SAT and SMT Solvers in Practice

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http://www.cs.cmu.edu/~mheule/15816-f21/
https://github.com/marijnheule/sat-examples.git

Automated Reasoning and Satisfiability
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DIMACS: SAT solver input format

The DIMACS format for SAT solvers has three types of lines:

- header: `p cnf n m` in which `n` denotes the highest variable index and `m` the number of clauses
- clauses: a sequence of integers ending with “0”
- comments: any line starting with “`c `”

$$
\begin{aligned}
&(a \lor b \lor \overline{c}) \land \\
&(\overline{a} \lor \overline{b} \lor c) \land \\
&(b \lor c \lor \overline{d}) \land \\
&(\overline{b} \lor \overline{c} \lor d) \land \\
&(a \lor c \lor d) \land \\
&(\overline{a} \lor \overline{c} \lor \overline{d}) \land \\
&(\overline{a} \lor b \lor d)
\end{aligned}
$$

```
c example
p cnf 4 7
 1  2 -3 0
-1 -2  3 0
 2  3 -4 0
-2 -3  4 0
 1  3  4 0
-1 -3 -4 0
-1  2  4 0
```

DIMACS: SAT solver output format

The solution line of a SAT solver starts with "`s `":

- `s SATISFIABLE`: The formula is satisfiable
- `s UNSATISFIABLE`: The formula is unsatisfiable
- `s UNKNOWN`: The solver cannot determine satisfiability

In case the formula is satisfiable, the solver emits a certificate:

- lines starting with "`v `"
- a list of integers ending with `0`
- e.g. `v -1 2 4 0`

In case the formula is unsatisfiable, then most solvers support emitting a proof of unsatisfiability to a separate file

CaDiCaL: download and install

Most SAT solvers are implemented in C/C++

CaDiCaL is one of the strongest SAT solvers. As the name suggests it is based on CDCL. Recommended for Linux and macOS users.

obtain CaDiCaL:

- `git clone https://github.com/arminbiere/cadical.git`
- cd cadical
- ./configure; make

to run: `./build/cadical` formula.cnf

SAT4J: download and install

SAT4J is a SAT solver in Java. It is also based on CDCL. Recommended for windows users.

obtain SAT4J:

- `git clone https://github.com/marijnheule/sat-examples.git`
- cd sat-examples

to run: `java -jar org.sat4j.core-2.3.1.jar` formula.cnf

UBCSAT: download and install

UBCSAT is a collection of local search SAT solvers.

obtain UBCSAT:

- download and unzip `http://ubcsat.dtompkins.com/downloads/ubcsat-beta-12-b18.tar.gz`
- cd ubcsat-beta-12-b18
- make clean; make

to run: `./ubcsat -alg ddfw -i` formula.cnf

there are many LS algorithms to choose from (`-alg`)

Many SAT solvers

Many SAT solvers have been developed

Lots of them participate in the annual SAT competition

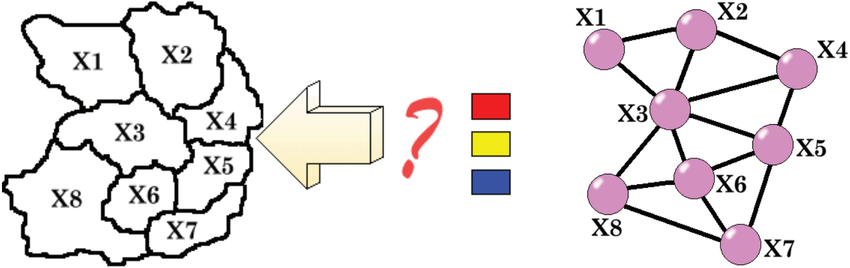
- All code of participants in open source
- Each solver is run on hundreds of benchmarks
- Large timeout 5000 seconds

For details and downloading more solvers visit
http://satcompetition.org/

Demo: SAT Solving

Graph coloring

Given a graph $G(V, E)$, can the vertices be colored with k colors such that for each edge $(v, w) \in E$, the vertices v and w are colored differently.



