10-607 Computational Foundations for Machine Learning

Machine Learning Department
School of Computer Science
Carnegie Mellon University

Recursion & Proof by Induction

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Lecture 7
Nov. 12, 2018

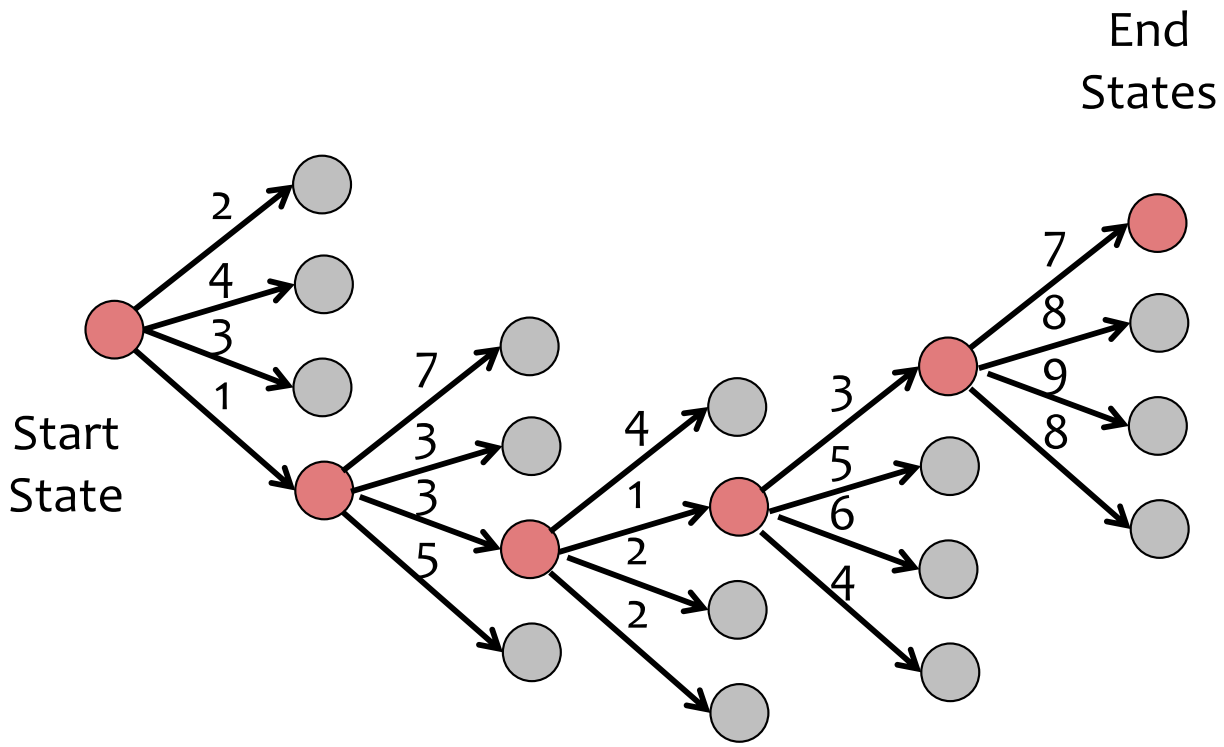
Reminders

- Homework B: Complexity & Recursion
  - Out: Thu, Nov. 8
  - Due: Tue, Nov. 20 at 11:59pm
- Quiz 1: Logic & Proofs; Computation
  - Mon, Nov. 19, in-class
  - Covers Lectures 1 – 6

Q&A

RECURSION

Example: Greedy Search



Goal:

- Search space consists of nodes and weighted edges
- Goal is to find the lowest (total) weight path from root to a leaf

Greedy Search:

- At each node, selects the edge with lowest (immediate) weight
- Heuristic method of search (i.e. does *not* necessarily find the best path)

