10-606 Mathematical Foundations for Machine Learning

Machine Learning Department
School of Computer Science
Carnegie Mellon University

Method of Lagrange Multipliers

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Lecture 10
Oct. 1, 2018

Reminders

- Homework 2: Linear Algebra + Calculus
  - Out: Tue, Sep. 25
  - Due: Tue, Oct. 2 at 11:59pm
- Quiz 1: Linear Algebra
  - In-class, Wed, Oct. 3
- Homework 3: Calculus + Probability
  - Out: Wed, Oct. 3
  - Due: Wed, Oct. 10 at 11:59pm
- Quiz 2: Matrix Calculus + Probability
  - In-class, Wed, Oct. 10

Q&A

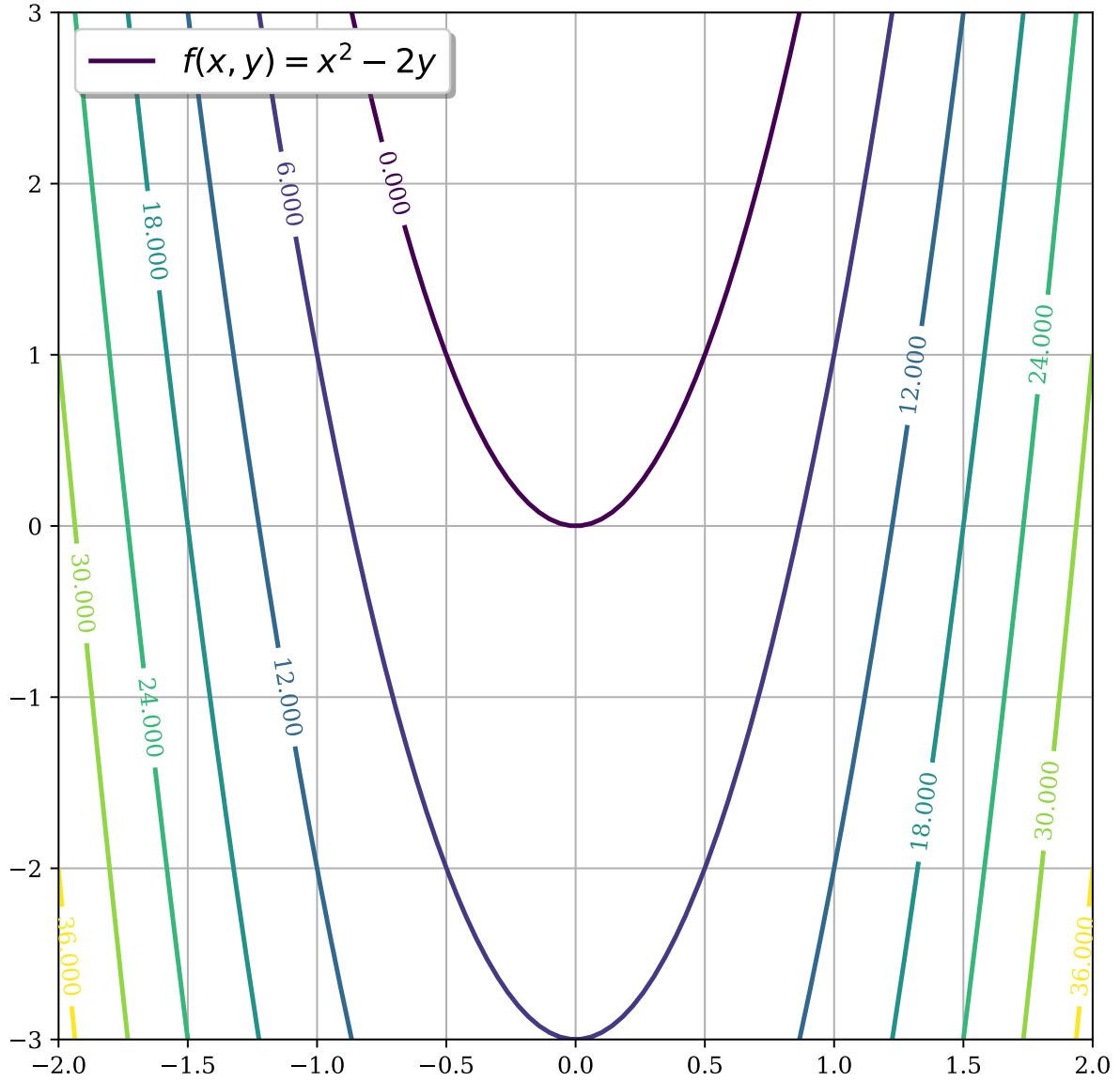
MATRIX CALCULUS

Method of Lagrange Multipliers

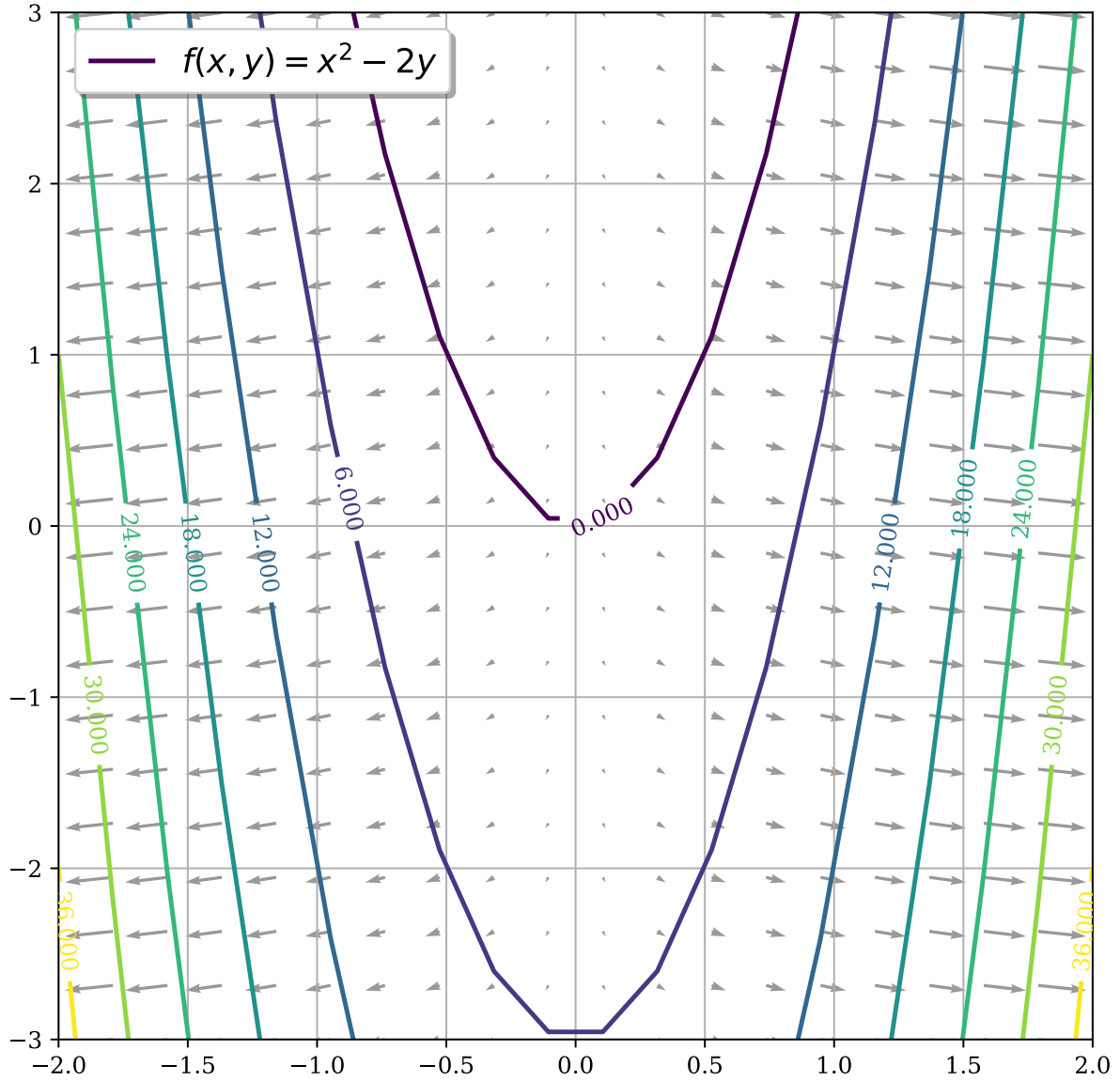