Graph coloring encoding

Variables	**Range**	**Meaning**
$x_{v,i}$	$i \in \{1, \ldots, c\}$ $v \in \{1, \ldots, \lvert V \rvert\}$	node v has color i

Clauses	**Range**	**Meaning**
$(x_{v,1} \lor x_{v,2} \lor \cdots \lor x_{v,c})$	$v \in \{1, \ldots, \lvert V \rvert\}$	v is colored
$(\overline{x}_{v,s} \lor \overline{x}_{v,t})$	$s \in \{1, \ldots, c-1\}$ $t \in \{s+1, \ldots, c\}$	v has at most one color
$(\overline{x}_{v,i} \lor \overline{x}_{w,i})$	$(v, w) \in E$	v and w have a different color

Graph coloring encoding code

```c
#include <stdio.h>
#include <stdlib.h>

int main (int argc, char** argv) {
  FILE* graph  = fopen (argv[1], "r");
  int i, j, a, b, nVertex, nEdge, nColor = atoi  (argv[2]);
  fscanf (graph, " p edge %i %i ", &nVertex, &nEdge);

  printf ("p cnf %i %i\n", nVertex * nColor, nVertex + nEdge * nColor);

  for (i = 0; i < nVertex; i++) {
    for (j = 1; j <= nColor; j++)
      printf ("%i ", i * nColor + j);
    printf ("0\n"); }

  while (1) {
    int tmp = fscanf (graph, " e %i %i ", &a, &b);
    if (tmp == 0 || tmp == EOF) break;
    for (j = 1; j <= nColor; j++)
      printf ("-%i -%i 0\n", (a-1) * nColor + j, (b-1) * nColor + j);
  }
}
```

Demo: Encode, Decode

Unsatisfiable cores

An unsatisfiable core of an unsatisfiable formula F is a subset of F that is unsatisfiable.

An minimal unsatisfiable core of an unsatisfiable formula such that the removal of any clause makes the formula satisfiable.

Extracting a minimal unsatisfiable core from a formula has many applications, but the computational costs could be high.

- maxSAT
- diagnosis
- formal verification

Proofs

A proof of unsatisfiability is a certificate that a given formula is unsatisfiable.

Various proof producing methods exists (another lecture).

Proof checking tools cannot only validate a proof but also produce additional information about the formula:

- unsatisfiable core
- optimized proof

DRAT-trim is a tool that validates proofs and produces such information

Demo: Core Extraction

StarExec

StarExec is a cross community logic solving service

- Great to evaluate solvers/heuristics in parallel
- Also used to run the SAT/SMT competitions

Register at `https://www.starexec.org/`

- select SAT as your community

Demo: StarExec

Tools for making SAT-based modeling easier

PySAT is a Python toolkit that makes it easier for users to call SAT solvers and build encodings using Python:

- `https://pysathq.github.io/`
- SAT solver is still written in C, C++
- Interface includes several encodings for linear constraints:
  - At-most-one constraints
  - Cardinality constraints
  - AIGER circuits to CNF
  - …
- Well documented
- Active development

Demo: PySAT

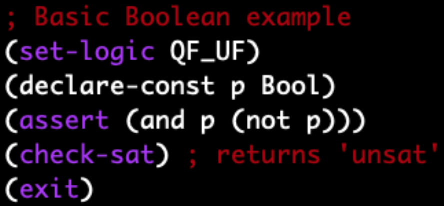
SMT-LIB: SMT solver input format (I)

http://smtlib.cs.uiowa.edu/

Language has similarities with functional languages and it is more readable than CNF. Theories:

