have a total energy comparable to that of the emitting primary, therefore these primaries can be expected to be lost during tracking after emitting just one photon. An important difference between the two codes is the speed of the simulation. Owing to the simplification of the electromagnetic field (soft-Gaussian approximation), `Xsuite` simulates the collision and the generation of the Bhabha photons orders of magnitude faster, while the accuracy remains sufficient to study most circular colliders thanks to the low disruption parameter.

SUMMARY

In this contribution we have discussed the implementation of a Monte Carlo event generator for Bhabha scattering in `Xsuite`, optimized for beam dynamics simulations. Our first benchmark results against `GUINEA-PIG` show good agreement in simplified and realistic FCC-ee scenarios. Since this process is one of the main limitations of beam lifetime in FCC-ee, we are now able to assess this important aspect of the machine design with `Xsuite`, by combining the beam-beam element with tracking through the accelerator lattice. Using a detailed model of the lattice and aperture we will also be able to make estimates on the location of lost particles and the impact of the background on the infrastructure.

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