Chalkboard

- Motivation: Constrained Optimization
- Method of Lagrange Multipliers
- Extending to multiple constraints
- Extending to inequality constraints

Method of Lagrange Multipliers



Method of Lagrange Multipliers



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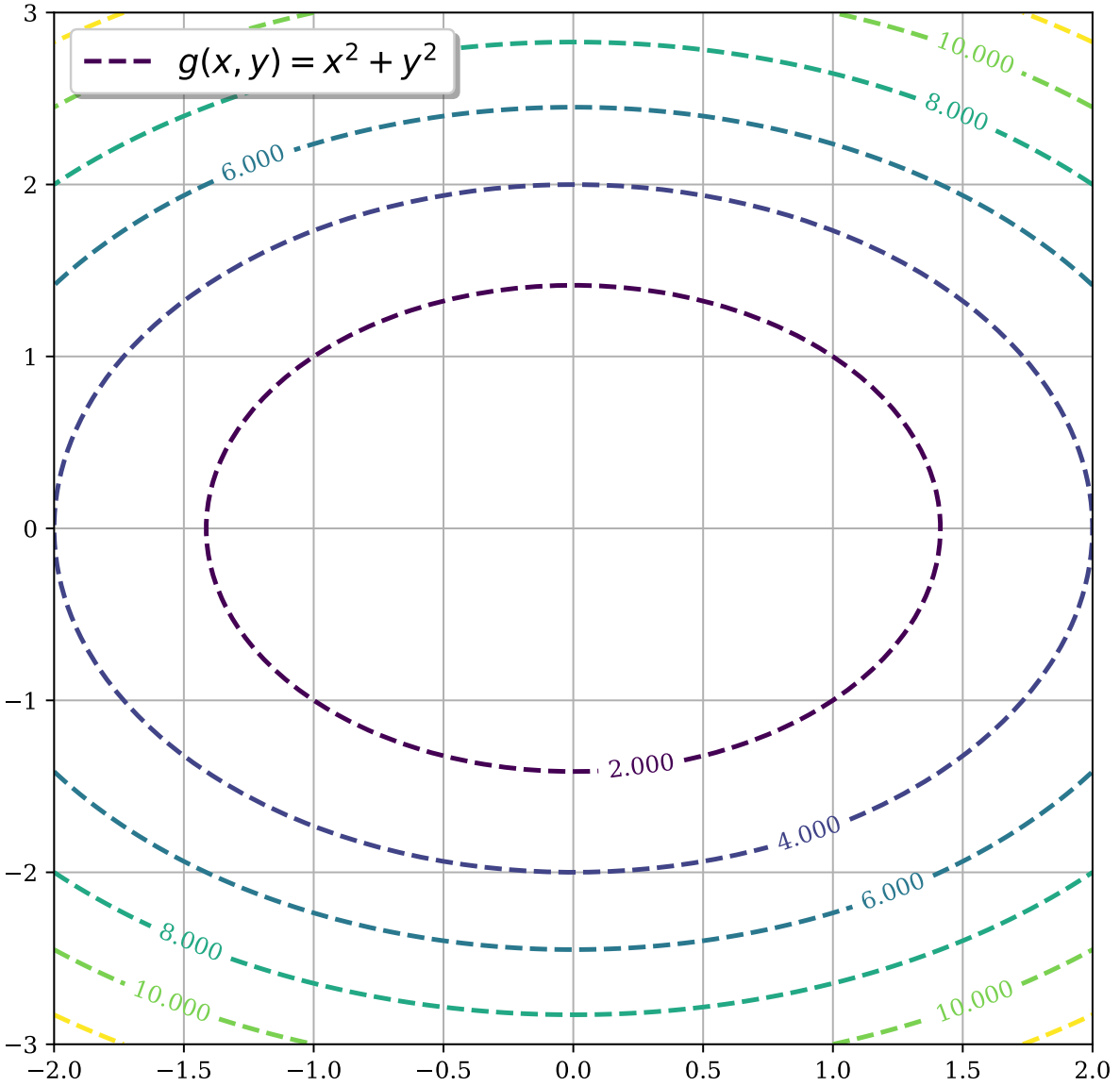