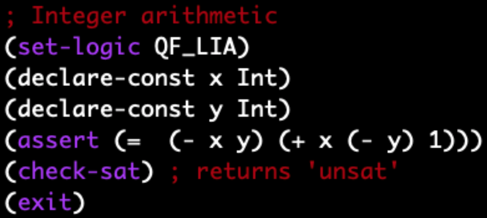
- Arrays,
- Bitvectors,
- Boolean predicates,
- Floating point,
- Ints,
- Reals

SMT-LIB: SMT solver input format (II)



```
; Basic Boolean example
(set-logic QF_UF)
(declare-const p Bool)
(assert (and p (not p)))
(check-sat) ; returns 'unsat'
(exit)
```

SMT-LIB: SMT solver input format (III)



```
; Integer arithmetic
(set-logic QF_LIA)
(declare-const x Int)
(declare-const y Int)
(assert (=  (- x y) (+ x (- y) 1)))
(check-sat) ; returns 'unsat'
(exit)
```

SMT Solvers

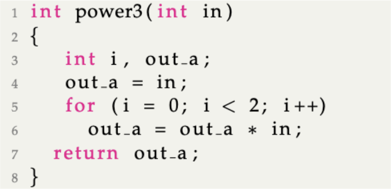
- Z3 (Microsoft):
  `https://github.com/Z3Prover/z3/wiki`
- CVC4 (Stanford):
  `http://cvc4.cs.stanford.edu/web/`
- Yices (SRI): `http://yices.csl.sri.com/`
- Boolector (JKU Austria):
  `https://boolector.github.io/`

SMT Solvers

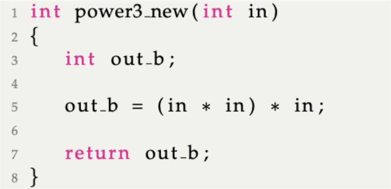
We recommend the use of Z3:

- Tutorial: `https://theory.stanford.edu/~nikolaj/programmingz3.html`
- APIs for Python, C++, Java
- MIT License: `https://github.com/Z3Prover/z3`
- Most used and cited SMT solver (>7,000 citations)

Proving program equivalence in SMT



```
int power3(int in)
{
   int i, out_a;
   out_a = in;
   for (i = 0; i < 2; i++)
     out_a = out_a * in;
  return out_a;
}
```

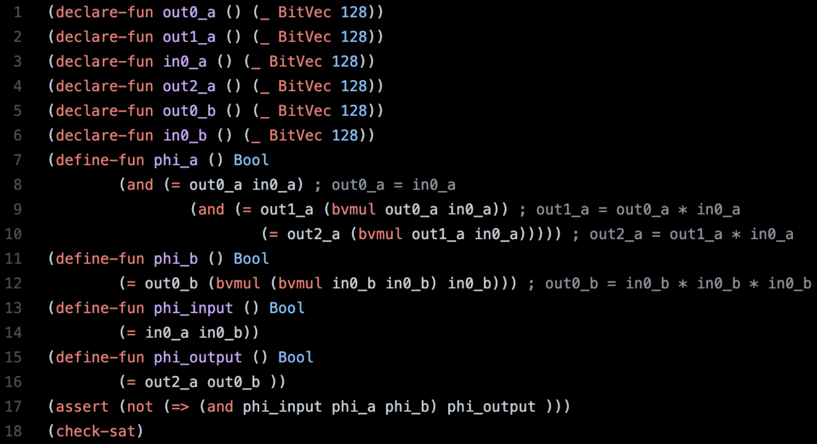


```
int power3_new(int in)
{
   int out_b;

   out_b = (in * in) * in;

   return out_b;
}
```

$$
\begin{aligned}
\varphi_a \equiv &(\mathrm{out0\_a} = \mathrm{in0\_a}) \wedge (\mathrm{out1\_a} = \mathrm{out0\_a} \times \mathrm{in0\_a}) \wedge \\
&(\mathrm{out2\_a} = \mathrm{out1\_a} \times \mathrm{in0\_a}) \\
\varphi_b \equiv &\mathrm{out0\_b} = (\mathrm{in0\_b} \times \mathrm{in0\_b}) \times \mathrm{in0\_b}
\end{aligned}
$$

