The Potential of Interference-Based Proof Systems

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Traditional Proofs vs. Interference-Based Proofs

- In traditional proof systems, everything that is inferred, is logically implied by the premises.

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- Different approach: Allow not only implied conclusions.
  - Require only that the addition of facts preserves satisfiability.
  - Reason also about the absence of facts.

  ➥ This leads to interference-based proof systems.

Interference-Based Proof Systems

- Interference-based proof systems generalize traditional proof systems.
- An interference-based proof is a sequence of clauses.
  - Idea: Clauses are added to the formula or deleted from it step-by-step.
  - Added clauses need not be implied, but their addition must preserve satisfiability:

➥ If the formula is satisfiable, then the formula obtained by adding a clause is also satisfiable.

➥ If the (unsatisfiable) empty clause, $\bot$, can be added, then the original formula must be unsatisfiable.

- The empty clause is unsatisfiable because it has no literal that could be true.

Interference-Based Proofs

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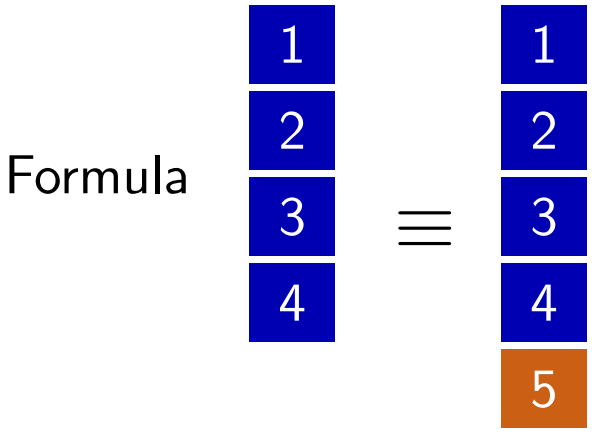

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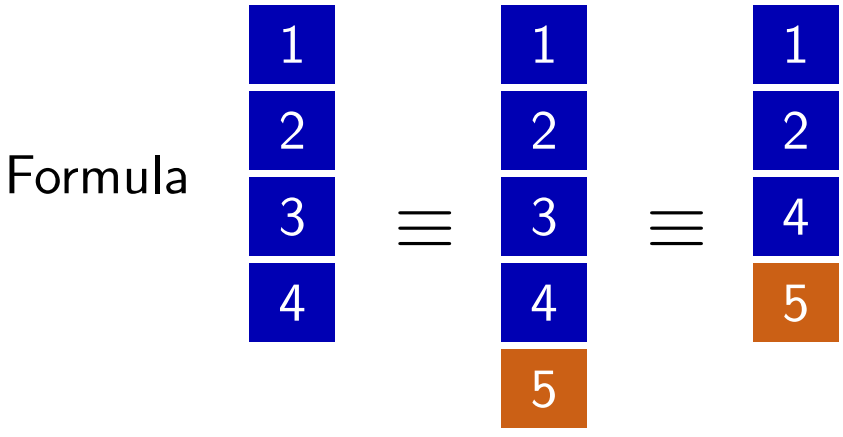

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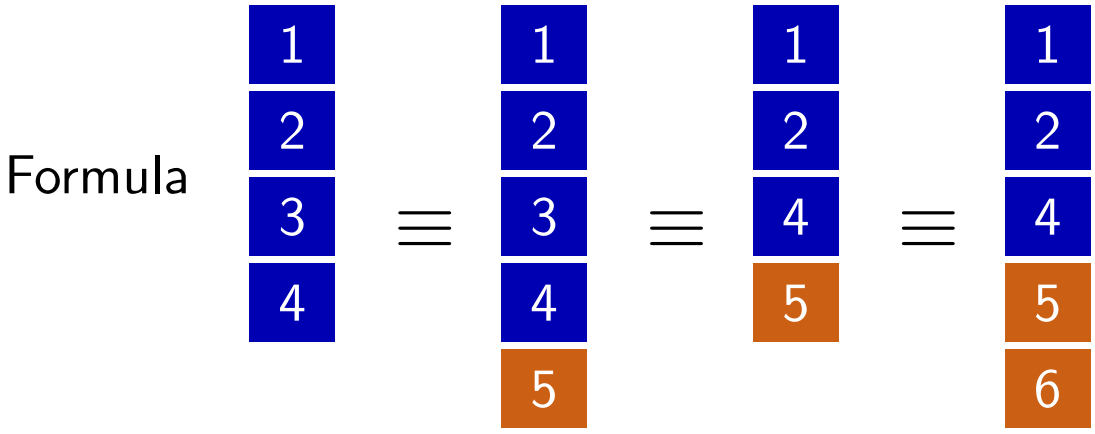
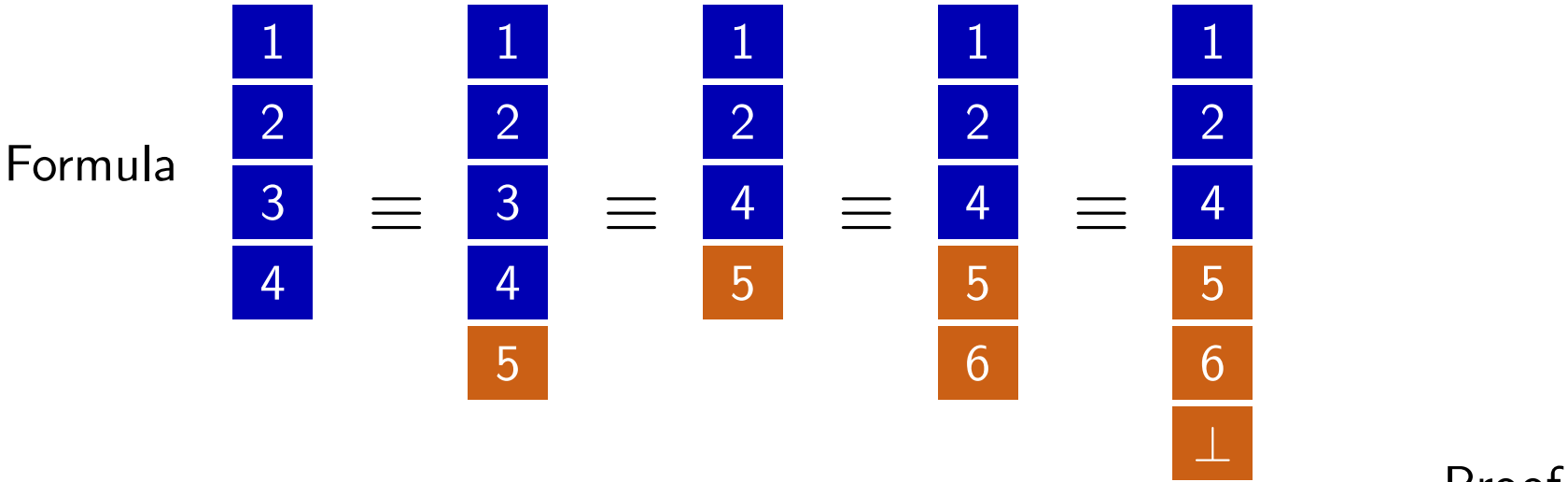