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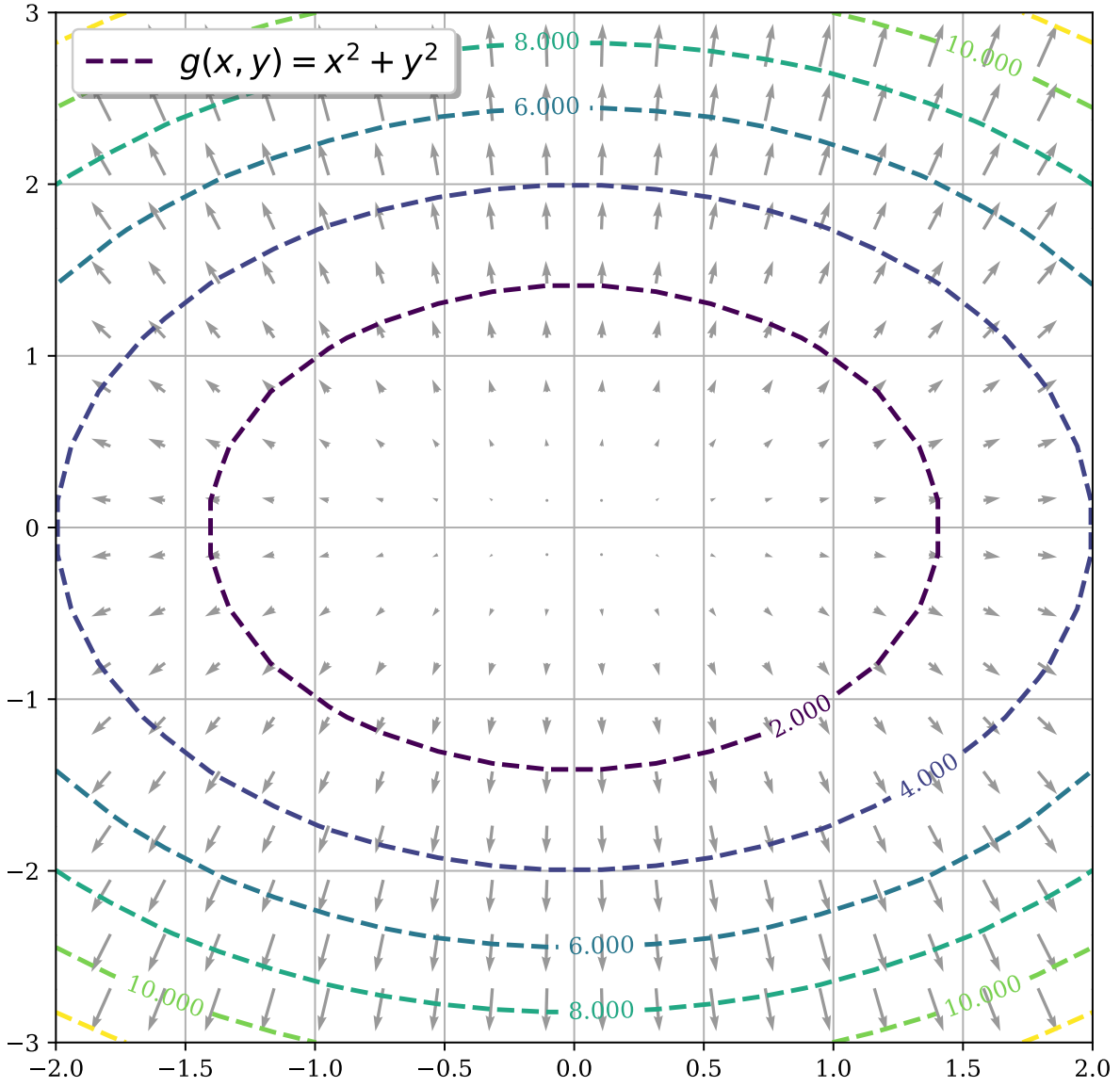