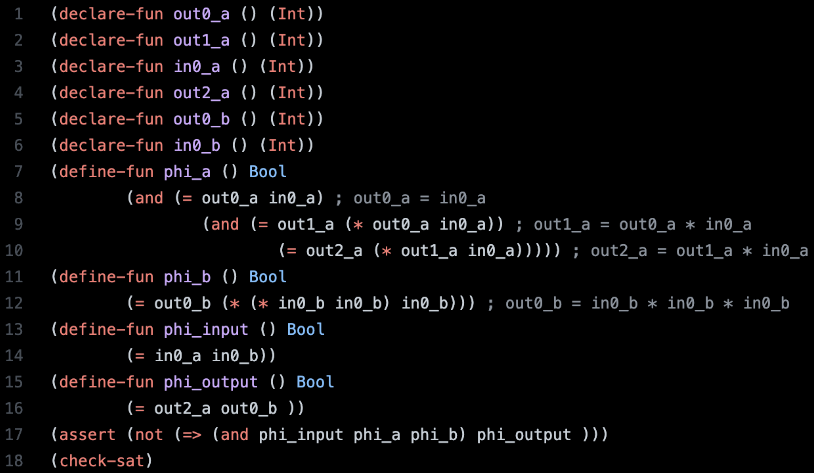
To show these programs are equivalent, we must show the following formula is valid: $\mathrm{in0\_a} = \mathrm{in0\_b} \wedge \varphi_a \wedge \varphi_b \implies \mathrm{out2\_a} = \mathrm{out0\_b}$

Demo: Program equivalence with SMT solving (BV)



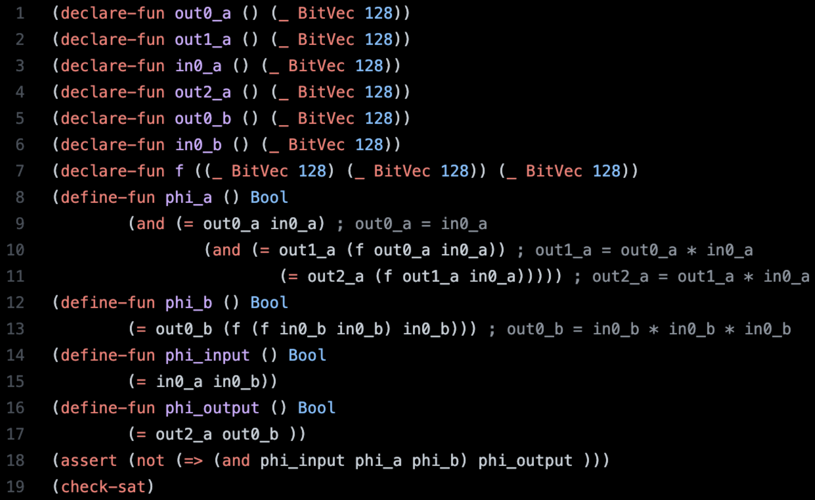
```
(declare-fun out0_a () (_ BitVec 128))
(declare-fun out1_a () (_ BitVec 128))
(declare-fun in0_a () (_ BitVec 128))
(declare-fun out2_a () (_ BitVec 128))
(declare-fun out0_b () (_ BitVec 128))
(declare-fun in0_b () (_ BitVec 128))
(define-fun phi_a () Bool
        (and (= out0_a in0_a) ; out0_a = in0_a
                (and (= out1_a (bvmul out0_a in0_a)) ; out1_a = out0_a * in0_a
                        (= out2_a (bvmul out1_a in0_a))))) ; out2_a = out1_a * in0_a
(define-fun phi_b () Bool
        (= out0_b (bvmul (bvmul in0_b in0_b) in0_b))) ; out0_b = in0_b * in0_b * in0_b
(define-fun phi_input () Bool
        (= in0_a in0_b))
(define-fun phi_output () Bool
        (= out2_a out0_b ))
(assert (not (=> (and phi_input phi_a phi_b) phi_output )))
(check-sat)
```

