Automated Reasoning and Satisfiability
Assignment 1

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The homework is due at 10am on Thursday, September 19. Please email your answers to `mheule@cs` and `rubenm@cs` with subject "Homework Assignment 1". The questions below are mostly encoding questions. We prefer answers that consist of a generator that produces the requested DIMCAS file in a common programming language, such as Python or C(++). Alternatively, you can submit the encoding answers as a latex document. However, question 1(d) and question 2 (b) can only be solved using a generated DIMACS file.

Question 1

(a) [10 points] Given the Boolean variables $x_1, \ldots, x_5$, construct two different encodings in conjunctive normal form (CNF) that express that at most two of them can be true: $x_1 + \ldots + x_5 \leq 2$. The first encoding uses only the variables $x_1, \ldots, x_5$, while the second encoding uses auxiliary variables.

(b) [10 points] Let us refer to the above encodings as AtMostTwoA (w/o auxiliary variables) and AtMostTwoB (with auxiliary variables). Encode in (CNF) $y_1 \leftrightarrow \text{AtMostTwoA}(x_1, \ldots, x_5)$ and $y_2 \leftrightarrow \text{AtMostTwoB}(x_1, \ldots, x_5)$.

(c) [5 points] Encode whether there exists an assignment to $x_1, \ldots, x_5$ that falsifies y_1 and satisfies y_2 by combining $y_1 \leftrightarrow \text{AtMostTwoA}(x_1, \ldots, x_5)$ and $y_2 \leftrightarrow \text{AtMostTwoB}(x_1, \ldots, x_5)$.

(d) [5 points] Solve the resulting formula using a SAT solver and show the output of the solver. (Hint: the formula should be unsatisfiable, so no local search solver can be used.)

Question 2

(a) [10 points] Consider a 10×10 grid of squares and all possible rectangles within the grid whose length and width are at least 2. Encode whether there exists a coloring of the grid using three colors so that no such rectangle has the same color for its four corners. (Hint: The encoding requires two types of constraints. First, each square needs to have at least one color. Second, if four squares form the corners of a rectangle, then they cannot have the same color.)

```
0 0 1 1 2 2 0 1 2
2 0 0 1 1 2 2 0 1
1 2 0 0 1 1 2 2 0
0 1 2 0 0 1 1 2 2
2 0 1 2 0 0 1 1 2
2 2 0 1 2 0 0 1 1
1 2 2 0 1 2 0 0 1
1 1 2 2 0 1 2 0 0
0 1 1 2 2 0 1 2 0
```

(b) [10 points] Solve the encoding using a SAT solver and decode the solution into a valid color. Show the output of the SAT solver and valid 3-coloring similar to the one above of the 9×9 grid.