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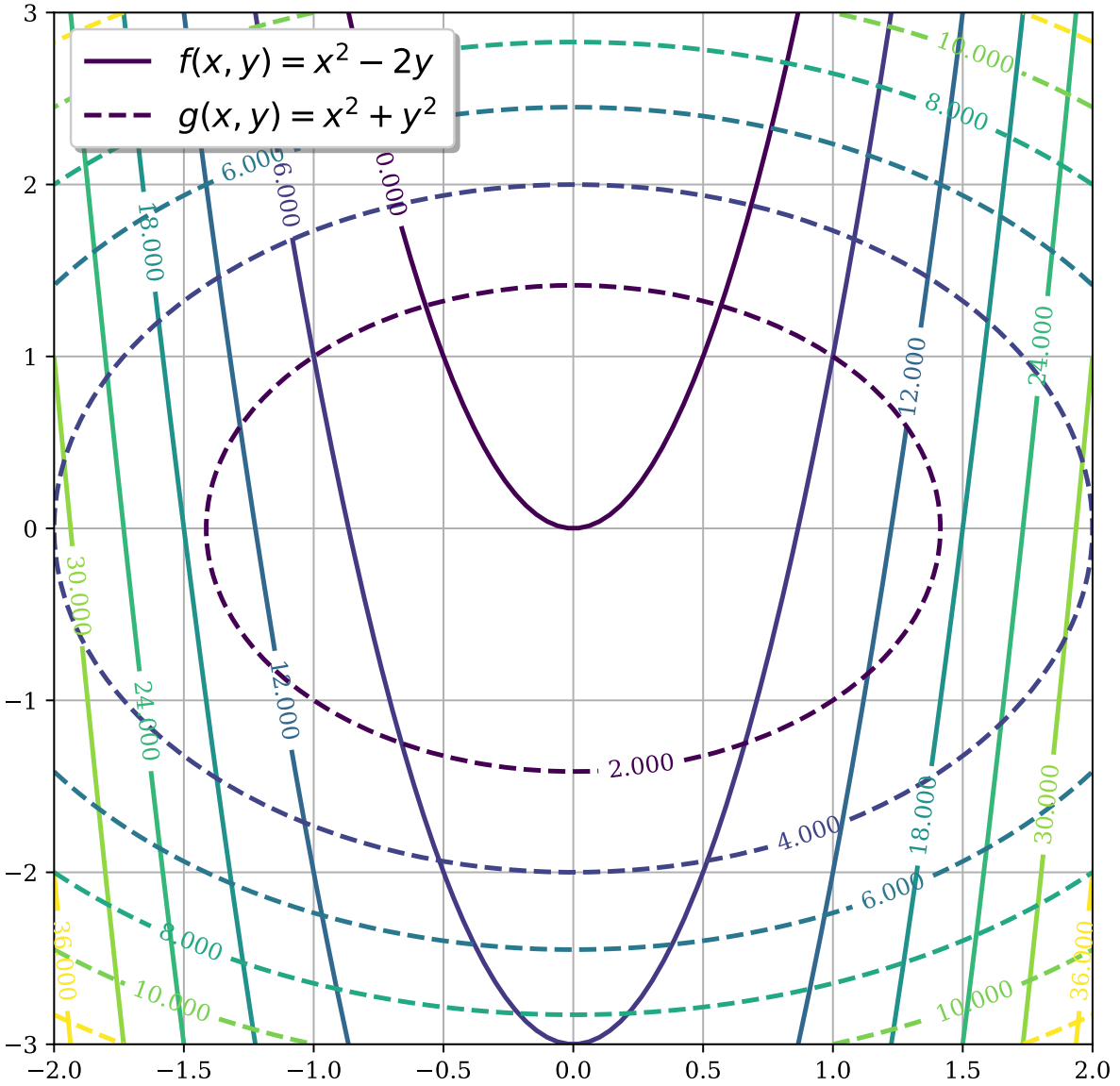