Demo: Program equivalence with SMT solving (Int)



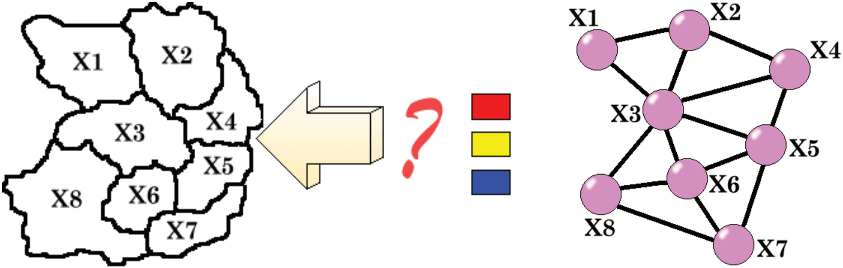
```
(declare-fun out0_a () (Int))
(declare-fun out1_a () (Int))
(declare-fun in0_a () (Int))
(declare-fun out2_a () (Int))
(declare-fun out0_b () (Int))
(declare-fun in0_b () (Int))
(define-fun phi_a () Bool
        (and (= out0_a in0_a) ; out0_a = in0_a
                (and (= out1_a (* out0_a in0_a)) ; out1_a = out0_a * in0_a
                        (= out2_a (* out1_a in0_a))))) ; out2_a = out1_a * in0_a
(define-fun phi_b () Bool
        (= out0_b (* (* in0_b in0_b) in0_b))) ; out0_b = in0_b * in0_b * in0_b
(define-fun phi_input () Bool
        (= in0_a in0_b))
(define-fun phi_output () Bool
        (= out2_a out0_b ))
(assert (not (=> (and phi_input phi_a phi_b) phi_output )))
(check-sat)
```

Demo: Program equivalence with SMT solving (UF)



```
(declare-fun out0_a () (_ BitVec 128))
(declare-fun out1_a () (_ BitVec 128))
(declare-fun in0_a () (_ BitVec 128))
(declare-fun out2_a () (_ BitVec 128))
(declare-fun out0_b () (_ BitVec 128))
(declare-fun in0_b () (_ BitVec 128))
(declare-fun f ((_ BitVec 128) (_ BitVec 128)) (_ BitVec 128))
(define-fun phi_a () Bool
        (and (= out0_a in0_a) ; out0_a = in0_a
                (and (= out1_a (f out0_a in0_a)) ; out1_a = out0_a * in0_a
                        (= out2_a (f out1_a in0_a))))) ; out2_a = out1_a * in0_a
(define-fun phi_b () Bool
        (= out0_b (f (f in0_b in0_b) in0_b))) ; out0_b = in0_b * in0_b * in0_b
(define-fun phi_input () Bool
        (= in0_a in0_b))
(define-fun phi_output () Bool
        (= out2_a out0_b ))
(assert (not (=> (and phi_input phi_a phi_b) phi_output )))
(check-sat)
```