Motivating Example

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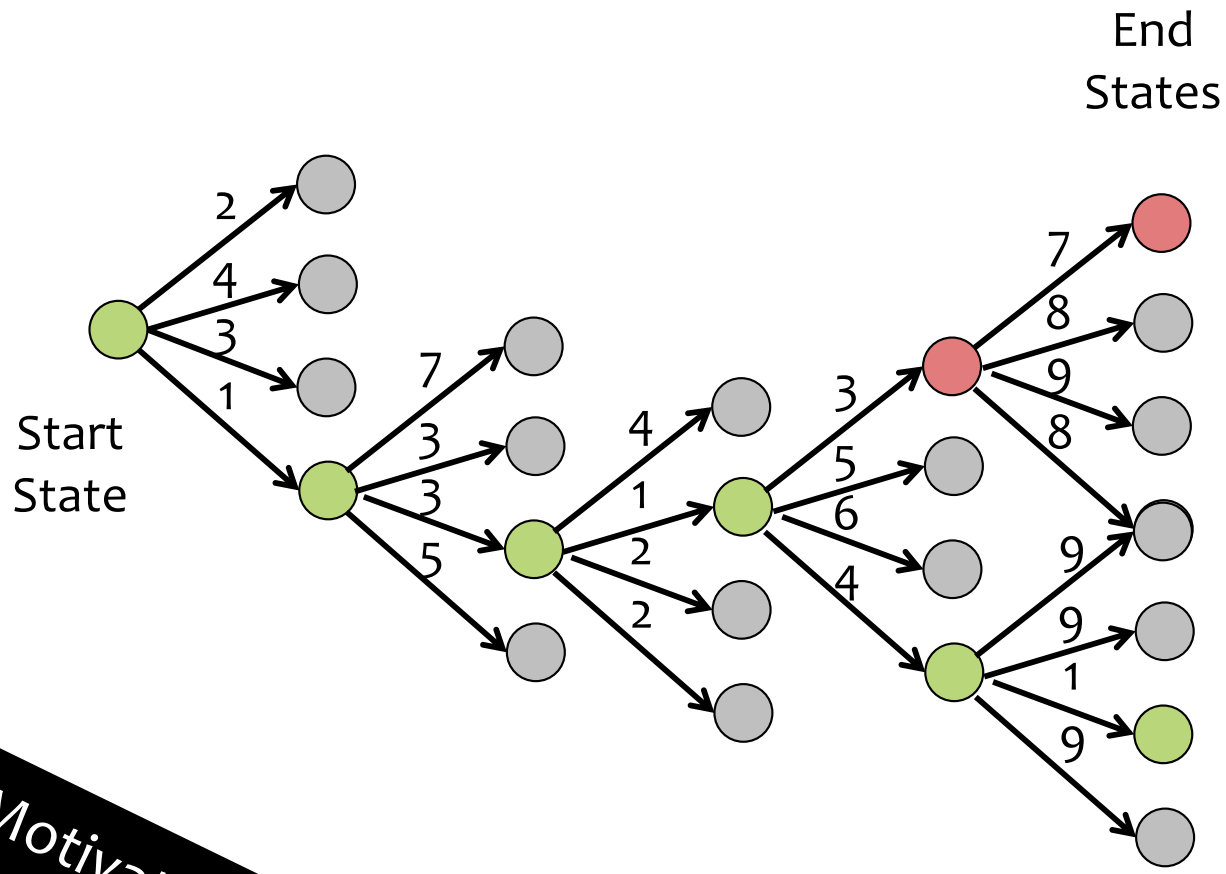