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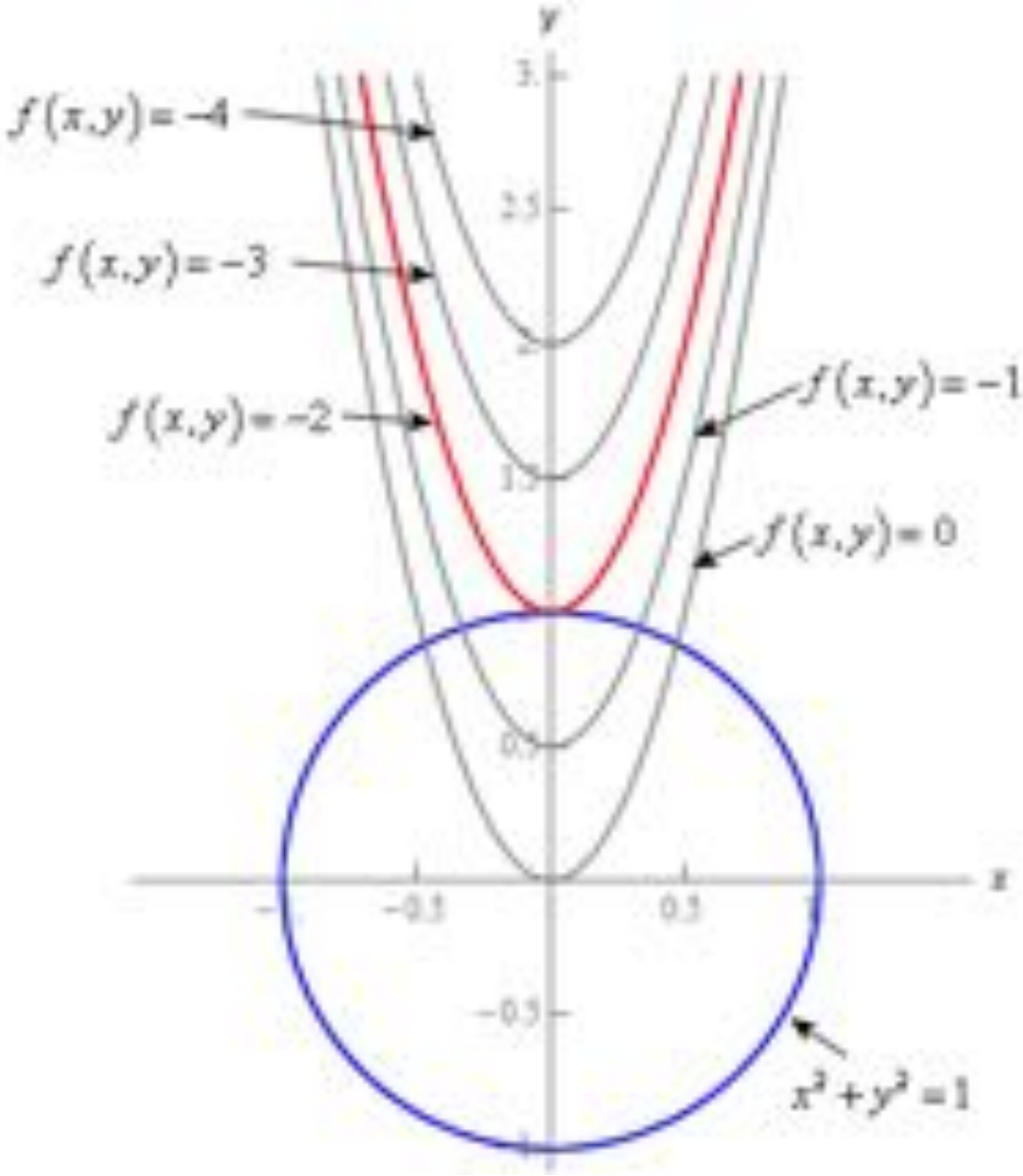

