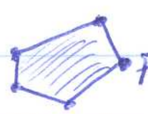
Some definitions (working in $\mathbb{R}^n$, $n \geq 0$)

- The <u>convex hull</u> of a set S is the minimal convex set containing S.

  Sometimes we mean the boundary of this minimal set, depending on context. (Overloaded terminology.)

- A point p of a convex set C is an <u>extreme point</u> of C if p does <u>not</u> lie on the line segment formed by any two other points of C.

  (A disk in 2D has infinitely many extreme points; a convex polygon has finitely many, namely all its vertices:



.)

- A <u>supporting hyperplane</u> h (<u>supporting line</u> ℓ in 2D) of a convex set C is a hyperplane ($(n-1)$-dim affine space) such that h contains at least one point of C and no two points of C lie on opposite sides of h.

  (E.g.:

.)

Some Theorems

In $\mathbb{R}^n$:

- A ray emanating from an interior point of a bounded convex set intersects the boundary of that set in exactly one point.

- A set is the convex hull of a finite set of points

  iff

  it is the intersection of a finite set of closed half-spaces and is bounded.

In 2D:

- Consecutive vertices of a convex polygon occur in sorted angular order about any interior point.

- A point p in a planar convex set C fails to be an extreme point of C iff p lies within some (possibly degenerate) triangle whose vertices are in $C \setminus \{p\}$.

- Let C be the boundary of the convex hull of a set S of points in $\mathbb{R}^2$. Let s_1 & s_2 be in S ($s_1 \neq s_2$). (s_1, s_2) is an edge (or subedge) of C iff S lies in one (closed) half-space determined by the line passing through s_1 & s_2.