Graph coloring encoding in SMT



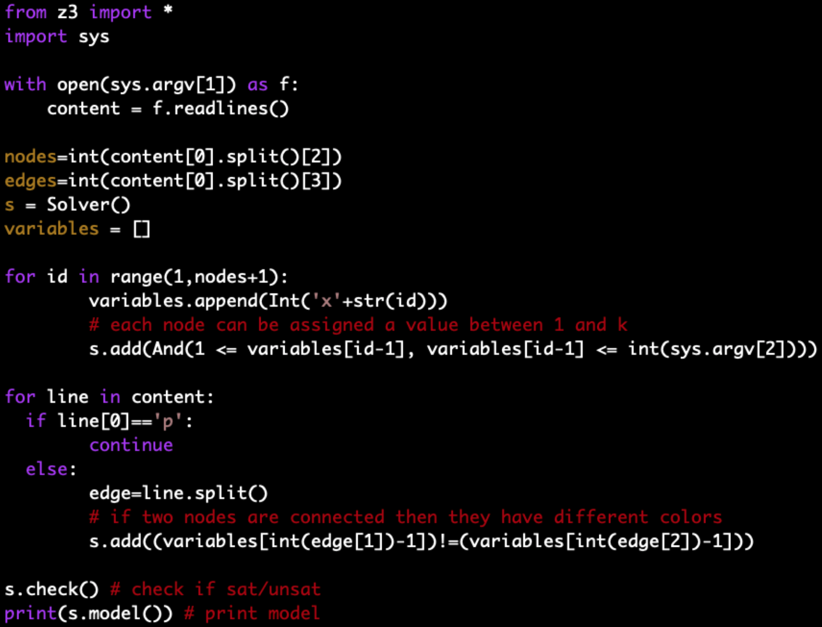
Variables:

- Integer variables x_i for each node

Constraints:

- $1 \leq x_i \leq c$
- $x_i \neq x_j$ for $(x_i, x_j) \in E$

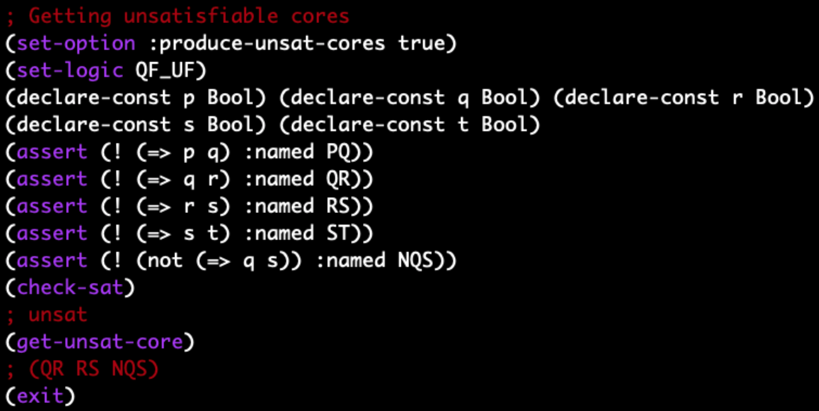
Graph coloring encoding code



```
from z3 import *
import sys

with open(sys.argv[1]) as f:
    content = f.readlines()

nodes=int(content[0].split()[2])
edges=int(content[0].split()[3])
s = Solver()
variables = []

for id in range(1,nodes+1):
        variables.append(Int('x'+str(id)))
        # each node can be assigned a value between 1 and k
        s.add(And(1 <= variables[id-1], variables[id-1] <= int(sys.argv[2])))

for line in content:
  if line[0]=='p':
        continue
  else:
        edge=line.split()
        # if two nodes are connected then they have different colors
        s.add((variables[int(edge[1])-1])!=(variables[int(edge[2])-1]))

s.check() # check if sat/unsat
print(s.model()) # print model
```

Demo: Encoding in SMT

Unsatisfiable cores in SMT



```
; Getting unsatisfiable cores
(set-option :produce-unsat-cores true)
(set-logic QF_UF)
(declare-const p Bool) (declare-const q Bool) (declare-const r Bool)
(declare-const s Bool) (declare-const t Bool)
(assert (! (=> p q) :named PQ))
(assert (! (=> q r) :named QR))
(assert (! (=> r s) :named RS))
(assert (! (=> s t) :named ST))
(assert (! (not (=> q s)) :named NQS))
(check-sat)
; unsat
(get-unsat-core)
; (QR RS NQS)
(exit)
```

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