Figure from http://tutorial.math.lamar.edu/Classes/CalcIII/LagrangeMultipliers.aspx

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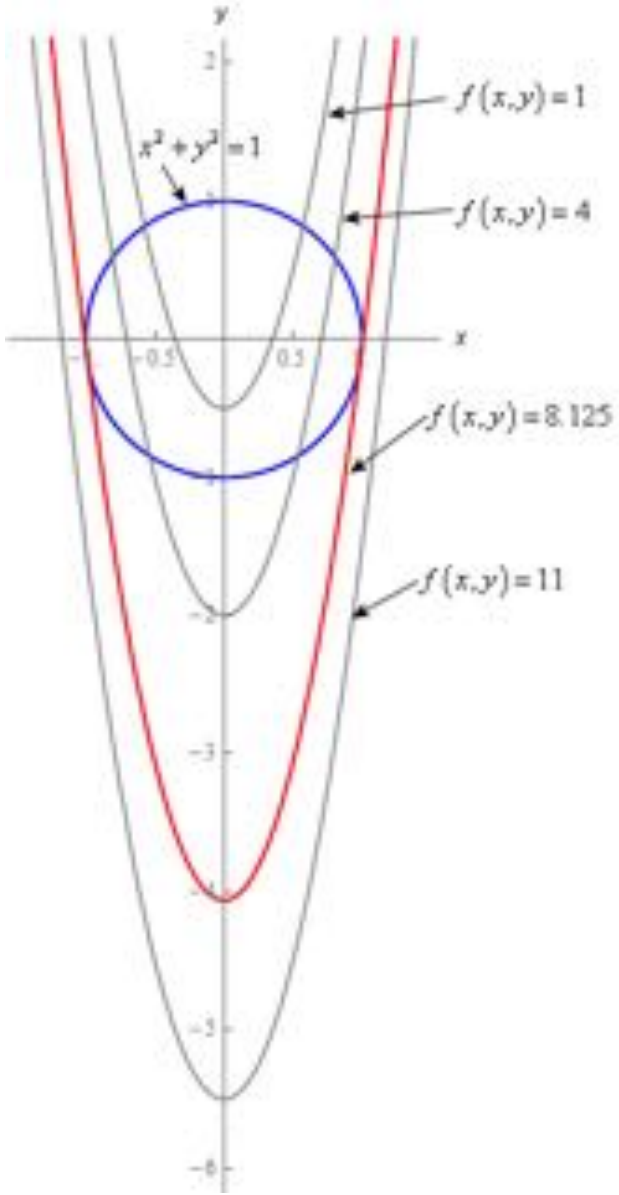

Figure from http://tutorial.math.lamar.edu/Classes/CalcIII/LagrangeMultipliers.aspx

DIMENSIONALITY REDUCTION

PCA Outline

- **Dimensionality Reduction**
  - High-dimensional data
  - Learning (low dimensional) representations
- **Principal Component Analysis (PCA)**
  - Examples: 2D and 3D
  - Data for PCA
  - PCA Definition
  - Objective functions for PCA
  - PCA, Eigenvectors, and Eigenvalues
  - Algorithms for finding Eigenvectors / Eigenvalues
- **PCA Examples**
  - Face Recognition
  - Image Compression