Proof

Interference-Based Proofs



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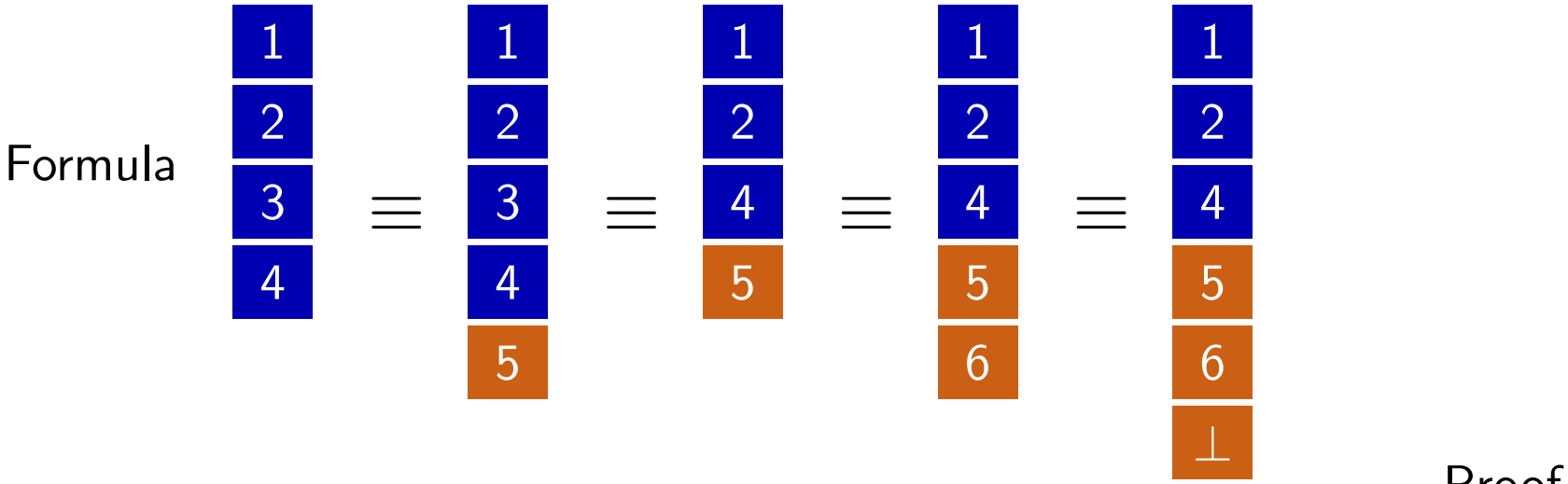

- Checking whether modifications preserve satisfiability should be efficient.
- Clauses that can be added or removed are called redundant.

↪ Idea: Allow only modifications that fulfill an efficiently checkable redundancy criterion.

↪ Idea: Showing satisfiability equivalence with the empty formula allows proving satisfiability.

Practical Usefulness of Interference-Based Proof Systems

- The interference-based proof system DRAT is the de-facto standard in SAT solving.
  - DRAT is based on the addition of RAT clauses and on deletion.
- QRAT, the extension of DRAT for the satisfiability problem of quantified Boolean formulas (QSAT), is very powerful.
  - It is the only QBF proof system that can succinctly express virtually all preprocessing techniques.
- At CADE we present new interference-based proof systems for SAT that allow for short proofs without new variables.

Question 1

- Almost all proof systems reason only about the presence of premises.
- *What prevents us, in particular theoreticians, from reasoning about their absence?*
- The best known interference-based system, extended resolution, allows exponentially smaller proofs compared to resolution.
- Other interference rules such as blocked-literal addition in QSAT also facilitate short proofs for hard problems.

Question 2

- Deletion of clauses can be a powerful technique.
- In SAT, clause deletion provides efficiency.
- In QSAT, it provides a way to prove satisfiability.
- *What can clause deletion offer in first-order logic*?

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