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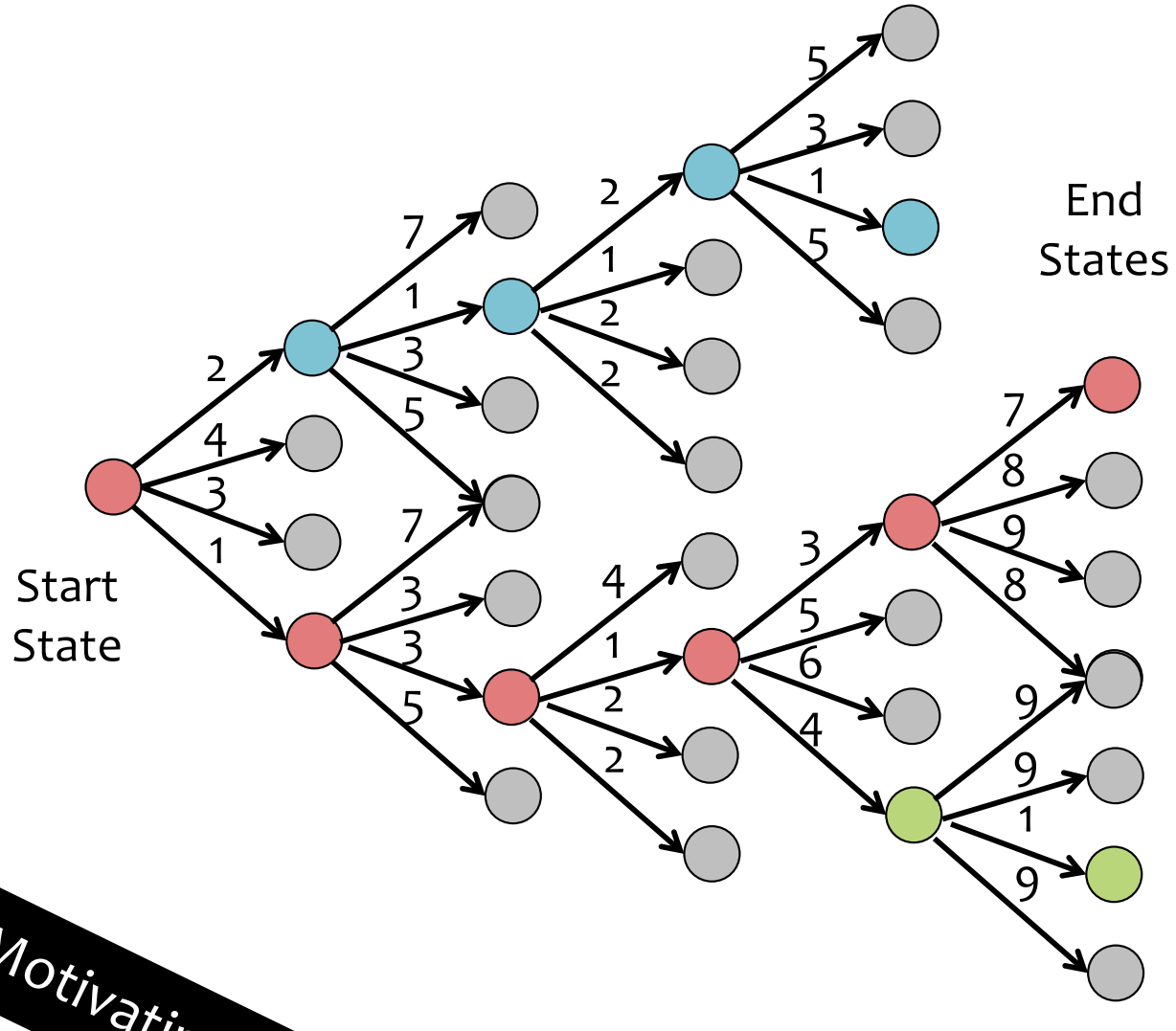