High Dimension Data

Examples of high dimensional data:

- High resolution images (millions of pixels)

High Dimension Data

Examples of high dimensional data:

- Multilingual News Stories
  (vocabulary of hundreds of thousands of words)

High Dimension Data

Examples of high dimensional data:

– Brain Imaging Data (100s of MBs per scan)

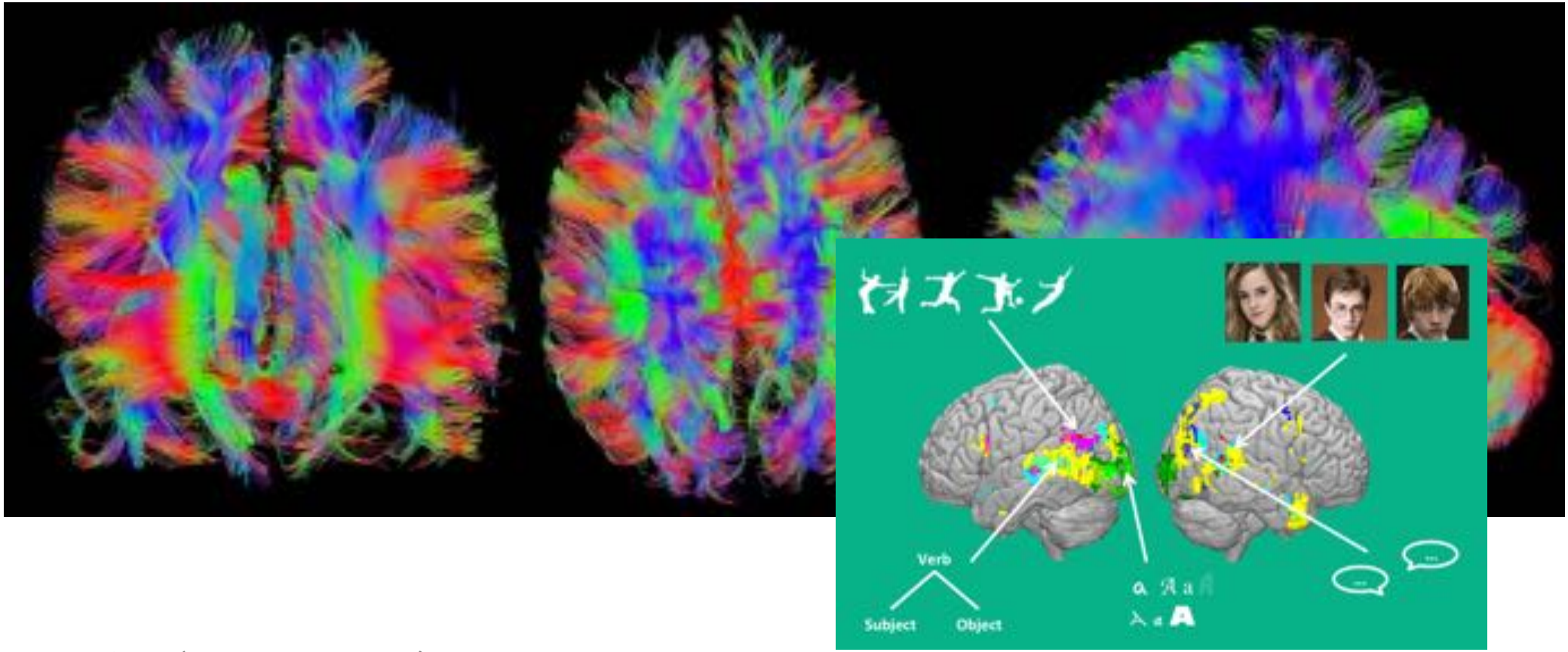

Image from (Wehbe et al., 2014)

Image from https://pixabay.com/en/brain-mrt-magnetic-resonance-imaging-1728449/

High Dimension Data

Examples of high dimensional data:

– Customer Purchase Data