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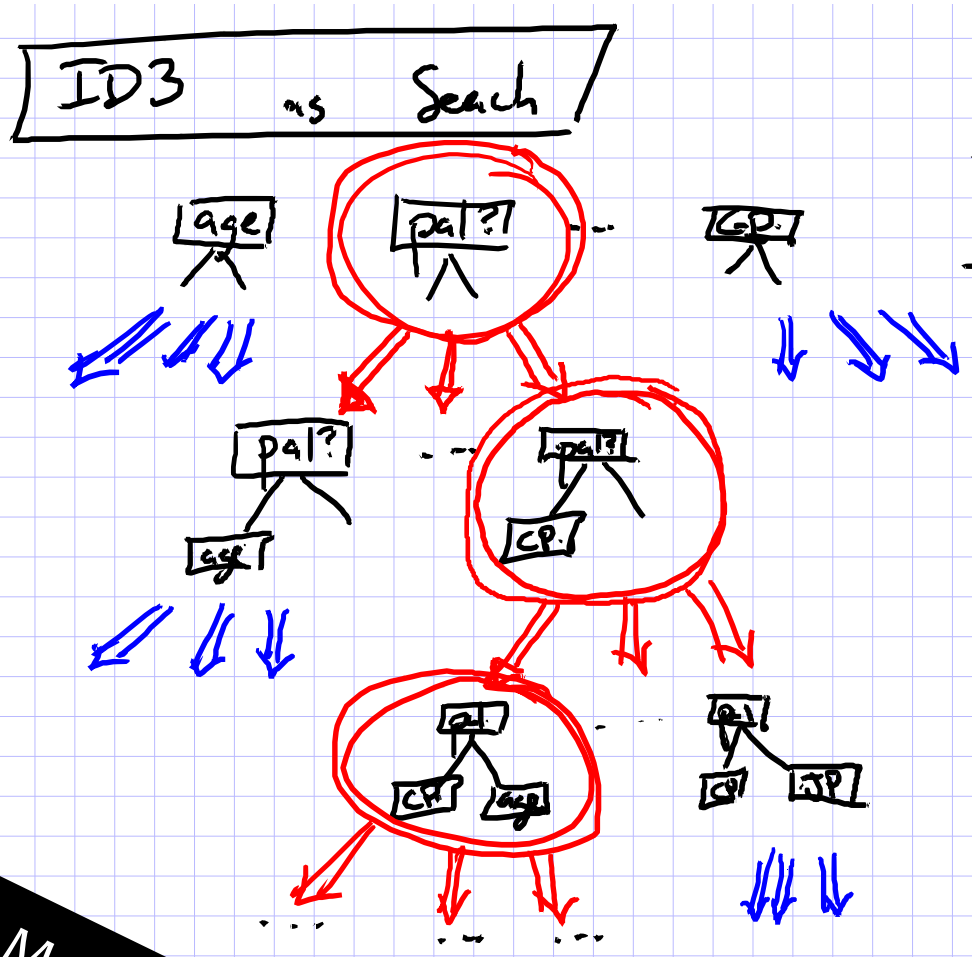
Motivating Example

Example: Decision Trees



Search space: all possible trees

ID3: greedy search, maximizing info. gain at each split

searches for smallest tree consistent with the training data

"inductive bias" of ID3

☆ Occam's Razor: prefers the simplest hypothesis that explains the data.
(i.e. 1300s smallest expl. is best)

Motivating Example

Proof by Induction

Chalkboard:

- Weak Induction
  - basis case
  - inductive hypothesis
  - inductive step
- Example: sum of powers of two
- Why does proof by induction work?
  - propositional logic interpretation

Proof by Induction

In-Class Exercise

Prove the following statement by induction.

$$\sum_{i=1}^{n} i = n(n+1)/2$$

Answer Here:

Recursion

Chalkboard:

- Example: Factorial (iterative implementation)
- Example: Factorial (recursive implementation)
- Strong Induction
  - multiple basis cases
  - complete assumption
- Proof of recursive factorial correctness

Recursion

Chalkboard:

- Definition: Sorted Array
- Example: Insertion Sort (iterative implementation)
- Example: Insertion Sort (recursive implementation)
- Big Idea: Divide and Conquer
- Example: Merge Sort